Part.1
Lateral Vehicle Dynamics

1. Vehicle Dynamic Model
2. Planar Model
3. Tire Models
4. **Bicycle Model**
5. Understeer/oversteer
6. Dynamic model in terms of error w.r.t. road
7. lane keeping model
8. Vehicle Stability Control

4. 2 DOF Bicycle Model

4.1 Dynamic Equation of 2 DOF Bicycle Model

4.2 Liner Lateral Tire Model

4.3 State Equation of 2 DOF Bicycle Model

4.4 Consideration of road bank angle

4.5 Consideration of cross wind

4.6 Consideration of both road bank angle and Crosswind

4.1 2DOF Bicycle Model

- Assumption of 2DOF Bicycle Model

1) Longitudinal speed is Constant ($a_x = 0, \;\; \lambda \simeq 0$)

2) Body slip angle is sufficiently small. ($\beta = v_y / v_x, \;\; \sin\delta_f = 0, \;\; \cos\delta_f = 1$)

3) Left and right slip angles are identical. ($\alpha_f = \alpha_{fL} = \alpha_{fR}, \quad \alpha_r = \alpha_{rL} = \alpha_{rR}$)

- y-axis Motion Dynamic Equation

$$\sum F_y = m \cdot a_y = m \cdot v_x \cdot (\dot{\beta} + \dot{\psi}) = 2 \cdot F_{yf} + 2 \cdot F_{yr}$$

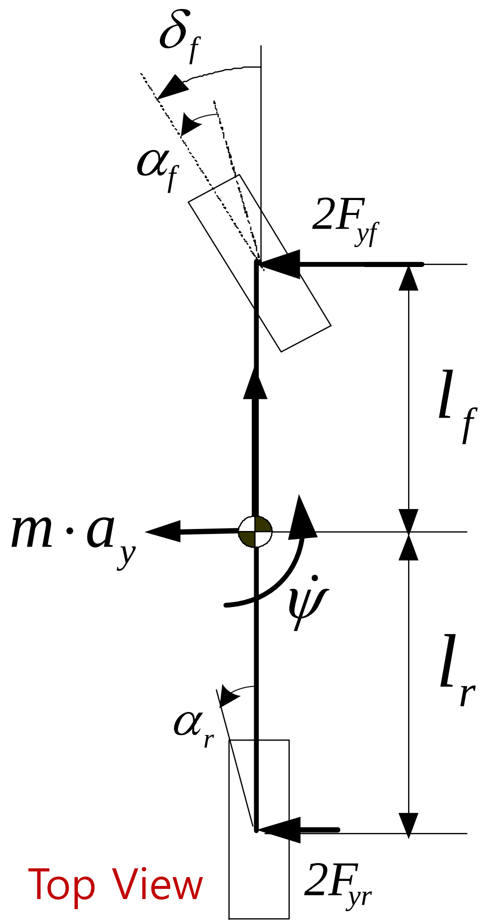
- yaw-axis Motion Dynamic Equation

$$\sum M_z = \dot{H}_z = I_z \cdot \ddot{\psi} = 2 \cdot l_f \cdot F_{yf} - 2 \cdot l_r \cdot F_{yr}$$

- Dynamic Equation of 2DOF Bicycle Model

$$\begin{bmatrix} \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \dfrac{2 \cdot F_{yf} + 2 \cdot F_{yr}}{m \cdot v_x} - \dot{\psi} \\ \dfrac{2 \cdot l_f \cdot F_{yf} - 2 \cdot l_r \cdot F_{yr}}{I_z} \end{bmatrix}$$

▲ Lateral Tire Force Model



Top View

4.2 Linear Lateral Tire Model

- Linear Tire Slip Angle at Low Slip Angle

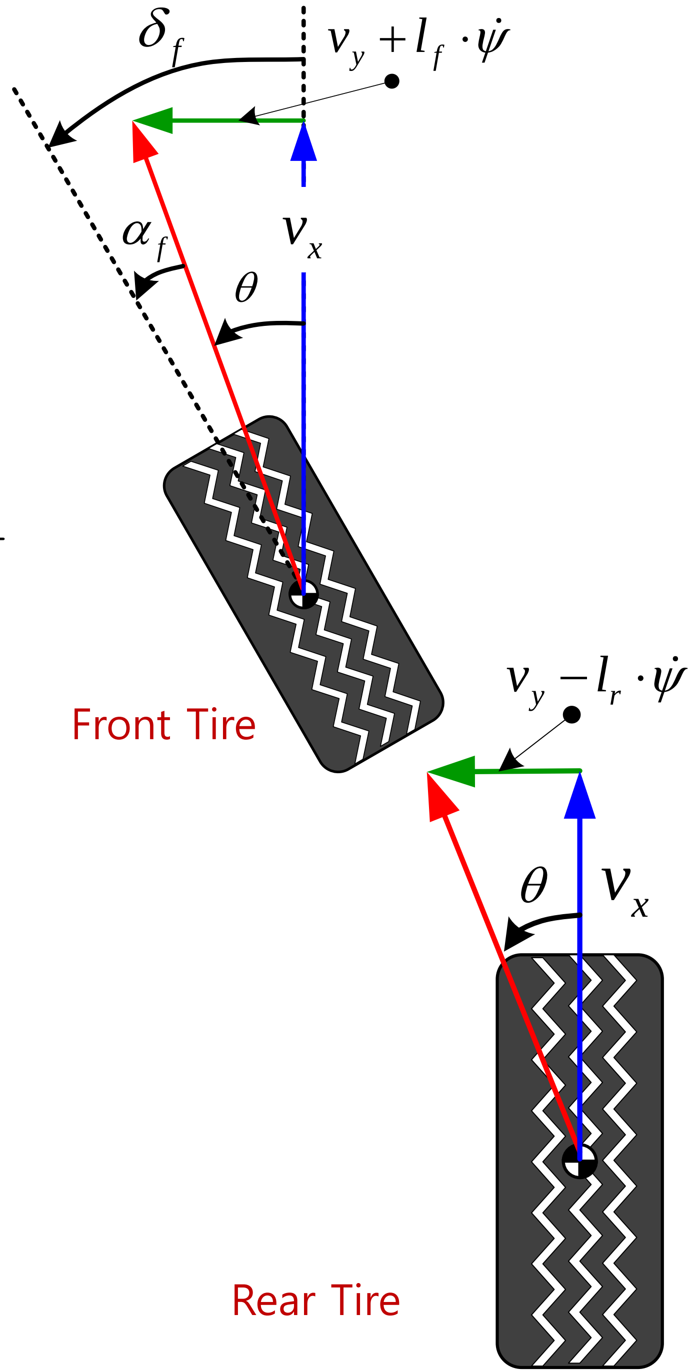
$$\tan\theta = \theta \quad if\left(\theta \ll 1\right)$$

Front Slip Angle

$$\alpha_f = \delta_f - \theta = \delta_f - \tan^1 \frac{v_y + l_f \cdot \dot{\psi}}{v_x} \simeq \delta_f - \frac{v_y + l_f \cdot \dot{\psi}}{v_x}$$

Rear Slip Angle

$$\alpha_r = -\theta = -\tan^1 \frac{v_y - l_r \cdot \dot{\psi}}{v_x} \simeq -\frac{v_y - l_r \cdot \dot{\psi}}{v_x}$$



4.2 Linear Lateral Tire Model

- Linear Lateral Tire Force at Low Slip Angle

Front Slip Angle

$$F_{yf} = \frac{\alpha_i^*}{\sigma_i^*} F_{ty0}(\sigma_i^* \times \alpha_m, \quad F_{tzi})$$

