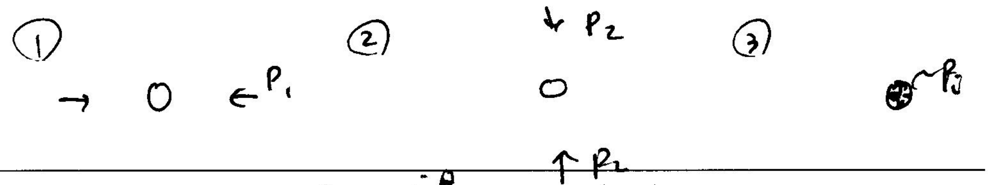
Our strategy.



error in 7 (2) $e^{i\theta}$ last lecture.

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§8.5

2) Circular hole with given far field principal stress.

Most holes drilled through rock → O

– immensely important.

internal pressure was already covered.

① uniaxial case → ② superposition → ③ internal pressure

① complex ~~variable~~ potential associated with a uniaxial stress aligned with x-axis → $\phi(z) = P_1 \frac{z}{4}$, $\psi(z) = -P_1 \cdot \frac{z}{2}$.

* $P_1 = \tau_1^{\infty}$ stress at infinity.

this doesn't work for ~~borehole~~ borehole wall.

we need to find some terms that satisfies BC.

$$\phi(z) = \frac{1}{4}P_1\left(z + \frac{A}{z}\right), \quad \psi(z) = -\frac{1}{2}P_1\left(z + \frac{B}{z} + \frac{C}{z^3}\right).$$

A, B, C are real.

Let's calculate some useful expression in advance.

$$\phi'(z) = \frac{1}{4}P_1\left(1 - \frac{A}{z^2}\right), \quad \phi''(z) = \frac{P_1 A}{2z^3},$$

$$\psi'(z) = -\frac{1}{2}P_1\left(1 - \frac{B}{z^2} - \frac{3C}{z^4}\right)$$

$$\tau_{yy} - \tau_{xx} + 2i\tau_{xy} = 2(\bar{z}\phi'' + \psi')$$
$$= AP_1 \bar{z}/z^3 - P_1\left(1 - \frac{B}{z^2} - \frac{3C}{z^4}\right)$$
$$= AP_1 \frac{1}{r^2} e^{-4i\theta} - P_1(1 - Br^{-2}e^{-2i\theta} - 3Cr^{-4}e^{-4i\theta})$$

in polar coordinates, $z = re^{i\theta}$.

$$\tau_{rr} + \tau_{\theta\theta} = 4\mathrm{Re}\{\phi'(z)\} = P_1 \mathrm{Re}\{1 - Ar^{-2}e^{-2i\theta}\}$$
$$= P_1(1 - Ar^{-2}\cos 2\theta)$$

using stress transformation,

from ⑦ ②

$\tau_{yy} - \tau_{xx} + 2i\tau_{xy} = 2(\bar{z}\phi'' + \psi')$

$$\tau_{\theta\theta} - \tau_{rr} + 2i\tau_{r\theta} = (\tau_{yy} - \tau_{xx} + 2i\tau_{xy})e^{2i\theta}$$
$$= P_1\{Br^{-2} - \underbrace{e^{2i\theta}}_{\cos 2\theta + i\sin 2\theta} + (Ar^{-2} + 3Cr^{-4})\underbrace{e^{-2i\theta}}_{\cos 2\theta - i\sin 2\theta}\}$$

real part of above equations,

$$\tau_{\theta\theta} - \tau_{rr} = P_1\{Br^{-2} - (1 - Ar^{-2} - 3Cr^{-4})\cos 2\theta\}$$

$$\tau_{\theta\theta} = \frac{1}{2}P_1\{1 + Br^{-2} + (3Cr^{-4} - 1)\cos 2\theta\}$$

$$\tau_{rr} = \frac{1}{2}P_1\{1 - Br^{-2} + (1 - 2Ar^{-2} - 3Cr^{-4})\cos 2\theta\}.$$

