Part.1

Lateral Vehicle Dynamics

1. Vehicle Dynamic Model
2. Planar Model
3. Tire Models
4. Bicycle Model
5. Understeer/oversteer
6. Dynamic model in terms of error w.r.t. road
7. lane keeping model
8. **Vehicle Stability Control**

8. Vehicle Stability Control

8.1 Bicycle model: Nonlinear and Linear models

8.2 Phase Plane Analysis

8.3 Phase Plane Analysis of Bicycle model

8.4 Electronic Stability Program (ESP)

8.5 Vehicle Stability Control Algorithm

8.1 2DOF Bicycle Model

- Assumption of 2DOF Bicycle Model

1) Longitudinal speed is Constant ($a_x = 0, \ \lambda \simeq 0$)

2) Body slip angle is sufficiently small. ($\beta = v_y / v_x, \quad \sin\delta_f = 0, \quad \cos\delta_f = 1$)

3) Left and right slip angles are identical. ($\alpha_f = \alpha_{fL} = \alpha_{fR}, \qquad \alpha_r = \alpha_{rL} = \alpha_{rR}$)

- y-axis Motion Dynamic Equation

$$\sum F_y = m \cdot a_y = m \cdot v_x \cdot (\dot{\beta} + \dot{\psi}) = 2 \cdot F_{yf} + 2 \cdot F_{yr}$$

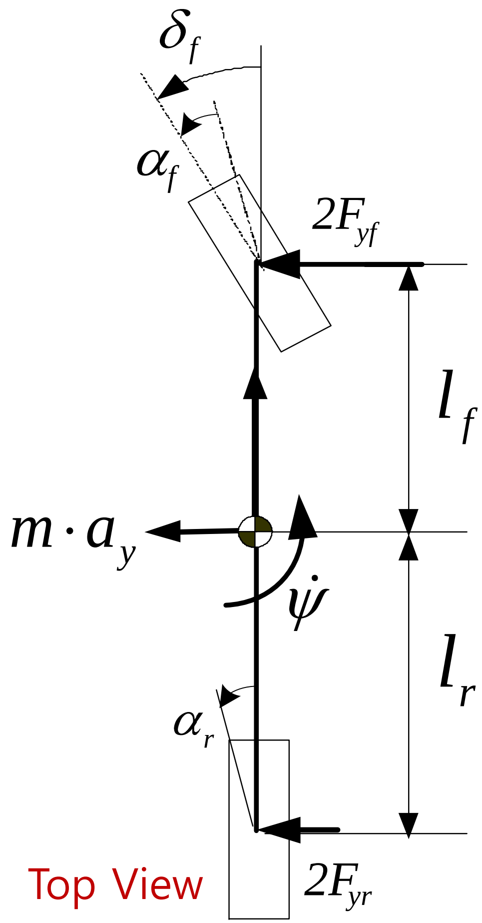
- yaw-axis Motion Dynamic Equation

$$\sum M_z = \dot{H}_z = I_z \cdot \ddot{\psi} = 2 \cdot l_f \cdot F_{yf} - 2 \cdot l_r \cdot F_{yr}$$

- Dynamic Equation of 2DOF Bicycle Model

$$\begin{bmatrix} \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \dfrac{2 \cdot F_{yf} + 2 \cdot F_{yr}}{m \cdot v_x} - \dot{\psi} \\ \dfrac{2 \cdot l_f \cdot F_{yf} - 2 \cdot l_r \cdot F_{yr}}{I_z} \end{bmatrix}$$

▲ Lateral Tire Force Model



Top View

Tire Slip Angle: small angle approximation

- Linear Tire Slip Angle at Low Slip Angle

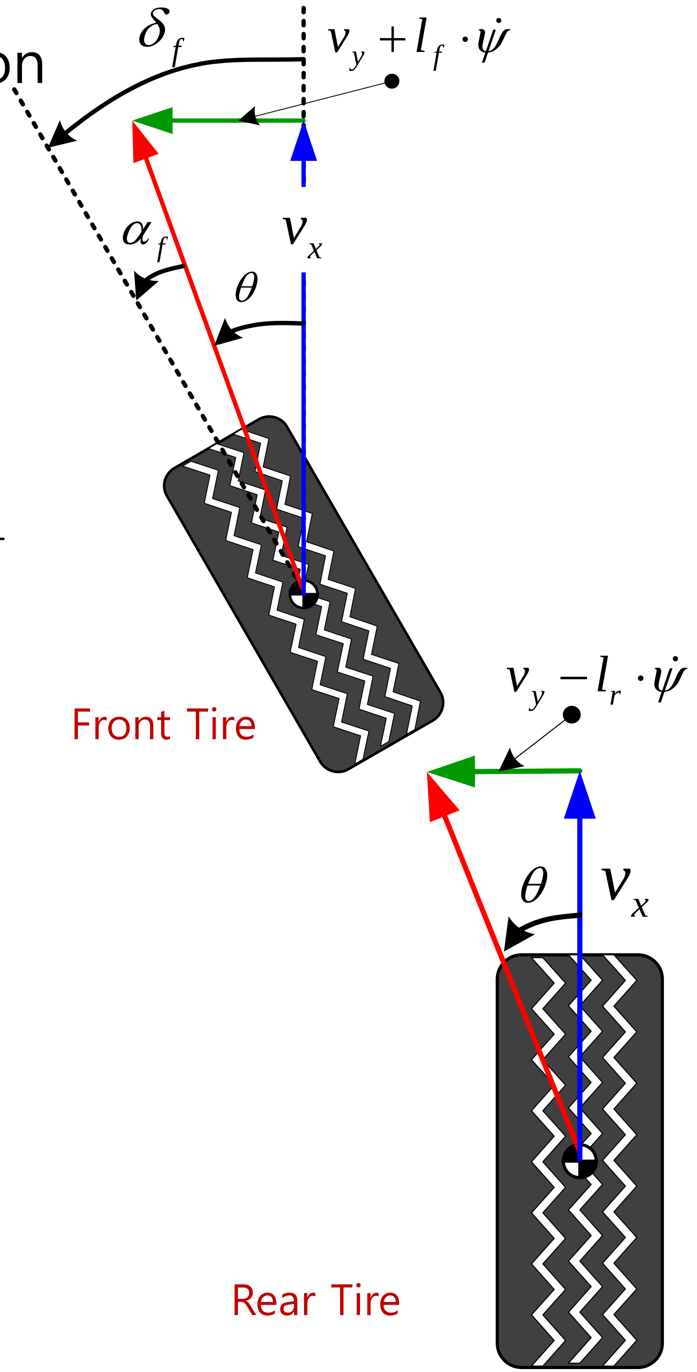
$$\tan\theta = \theta \quad if\ (\theta \ll 1)$$

Front Slip Angle

$$\alpha_f = \delta_f - \theta = \delta_f - \tan^{1}\frac{v_y + l_f \cdot \dot{\psi}}{v_x} \simeq \delta_f - \frac{v_y + l_f \cdot \dot{\psi}}{v_x}$$

Rear Slip Angle

$$\alpha_r = -\theta = -\tan^{1}\frac{v_y - l_r \cdot \dot{\psi}}{v_x} \simeq -\frac{v_y - l_r \cdot \dot{\psi}}{v_x}$$



Linear Lateral Tire Model

- Linear Lateral Tire Force at Low Slip Angle

Front Slip Angle

$$F_{yf} = \frac{\alpha_i^*}{\sigma_i^*} F_{ty0}(\sigma_i^* \times \alpha_m, \quad F_{tzi})$$

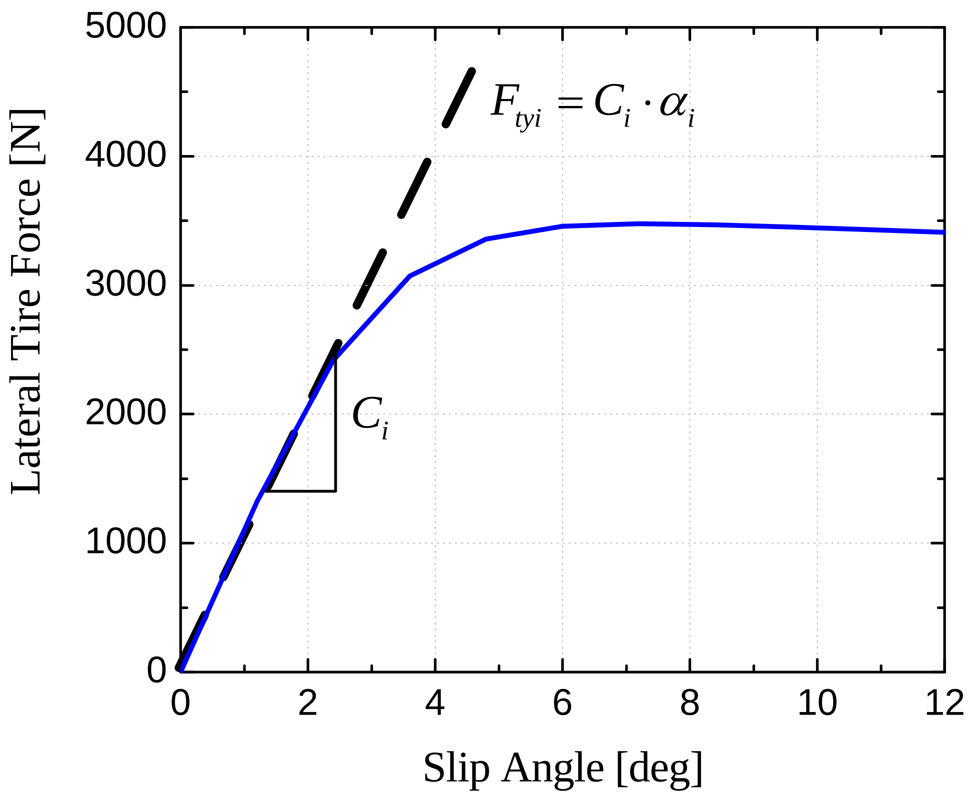
$$\simeq C_f \cdot \alpha_f = C_f \cdot \left(\delta_f - \frac{v_y + l_f \cdot \dot{\psi}}{v_x} \right)$$

Rear Slip Angle

$$F_{yr} = \frac{\alpha_i^*}{\sigma_i^*} F_{ty0}(\sigma_i^* \times \alpha_m, \quad F_{tzi})$$

$$\simeq C_r \cdot \alpha_r = C_r \cdot \left(- \frac{v_y - l_r \cdot \dot{\psi}}{v_x} \right)$$

Where, $C_i =$ Cornering Stiffness



Linear Lateral Tire Force

State Equation of 2 DOF Linear Bicycle Model

- Dynamic Equation of 2DOF Bicycle Model
  - y-axis Motion Dynamic Equation

$$\sum F_y = m \cdot a_y = m \cdot v_x \cdot (\dot{\beta} + \dot{\psi}) = 2 \cdot F_{yf} + 2 \cdot F_{yr}$$

$$= 2 \cdot C_f \cdot \left(\delta_f - \frac{v_y + l_f \cdot \dot{\psi}}{v_x} \right) + 2 \cdot C_r \cdot \left(-\frac{v_y - l_r \cdot \dot{\psi}}{v_x} \right)$$

  - yaw-axis Motion Dynamic Equation

$$\sum M_z = \dot{H}_z = I_z \cdot \ddot{\psi} = 2 \cdot l_f \cdot F_{yf} - 2 \cdot l_r \cdot F_{yr}$$

$$= 2 \cdot l_f \cdot C_f \cdot \left(\delta_f - \frac{v_y + l_f \cdot \dot{\psi}}{v_x} \right) - 2 \cdot l_r \cdot C_r \cdot \left(-\frac{v_y - l_r \cdot \dot{\psi}}{v_x} \right)$$

- Define state = [Body Side Slip Angle, Yaw Rate]

$$x = \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix}$$

State Equation of 2 DOF Bicycle Model

- Define state = [Body Side Slip Angle, Yaw Rate]

$$x = \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix}$$

- State Equation of 2DOF Bicycle Model

$$\dot{x} = \begin{bmatrix} \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} -\dfrac{2(C_f + C_r)}{m \cdot v_x} \cdot \beta + \left(-1 - \dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{m \cdot {v_x}^2} \right) \cdot \dot{\psi} + \dfrac{2 \cdot C_f}{m v_x} \cdot \delta_f \\ -\dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} \cdot \beta - \dfrac{2({l_f}^2 \cdot C_f + {l_r}^2 \cdot C_r)}{I_z \cdot v_x} \cdot \dot{\psi} + \dfrac{2 \cdot l_f \cdot C_f}{I_z} \cdot \delta_f \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{2(C_f + C_r)}{m \cdot v_x} & -1 - \dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{m \cdot {v_x}^2} \\ -\dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} & -\dfrac{2({l_f}^2 \cdot C_f + {l_r}^2 \cdot C_r)}{I_z \cdot v_x} \end{bmatrix} \cdot \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} \dfrac{2 \cdot C_f}{m v_x} \\ \dfrac{2 \cdot l_f \cdot C_f}{I_z} \end{bmatrix} \cdot \delta_f$$

$$= Ax + B\delta_f$$

8. Vehicle Stability Control

8.2 Phase Plane Analysis

▪ a graphical method for analysis of dynamic behavior of a systems

▪ Phase plane analysis is limited to second-order systems.

▪ For second order systems, solution trajectories can be represented by curves in the plane, which allows for visualization of the qualitative behavior of the system.