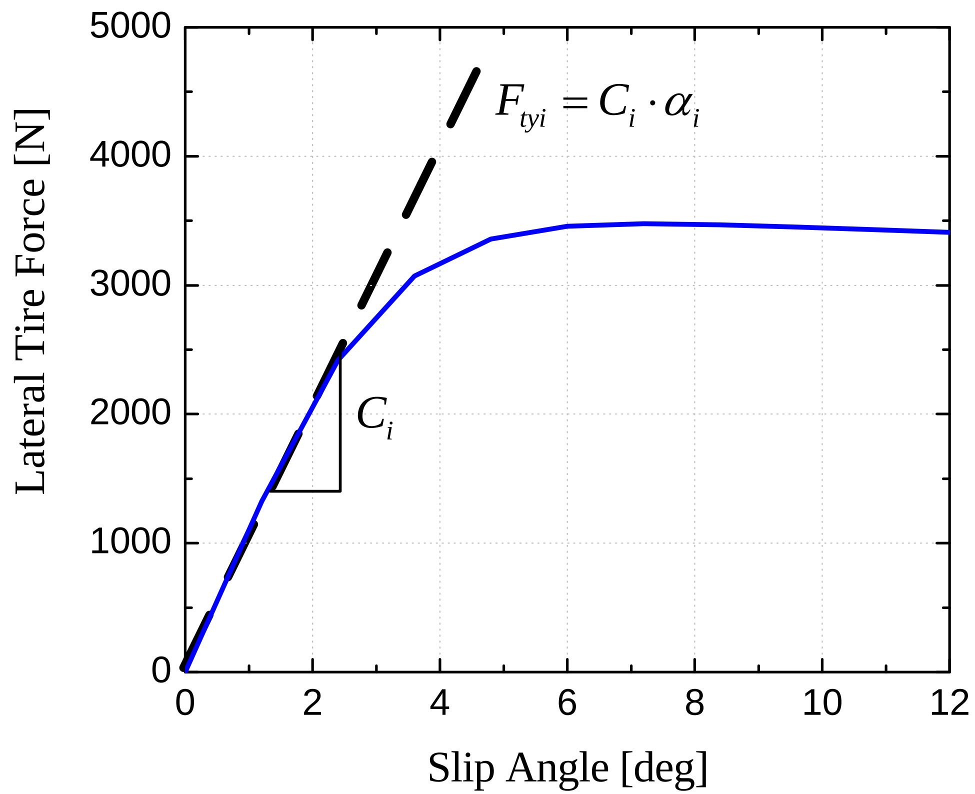
$$\simeq C_f \cdot \alpha_f = C_f \cdot \left(\delta_f - \frac{v_y + l_f \cdot \dot{\psi}}{v_x} \right)$$

Rear Slip Angle

$$F_{yr} = \frac{\alpha_i^*}{\sigma_i^*} F_{ty0}(\sigma_i^* \times \alpha_m, \quad F_{tzi})$$

$$\simeq C_r \cdot \alpha_r = C_r \cdot \left(- \frac{v_y - l_r \cdot \dot{\psi}}{v_x} \right)$$

Where, $C_i =$ Cornering Stiffness



Linear Lateral Tire Force

4.3 State Equation of 2 DOF Bicycle Model

- Dynamic Equation of 2DOF Bicycle Model

  - y-axis Motion Dynamic Equation

$$\sum F_y = m \cdot a_y = m \cdot v_x \cdot (\dot{\beta} + \dot{\psi}) = 2 \cdot F_{yf} + 2 \cdot F_{yr}$$

$$= 2 \cdot C_f \cdot \left(\delta_f - \frac{v_y + l_f \cdot \dot{\psi}}{v_x} \right) + 2 \cdot C_r \cdot \left(- \frac{v_y - l_r \cdot \dot{\psi}}{v_x} \right)$$

  - yaw-axis Motion Dynamic Equation

$$\sum M_z = \dot{H}_z = I_z \cdot \ddot{\psi} = 2 \cdot l_f \cdot F_{yf} - 2 \cdot l_r \cdot F_{yr}$$

$$= 2 \cdot l_f \cdot C_f \cdot \left(\delta_f - \frac{v_y + l_f \cdot \dot{\psi}}{v_x} \right) - 2 \cdot l_r \cdot C_r \cdot \left(- \frac{v_y - l_r \cdot \dot{\psi}}{v_x} \right)$$

- Define state = [Body Side Slip Angle, Yaw Rate]

$$x = \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix}$$

4.3 State Equation of 2 DOF Bicycle Model

- Define state = [Body Side Slip Angle, Yaw Rate]

$$x = \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix}$$

- State Equation of 2DOF Bicycle Model

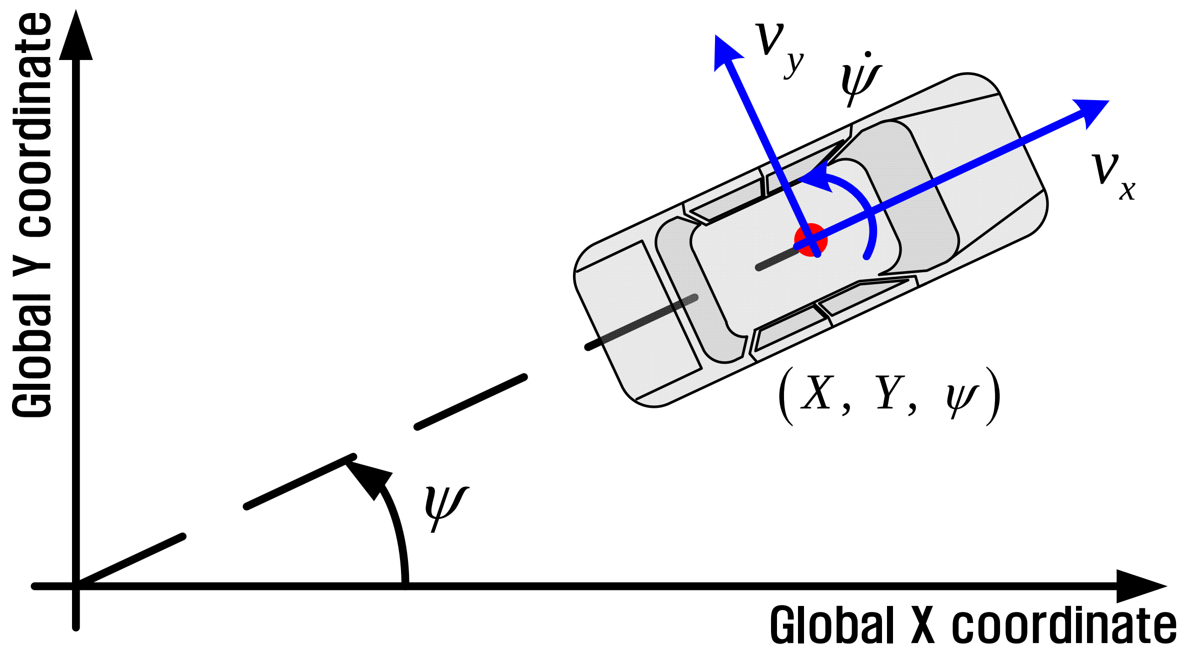
$$\dot{x} = \begin{bmatrix} \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} -\dfrac{2(C_f + C_r)}{m \cdot v_x} \cdot \beta + \left(-1 - \dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{m \cdot {v_x}^2} \right) \cdot \dot{\psi} + \dfrac{2 \cdot C_f}{m v_x} \cdot \delta_f \\ -\dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} \cdot \beta - \dfrac{2({l_f}^2 \cdot C_f + {l_r}^2 \cdot C_r)}{I_z \cdot v_x} \cdot \dot{\psi} + \dfrac{2 \cdot l_f \cdot C_f}{I_z} \cdot \delta_f \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{2(C_f + C_r)}{m \cdot v_x} & -1 - \dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{m \cdot {v_x}^2} \\ -\dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} & -\dfrac{2({l_f}^2 \cdot C_f + {l_r}^2 \cdot C_r)}{I_z \cdot v_x} \end{bmatrix} \cdot \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} \dfrac{2 \cdot C_f}{m v_x} \\ \dfrac{2 \cdot l_f \cdot C_f}{I_z} \end{bmatrix} \cdot \delta_f$$

$$= Ax + B\delta_f$$

4.3 State Equation of 2 DOF Bicycle Model

- state = [Body Side Slip Angle, Yaw Rate] $x = \begin{bmatrix} \beta & \dot{\psi} \end{bmatrix}^T$
- Lateral Speed: $v_y = v_x \cdot \beta$
- Yaw Angle: $\psi = \int \dot{\psi} \, dt$
- X Position: $X = X_0 + \int v_x \cdot \cos\psi - v_y \cdot \sin\psi \, dt$
- Y Position: $Y = Y_0 + \int v_x \cdot \sin\psi + v_y \cdot \cos\psi \, dt$



