Lecture Note 9

Nuclear Design Analysis Procedure and Methods

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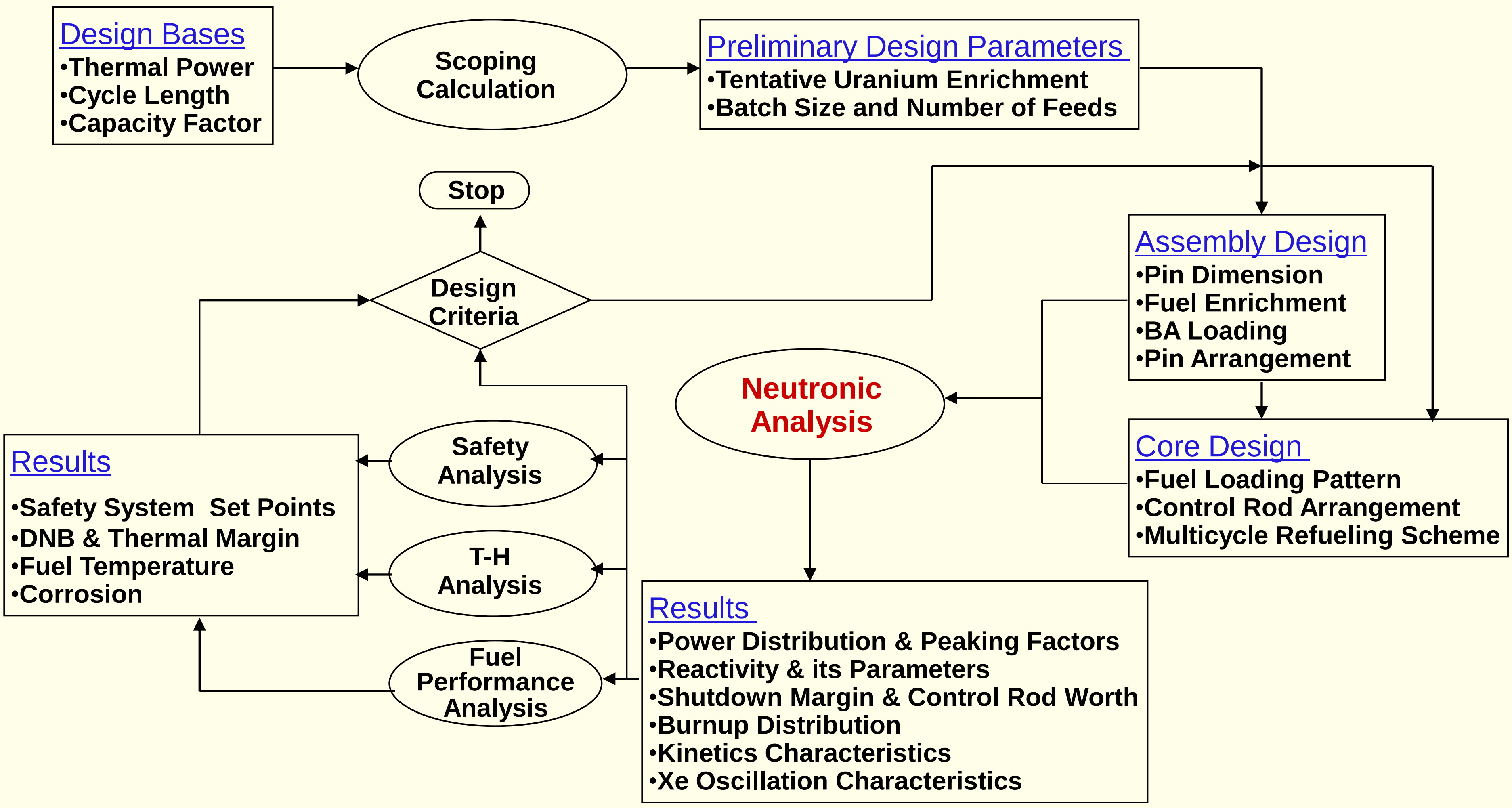
Overall Nuclear Design Procedure



Boltzmann Transport Equation

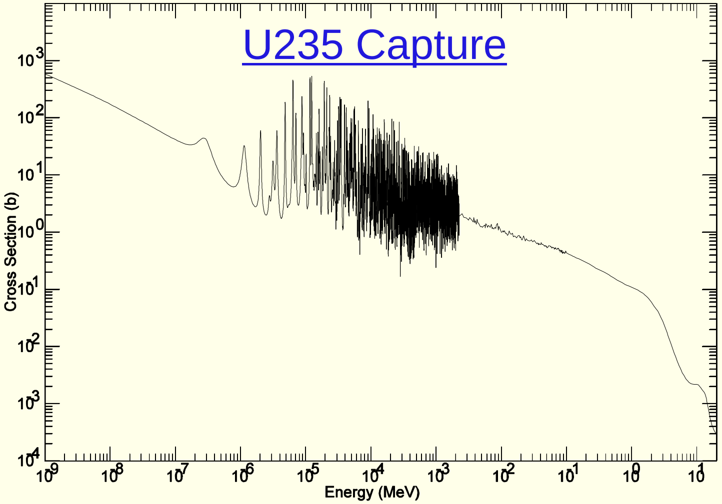
Balance Equation for Angular, Energy, and Space Dependent Flux

$$\Omega \bullet \nabla \varphi(r,E,\Omega) + \Sigma_t(r,E)\varphi(r,E,\Omega)$$
$$= \int_{\Omega'}\int_{E'} \Sigma_s(\Omega' \to \Omega, E' \to E)\varphi(r,E,\Omega)\,dE'\,d\Omega' + \frac{1}{4\pi}\chi(E)\psi(r)$$