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imaginary part,

$$\tau_{r\theta} = -\frac{1}{2}P_1\left\{ (1 + Ar^{-2} + 3Cr^{-4})\sin 2\theta \right\}$$

In order for the hole boundary to be traction free,

$$\tau_{rr} = \tau_{r\theta} = 0$$

$$1 - Ba^{-2} = 0, \quad 1 - 2Aa^{-2} - 3Ca^{-4} = 0, \quad 1 + Aa^{-2} + 3Ca^{-4} = 0.$$

$$A = 2a^2, \quad B = a^2, \quad C = -a^4.$$

$\therefore$

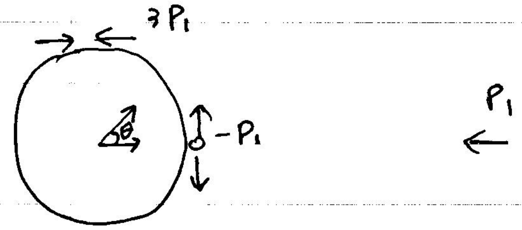
$$\tau_{\theta\theta} = \frac{1}{2}P_1\left\{1 + \left(\frac{a}{r}\right)^2\right\} - \frac{1}{2}P_1\left\{1 + 3\left(\frac{a}{r}\right)^4\right\}\cos 2\theta$$

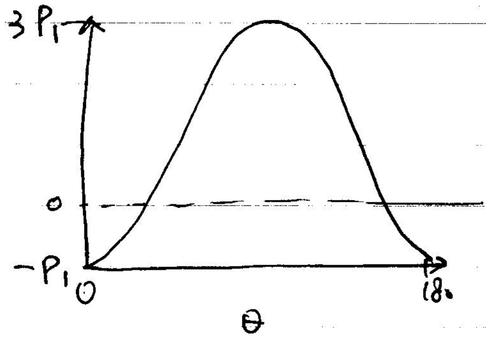
$$\tau_{rr} = \frac{1}{2}P_1\left\{1 - \left(\frac{a}{r}\right)^2\right\} + \frac{1}{2}P_1\left\{1 - 4\left(\frac{a}{r}\right)^2 + 3\left(\frac{a}{r}\right)^4\right\}\cos 2\theta$$

$$\tau_{r\theta} = -\frac{1}{2}P_1\left\{1 + 2\left(\frac{a}{r}\right)^2 - 3\left(\frac{a}{r}\right)^4\right\}\sin 2\theta.$$

At the surface of the hole, $r = a$,

$$\tau_{\theta\theta} = P_1(1 - 2\cos\theta).$$





② stresses distribution due to a second far-field principal normal stress can be found by replacing θ with $(\theta + \frac{\pi}{2})$.

...