▪ In particular, it is interesting to consider the behavior of systems around equilibrium points

▪ Consider the second-order system described by the following equations:

$$\dot{x}_1 = P(x_1, x_2)$$
$$\dot{x}_2 = Q(x_1, x_2)$$

where,

x_1 and x_2 are states of the system.

P and Q are nolinear functions of states.

▪ Slope of the phase plane trajectory

$$\frac{dx_2}{dx_1} = \frac{dx_2}{dt} \cdot \frac{dt}{dx_1} = \frac{Q(x_1, x_2)}{P(x_1, x_2)}$$

1) Equilibrium points of the system

$$0 = P(x_{1e}, x_{2e})$$
$$0 = Q(x_{1e}, x_{2e})$$

Example:

- Find the equilibrium points of the system described by the following equation:

$$\ddot{x} = (x - a)^2 + \dot{x}^3$$

- Put the system in the standard form by setting $x = x_1, \quad \dot{x} = x_2$

$$\dot{x}_1 = x_2 = P(x_1, x_2)$$
$$\dot{x}_2 = (x_1 - a)^2 + x_2^3 = Q(x_1, x_2)$$

- The slope of the phase plane trajectory

$$\frac{dx_2}{dx_1} = \frac{Q(x_1, x_2)}{P(x_1, x_2)} = \frac{(x_1 - a)^2 + x_2^3}{x_2}$$

- Equilibrium Points:

$$\begin{aligned} 0 &= P(x_{1e}, x_{2e}) = x_{2e} \\ 0 &= Q(x_{1e}, x_{2e}) = (x_{1e} - a)^2 + x_{2e}^3 \end{aligned} \Rightarrow \begin{aligned} x_{2e} &= 0 \\ x_{1e} &= a \end{aligned} \Rightarrow x_{eq} = \begin{pmatrix} a & 0 \end{pmatrix}$$

2) Investigate the linear behavior about an equilibrium points

$$x_1 = x_{1e} + \delta x_1$$

$$x_2 = x_{2e} + \delta x_2$$

- From above equation,

$$\begin{bmatrix} \delta \dot{x}_1 \\ \delta \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \left.\frac{\partial P}{\partial x_1}\right|_e \cdot \delta x_1 + \left.\frac{\partial P}{\partial x_2}\right|_e \cdot \delta x_2 \\ \left.\frac{\partial Q}{\partial x_1}\right|_e \cdot \delta x_1 + \left.\frac{\partial Q}{\partial x_2}\right|_e \cdot \delta x_2 \end{bmatrix} = \begin{bmatrix} a \cdot \delta x_1 + b \cdot \delta x_2 \\ c \cdot \delta x_1 + d \cdot \delta x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} \delta x_1 \\ \delta x_2 \end{bmatrix}$$

- Characteristic equation

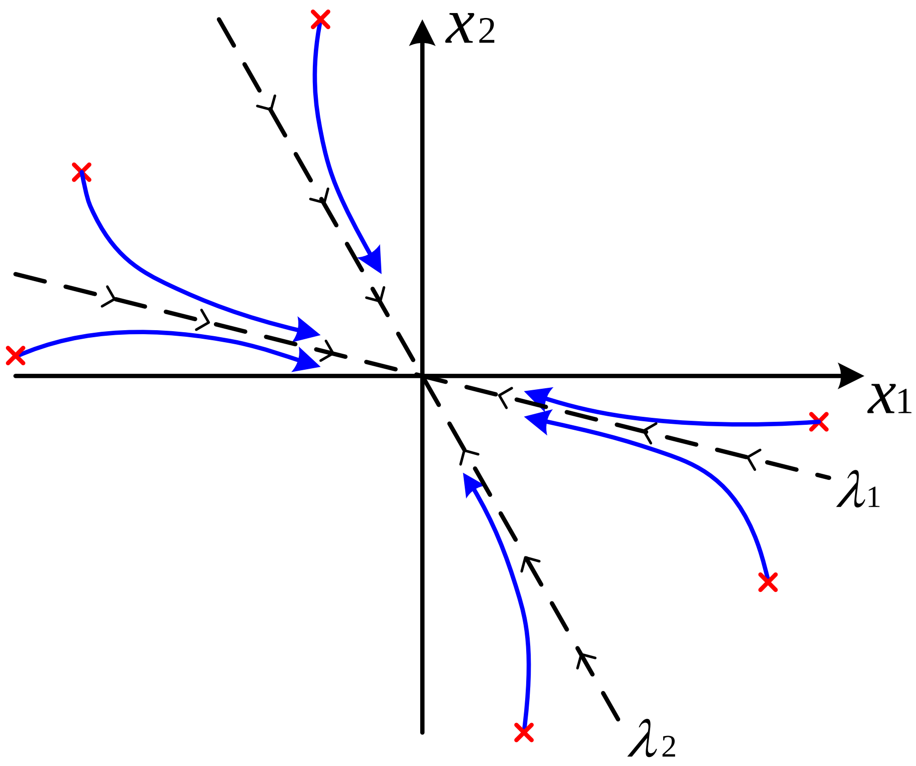
$$(s-a)\cdot(s-d) - bc = s^2 - (a+d)\cdot s + ad - bc = 0$$

- Eigen value

$$\lambda_{1,2} = \frac{(a+d) \pm \sqrt{(a+d)^2 - 4\cdot(ad-bc)}}{2}$$

3) Possible Case

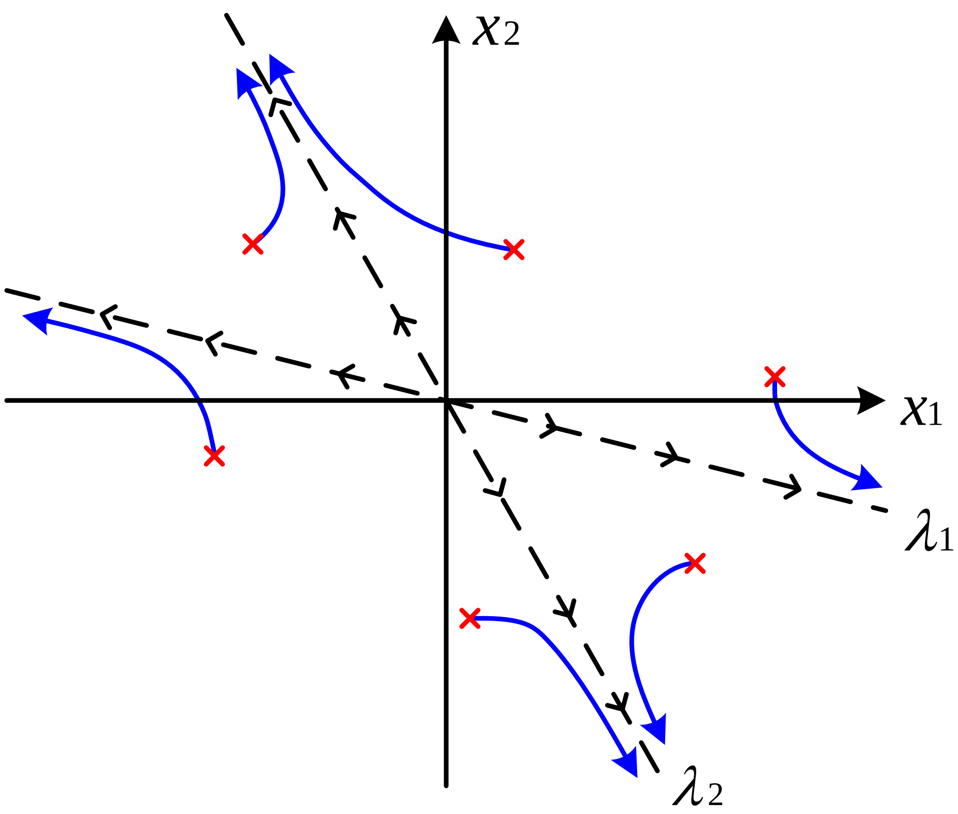
▼ λ_1 and λ_2 are real and negative.



< Stable Node >

where, $|\lambda_1| \leq |\lambda_2|$

▼ λ_1 and λ_2 are real and positive.



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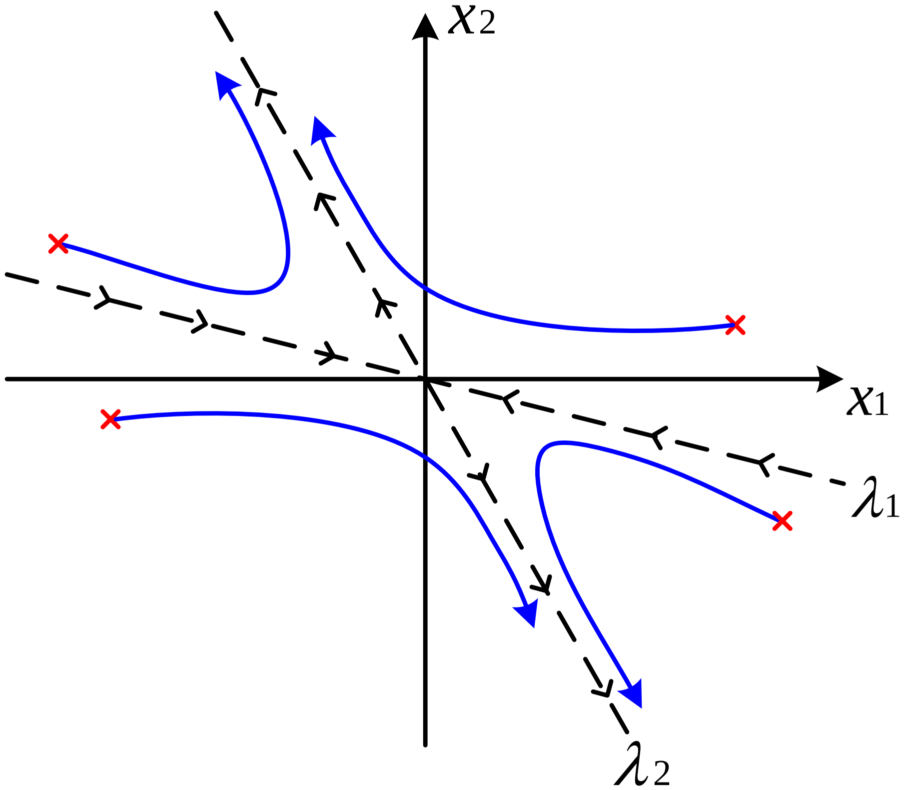
where, $|\lambda_1| \leq |\lambda_2|$

▼ λ_1 and λ_2 are real and opposite signs.



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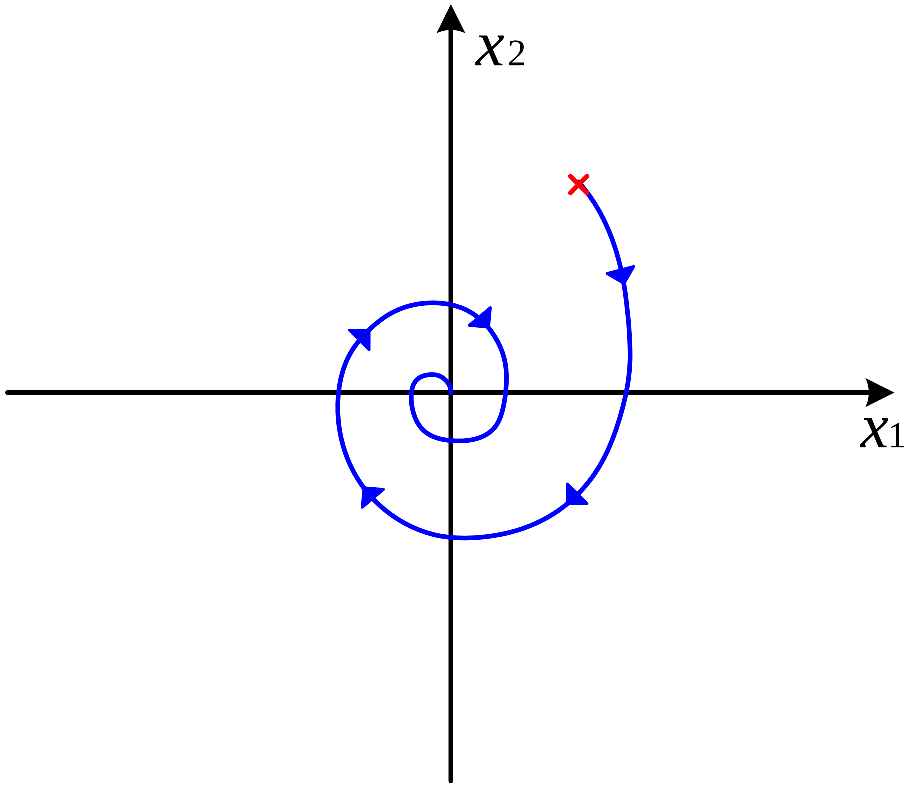
▼ λ_1 and λ_2 are complex and negative real parts.