Condition Dependence of Macroscopic Cross Section

$$\Sigma(r,E) = \sum_i N_i(r)\sigma_i(T(r),E)$$

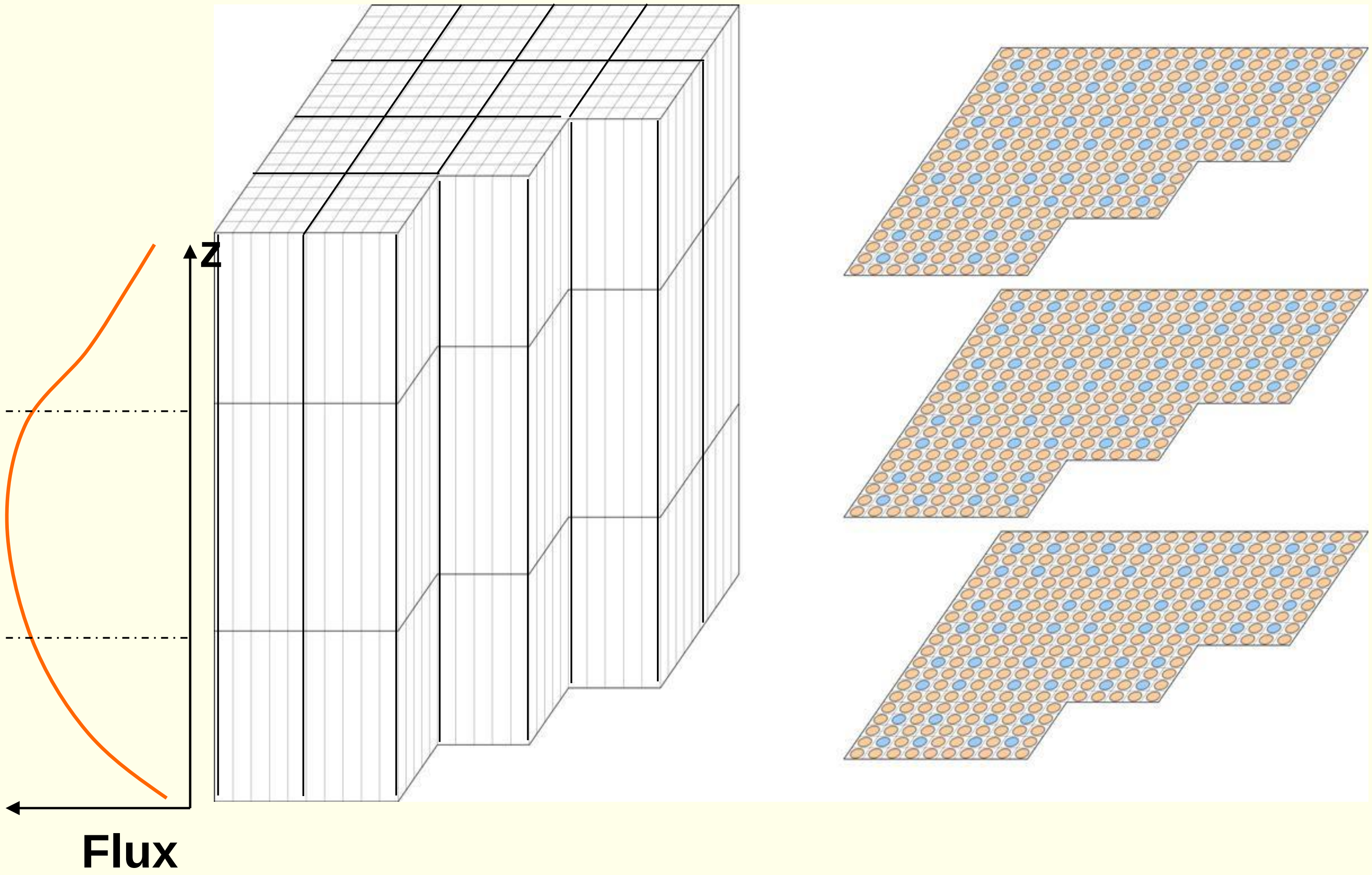
- **Energy of Incident Neutrons**
  - 1/v dependence
  - Resonance behavior
- **Temperature (T(r)) of Medium**
  - Power or flux dependent
- **Number Densities of Constituent Isotopes**
  - Burnup dependent
  - Coolant density dependent



Nonlinear Exhaustive Problem

- **Impractical to use Boltzmann equation for real core problems**

Problem Domain and Mesh Structures



Simplifications for Practical Core Calc.

Multigroup Approach

- **Define group cross sections with reaction rate conservation**

$$\bar{\sigma}_g(r) = \frac{\int_{E_g}^{E_{g-1}} \sigma(r,E)\varphi(r,E)dE}{\int_{E_g}^{E_{g-1}} \varphi(r,E)dE}$$

- **Use smaller geometry (e.g. Single Assembly) with sufficient details and fine energy groups to determine spectrum for use in above definition**
- **Assume that local spectrum won't change much by presence of other materials in the core**