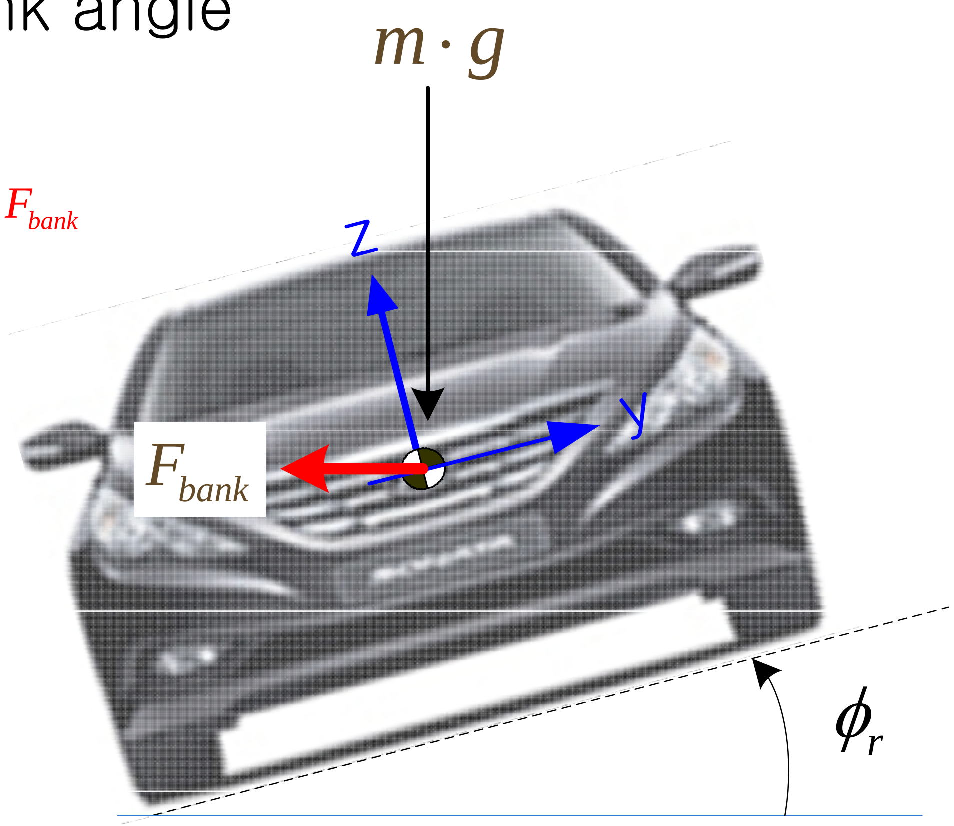
4.4 Consideration of road bank angle

- y-axis Motion Dynamic Equation

$$\sum F_y = m \cdot a_y = m \cdot v_x \cdot (\dot{\beta} + \dot{\psi}) = 2 \cdot F_{yf} + 2 \cdot F_{yr} - F_{bank}$$

Where, $F_{bank} = m \cdot g \cdot \sin(\phi_r)$

$\phi_r =$ Road Bank Angle



- yaw-axis Motion Dynamic Equation

$$\sum M_z = \dot{H}_z = I_z \cdot \ddot{\psi} = 2 \cdot l_f \cdot F_{yf} - 2 \cdot l_r \cdot F_{yr}$$

- State Equation considering of road bank angle

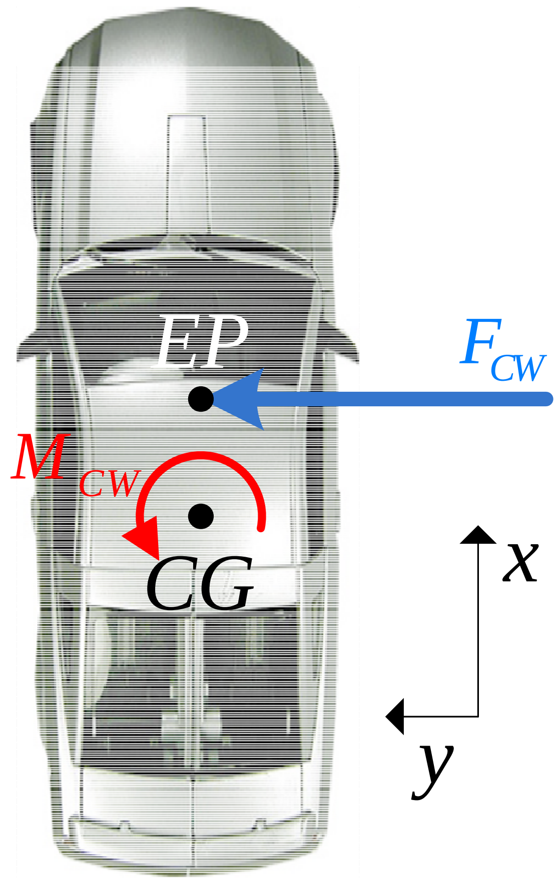
$$\dot{x} = A \cdot \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix} + B \cdot \delta_f + \underbrace{\begin{bmatrix} -\dfrac{g}{v_x} \cdot \sin(\phi_r) \\ 0 \end{bmatrix}}_{w_d(\phi_r)} = A \cdot \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix} + B \cdot \delta_f + w_d(\phi_r)$$

Where, $w_d(\phi_r)$ denotes the disturbance term by road bank angle.

4.5 Consideration of Crosswind disturbance

- Factors of Crosswind Disturbance

1) Air density
2) Relative velocity of crosswind to vehicle
3) Vehicle frontal area
4) Crosswind lateral force coefficient
   / Crosswind yaw moment coefficient



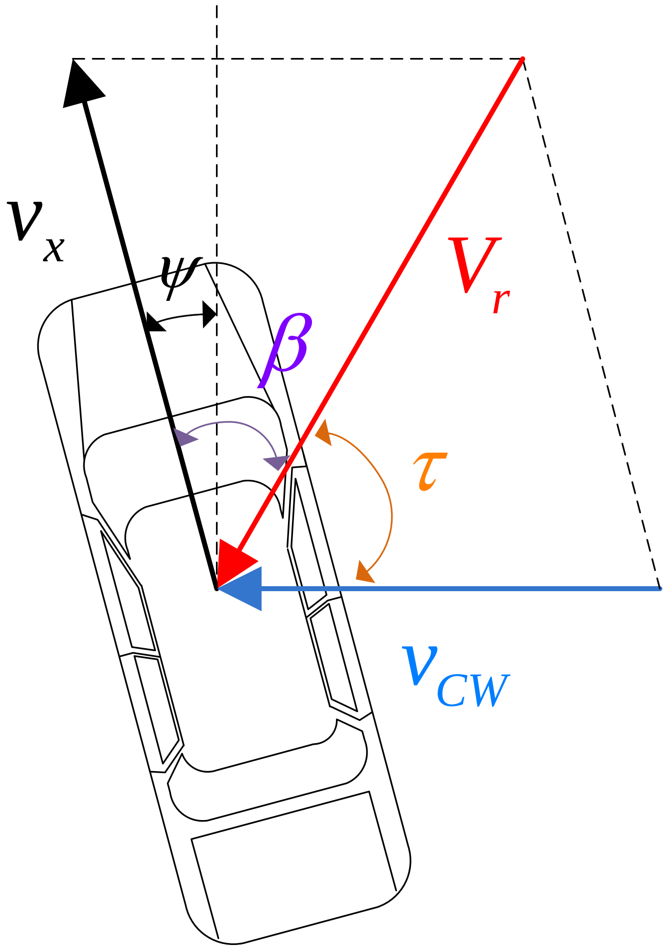
1) Air density

: 1.225 kgm^{-3}

under 1 atm, 20 degrees Celsius

4.5 Consideration of Crosswind disturbance

2) Relative velocity of crosswind to vehicle



$$\vec{v}_r = \vec{v}_{CW} - \vec{v}_x$$

$$v_r^2 = v_x^2 + v_{CW}^2 - 2 \cdot v_x \cdot v_{CW} \cdot \cos\left(\frac{\pi}{2} - \psi\right)$$