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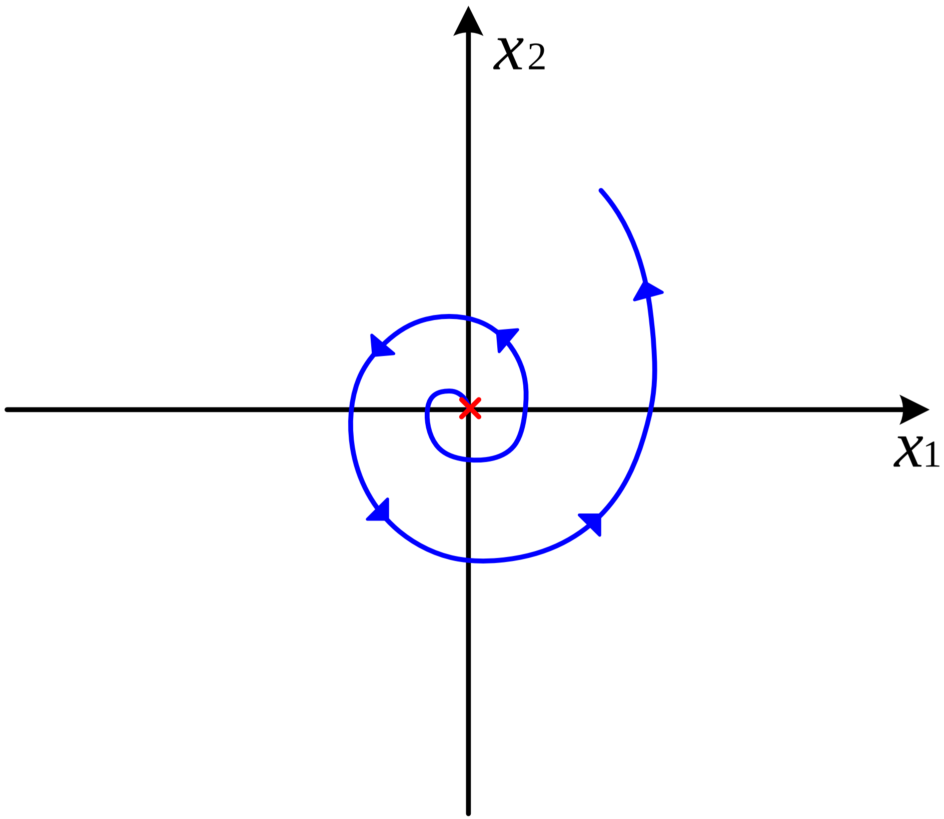
where, $\lambda_1 < 0 < \lambda_2$

▼ λ_1 and λ_2 are complex and positive real parts.



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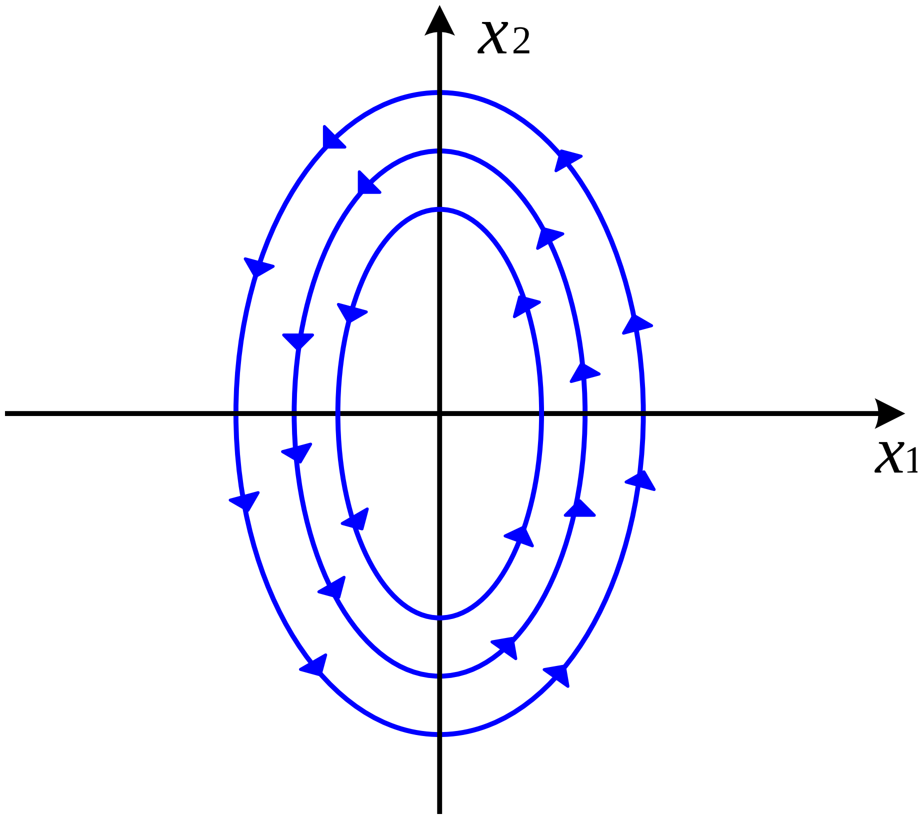
▼ λ_1 and λ_2 are complex and zero real parts.



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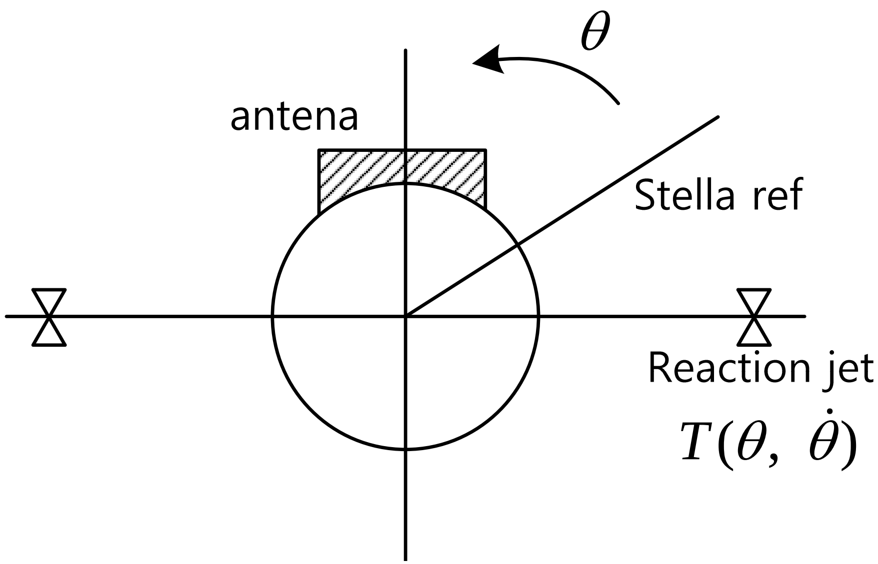
Example: Satellite Control

- Consider a satellite control system as shown in the figure shown below:



< Satellite Control System >

- The equation of motion can be written as follows:

$$I \cdot \ddot{\theta} = T$$

- Rewriting the equation

$$\ddot{\theta} = \frac{T}{I} = u$$

1. PD Control

- Controller

$$u(\theta,\ \dot{\theta}) = -k_1 \cdot \theta - k_2 \cdot \dot{\theta}$$

- Closed system

$$\ddot{\theta} = u = -k_1 \cdot \theta - k_2 \cdot \dot{\theta}$$

$$\Downarrow$$

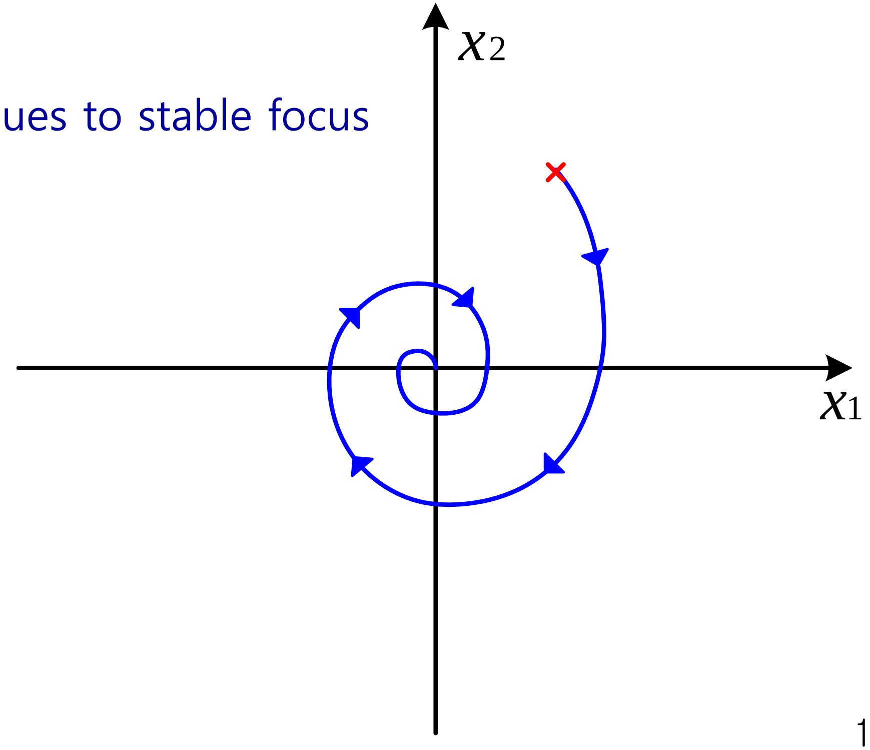
$$\ddot{\theta} + k_2 \cdot \dot{\theta} + k_1 \cdot \theta = 0$$

- Controller can be designed to make eigen values to stable focus

$$\begin{aligned} x_1 &= \theta \\ x_2 &= \dot{\theta} \end{aligned} \quad \Rightarrow \quad \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u = -k_1 \cdot x_1 - k_2 \cdot x_2 \end{aligned}$$

$$\Downarrow$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



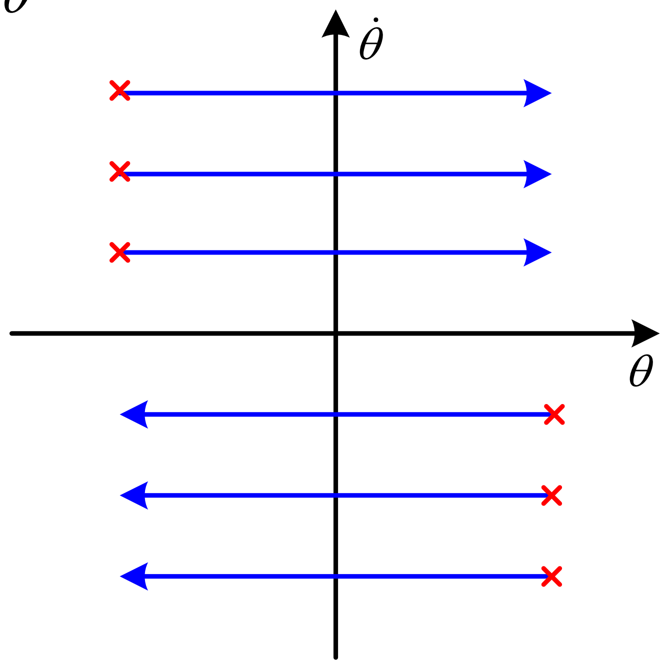
2. On/Off Controller

- The slope of the phase plane trajectory

$$\ddot{\theta} = \frac{T}{I} = u \quad \Rightarrow \quad \frac{d\dot{\theta}}{dt} = u = \frac{d\dot{\theta}}{d\theta} \cdot \frac{d\theta}{dt} = \frac{d\dot{\theta}}{d\theta} \cdot \dot{\theta}$$

$$\frac{d\dot{\theta}}{d\theta} = \frac{u}{\dot{\theta}}$$

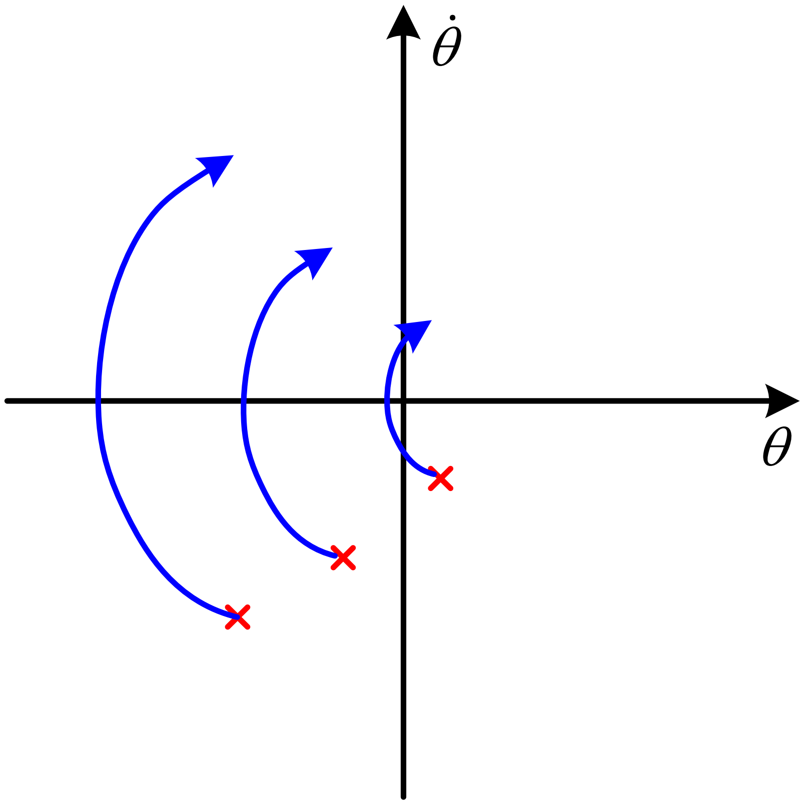
Case.1 $u = 0 \quad \Rightarrow \quad \frac{d\dot{\theta}}{d\theta} = 0$