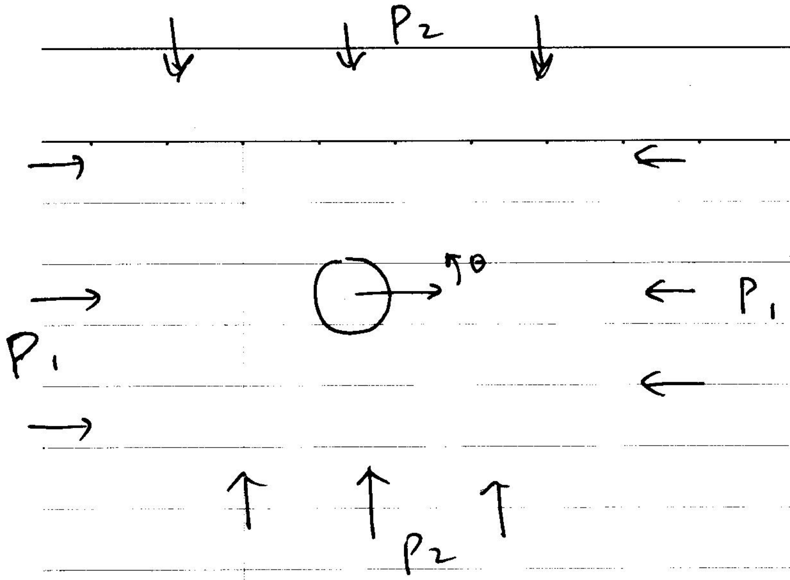
- By superposition,

$$\tau_{\theta\theta} = \frac{1}{2}(P_1 + P_2)\left\{1 + \left(\frac{a}{r}\right)^2\right\} - \frac{1}{2}(P_1 - P_2)\left\{1 + 3\left(\frac{a}{r}\right)^4\right\}\cos 2\theta$$

$$\tau_{rr} = \frac{1}{2}(P_1 + P_2)\left\{1 - \left(\frac{a}{r}\right)^2\right\} + \frac{1}{2}(P_1 - P_2)\left\{1 - 4\left(\frac{a}{r}\right)^2 + 3\left(\frac{a}{r}\right)^4\right\}\cos 2\theta$$

$$\tau_{r\theta} = -\frac{1}{2}(P_1 - P_2)\left\{1 + 2\left(\frac{a}{r}\right)^2 - 3\left(\frac{a}{r}\right)^4\right\}\sin 2\theta.$$

* use compression positive for convenience.



at $r = a$,

$$\tau_{\theta\theta}(a, \theta) = (P_1 + P_2) - 2(P_1 - P_2)\cos 2\theta .$$

when $|P_1| > |P_2|$,

minimum $= 3P_2 - P_1$.

maximum $= 3P_1 - P_2$.

$\frac{P_1}{P_3} < 3$, $\longrightarrow$ $\tau_{\theta\theta} > 0$ (compressive) at all points on the boundary of the hole.

$\frac{P_1}{P_3} > 3$, $\longrightarrow$ $\cos 2\theta > \frac{(P_1 + P_2)}{2(P_1 - P_2)}$ $\rightarrow$ condition for $\tau_{\theta\theta}$ to be tensile.

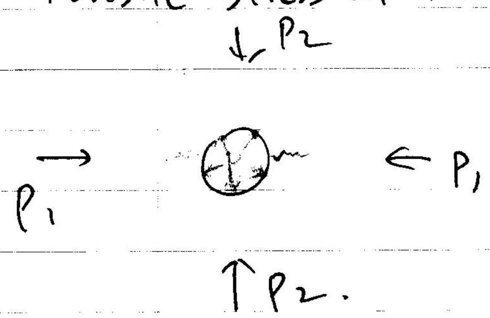
③ inclusion of internal pressure p.

$$\tau_{\theta\theta} = \frac{1}{2}(P_1 + P_2)\left[1 + \left(\frac{a}{r}\right)^2\right] - p\left(\frac{a}{r}\right)^2 - \frac{1}{2}(P_1 - P_2)\left[1 + 3\left(\frac{a}{r}\right)^4\right]\cos 2\theta$$

$$\tau_{rr} = \frac{1}{2}(P_1 + P_2)\left[1 - \left(\frac{a}{r}\right)^2\right] + p\left(\frac{a}{r}\right)^2 + \frac{1}{2}(P_1 - P_2)\left[1 - 4\left(\frac{a}{r}\right)^2 + 3\left(\frac{a}{r}\right)^4\right]\cos 2\theta$$

$$\tau_{r\theta} = -\frac{1}{2}(P_1 - P_2)\left[1 + 2\left(\frac{a}{r}\right)^2 - 3\left(\frac{a}{r}\right)^4\right]\sin 2\theta$$

tensile stress at the wall $\rightarrow$ $p > 3P_2 - P_1$.