From cosine 2'nd law

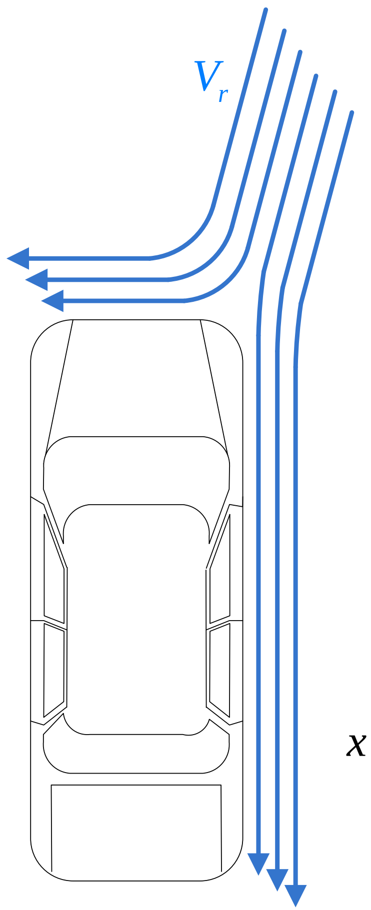
$$\tau = \sin^{-1}\left(\frac{v_x \sin\left(\frac{\pi}{2} - \psi\right)}{v_r}\right)$$

airflow side slip β

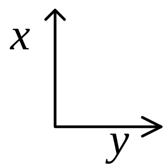
$$\beta = \frac{\pi}{2} + \psi - \tau$$

4.5 Consideration of Crosswind disturbance

3) Vehicle frontal area



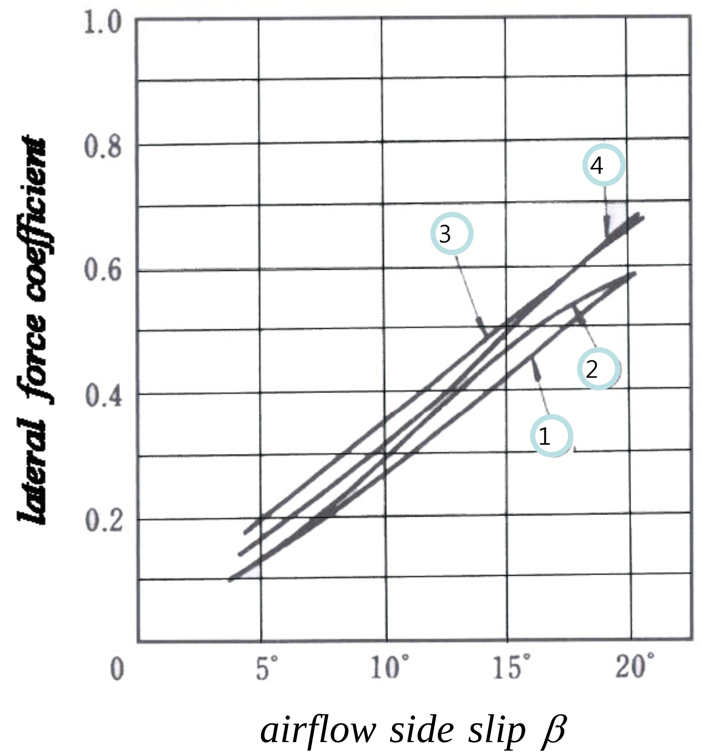
Crosswind drag force acts on the vehicle in lateral direction

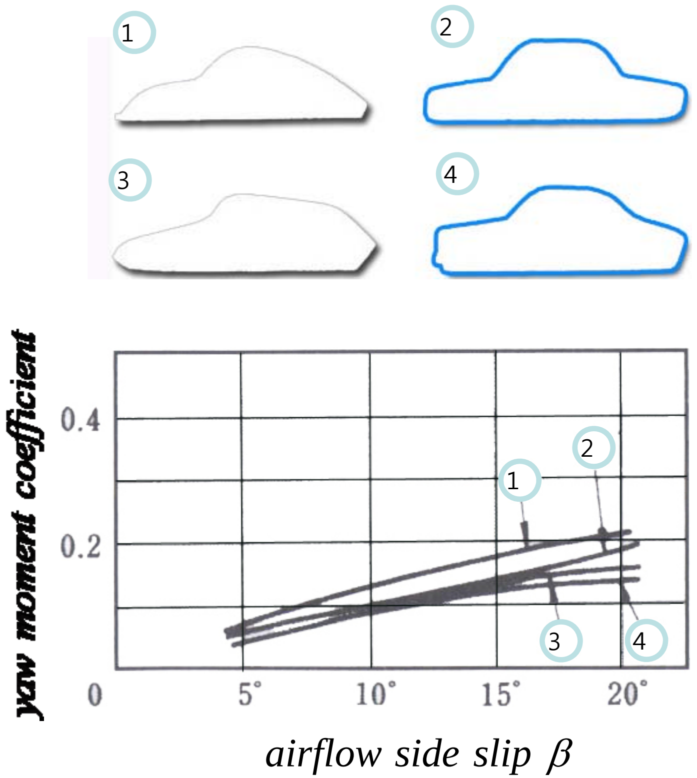


4.5 Consideration of Crosswind disturbance

4) Crosswind lateral force coefficient / Crosswind yaw moment coefficient

Lateral force/yaw moment coefficient due to vehicle side shape





2009 Abe, Masato 『Vehicle Handling Dynamics : Theory and Application』

4.5 Consideration of Crosswind disturbance

- Crosswind Lateral Force / Yaw Moment

$$F_{CW} = \frac{1}{2}\rho C_f {v_r}^2 A$$

$$M_{z,CW} = C_n \frac{\rho}{2} l A v_r^2$$

$v_r = crosswind\ velocity\ relative\ to\ vehicle$

$\rho = density\ of\ air$

$C_f = lateral\ force\ coefficient$

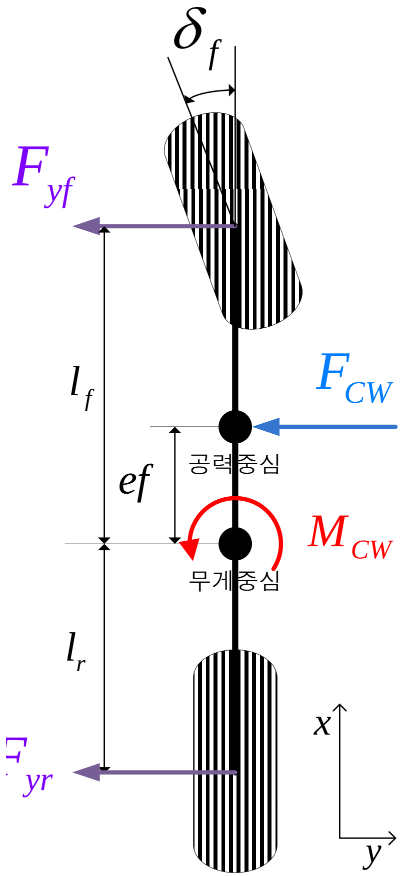
$C_n = yaw\ moment\ coefficient$

$A = front\ area\ of\ the\ vehicle$

$l = wheel\ base$

4.5 Consideration of Crosswind disturbance

- Total Lateral Effects including Crosswind Disturbance acting on Bicycle Model



- y-axis Motion Dynamic Equation

$$\sum F_y = m \cdot v_x(\dot{\beta} + \dot{\psi}) = 2F_{yf} + 2F_{yr} + F_{CW}$$

- yaw-axis Motion Dynamic Equation

$$\sum M_z = I_z \cdot \ddot{\psi} = 2F_{yf} \cdot l_f - 2F_{yr} \cdot l_r + M_{CW}$$

Linear Slip angle

$$\alpha_f = \delta_f - (\beta + \frac{l_f \cdot \dot{\psi}}{v_x}) \qquad \alpha_r = -\beta + \frac{l_r \cdot \dot{\psi}}{v_x}$$

Linear Lateral forces

$$F_{yf} = C_f \alpha_f \qquad F_{yr} = C_r \alpha_r$$

4.5 Consideration of Crosswind disturbance

- Modified State Equation of 2DOF Bicycle Model under crosswind disturbance

$$\dot{x} = \begin{bmatrix} \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} -\dfrac{2(C_f + C_r)}{m \cdot v_x} \cdot \beta + \left(-1 - \dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{m \cdot v_x{}^2} \right) \cdot \dot{\psi} + \dfrac{2 \cdot C_f}{m v_x} \cdot \delta_f + \dfrac{F_{CW}}{m v_x} \\ -\dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} \cdot \beta - \dfrac{2(l_f{}^2 \cdot C_f + l_r{}^2 \cdot C_r)}{I_z \cdot v_x} \cdot \dot{\psi} + \dfrac{2 \cdot l_f \cdot C_f}{I_z} \cdot \delta_f + \dfrac{M_{CW}}{I_z} \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{2(C_f + C_r)}{m \cdot v_x} & -1 - \dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{m \cdot v_x{}^2} \\ -\dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} & -\dfrac{2(l_f{}^2 \cdot C_f + l_r{}^2 \cdot C_r)}{I_z \cdot v_x} \end{bmatrix} \cdot \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} \dfrac{2 \cdot C_f}{m v_x} \\ \dfrac{2 \cdot l_f \cdot C_f}{I_z} \end{bmatrix} \cdot \delta_f + \begin{bmatrix} \dfrac{1}{m v_x} & 0 \\ 0 & \dfrac{1}{I_z} \end{bmatrix} \cdot \begin{bmatrix} F_{CW} \\ M_{CW} \end{bmatrix}$$

$$= Ax + B\delta_f + B_{CW} \cdot \begin{bmatrix} F_{CW} \\ M_{CW} \end{bmatrix}$$