Case.2 $u = u_m \quad \Rightarrow \quad \frac{d\dot{\theta}}{d\theta} = \frac{u_m}{\dot{\theta}}$

$$\dot{\theta} \cdot d\dot{\theta} = u_m \cdot d\theta$$

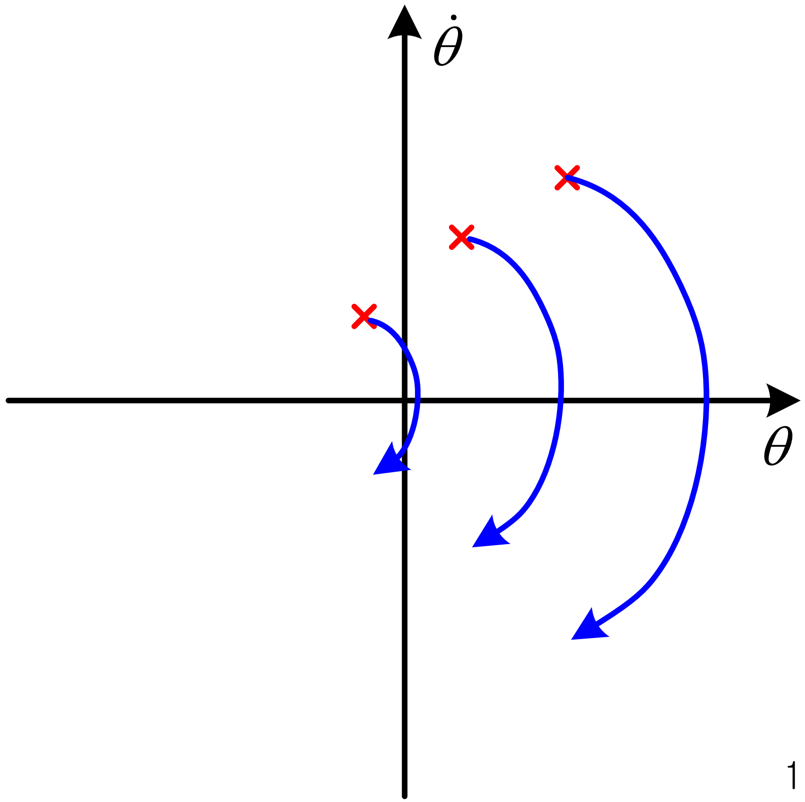
$$\frac{1}{2}\dot{\theta}^2 - u_m \cdot \theta = \text{constant}$$



Case.3 $u = -u_m \quad \Rightarrow \quad \frac{d\dot{\theta}}{d\theta} = -\frac{u_m}{\dot{\theta}}$

$$\dot{\theta} \cdot d\dot{\theta} = -u_m \cdot d\theta$$

$$\frac{1}{2}\dot{\theta}^2 + u_m \cdot \theta = \text{constant}$$

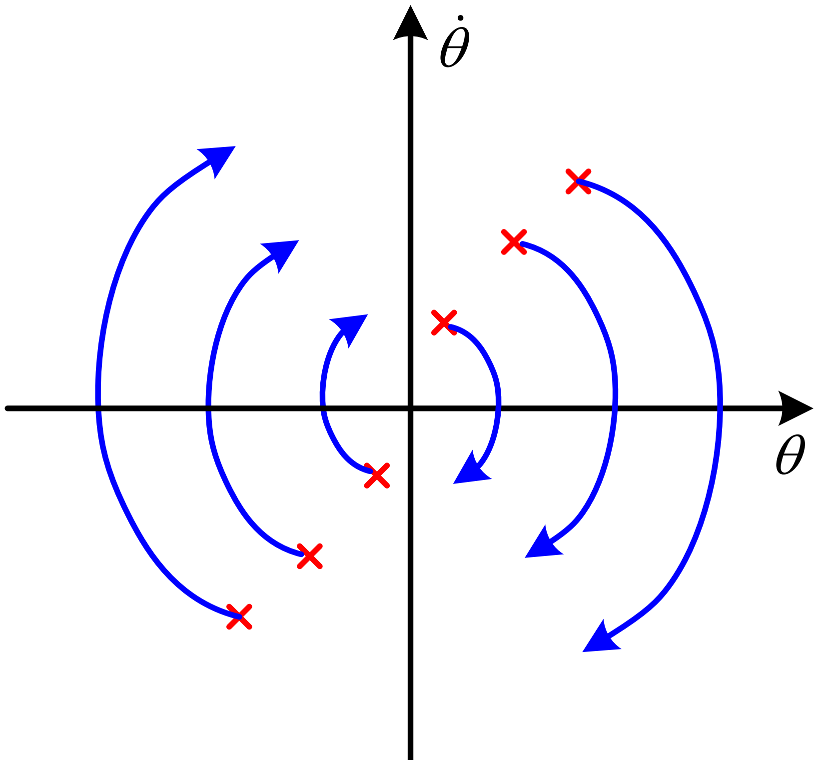


Case.4

$$u = -u_m \cdot \mathrm{sgn}(\theta) \quad \Rightarrow \quad \begin{array}{ll} \dfrac{d\dot{\theta}}{d\theta} = -\dfrac{u_m}{\dot{\theta}} & \textit{if}\ (\theta > 0) \\ \dfrac{d\dot{\theta}}{d\theta} = +\dfrac{u_m}{\dot{\theta}} & \textit{elsewhere} \end{array}$$

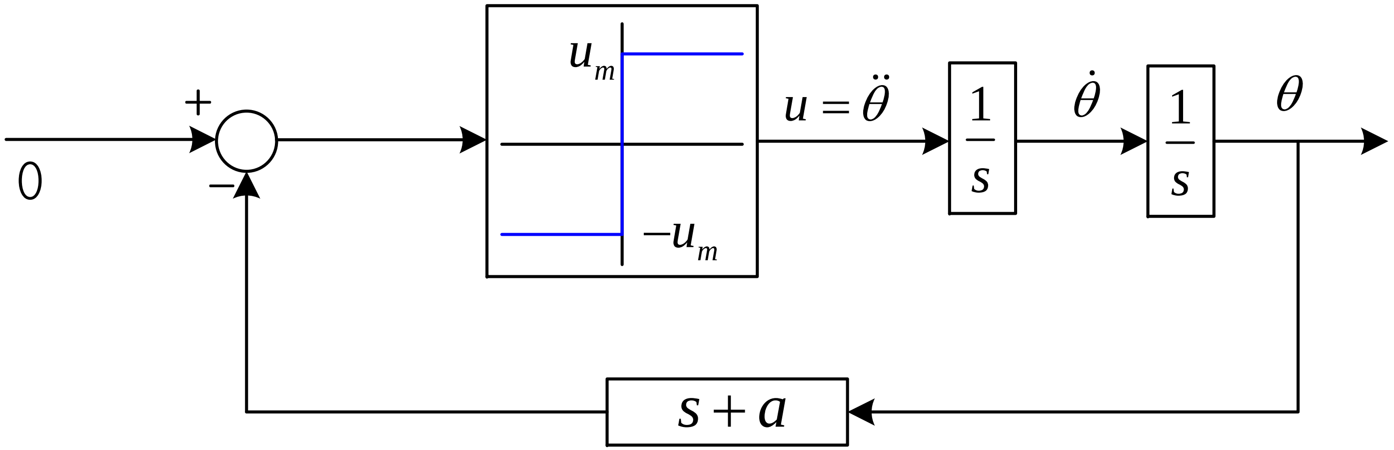
$$\frac{1}{2}\dot{\theta}^2 + u_m \cdot \theta = \text{constantt} \quad \textit{if}\ (\theta > 0)$$

$$\frac{1}{2}\dot{\theta}^2 - u_m \cdot \theta = \text{constant} \quad \textit{elsewhere}$$



Case.5 Lead Compensation/On-Off Controller

$$u = -u_m \cdot \mathrm{sgn}(\dot{\theta} + a \cdot \theta)$$



< Block Diagram of Closed System >

- Sliding Region

$$s = \dot{\theta} + a \cdot \theta$$

$$\textit{if}\ (s = 0), \quad \dot{\theta} + a \cdot \theta = 0 \quad \Rightarrow \quad \theta(t) = \theta_0 \cdot e^{-a \cdot t}$$

- Derivative of sliding surface

$$\dot{s} = \ddot{\theta} + a \cdot \dot{\theta} = u + a \cdot \dot{\theta}$$

$$= -u_m \cdot \mathrm{sgn}(\dot{\theta} + a \cdot \theta) + a \cdot \dot{\theta}$$

Case.5 Lead Compensation/On-Off Controller

- From Lyapunov Theory

$$V = \frac{1}{2}s^2 > 0 \quad \text{and} \quad \dot{V} = s \cdot \dot{s} < 0$$

System is stable.

- if (s>0)

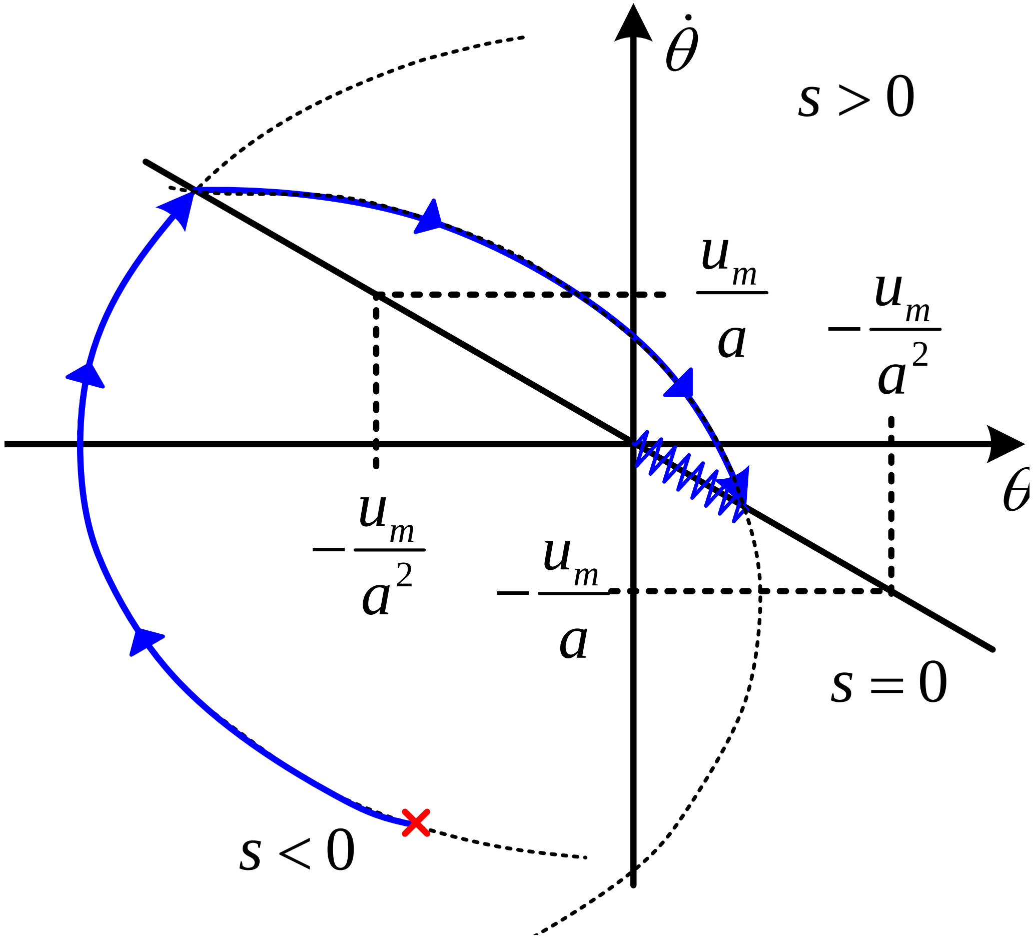
$$\dot{s} = -u_m \cdot \text{sgn}(s) + a \cdot \dot{\theta} = -u_m + a \cdot \dot{\theta} < 0$$

$$\dot{\theta} < \frac{u_m}{a}$$

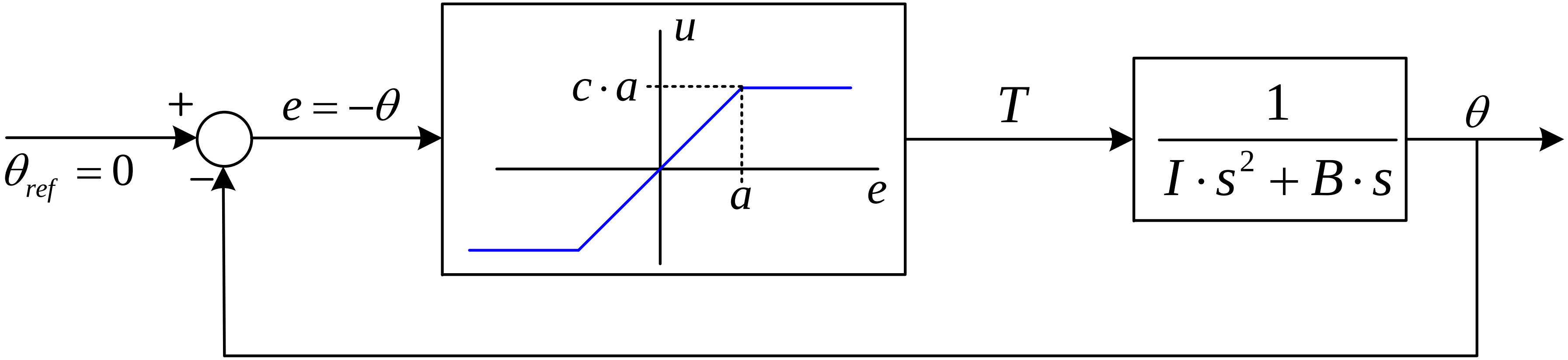
- if (s<0)

$$\dot{s} = -u_m \cdot \text{sgn}(s) + a \cdot \dot{\theta} = u_m + a \cdot \dot{\theta} > 0$$

$$\dot{\theta} > -\frac{u_m}{a}$$



Case.6 Servo with Saturation



- The equation of motion

$$\theta_{ref} = 0$$

$$I \cdot \ddot{\theta} + B \cdot \dot{\theta} = T = u = \begin{cases} -c \cdot a & \text{if } (\theta > a) \\ -c \cdot \theta & \text{if } (|\theta| \leq a) \\ c \cdot a & \text{if } (\theta < -a) \end{cases}$$