$A \quad \frac{1}{r^2}(\cancel{e^{-2i\theta}} + i e^{2i\theta})$

$\frac{1}{r^2} e^{-2i\theta}$

$\frac{1}{4} P_1 \left(1 - \frac{A\cancel{\theta}}{z^2}\right)$

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*) Displacements, $\quad K\phi(z) - z\overline{\phi'(z)} - \overline{\psi(z)}$

$$\frac{8G(u+iv)}{P_1} = K(re^{i\theta} + Ar^{-1}e^{-i\theta}) - (re^{i\theta} - Ar^{-1}e^{3i\theta}) + 2(re^{-i\theta} + Br^{-1}e^{i\theta} + Cr^{-3}e^{3i\theta}).$$

from $u' + iv' = (u+iv)e^{-i\theta}$,

$$\frac{8G(u_r + iu_\theta)}{P_1} = K(r + Ar^{-1}e^{-2i\theta}) - (r - Ar^{-1}e^{2i\theta}) + 2(re^{-2i\theta} + Br^{-1} + Cr^{-3}e^{2i\theta})$$

using the values of A, B and C.

$$\frac{8G(u_r + iu_\theta)}{P_1} = K\cdot\left[\frac{r}{a} + 2\left(\frac{a}{r}\right)e^{-2i\theta}\right] - \left[\frac{r}{a} - 2\left(\frac{a}{r}\right)e^{2i\theta}\right] + 2\left[\left(\frac{r}{a}\right)e^{-2i\theta} + \left(\frac{a}{r}\right) - \left(\frac{a}{r}\right)^3 e^{2i\theta}\right]$$

$K = 3-4\nu$ plane strain

$\frac{3-\nu}{1+\nu}$ plane stress

Separating real & imaginary parts.

$$u_r = \frac{P_1}{8G}a\left[\left\{(K-1)\left(\frac{r}{a}\right) + 2\left(\frac{a}{r}\right)\right\} + 2\left\{\left(\frac{r}{a}\right) + (K+1)\left(\frac{a}{r}\right) - \left(\frac{a}{r}\right)^3\right\}\cos 2\theta\right]$$

$$u_\theta = -\frac{P_1}{8G}a\left[2\left\{\left(\frac{r}{a}\right) + (K-1)\left(\frac{a}{r}\right) + \left(\frac{a}{r}\right)^3\right\}\sin 2\theta\right].$$

By considering additional P_2 in perpendicular direction,

in a more general case in which there is an axial strain ε in the longitudinal direction,

put $K = 3-4\nu$, and add $-\varepsilon\nu r$ to the radial displ.

$$u_r = \frac{P_1}{4G}a\left[\left\{(1-2\nu)\frac{r}{a} + \frac{a}{r}\right\} + \left\{\frac{r}{a} + 4(1-\nu)\left(\frac{a}{r}\right) - \left(\frac{a}{r}\right)^3\right\}\cos 2\theta - \frac{4G\nu\varepsilon r}{P_1 a}\right]$$

the change in the radius of the hole $\Leftarrow r = a$

$$\frac{u_r}{a} = \frac{(1-\nu)P_1}{2G}(1 + 2\cos 2\theta) - \nu\varepsilon = \frac{1-\nu^2}{E}P_1(1+2\cos 2\theta) - \nu\varepsilon.$$

considering additional P_2 acting in the y-direction.
the displacement at the borehole is given by

$$u_r = \frac{1-\nu^2}{E}\cdot a\left[(P_1+P_2) + 2(P_1-P_2)\cos 2\theta\right] - a\nu\varepsilon.$$

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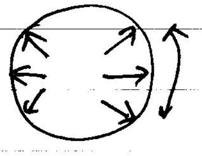
this can be expressed solely in terms of stresses,

from $E\varepsilon = \bar{\sigma}_{zz} - \nu(\tau_{xx} + \tau_{yy})$.

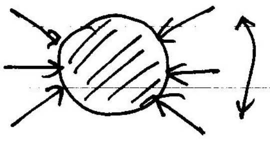
$$u_r = \frac{a}{E}\left\{ (P_1 + P_2 - \nu P_3) + 2(1-\nu^2)(P_1 - P_2)\cos 2\theta \right\}$$

§ 8.6 Stresses applied to a circular hole with an arbitrary distribution of tractions along the hole boundary.