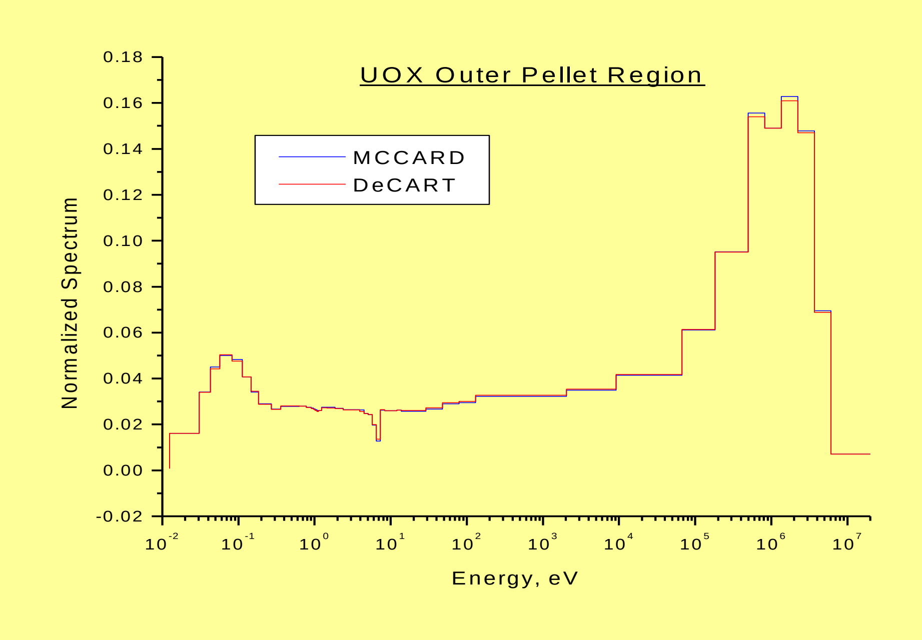
Diffusion Approximation in Core Calculation

- **Neglect angular dependence of flux and approximate neutron leakage by**

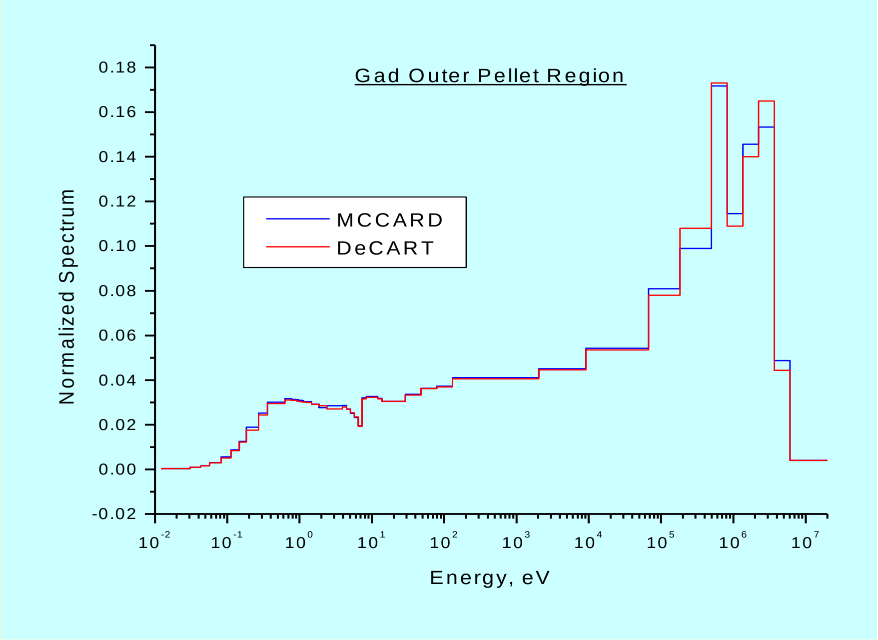
$$\nabla \cdot J_g(r) \quad \text{where} \quad J_g(r) = -D\nabla\varphi_g(r)$$

Material Dependence of Spectrum

Uranium Fuel Pin



Gadolinia Burnable Absorber Pin

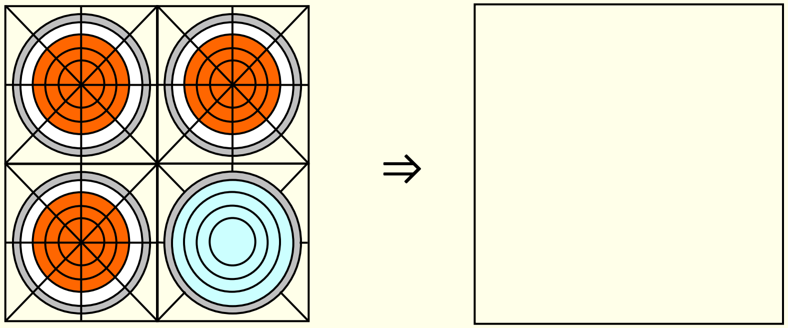


Spectrum

Further Simplifications in Core Calculation

Homogenization of Different Material Regions

- **Neglect details of actual geometrical configuration in local regions having different compositions**
- **Use a homogenized mixture for homogenized region**



