

# System Control

## 6-1. Electrical Motor

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# Electrical Motor



# Development of Integrated Vehicle Control System of “Fine-X” Which Realized Freer Movement.



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Integrated System Engineering Div.  
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**TOYOTA**



# TOYOTA Freer Movement Control System



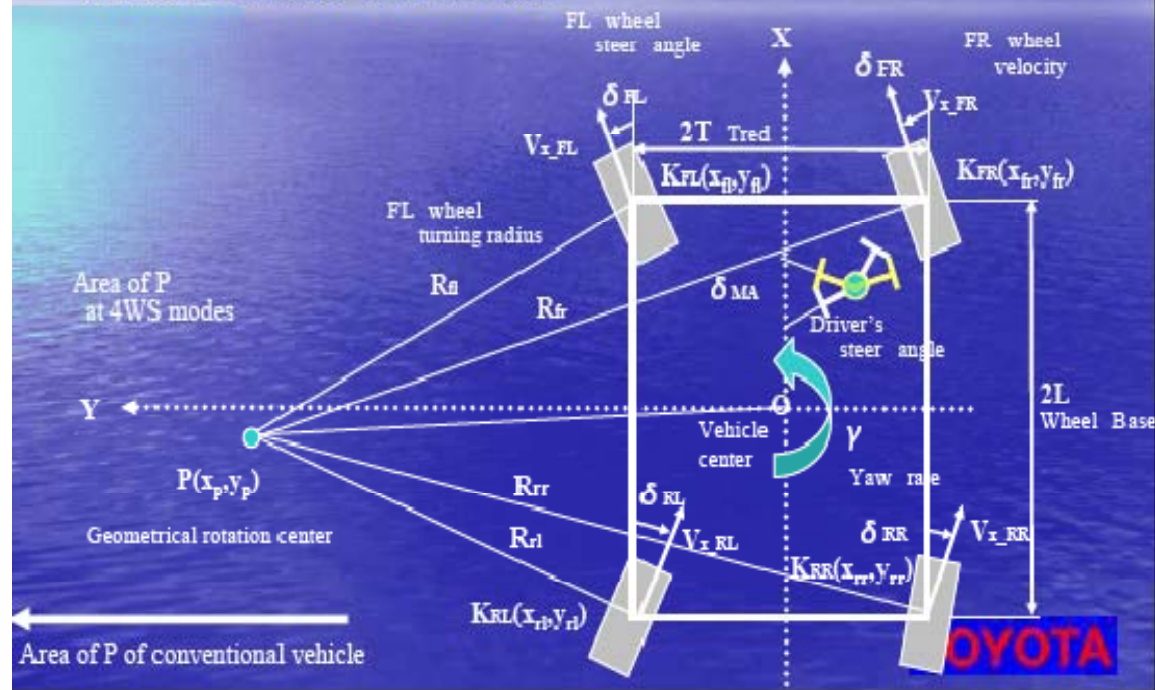
4Wheel independent drive  
4wheel independent steering  
4wheel independent braking  
By 'wheel-in-motor'