Case.6 Servo with Saturation

- phase plane trajectory

$if\ (|\theta| \le a)$

$$I \cdot \ddot{\theta} + B \cdot \dot{\theta} + c \cdot \theta = 0$$

$if\ (\theta > a)$

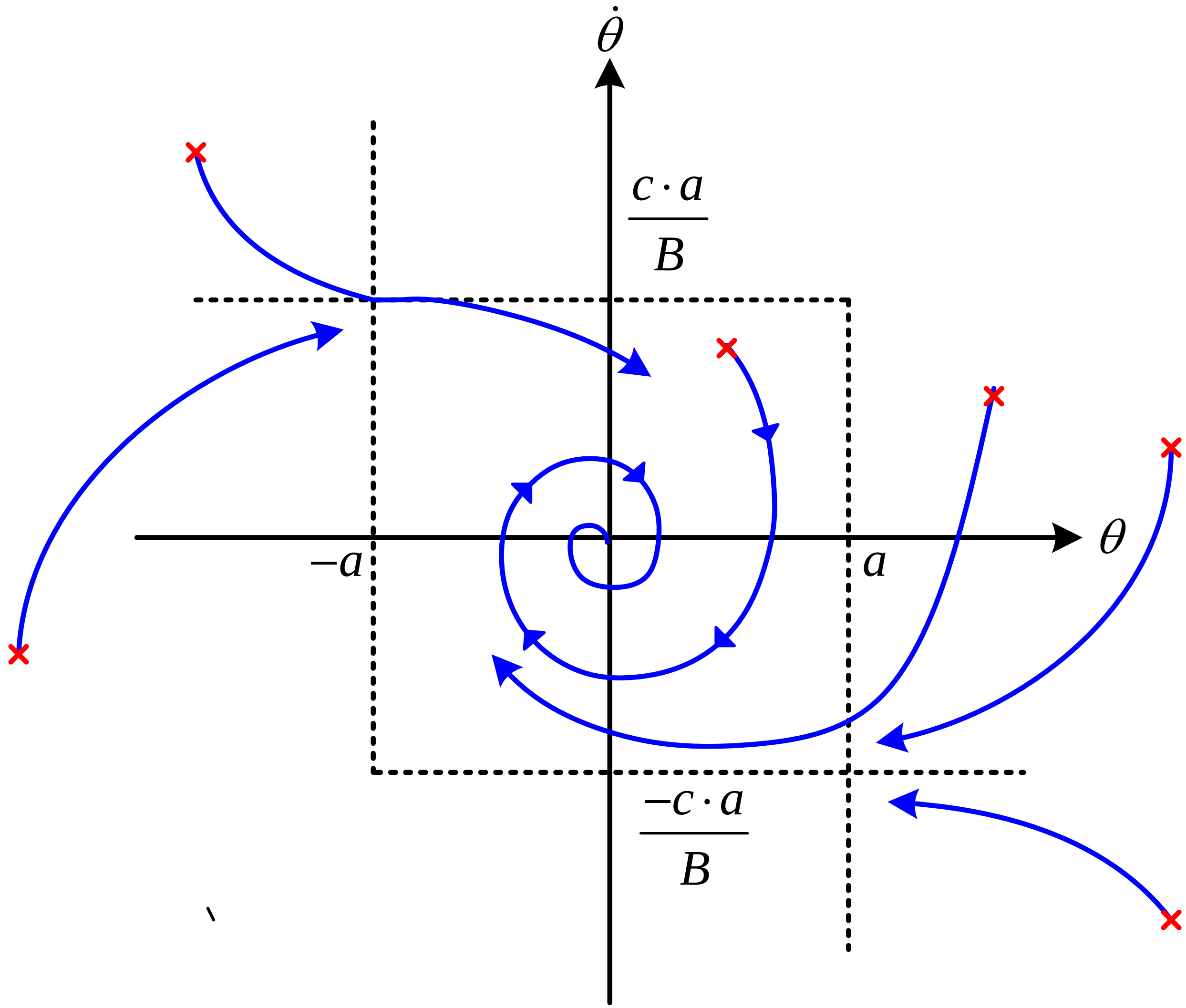
$$I \cdot \ddot{\theta} + B \cdot \dot{\theta} = -c \cdot a$$

$$\dot{\theta}_{ss} = \frac{-c \cdot a}{B}$$

$if\ (\theta < -a)$

$$I \cdot \ddot{\theta} + B \cdot \dot{\theta} = c \cdot a$$

$$\dot{\theta}_{ss} = \frac{c \cdot a}{B}$$



8. Vehicle Stability Control

8.3 Phase Plane Analysis of Bicycle model

- Assumption of 2DOF Bicycle Model

1) Longitudinal speed is Constant ($a_x = 0,\ \lambda \simeq 0$)
2) Body slip angle is sufficiently small. ($\beta = v_y / v_x,\quad \sin\delta_f = 0,\quad \cos\delta_f = 1$)
3) Left and right slip angles are identical. ($\alpha_f = \alpha_{fL} = \alpha_{fR},\qquad \alpha_r = \alpha_{rL} = \alpha_{rR}$)

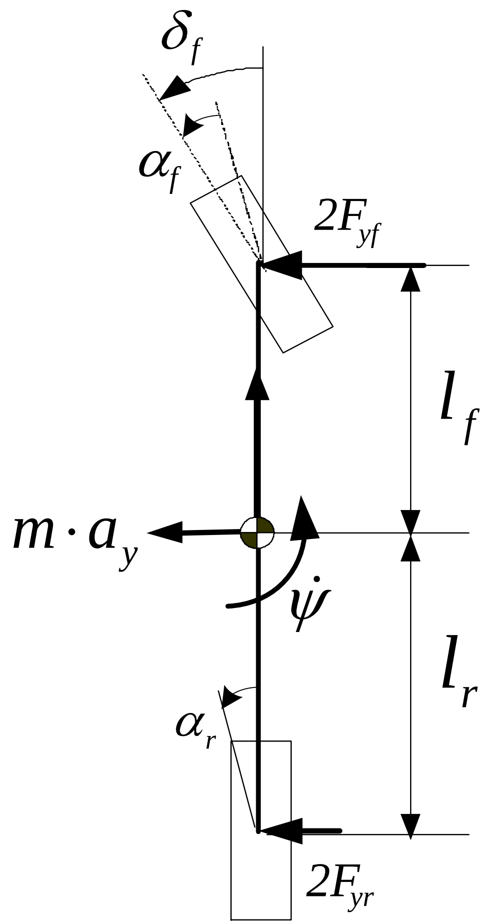
- y-axis Motion Dynamic Equation

$$\sum F_y = m \cdot a_y = m \cdot v_x \cdot (\dot{\beta} + \dot{\psi}) = 2 \cdot F_{yf} + 2 \cdot F_{yr}$$

- yaw-axis Motion Dynamic Equation

$$\sum M_z = \dot{H}_z = I_z \cdot \ddot{\psi} = 2 \cdot l_f \cdot F_{yf} - 2 \cdot l_r \cdot F_{yr} + \sum_{i=1}^{4} M_{tzi}$$

- Lateral Tire Force and Self Aligning Moment
  - Non-linear Characteristic using Pacejka Tire Model



8.3 Phase Plane Analysis of Bicycle model

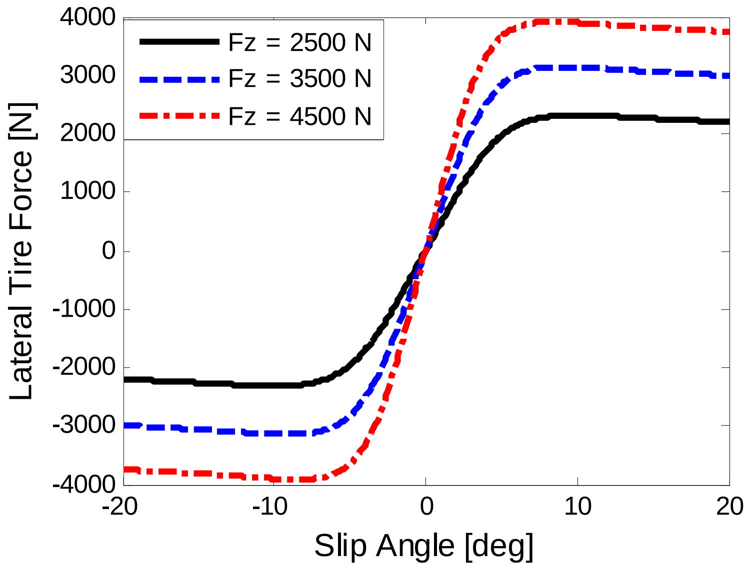
- Define state = [Body Side Slip Angle, Yaw Rate]

$$x = \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix}$$

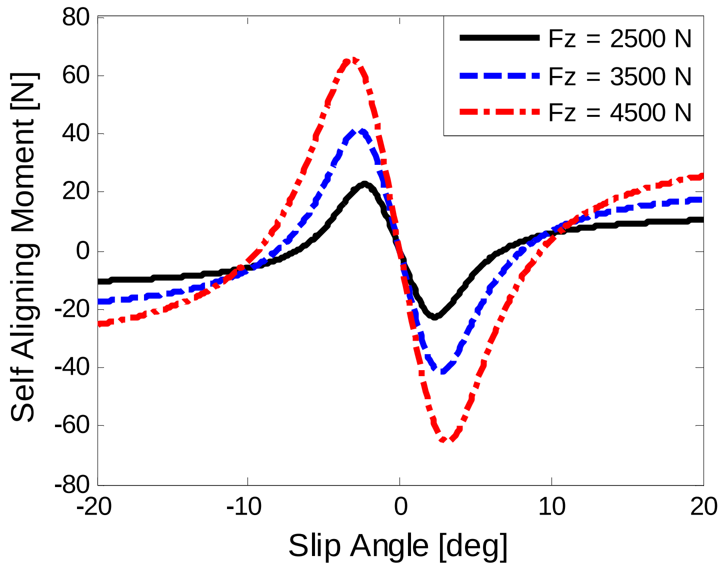
- Dynamic Equation

$$\begin{bmatrix} \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \dfrac{2 \cdot F_{tyf} + 2 \cdot F_{tyr}}{m \cdot v_x} - \dot{\psi} \\ \dfrac{2 \cdot l_f \cdot F_{tyf} - 2 \cdot l_r \cdot F_{tyr} + \sum_{i=1}^{4} M_{tzi}}{I_z} \end{bmatrix}$$