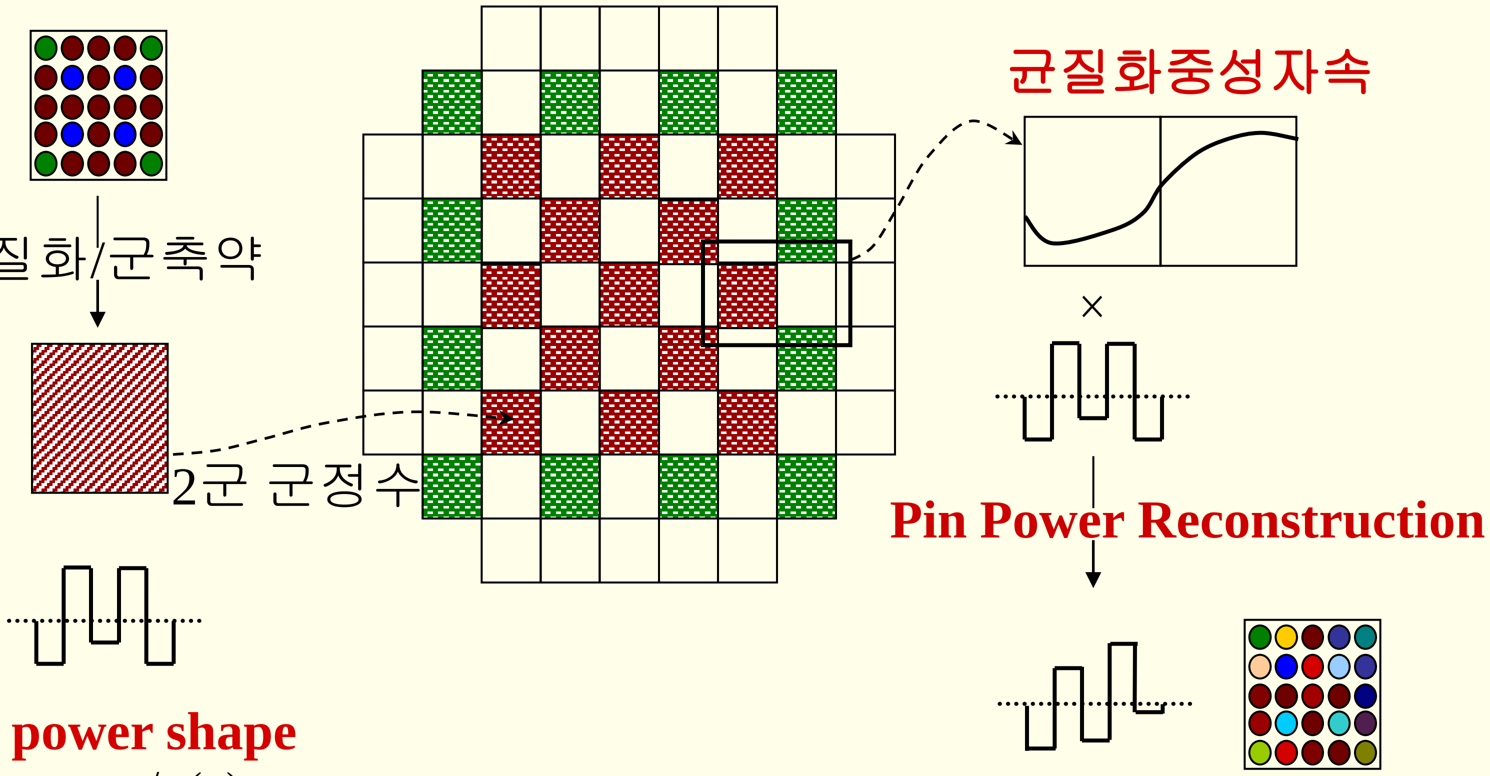
- **Allow flux variation within Node**

Reconstruction of Local Pin Powers

- **Multiply smooth inter-assembly power shape by discrete pin power form shape**

Two-Step Neutronics Calc. Procedure

1.군정수 생산 2-1. 노심 중성자 확산 계산 2-2. 봉출력재합성



pin power shape

$$f_g(\mathbf{r}) \equiv \frac{\phi_g(\mathbf{r})}{\bar{\phi}_g}$$

Group Constant Generation

Lattice Transport Calculation

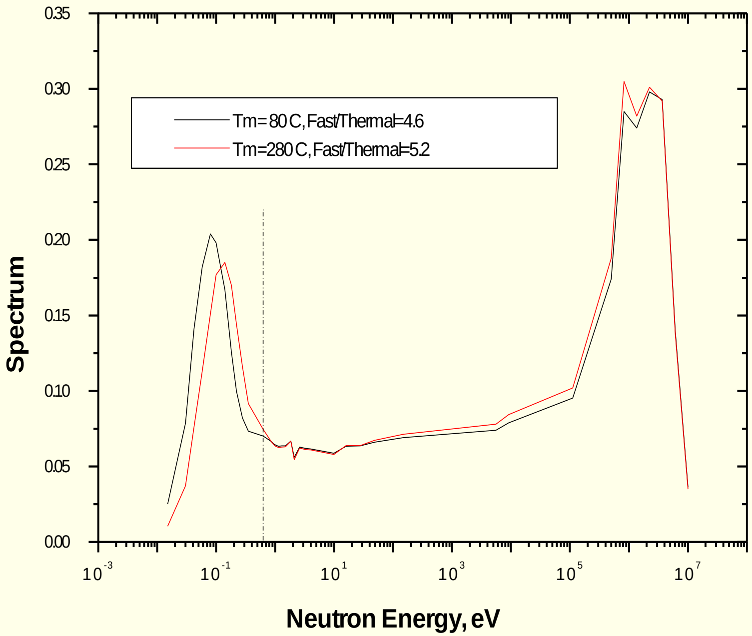
- Performed on fuel assembly base and retain heterogeneous geometrical configuration within assembly
- Use a multigroup cross section library (e.g. 45 Groups)
- Incorporate composition and geometry dependent resonance self-shielding into multigroup cross section
- Employs discrete integral transport method to obtain multigroup flux solution in each region
- Generate assembly homogenized Xsec and pin power shape
- Perform depletion calculation to determine spectrum change due to fuel burnout and fission product buildup

Restart Calculations

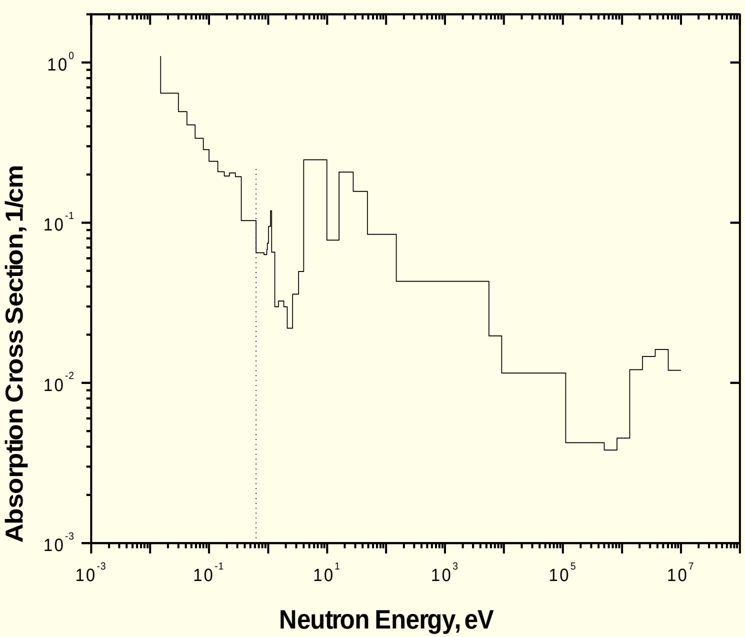
- Generate cross change for condition dependence of cross sections at selected burnup points
  - Enrichment, Fuel Temperature, Coolant Temperature, etc