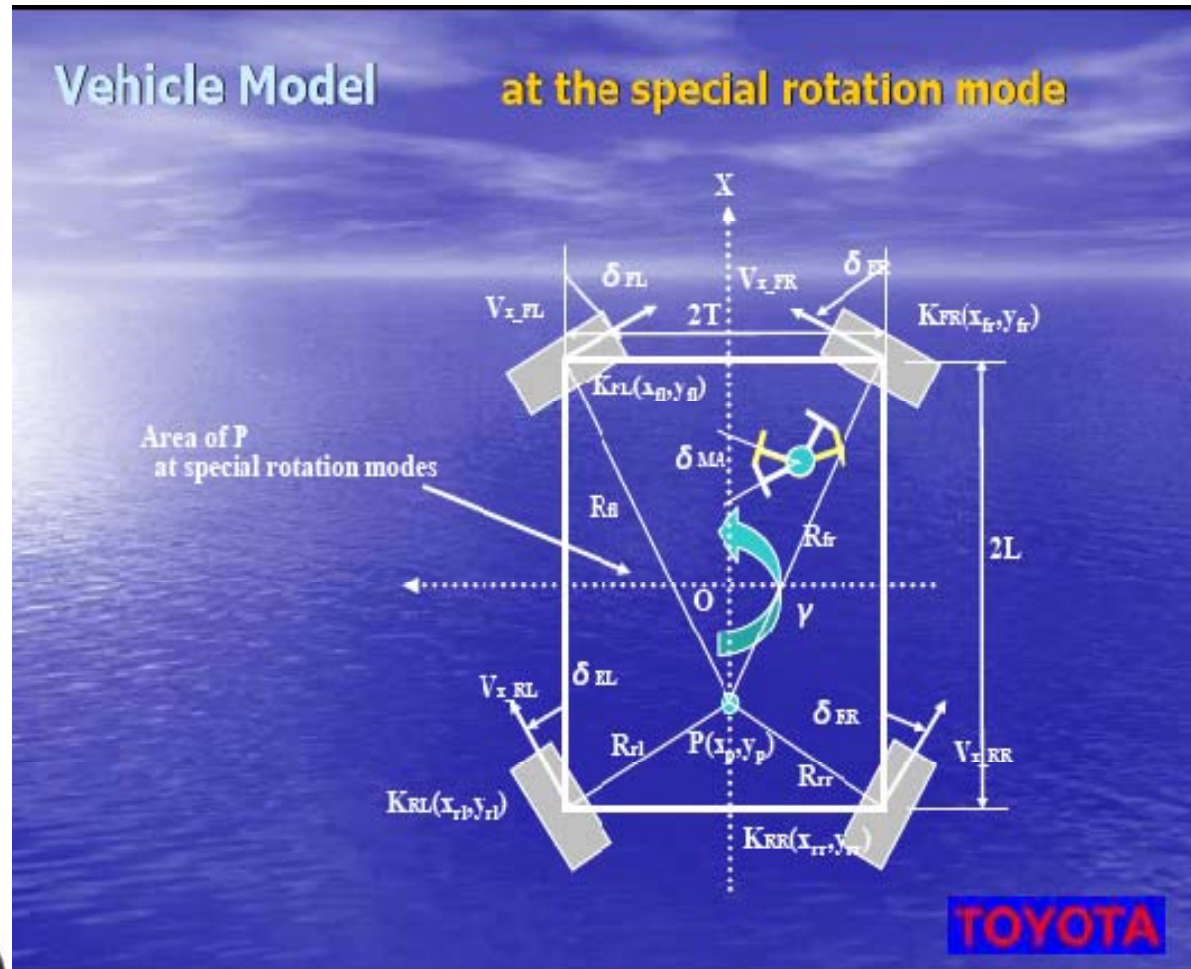
# TOYOTA Freer Movement Control System

## Integrated vehicle control

**Vehicle Model** <geometric model > only focuses on low-speed driving  
**at the normal 4WS mode** neglects the side slip of tires