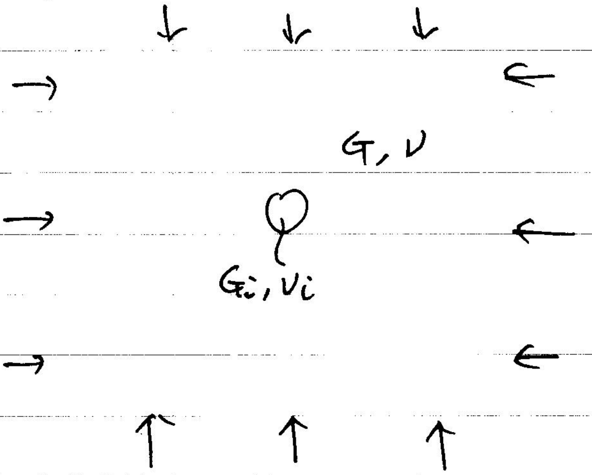
→ to determine elastic moduli of rock.

§ 8.7 Stresses applied to the surface of a solid cylinder.



→ Brazilian test

§ 8.8 inclusions in an infinite region.



$\left\langle \begin{array}{ll} G_i > G & \text{stiffer} \\ \tau^i > \tau & \text{higher} \end{array} \right.$

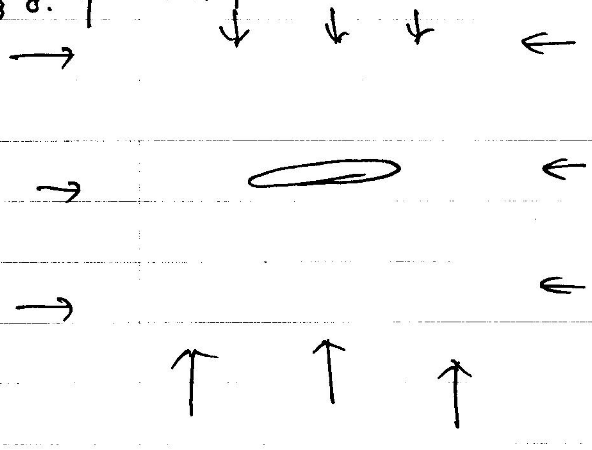
$\left\langle \begin{array}{l} G_i < G \\ \tau^i < \tau. \end{array} \right\rangle$ less stiff, lower.

inclusion itself is in a state of homogeneous stress.

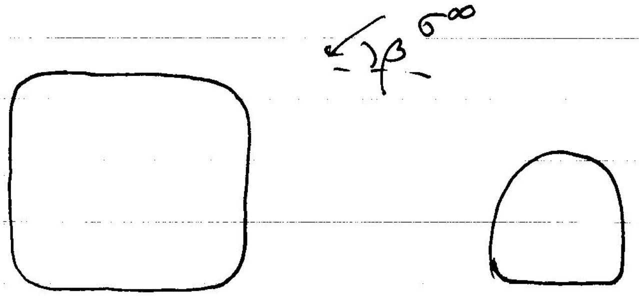
$$\sigma_1^i = \frac{3\beta}{2\beta+1}\sigma_1^\infty \quad \left\langle \begin{array}{l} 0 \text{ . for very compliant} \\ 1.5\sigma_1^\infty \text{ for very stiff.} \end{array} \right.$$

§ 8.9 Elliptical hole in an infinite rock mass. & § 8.10.

Stresses near a crack tip.



§8.11 Nearly rectangular hole.

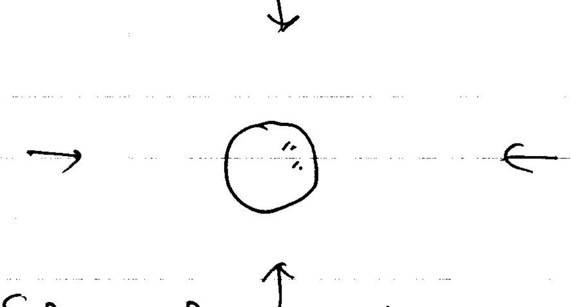


tunnel.