4.6 Consideration of both road bank angle and Crosswind

- y-axis Motion Dynamic Equation

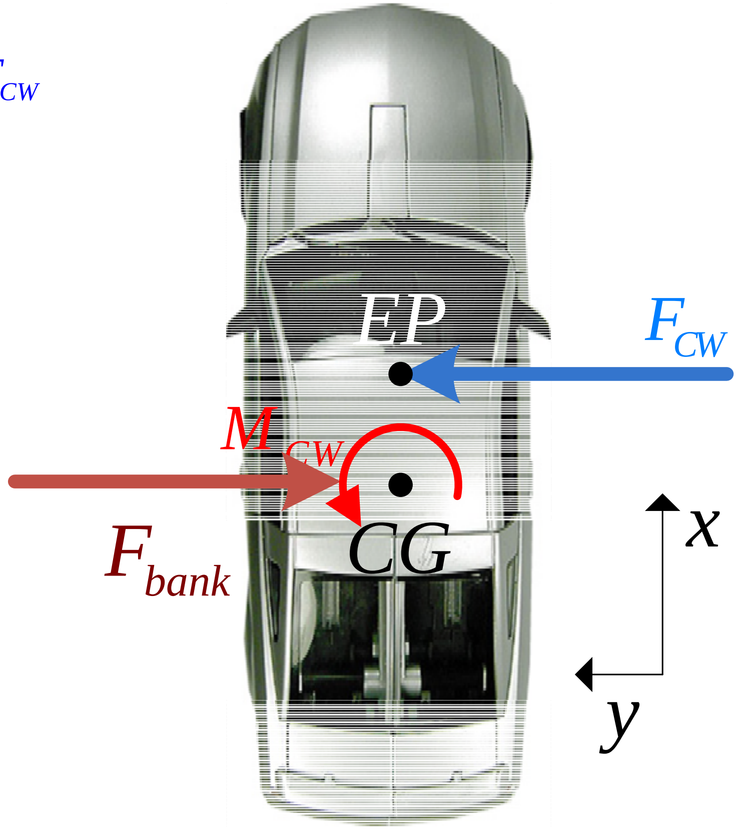
$$\sum F_y = m \cdot a_y = m \cdot v_x \cdot (\dot{\beta} + \dot{\psi}) = 2 \cdot F_{yf} + 2 \cdot F_{yr} - F_{bank} + F_{CW}$$

Where, $F_{bank} = \quad m \cdot g \cdot \sin(\phi_r)$

$\phi_r = \quad$ Road Bank Angle

$F_{CW} = \quad \frac{1}{2} \rho C_f v_r^{\,2} A$

$v_r = \quad$ *crosswind velocity relative to vehicle*



- yaw-axis Motion Dynamic Equation

$$\sum M_z = \dot{H}_z = I_z \cdot \ddot{\psi} = 2 \cdot l_f \cdot F_{yf} - 2 \cdot l_r \cdot F_{yr} + M_{CW}$$

Where, $M_{z,CW} = C_n \frac{\rho}{2} l A v_r^2$

- State Equation considering of both road bank angle and cross wind

$$\dot{x} = Ax + B\delta_f + \begin{bmatrix} -\frac{g}{v_x} & \frac{1}{mv_x} & 0 \\ \frac{g}{v_x} & 0 & \frac{1}{I_z} \end{bmatrix} \cdot \begin{bmatrix} \sin(\phi_r) \\ F_{CW} \\ M_{CW} \end{bmatrix} = Ax + B\delta_f + B_W \cdot w_d$$

Where, w_d denotes the disturbance term by both road bank angle and cross wind.

5. Understeer/oversteer

5. Understeer/oversteer

- Lateral Force and Yaw Moment Balance

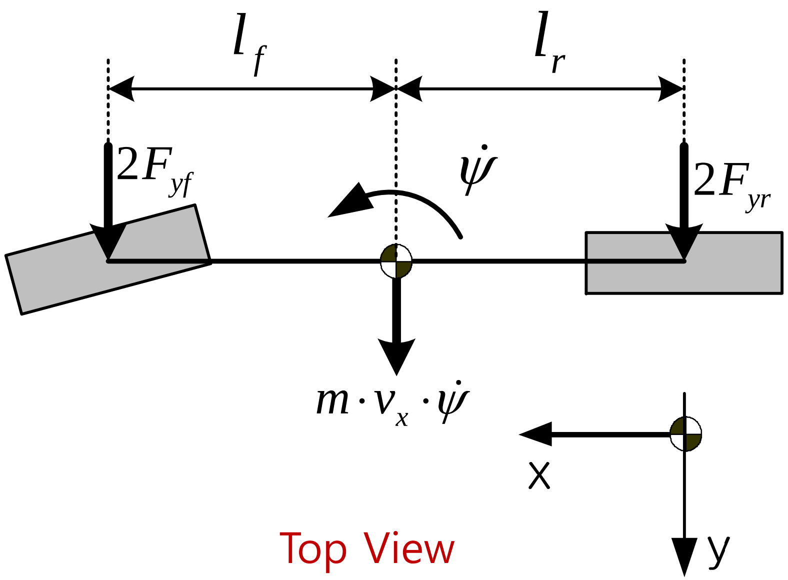
  Dynamic Equation for steady state ($\dot{v}_y = \ddot{\psi} = 0$)

$$m \cdot v_x \cdot \dot{\psi} = 2 \cdot F_{yf} + 2 \cdot F_{yr}$$

$$0 = 2 \cdot l_f \cdot F_{yf} - 2 \cdot l_r \cdot F_{yr}$$

$$\Rightarrow F_{yf} = \frac{l_r}{2(l_f + l_r)} m \cdot v_x \cdot \dot{\psi}, \quad F_{yr} = \frac{l_f}{2(l_f + l_r)} m \cdot v_x \cdot \dot{\psi}$$

▲ Lateral Tire Force for steady state



Top View

5. Understeer/oversteer

- Vertical Force Balance (No Suspension Dynamics)

Dynamic Equation for steady state ($\dot{v}_z = \ddot{\theta} = 0$)

$$m \cdot g = 2 \cdot F_{zf} + 2 \cdot F_{zr}$$

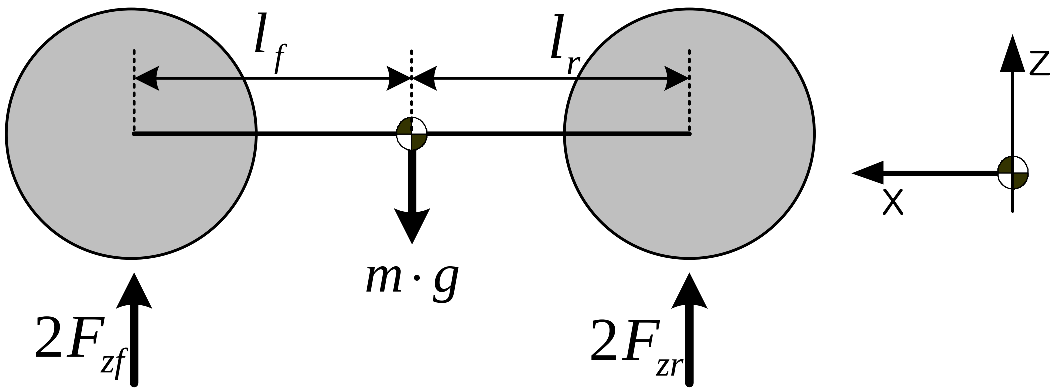
$$0 = 2l_f \cdot F_{zf} - 2l_f \cdot F_{zr}$$

$$\Rightarrow F_{zf} = \frac{l_r}{2\left(l_f + l_r\right)} m \cdot g, \quad F_{zr} = \frac{l_f}{2\left(l_f + l_r\right)} m \cdot g$$

▲ Vertical Tire Force for steady state

- Relation between Lateral and Vertical Tire Force for steady state

$$\Rightarrow \frac{F_{yf}}{F_{zf}} = \frac{F_{yr}}{F_{zr}} = \frac{v_x \cdot \dot{\psi}}{g}$$