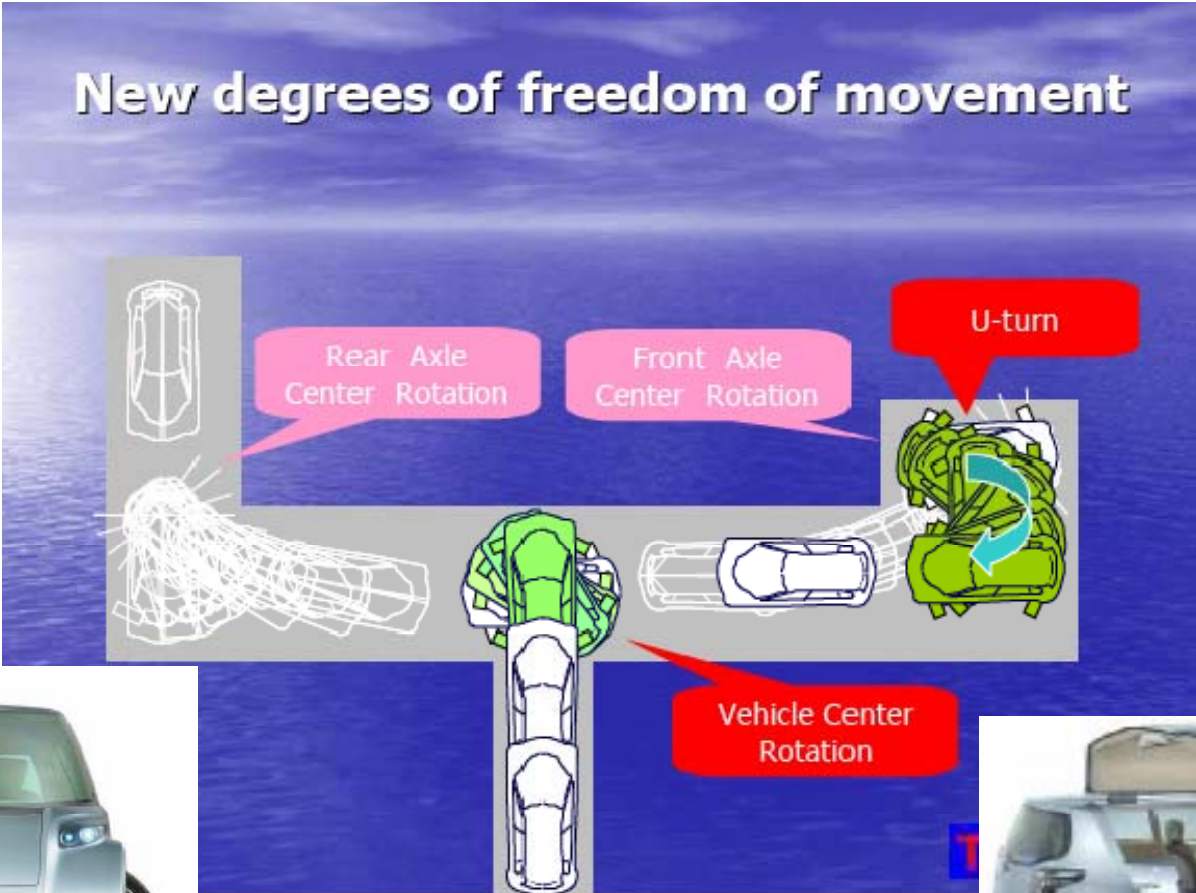


# TOYOTA Freer Movement Control System

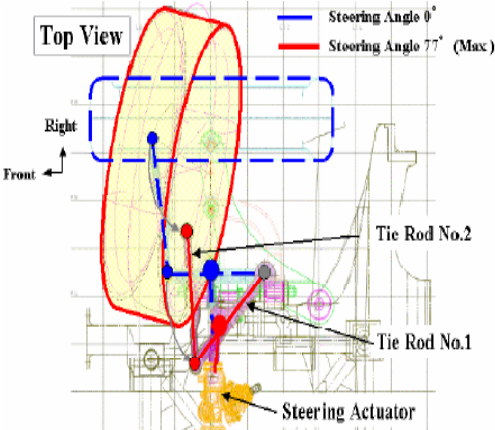
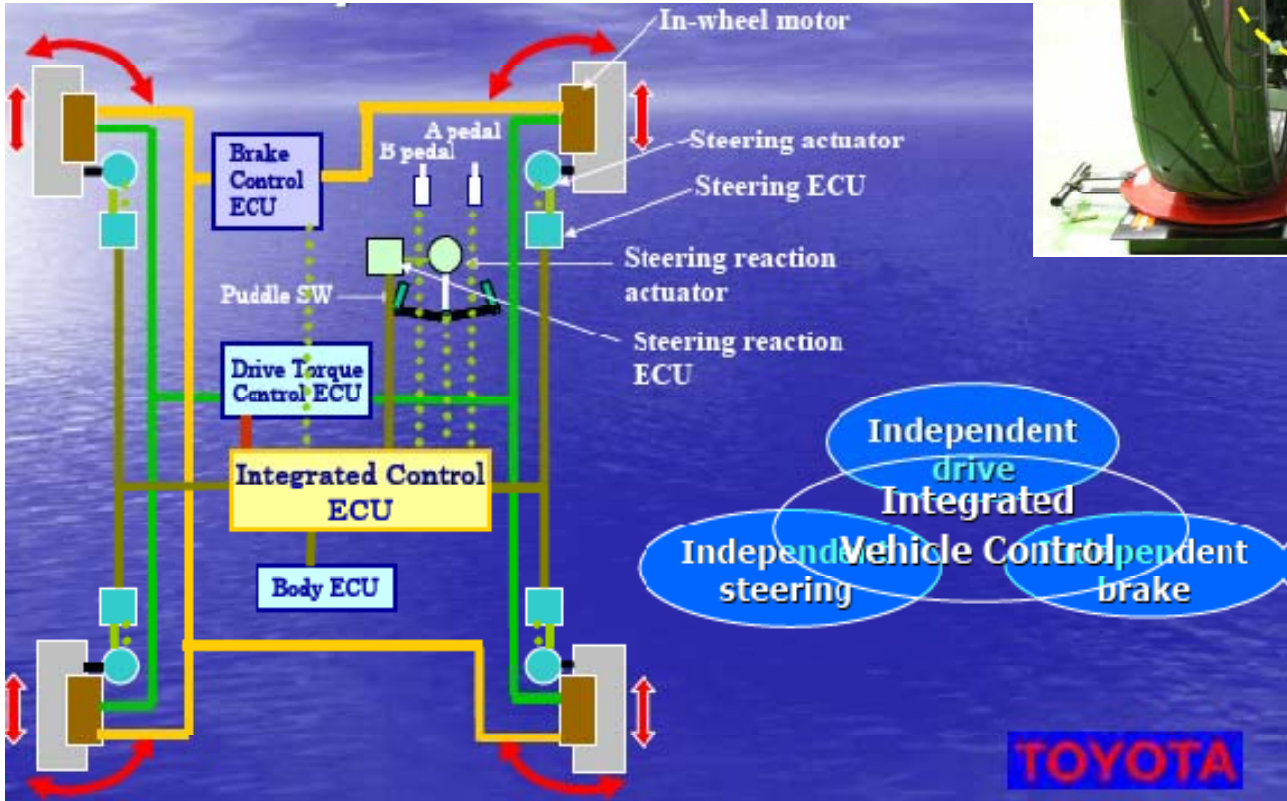