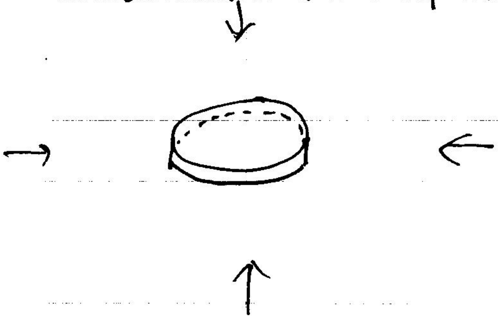
solution.

§.8.12 Spherical cavity.

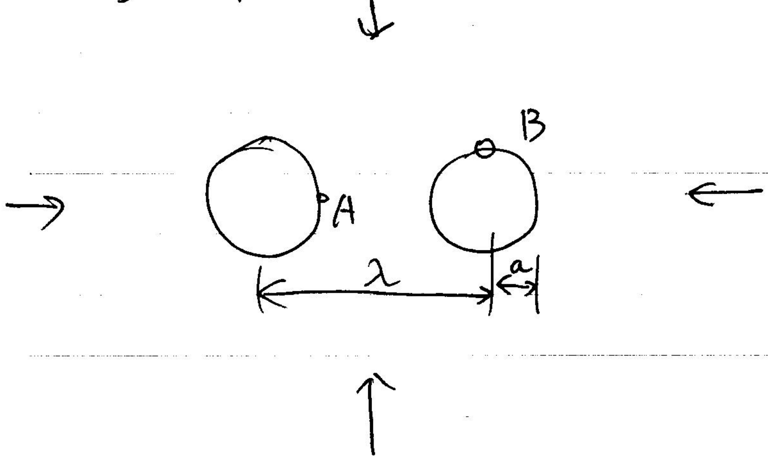
underground nuclear testing.

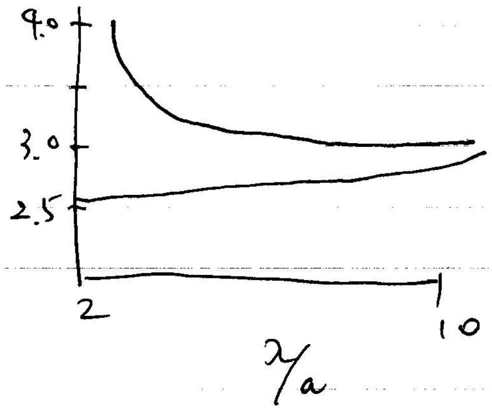


§8.13 Penny-shaped crack.



§8.14 Interactions between nearby cavities.





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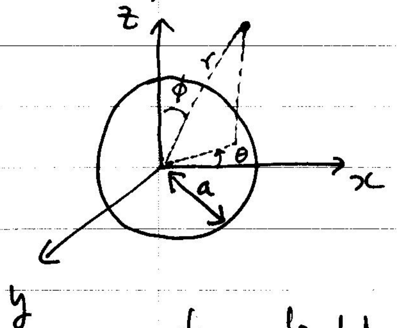
Spherical cavities §8.12



ϕ: angle of rotation from z-axis.

θ : " x-axis.

$x = r\cos\theta\sin\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\phi$.

far field stress can be aligned with the z-axis.

→ resulting stress & displacement will be axisymmetric about the z-axis.

solution for three principal stresses can be obtained by superposition.

BC, as $r \to \infty$ $\tau_{zz} \to \tau_{zz}^{\infty} = T$, all Other Stresses ⇒ 0.

all the traction must vanish at $r = a$.

In terms of spherical coordinate system, stress state corresponds to the following non-zero stress components.

$\tau_{rr} = T\cos^2\phi$, $\tau_{\phi\phi} = T\sin^2\phi$, $\tau_{r\phi} = -T\sin\phi\cos\phi$