Side View

5. Understeer/oversteer

- Relation of front and rear slip angles
  - Linear Tire Slip Angle at Low Slip Angle

$$\alpha_f = \delta_f - \frac{v_y + l_f \cdot \dot{\psi}}{v_x}, \quad \alpha_r = -\frac{v_y - l_r \cdot \dot{\psi}}{v_x}$$

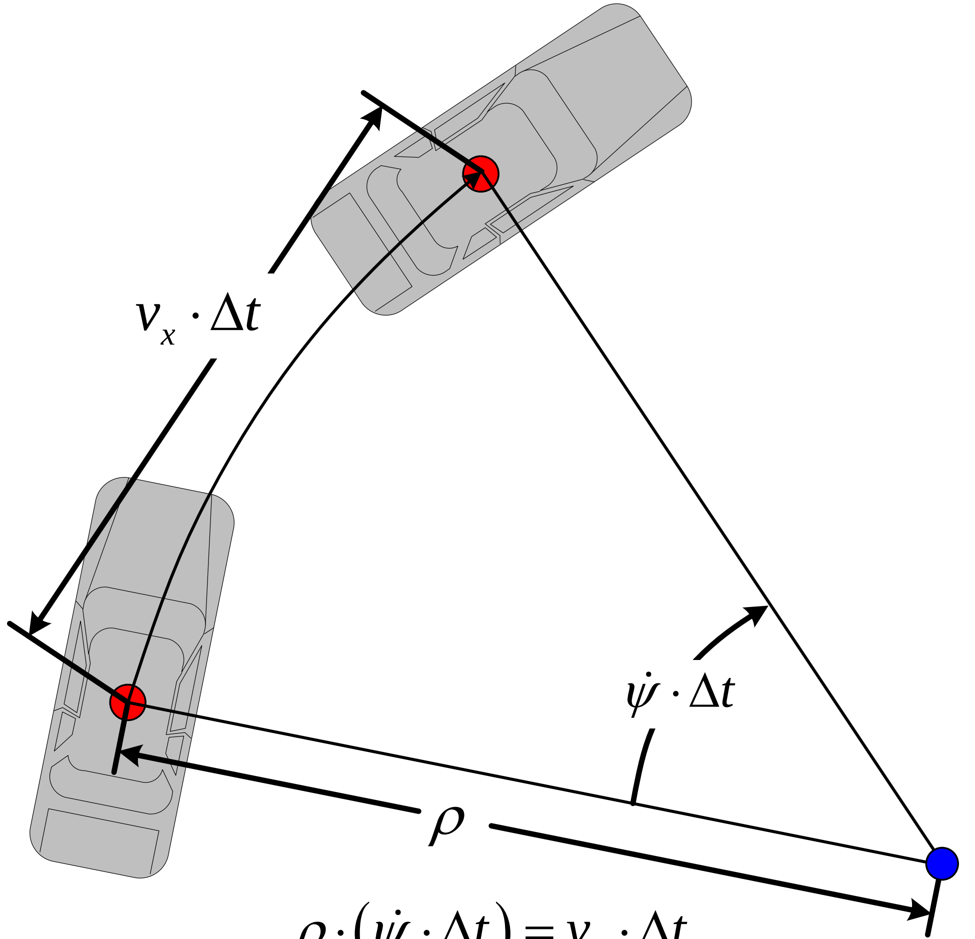
  - Relation of front and rear slip angles

$$\alpha_f - \alpha_r = \delta_f - \frac{v_y + l_f \cdot \dot{\psi}}{v_x} + \frac{v_y - l_r \cdot \dot{\psi}}{v_x}$$

$$= \delta_f - (l_f + l_r)\frac{\dot{\psi}}{v_x}$$

$$= \delta_f - \frac{l_f + l_r}{\rho}$$

$$\Rightarrow \delta_f = \frac{l_f + l_r}{\rho} + \alpha_f - \alpha_r$$



$$\rho \cdot (\dot{\psi} \cdot \Delta t) = v_x \cdot \Delta t$$

$$\rho \cdot \dot{\psi} = v_x$$

- Slip Angle for Steady State

$$F_{yi} = C_i \cdot \alpha_i \quad \Rightarrow \begin{cases} \alpha_f = \dfrac{F_{yf}}{C_f} = \dfrac{l_r}{2(l_f + l_r)} \dfrac{m \cdot v_x \cdot \dot{\psi}}{C_f} \\ \alpha_r = \dfrac{F_{yr}}{C_r} = \dfrac{l_f}{2(l_f + l_r)} \dfrac{m \cdot v_x \cdot \dot{\psi}}{C_r} \end{cases} \Leftarrow \begin{cases} F_{yf} = \dfrac{l_r}{2(l_f + l_r)} m \cdot v_x \cdot \dot{\psi} \\ F_{yr} = \dfrac{l_f}{2(l_f + l_r)} m \cdot v_x \cdot \dot{\psi} \end{cases}$$

▲ Linear Tire Model

▲ Lateral Tire Force for steady state

- Under Steer Coefficient

$$\begin{aligned} \delta_f &= \frac{l_f + l_r}{\rho} + \alpha_f - \alpha_r \\ &= \frac{l_f + l_r}{\rho} + \frac{l_r}{2(l_f + l_r)} \frac{m \cdot v_x \cdot \dot{\psi}}{C_f} - \frac{l_f}{2(l_f + l_r)} \frac{m \cdot v_x \cdot \dot{\psi}}{C_r} \\ &= \frac{l_f + l_r}{\rho} + \left(\frac{l_r \cdot m}{2(l_f + l_r) \cdot C_f} - \frac{l_f \cdot m}{2(l_f + l_r) \cdot C_r} \right) \cdot v_x \cdot \dot{\psi} \\ &= \frac{l_f + l_r}{\rho} + \left(\frac{F_{zf}}{C_f} - \frac{F_{zr}}{C_r} \right) \cdot \frac{v_x \cdot \dot{\psi}}{g} \\ &= \frac{l_f + l_r}{\rho} + \underbrace{\left(\frac{F_{zf}}{C_f} - \frac{F_{zr}}{C_r} \right)}_{K_{us}} \cdot \frac{v_x^2}{g \cdot \rho} \end{aligned}$$

$$\Rightarrow \delta_f = \frac{l_f + l_r}{\rho} + K_{us} \cdot \frac{v_x^2}{g \cdot \rho}$$

Where, K_{us} *is* under steer coefficient.

5. Understeer/oversteer

- Under Steer Coefficient

$$\delta_f = \frac{L}{\rho} + K_{us} \cdot \frac{v_x^2}{g \cdot \rho}$$

Where, $K_{us} = \frac{F_{zf}}{C_f} - \frac{F_{zr}}{C_r}$

Neutral Steer ($K_{us} = 0$)