# TOYOTA Freer Movement Control System for Auto-Parking

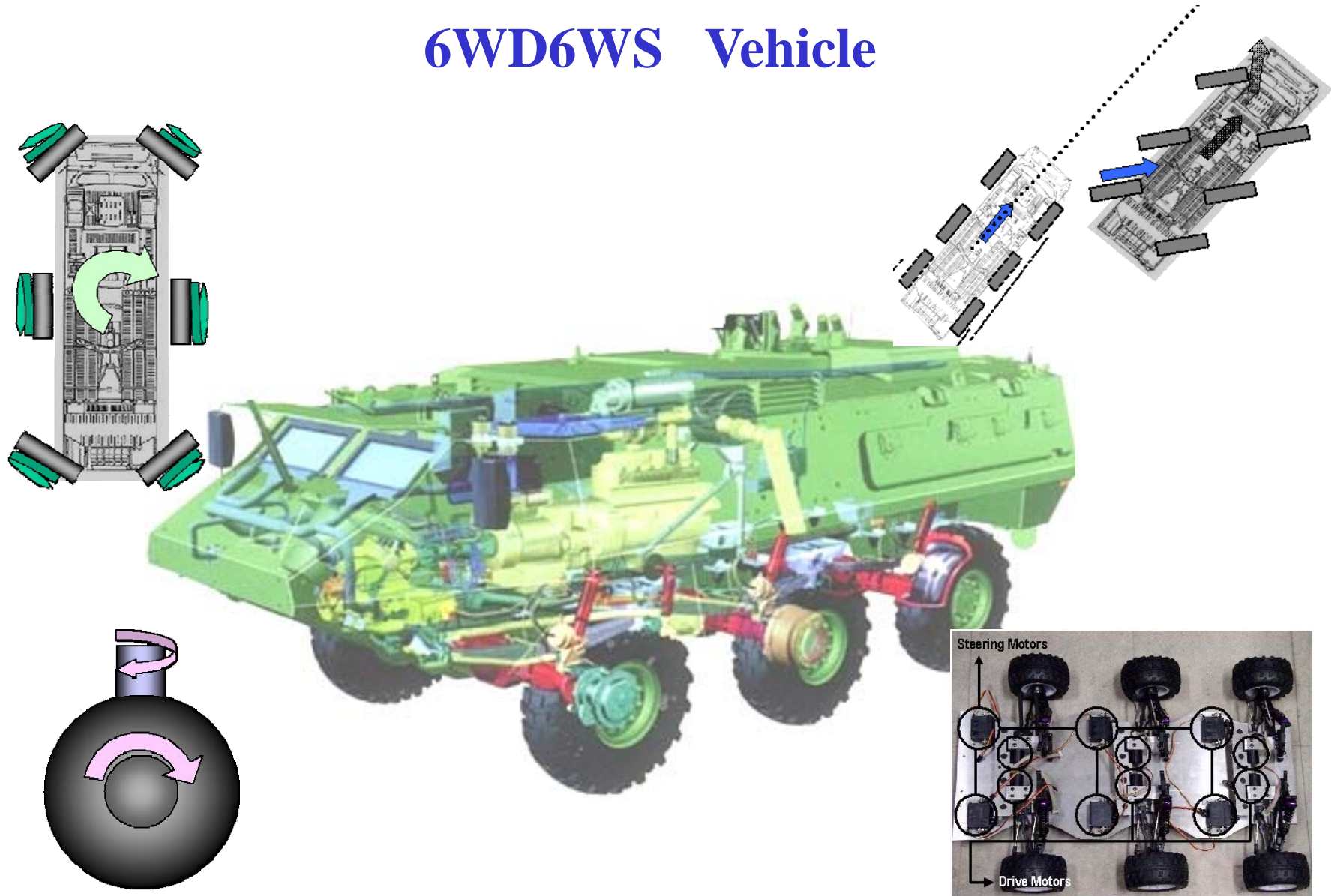


# TOYOTA Freer Movement Control System for Auto-Parking

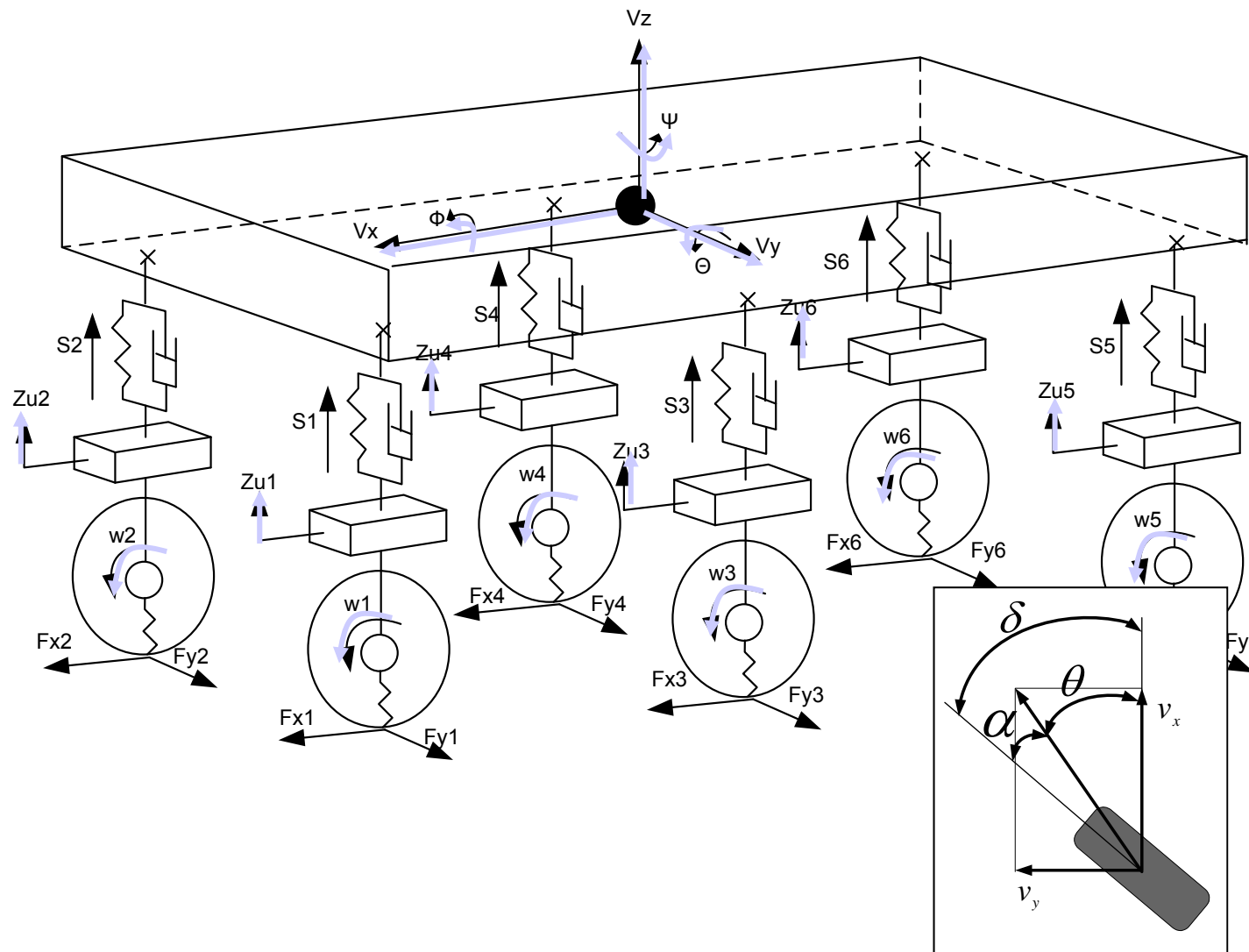