Temperature Dependence of Spectrum

Spectrum

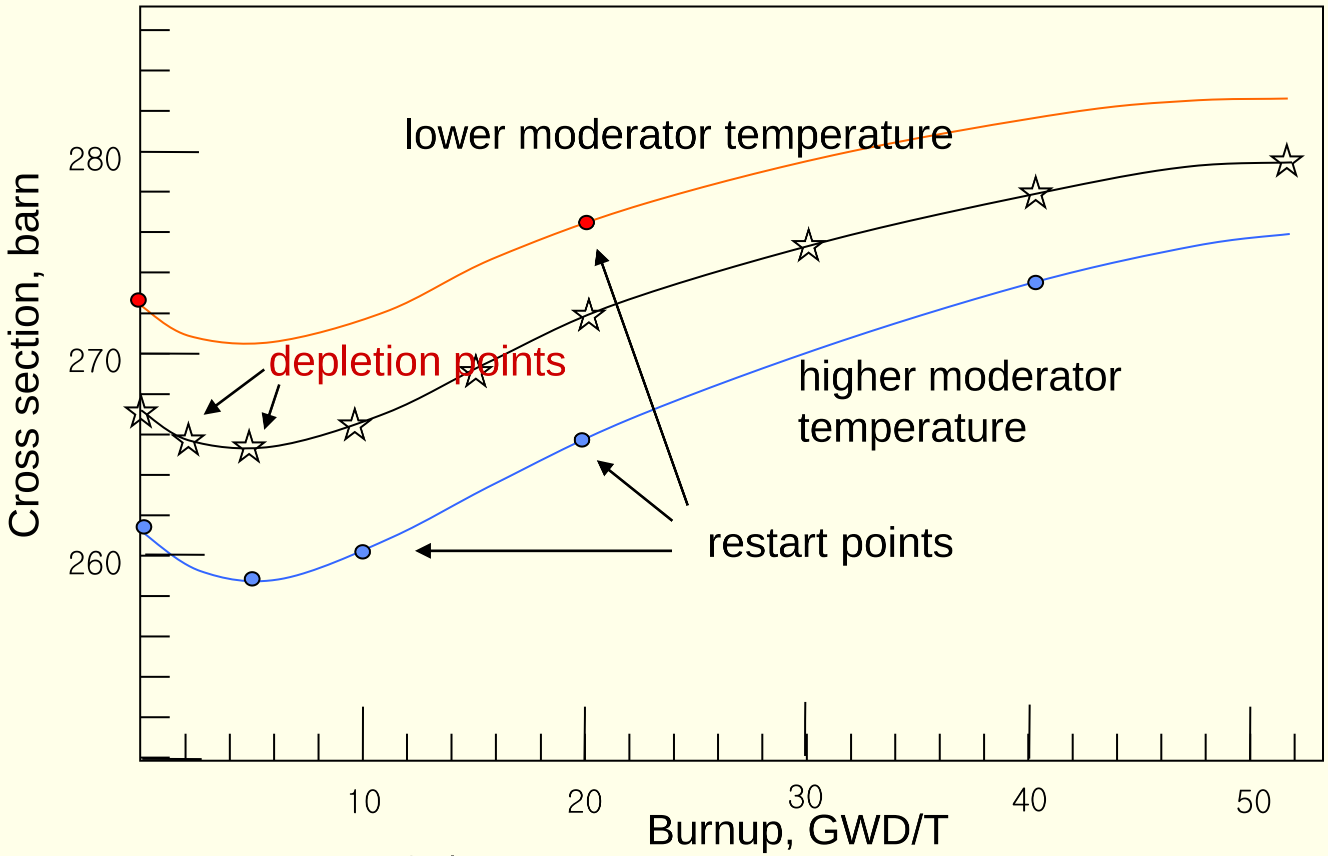


Fuel Cross Section



- $T_m \uparrow \rightarrow D_m \downarrow \rightarrow$ Less Moderation $\rightarrow$ More Resonance Absorption
- Spectrum Shifted toward High Energy Region $\rightarrow$ Thermal Xsec $\downarrow$
- Causes a decrease in thermal fission in fuel (1/v Xsec)
- Reactivity reduces → Negative MTC

Functional Dependence of Cross Section
(u-235 thermal absorption)