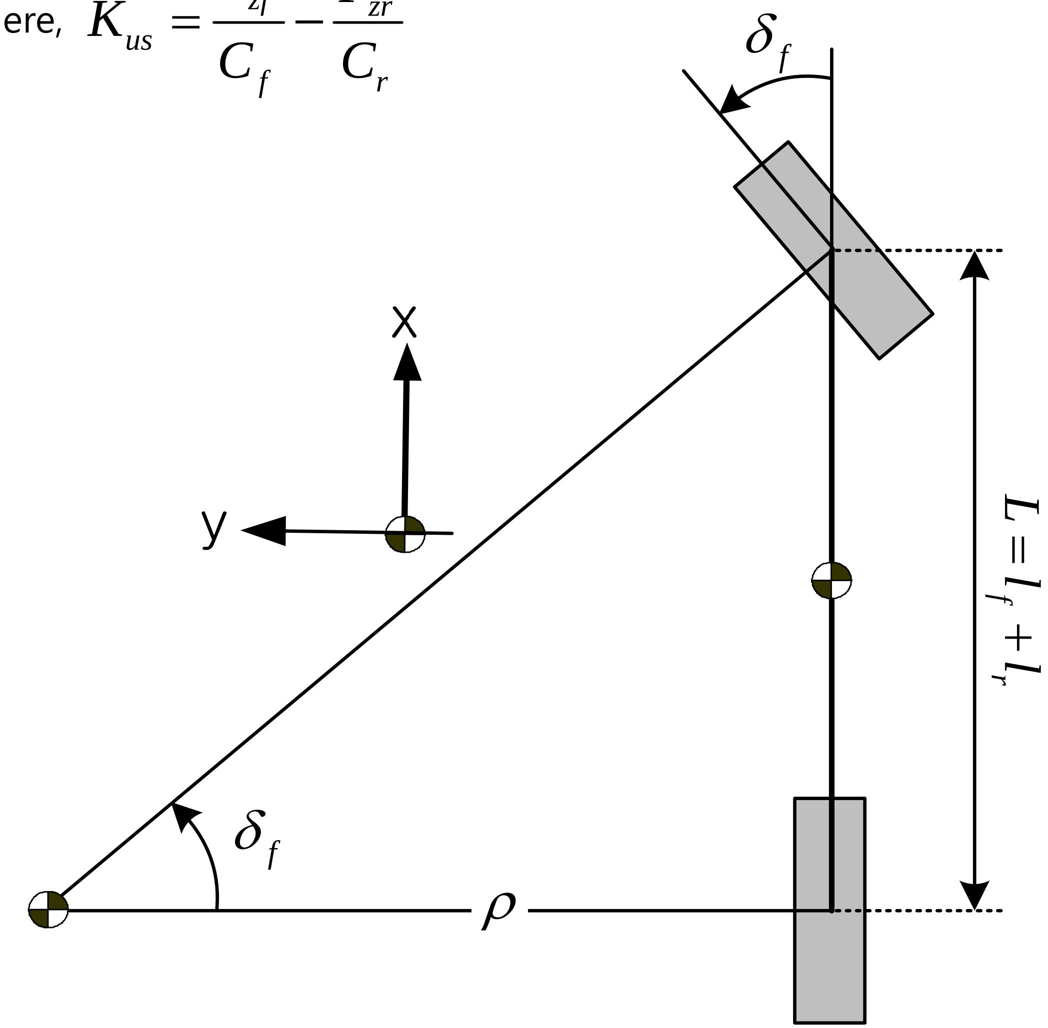
$$\rho = \frac{L}{\delta_f}$$

Under Steer ($K_{us} > 0$)

$$\rho = \frac{1}{\delta_f} \cdot \left(L + K_{us} \cdot \frac{v_x^2}{g} \right) > \frac{L}{\delta_f}$$

Over Steer ($K_{us} < 0$)

$$\rho = \frac{1}{\delta_f} \cdot \left(L + K_{us} \cdot \frac{v_x^2}{g} \right) < \frac{L}{\delta_f}$$



5. Understeer/oversteer

Neutral Steer ($K_{us} = 0$)

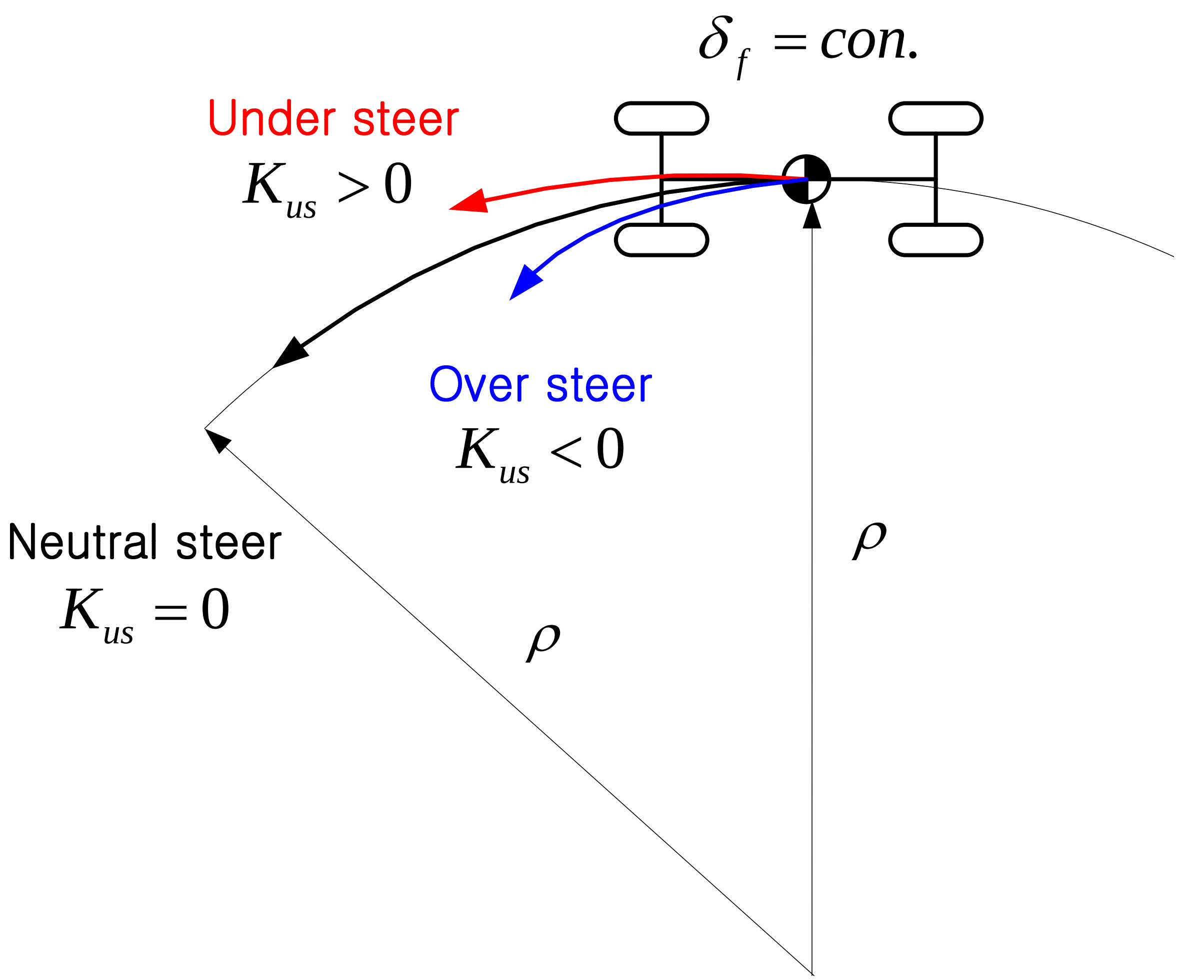
$$\rho = \frac{L}{\delta_f}$$

Under Steer ($K_{us} > 0$)

$$\rho = \frac{1}{\delta_f} \cdot \left(L + K_{us} \cdot \frac{v_x^2}{g} \right) > \frac{L}{\delta_f}$$

Over Steer ($K_{us} < 0$)

$$\rho = \frac{1}{\delta_f} \cdot \left(L + K_{us} \cdot \frac{v_x^2}{g} \right) < \frac{L}{\delta_f}$$



- Under Steer Coefficient

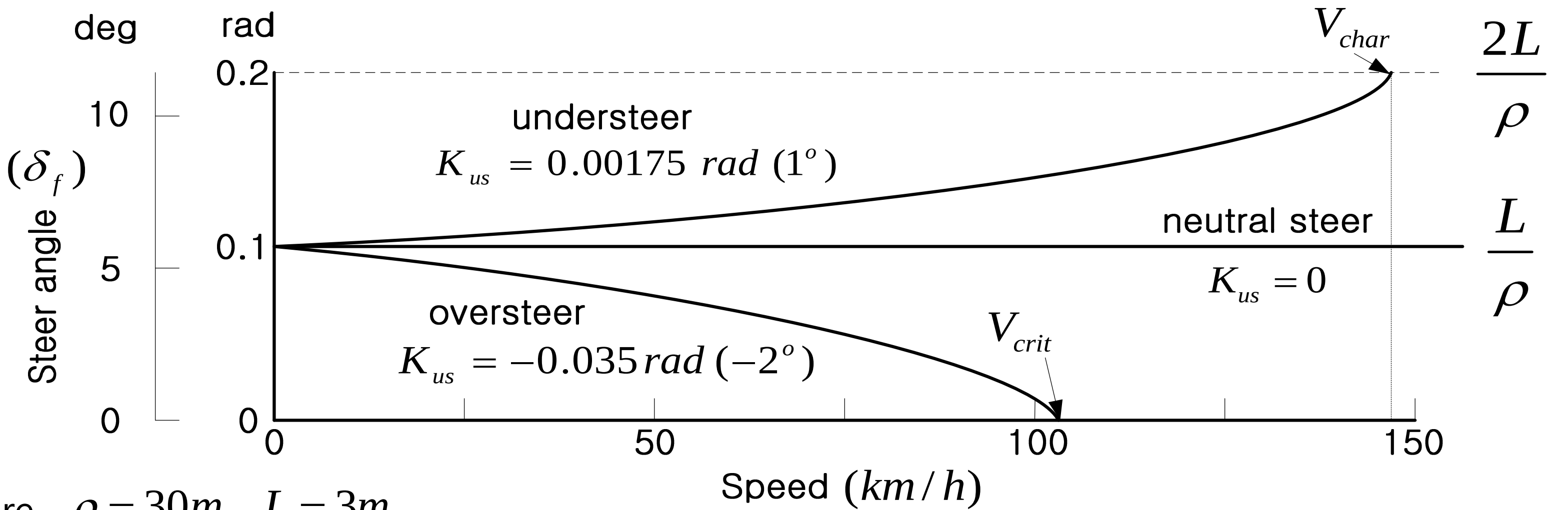
$$\delta_f = \frac{L}{\rho} + K_{us} \cdot \frac{v_x^2}{g \cdot \rho} \qquad \text{Where, } K_{us} = \frac{F_{zf}}{C_f} - \frac{F_{zr}}{C_r}$$