# 6WD6WS Vehicle



# 6WD6WS Vehicle

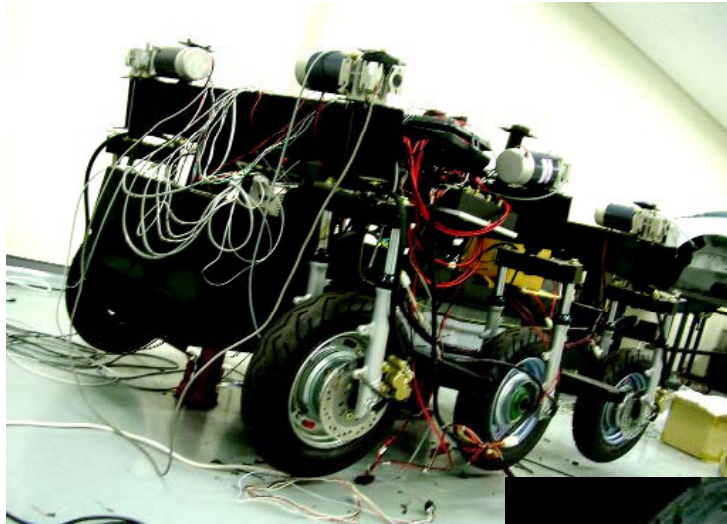


$$u = \begin{bmatrix} \Delta\delta_f \\ \delta_r \end{bmatrix}$$

$$\delta_m = X_1\beta + X_2r$$



# BLDC Wheel-in-Motor of 6WD6WS Vehicle