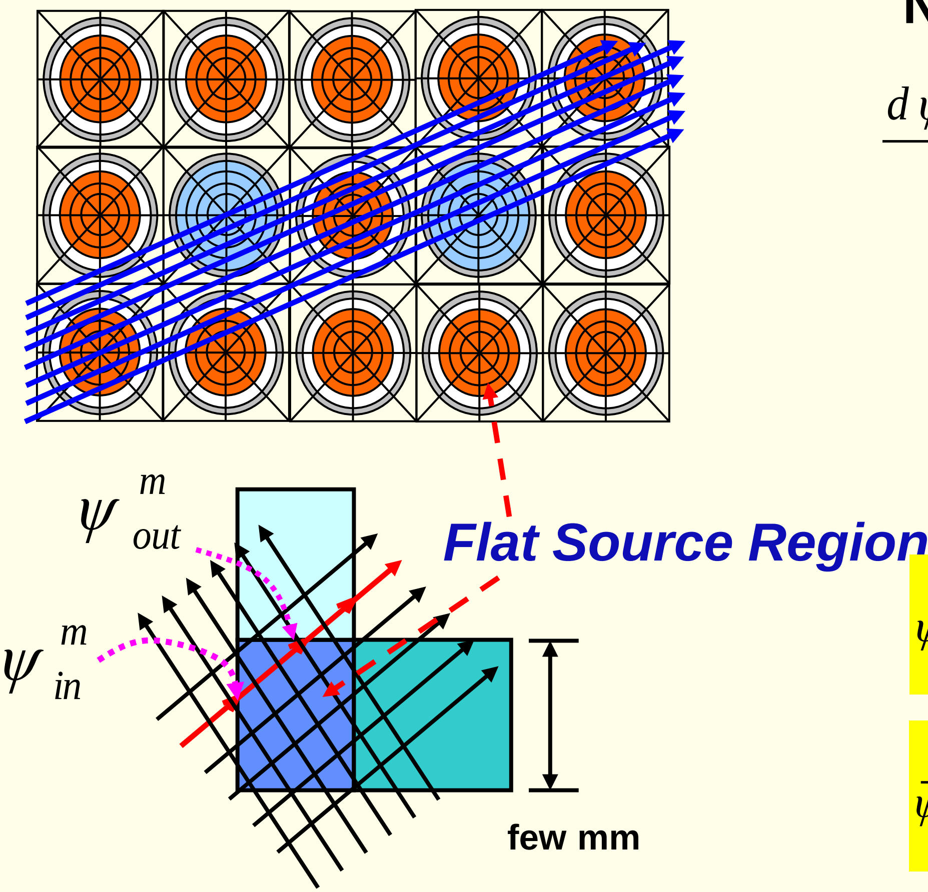
Cross Section Function

fuel temperature

moderator density

$$\sigma(BU, ppm, T_f, \rho_m) = \sigma(BU, ppm_0, T_{f0}, \rho_{m0}) + \frac{\partial\sigma}{\partial ppm}(ppm - ppm_0) + \frac{\partial\sigma}{\partial\sqrt{T_f}}(\sqrt{T_f} - \sqrt{T_{f0}}) + \frac{\partial\sigma}{\partial\rho_m}(\rho_m - \rho_{m0})$$

MOC Lattice Transport Calculation Method



(s = segment length)

Neutron Balance along Ray

$$\frac{d\psi(\xi)}{d\xi} + \Sigma\psi(\xi) = q \;\; ; \quad \psi(0) = \psi_{\text{in}}$$

Assume Constant Xsec and Flat Source within a Micro Region

$$\psi^m_{out} = \psi^m_{in} e^{-\Sigma s} + \frac{q}{\Sigma}\left(1 - e^{-\Sigma s}\right)$$

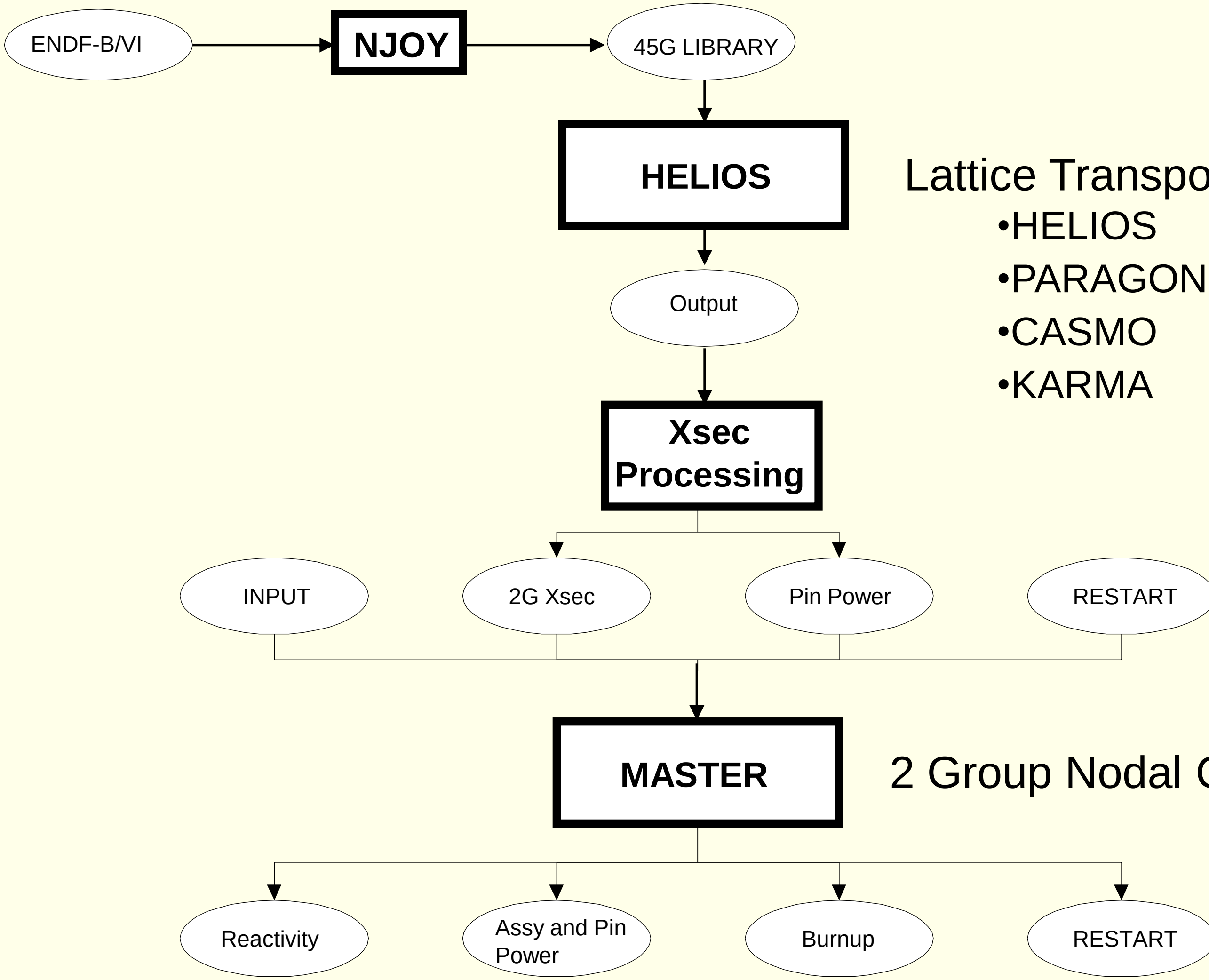
$$\overline{\psi}_m = \frac{\psi^m_{in} - \psi^m_{out}}{\Sigma s} + \frac{q}{\Sigma}$$