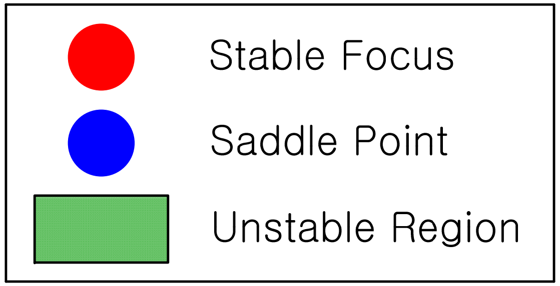
- Lateral Tire Force



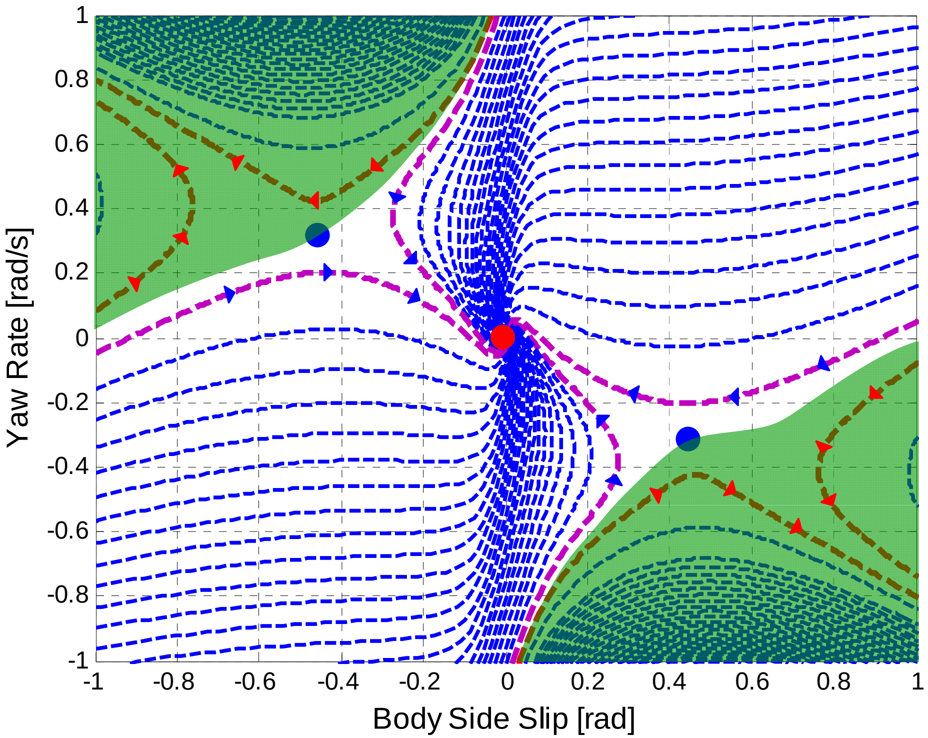
- Self Aligning Moment

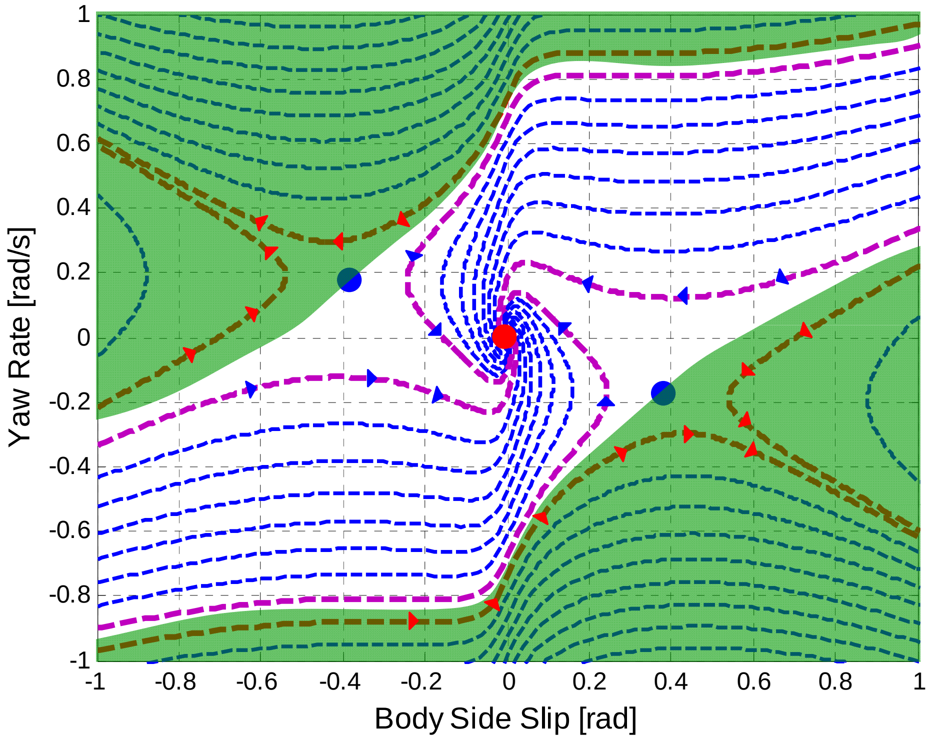


$\beta - \gamma$ Phase Plane Trajectory

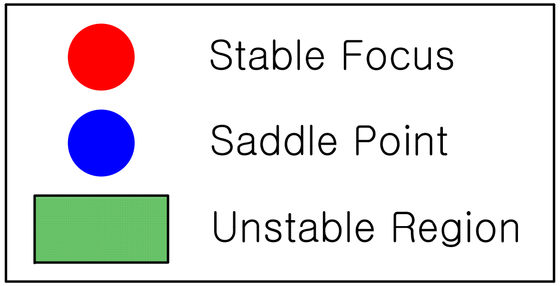


- $v_x = 70kph, \quad \delta_f = 0\deg$
- $v_x = 150kph, \quad \delta_f = 0\deg$

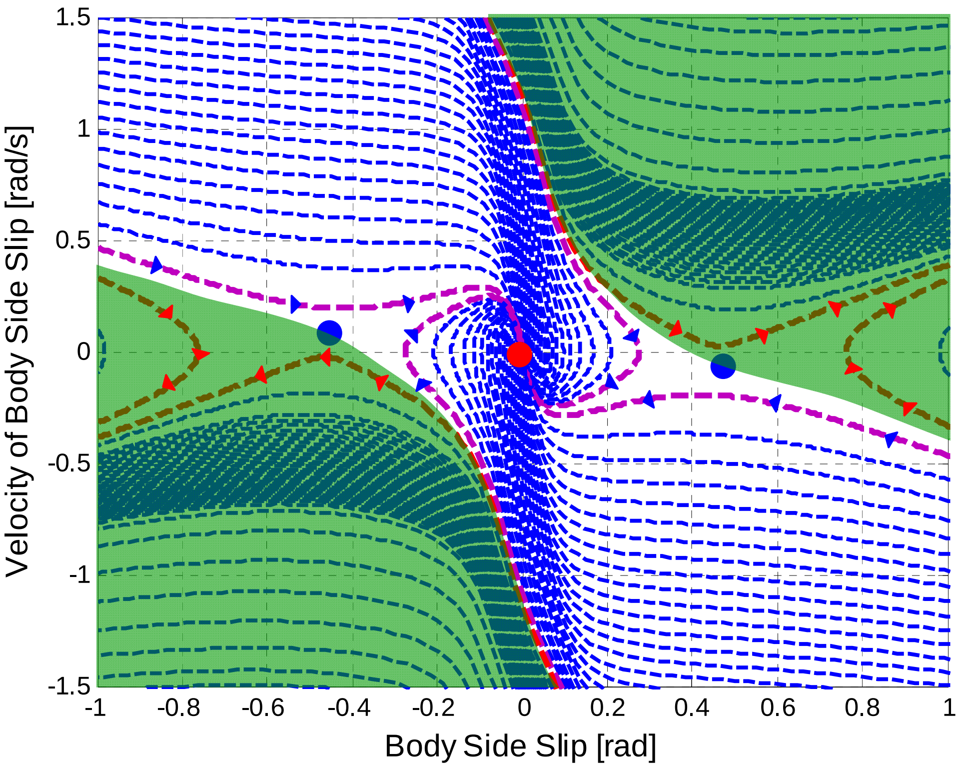




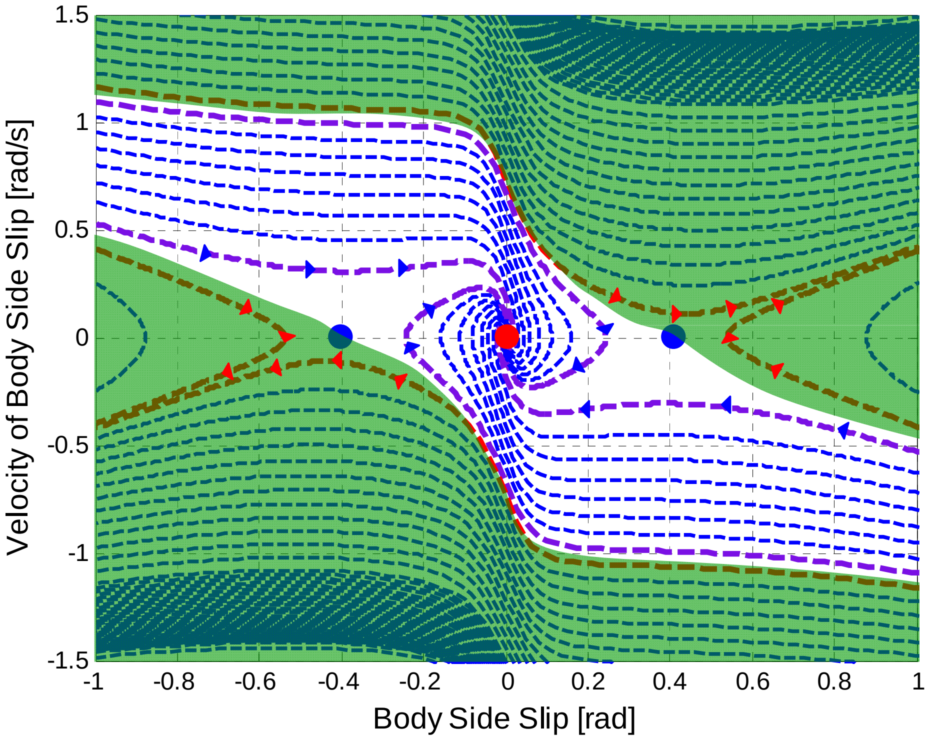
$\beta - \dot{\beta}$ Phase Plane Trajectory



- $v_x = 70kph, \quad \delta_f = 0\,\deg$

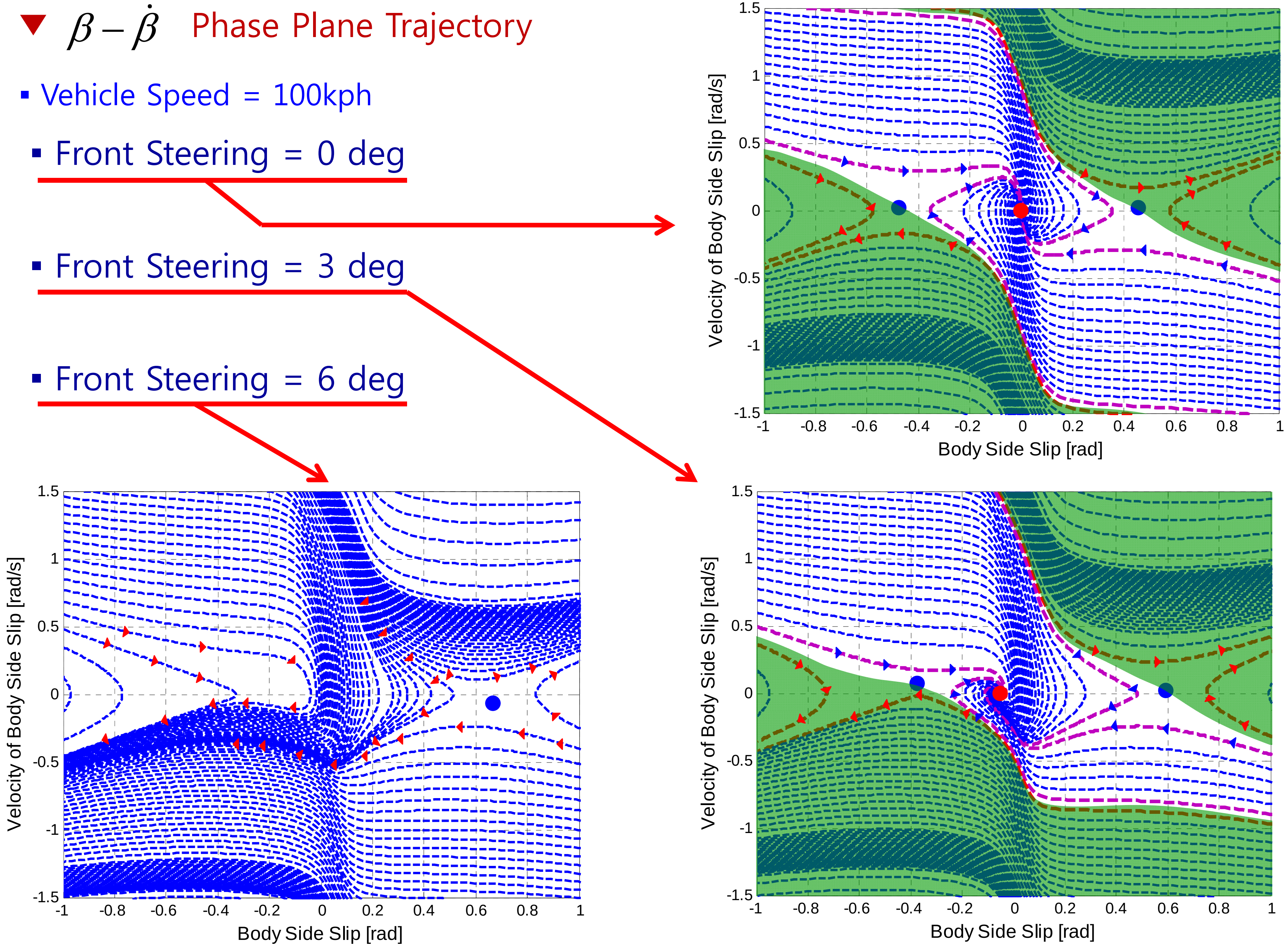


- $v_x = 150kph, \quad \delta_f = 0\,\deg$



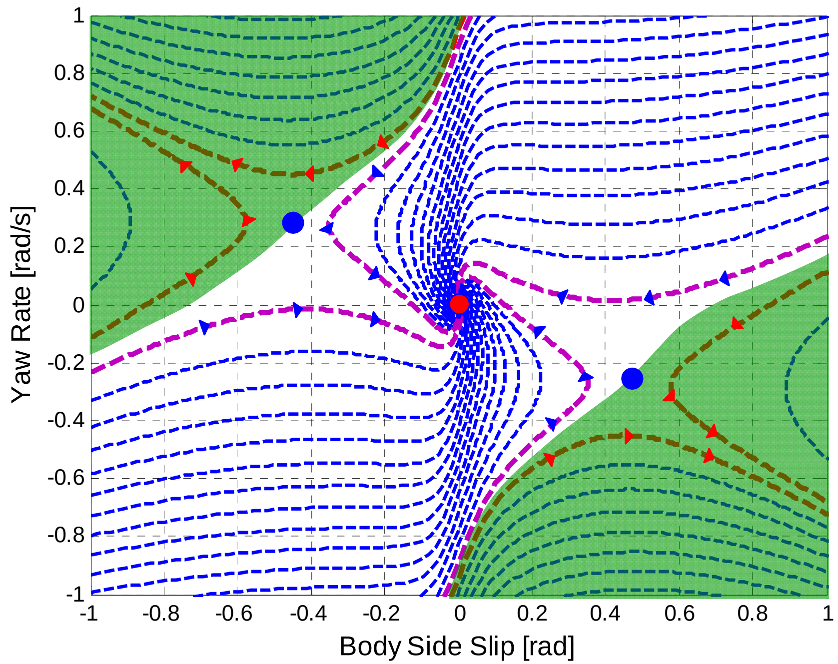
▼ $\beta - \dot{\beta}$ Phase Plane Trajectory