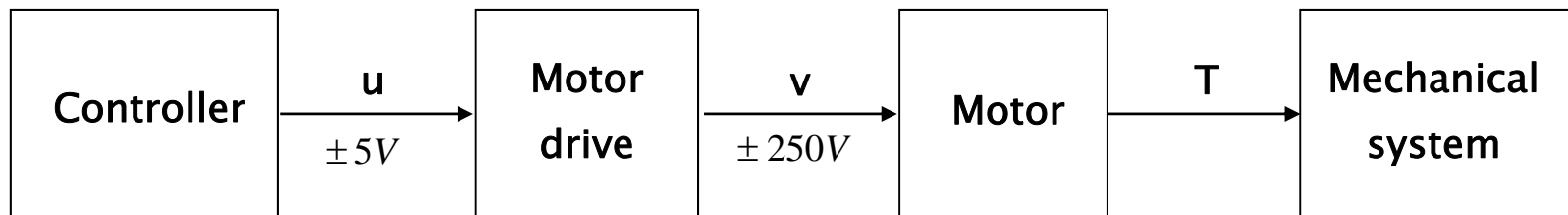


# Sectional View of BLDC Motor

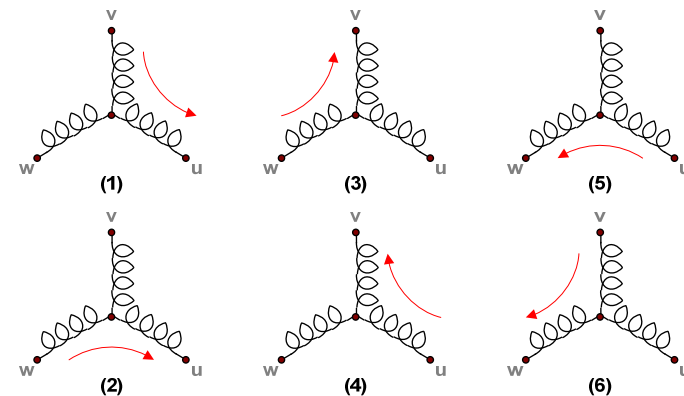
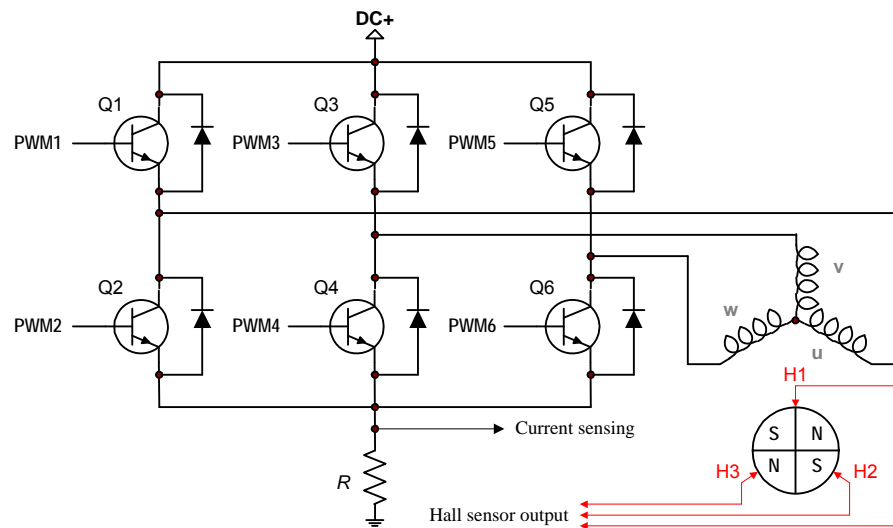


# Constitution of DC Servomotor System

Motor drive system :