Segment Average Angular Flux

$$\phi = \sum_m \omega_m \overline{\psi}_m$$

Scalar Flux

Neutronics Calculation Procedure



Lattice Transport Code

- HELIOS
- PARAGON
- CASMO
- KARMA

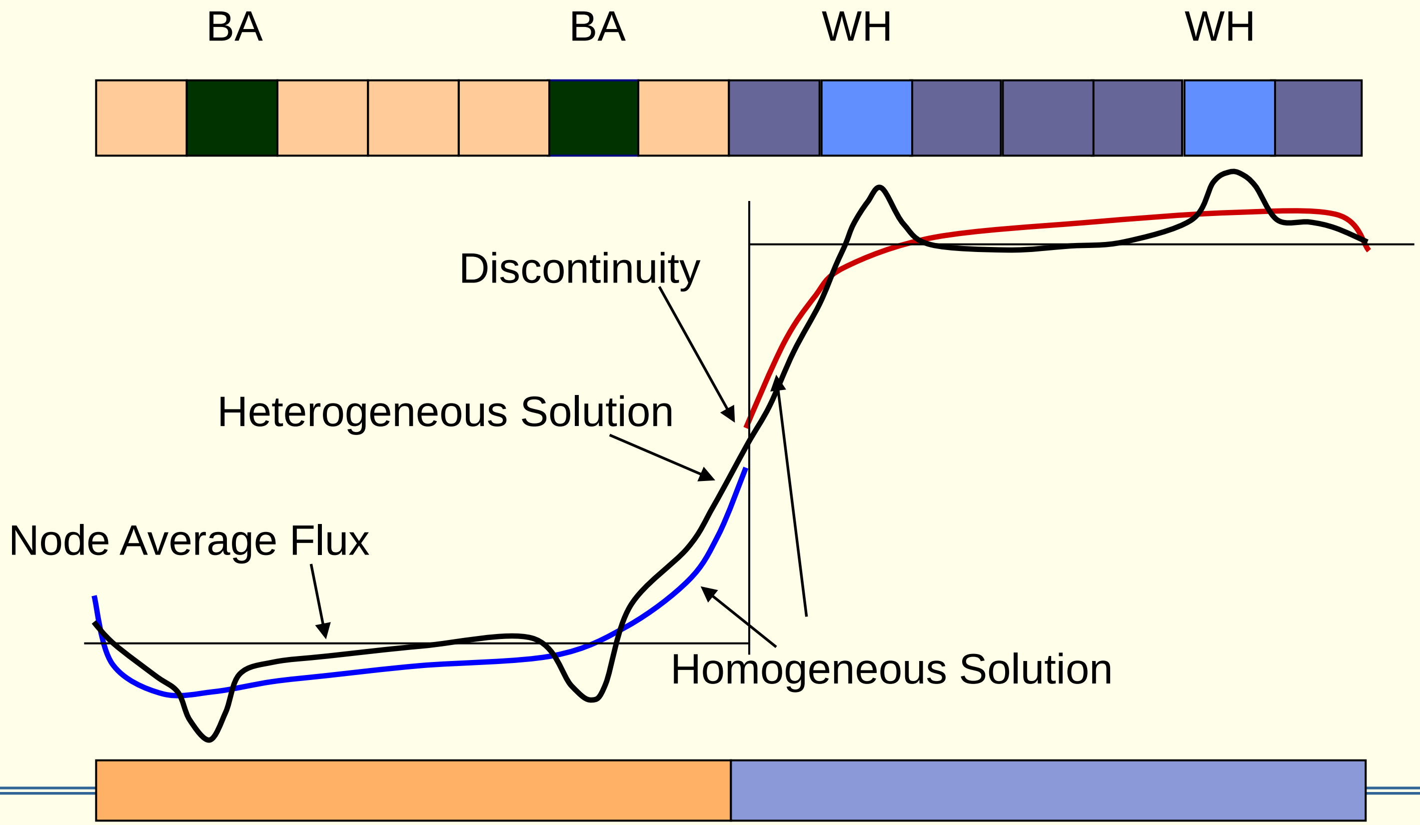
2 Group Nodal Code

- MASTER
- ANC
- SIMULATE
- ASTRA

Equivalence Theory in Homogenization - 1

Discontinuity in Homogeneous solution

- **It is possible to construct a homogenized problem fully consistent with the heterogeneous problem by Introducing a new degree of freedom, Flux Discontinuity**