- Characteristic Speed
  - The speed at which the steer angle required to negotiate a turn is equal to $\frac{2L}{\rho}$

$$\frac{2L}{\rho} = \frac{L}{\rho} + K_{us} \cdot \frac{V_{char}^2}{g \cdot \rho} \qquad \Rightarrow V_{char} = \sqrt{\frac{Lg}{K_{us}}}$$

- Critical Speed
  - The speed at which the steer angle required to negotiate any turn is zero

$$0 = \frac{L}{\rho} + K_{us} \cdot \frac{V_{char}^2}{g \cdot \rho} \qquad \Rightarrow V_{crit} = \sqrt{\frac{Lg}{-K_{us}}}$$



Where, $\rho = 30m, \ L = 3m$

Understeer/oversteer according to Vehicle Parameter

- Under Steer Coefficient

$$K_{us} = \frac{F_{zf}}{C_f} - \frac{F_{zr}}{C_r} = \left(\frac{l_r}{C_f} - \frac{l_f}{C_r} \right) \cdot \frac{m \cdot g}{2L}$$

▼ Vertical Tire Force for steady state

$$\begin{cases} F_{zf} = \dfrac{l_r}{2L} m \cdot g \\ F_{zr} = \dfrac{l_f}{2L} m \cdot g \end{cases}$$

- Assuming that $C_f = C_r = C_s$

$$K_{us} = \frac{F_{zf}}{C_s} - \frac{F_{zr}}{C_s} = \left(l_r - l_f \right) \cdot \frac{m \cdot g}{2 \cdot C_s \cdot L}$$

$if\,(l_f = l_r)$

$$K_{us} = \left(l_r - l_f \right) \cdot \frac{m \cdot g}{2 \cdot C_s \cdot L} = 0$$

➡ Neutral Steer Vehicle

$if\,(l_f < l_r)$

$$K_{us} = \left(l_r - l_f \right) \cdot \frac{m \cdot g}{2 \cdot C_s \cdot L} > 0$$

➡ Under Steer Vehicle

$if\,(l_f > l_r)$

$$K_{us} = \left(l_r - l_f \right) \cdot \frac{m \cdot g}{2 \cdot C_s \cdot L} < 0$$

➡ Over Steer Vehicle

Understeer/oversteer according to Vehicle Parameter

- Under Steer Coefficient

$$K_{us} = \left(\frac{l_r}{C_f} - \frac{l_f}{C_r} \right) \cdot \frac{m \cdot g}{2L}$$

- Assuming that: (i) $l_f = l_r = l_i$,

  (ii) $C_f \neq C_r$

$if\,(C_f = C_r)$

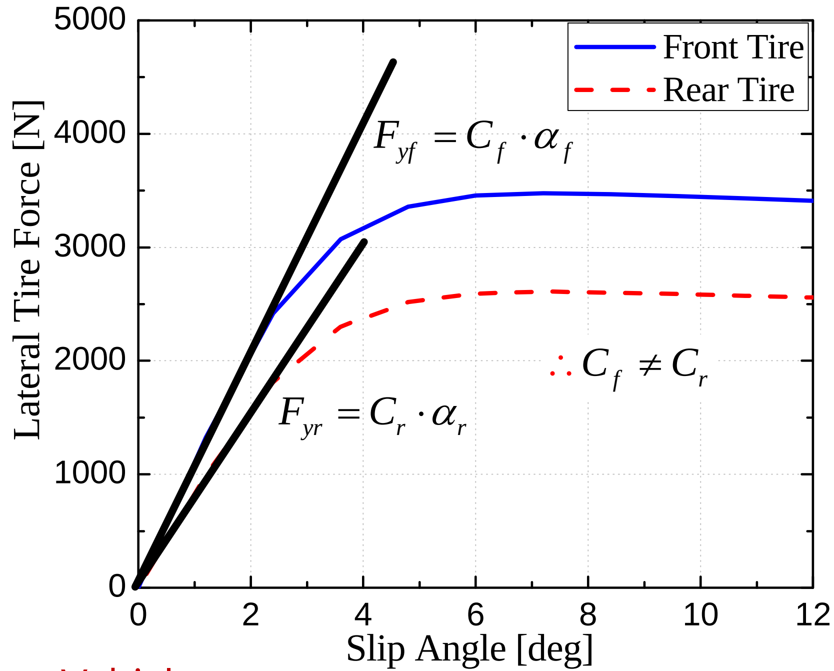
$$K_{us} = \left(\frac{C_r - C_f}{C_f \cdot C_r} \right) \cdot \frac{l_i \cdot m \cdot g}{2L} = 0$$ ⇨ Neutral Steer Vehicle

$if\,(C_f < C_r)$

$$K_{us} = \left(\frac{C_r - C_f}{C_f \cdot C_r} \right) \cdot \frac{l_i \cdot m \cdot g}{2L} > 0$$ ⇨ Under Steer Vehicle

$if\,(C_f > C_r)$

$$K_{us} = \left(\frac{C_r - C_f}{C_f \cdot C_r} \right) \cdot \frac{l_i \cdot m \cdot g}{2L} < 0$$ ⇨ Over Steer Vehicle



▲ Linear Lateral Tire Force

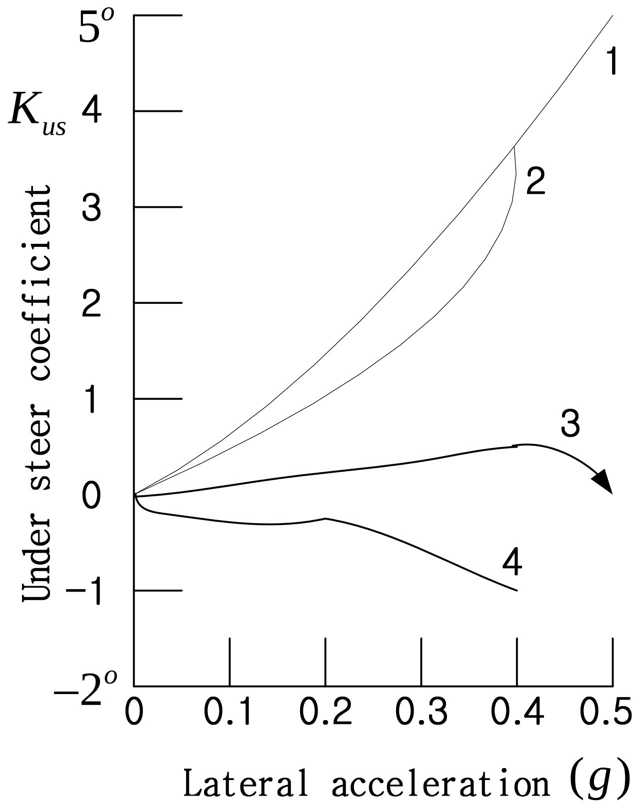
5. Understeer/oversteer

- Over Steer Vehicle

  - not desirable from a directional stability point of view

- Under Steer Vehicle

  - It is considered desirable for a road vehicle to have a small degree of understeer up to a certain level of lateral acceleration, such as 0.4g.



Variation of understeer coefficient with lateral acceleration of various types of car.

1 A conventinal front engine/rear-wheel-drive car

2 A European front engine/front-wheel-drive car

3 A European rear engine rear-wheel-drive car

4 An American rear engine/rear-wheel-drive car