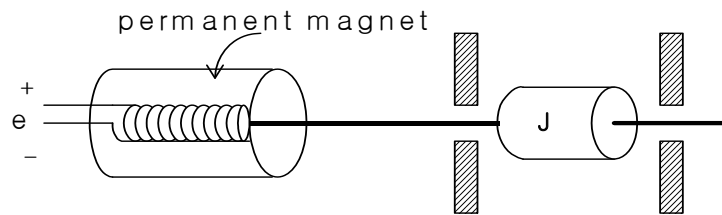
3 phase BLDC motor driver :





# Constitution of DC Servomotor System

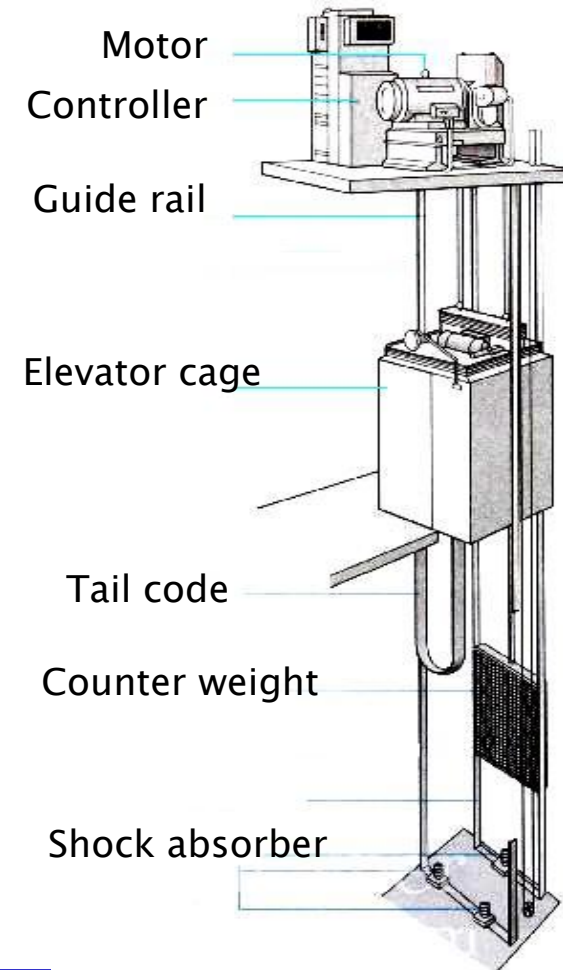
DC servo motor :



Electric Energy → Mechanical energy



ex) Elevator structure :



# Armature Control of DC Servomotors

## Variables :

$R_a$  : armature resistance,  $\Omega$

$L_a$  : armature inductance, H

$i_a$  : armature current, A

$i_f$  : field current, A

$e_a$  : applied armature voltage, V

$e_b$  : back emf, V

$\theta$  : angular displacement of the motor shaft, rad

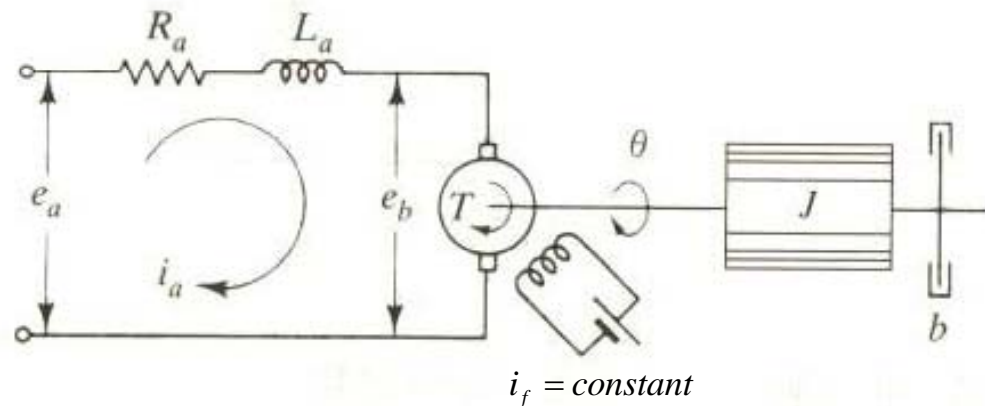
$T$  : torque developed by the motor, N-m