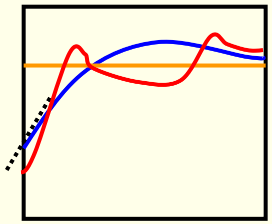
Equivalence Theory in Homogenization - 2

Heterogeneous Reference Solution

- **Heterogeneous reference solution is obtained for two homogenized nodes**

Consistent Homogeneous Solution

- **Identical node average flux to the heterogeneous Solution**
  - Same reaction rate, eventually same power
- **Identical Interface Current**
  - Same leakage out of the node, eventually same reactivity



Constraints in the Two-Node Problem

- **2 Fluxes and 1 Current**
- **Overdetermined for 2nd order differential equation**

$$\zeta = \frac{\phi_s^{het}}{\phi_s^{hom}}$$

Additional Degree of Freedom of Flux Discontinuity

- **Introduced to match homogeneous current with heterogeneous Current at the Interface**

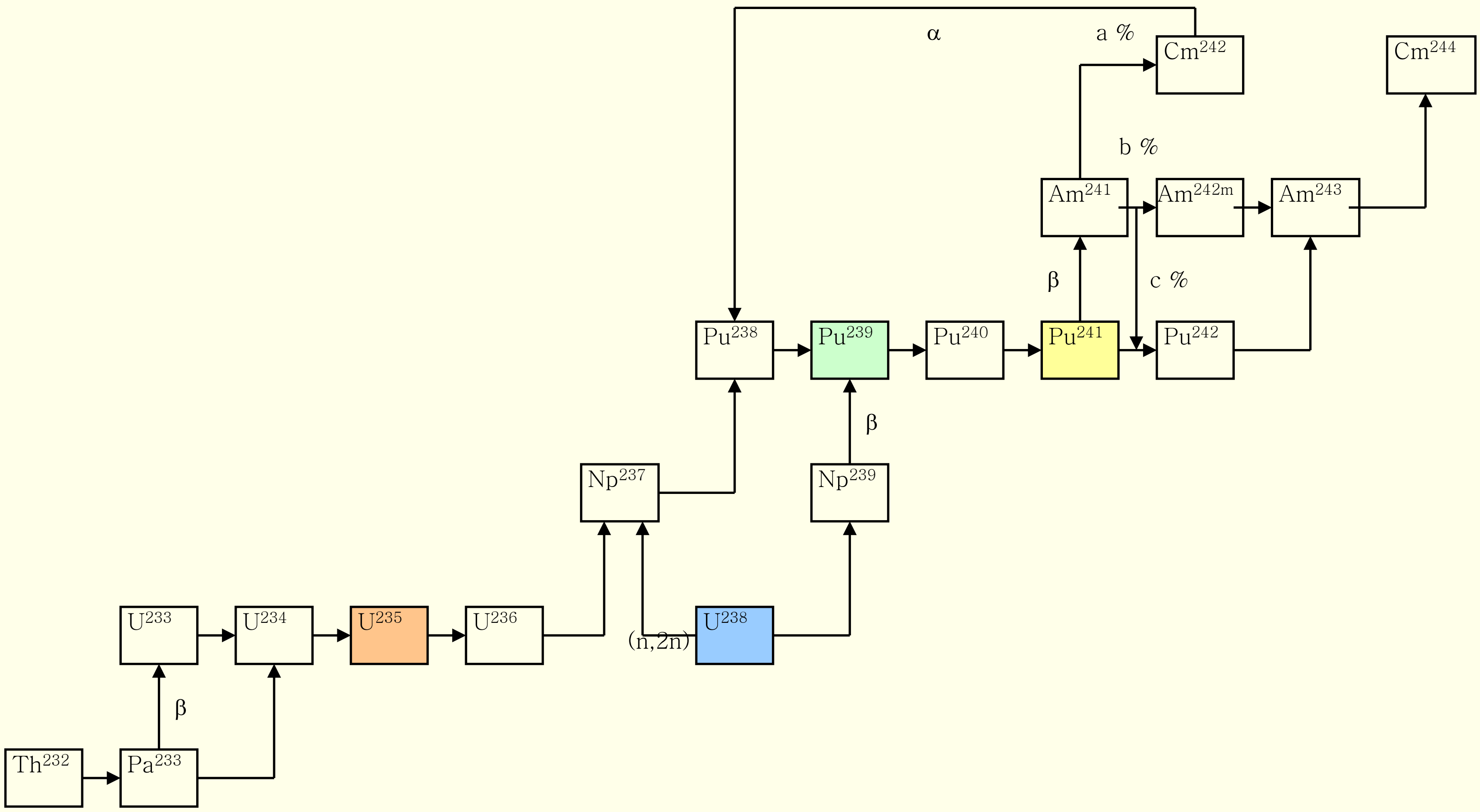
Practical Solution to Determine DF

- **Solve one node problem given the net current determined from heterogeneous calculation**

Assembly Discontinuity Factors

- **Generated by Lattice Physics Codes**

Depletion Chain



Microscopic Depletion

Depletion Equation

$$\frac{dN_i}{dt} = P_{i\text{-}1} N_{i\text{-}1} - R_i N_i$$

where

$$P_{i\text{-}1} = \lambda_{i\text{-}1} + \sum_{i'} y_{ii'} \sum_g \sigma_{cgi'} \varphi_g \qquad \text{Production rate}$$

$$R_i = \lambda_i + \sum_g \sigma_{agi} \varphi_g \qquad \text{Removal Rate}$$

Analytic Solution

$$N_i = \sum_{j=1}^{i} a_{ij} \exp(-R_j \Delta t)$$

where

$$a_{ij} = \frac{P_{i\text{-}1} a_{i\text{-}1\,j}}{R_i - R_j}$$

$$a_{ii} = N_i \Delta t - \sum_{j=1}^{i-1} a_{ij}$$