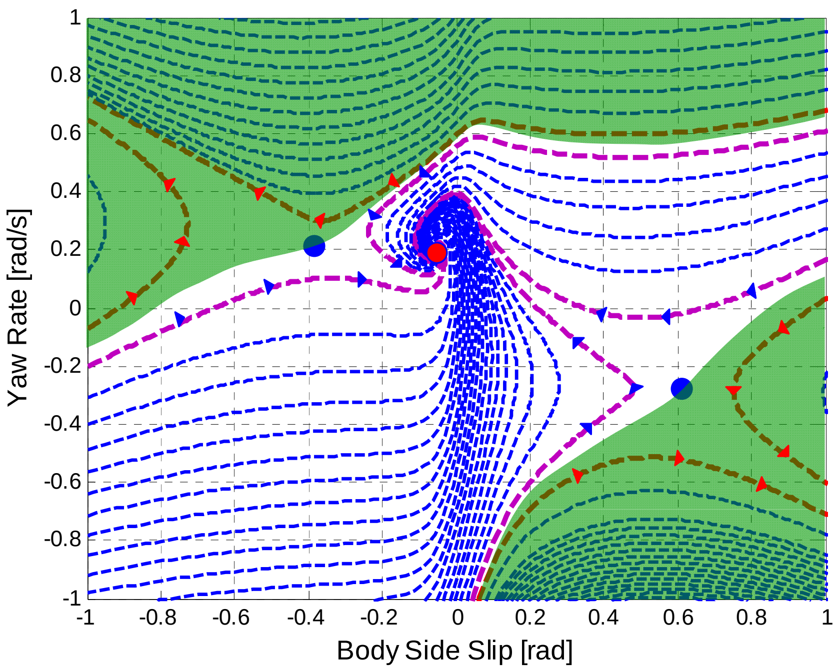
- Vehicle Speed = 100kph
- Front Steering = 0 deg
- Front Steering = 3 deg
- Front Steering = 6 deg

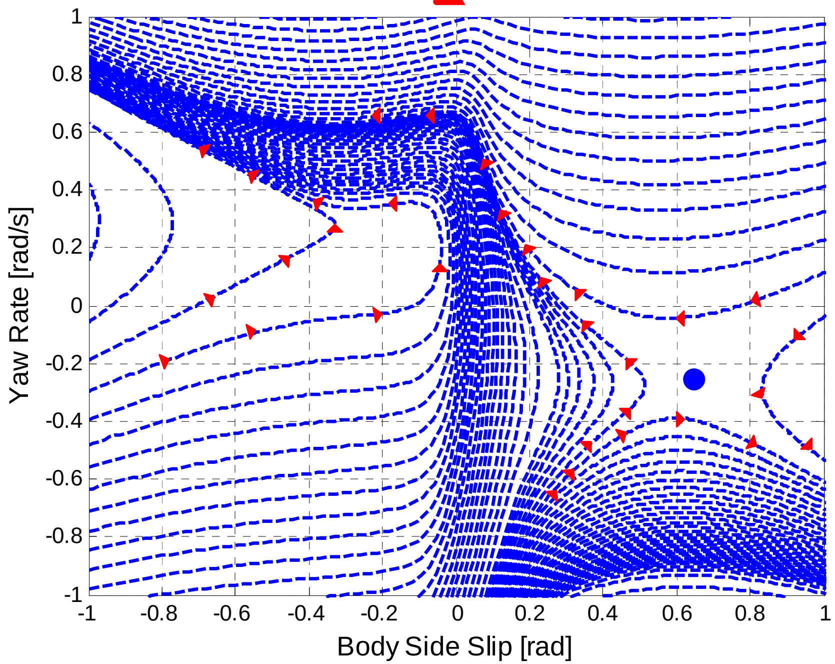


▼ $\beta - \gamma$ Phase Plane Trajectory

- Vehicle Speed =100 kph
  - Front Steering = 0 deg
  - Front Steering = 3 deg
  - Front Steering = 6 deg

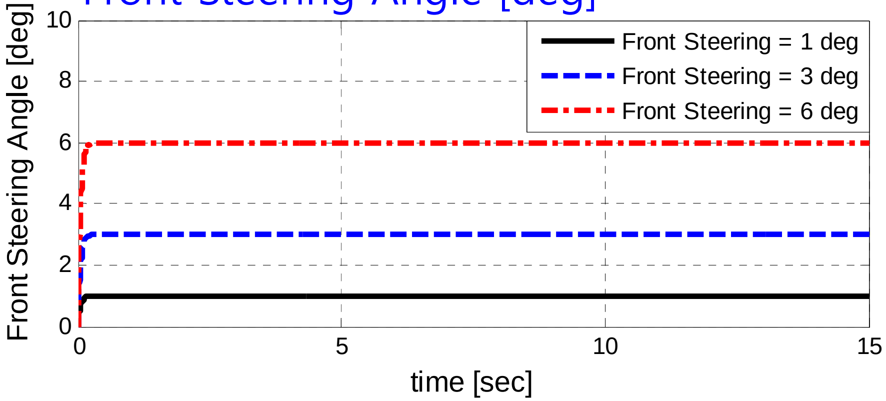




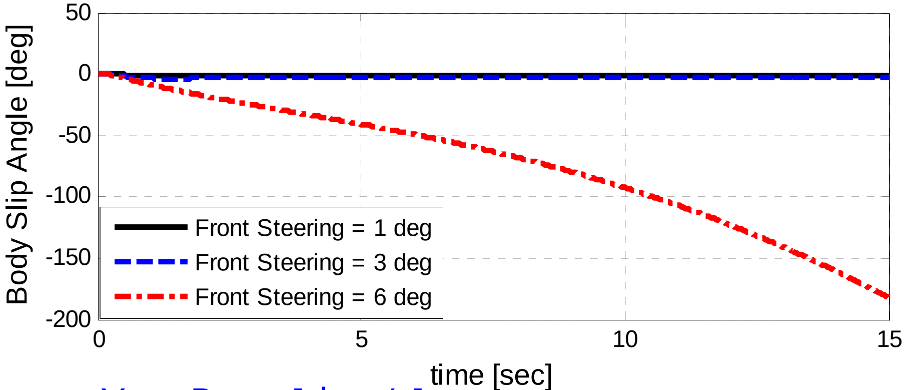


Time Domain Simulation-1: Vehicle Speed =100 kph

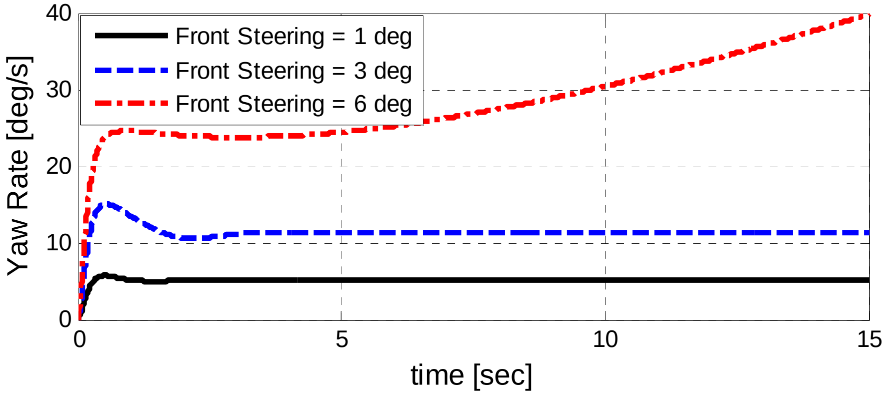
- Front Steering Angle [deg]



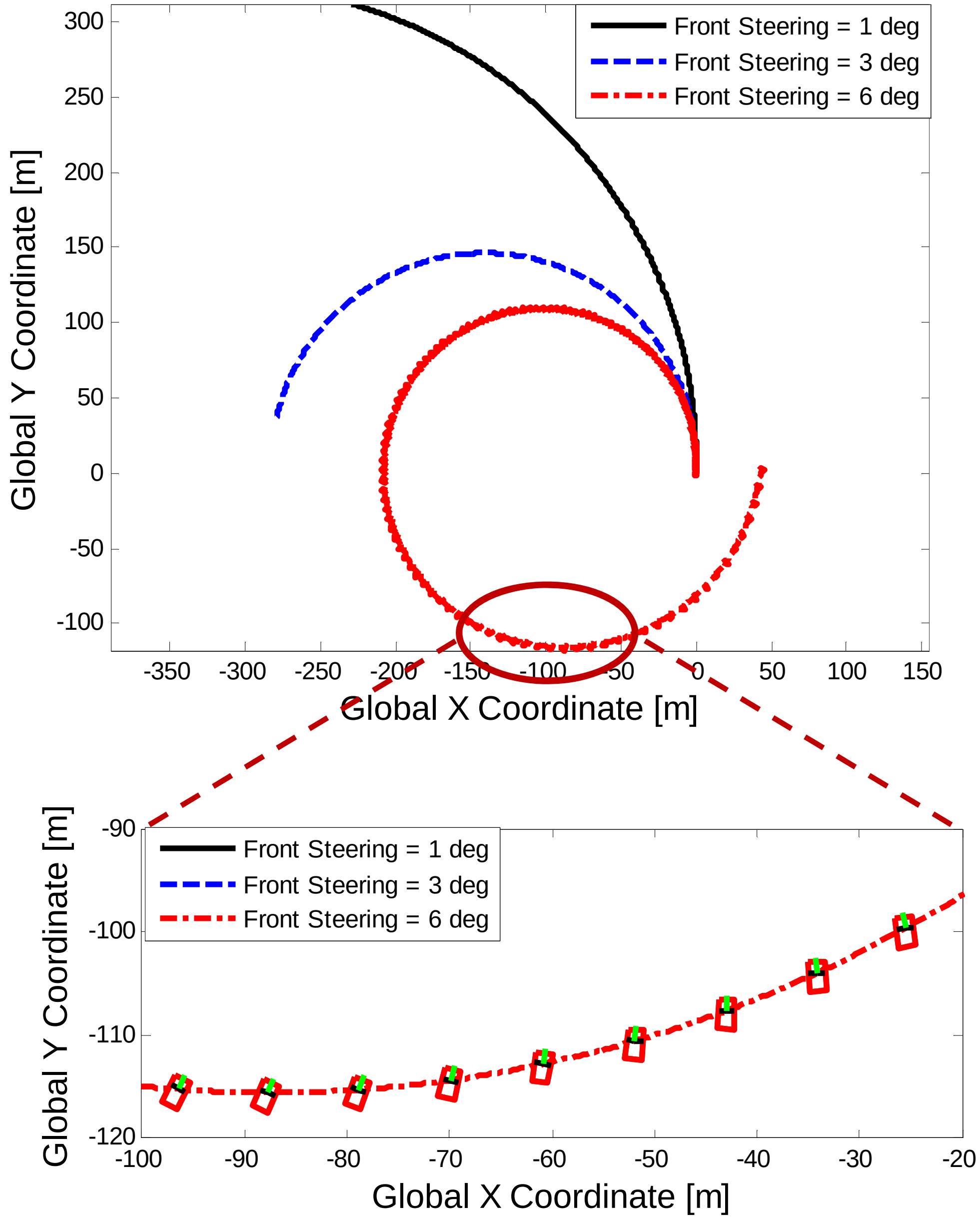
- Body Slip Angle [deg]



- Yaw Rate [deg/s]