$J$  : equivalent moment of inertia of the motor

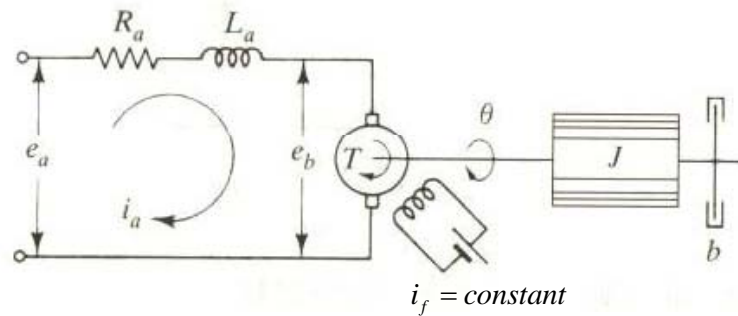
and load referred to the motor shaft,  $\text{kg-m}^2$

$b$  : equivalent viscous-friction coefficient of the motor

and load referred to the motor shaft, N-m/rad/s



# Armature Control of DC Servomotors



The torque of motor :  $T = K i_a$   $K$  : motor – torque constant

For a constant flux, the induced voltage :  $e_b = K_b \frac{d\theta}{dt}$   $K_b$  : back emf constant

Armature circuit D.E :  $L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$

Inertia and friction :  $J \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} = T = K i_a$



# Armature Control of DC Servomotors

Laplace transforms of equations :

$$e_b = K_b \frac{d\theta}{dt}$$

$$K_b s \Theta(s) = E_b(s)$$

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$

→

$$(L_a s + R_a) I_a(s) + E_b(s) = E_a(s)$$

$$J \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} = T = K i_a$$

$$(J s^2 + b s) \Theta(s) = T(s) = K I_a(s)$$

$$T.F = \frac{\Theta(s)}{E_a(s)} = \frac{K}{s(R_a J s + R_a b + K K_b)} = \frac{\frac{K}{R_a J}}{s \left( s + \frac{R_a b + K K_b}{R_a J} \right)}$$

$$= \frac{K_m}{s(T_m s + 1)}$$

$$K_m = K / (R_a b + K K_b) = \text{motor gain constant}$$

$$T_m = R_a J / (R_a b + K K_b) = \text{motor time constant}$$



# Example of a DC Servomotor System

ex) servo-motor system

$R_a$  : armature resistance,  $\Omega$

$i_a$  : armature current, A

$i_f$  : field current, A

$e_a$  : applied armature voltage, V

$e_b$  : back emf, V

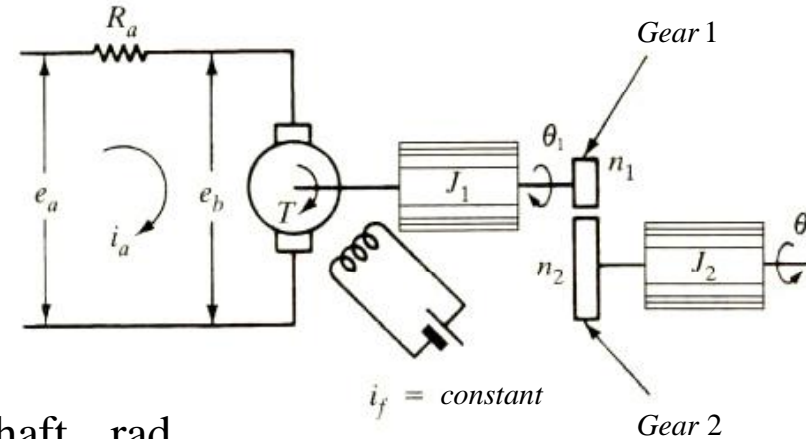
$\theta_1$  : angular displacement of the motor shaft, rad

$\theta_2$  : angular displacement of the load shaft, rad

$T$  : torque developed by the motor, N-m

$J_1$  : equivalent moment of inertia of the motor,  $\text{kg-m}^2$

$J_2$  : equivalent moment of inertia of the load,  $\text{kg-m}^2$



The torque of motor :  $T = K i_a$

For a constant flux, the induced voltage :  $e_b = K_b \frac{d\theta}{dt}$        $K_b$  : back emf constant





# Example of a DC Servomotor System

Armature circuit D.E :  $R_a i_a + e_b = e_a$       Inertia and friction :  $J_{1eq} = J_1 + \left(\frac{n_1}{n_2}\right)^2 J_2$

Laplace transforms of these equations :

$$K_b s \Theta(s) = E_b(s), \quad (L_a s + R_a) I_a(s) + E_b(s) = E_a(s), \quad (J s^2 + b s) \Theta(s) = T(s) = K I_a(s)$$

$$\begin{aligned} T.F = \frac{\Theta(s)}{E_a(s)} &= \frac{K}{s(R_a J s + R_a b + K K_b)} = \frac{\frac{K}{R_a J}}{s \left( s + \frac{R_a b + K K_b}{R_a J} \right)} \\ &= \frac{K_m}{s(T_m s + 1)} \end{aligned}$$