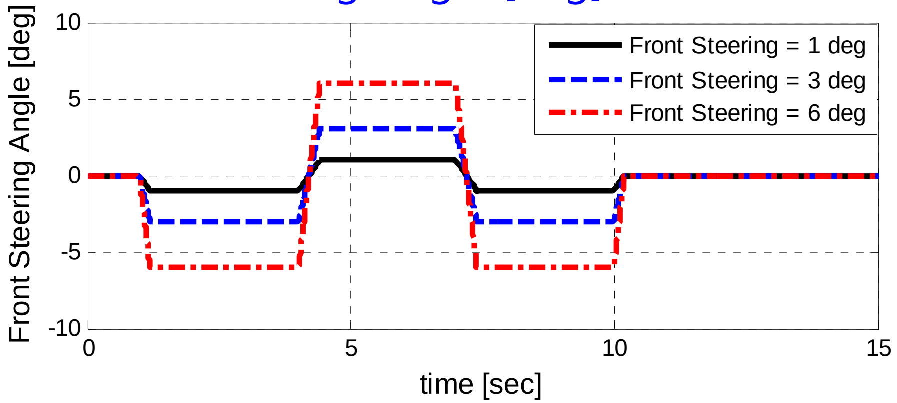


- Vehicle Trajectory

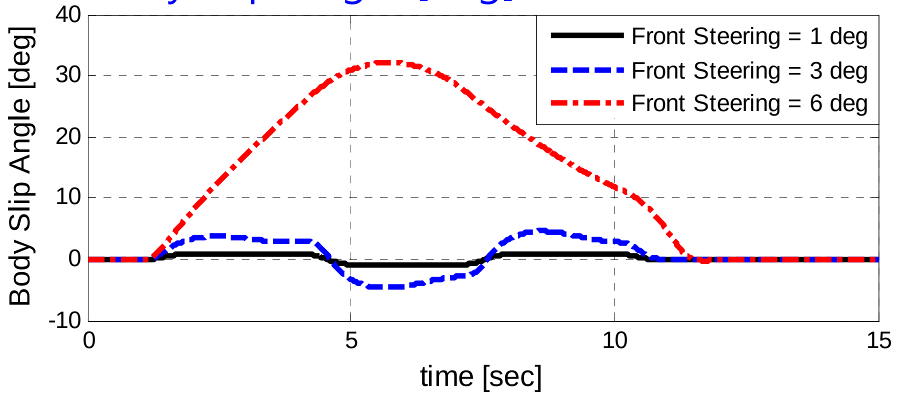


Time Domain Simulation-2: Vehicle Speed =100 kph

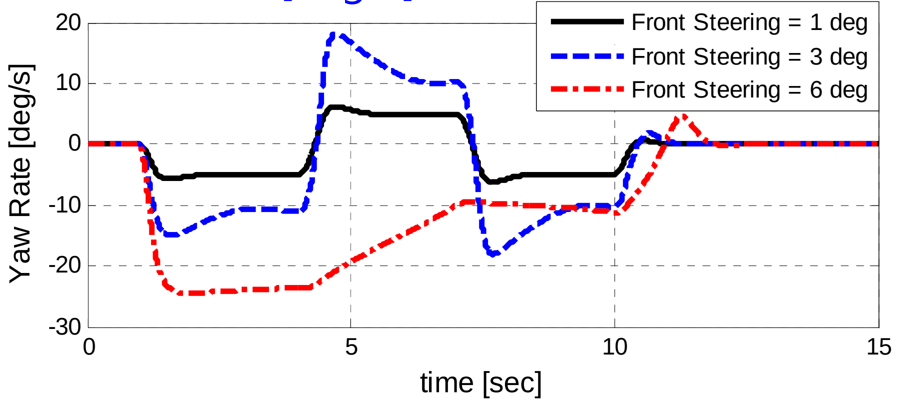
- Front Steering Angle [deg]



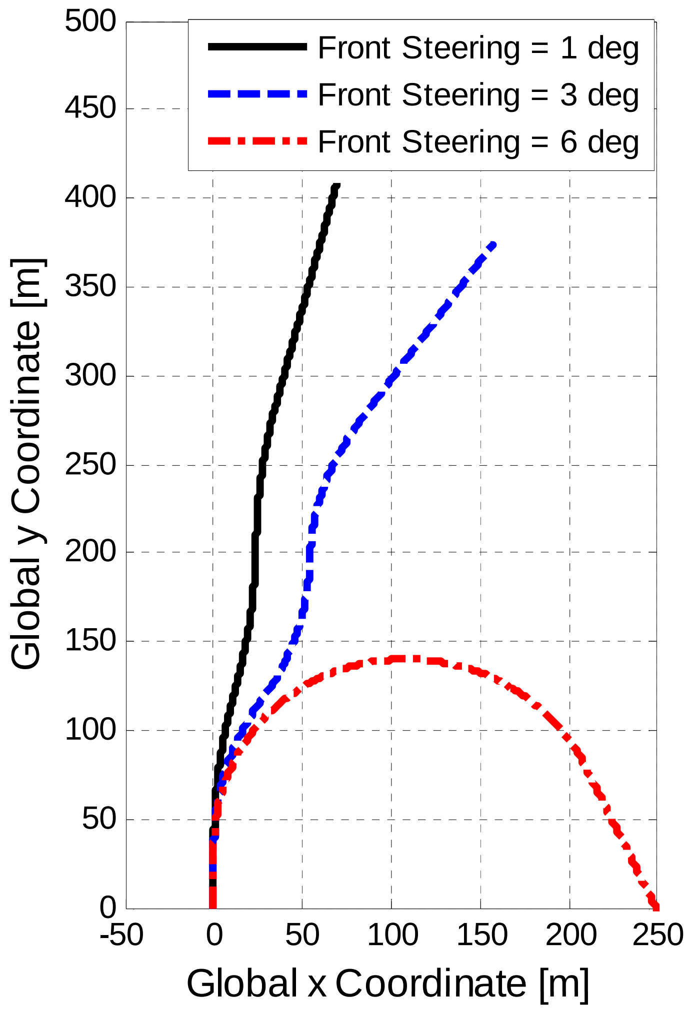
- Body Slip Angle [deg]



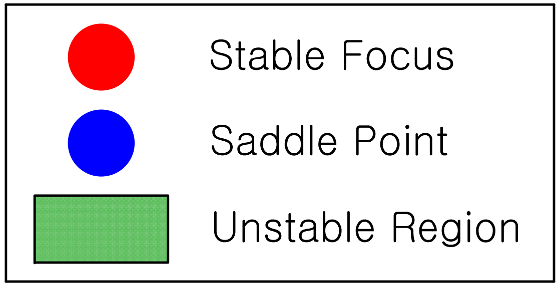
- Yaw Rate [deg/s]



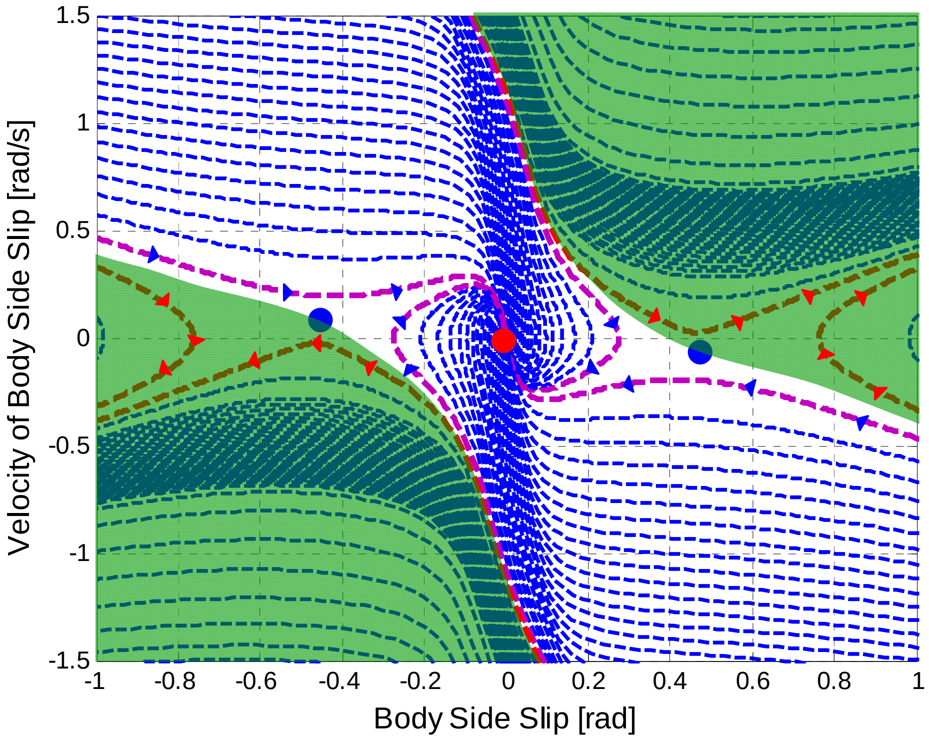
- Vehicle Trajectory



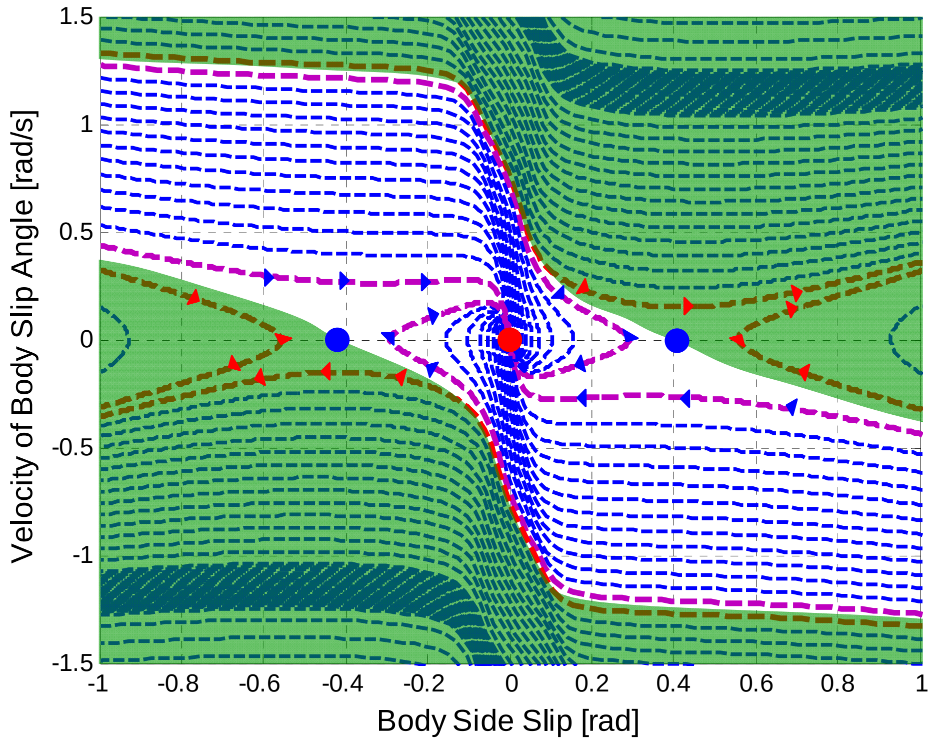
$\beta - \dot{\beta}$ Phase Plane Trajectory



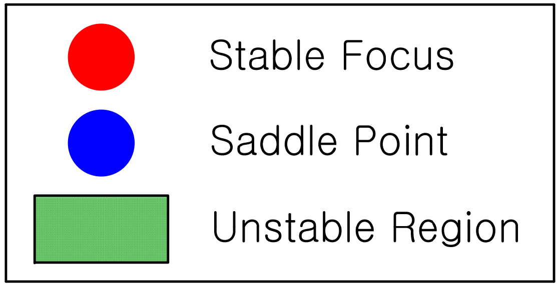
- $v_x = 70kph, \quad \delta_f = 0\,\text{deg}, \quad \mu=0.85$



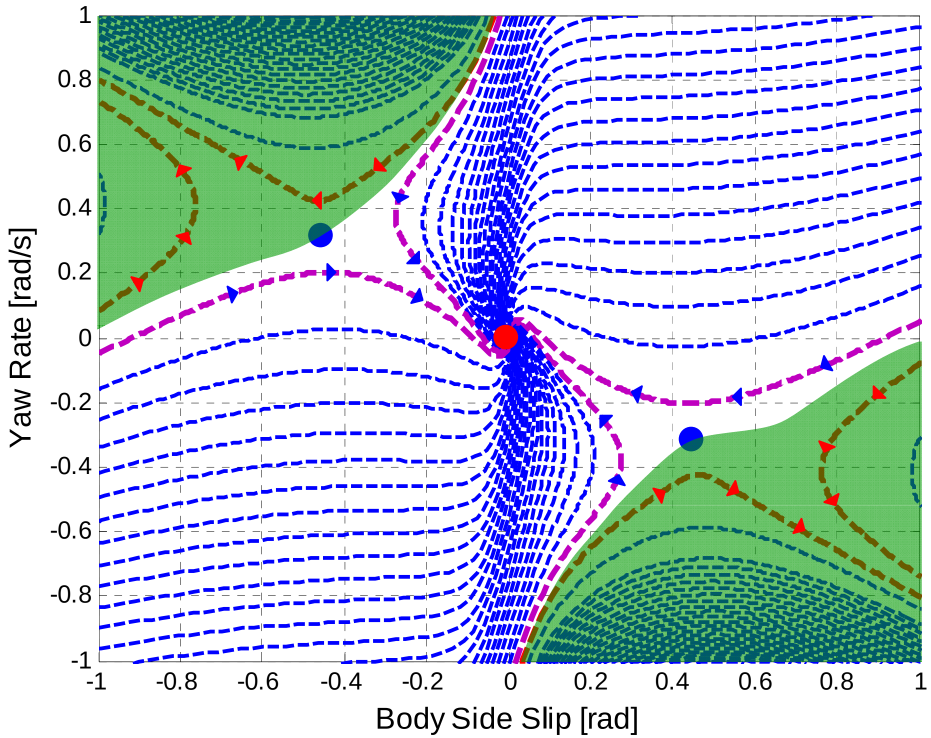
- $v_x = 70kph, \quad \delta_f = 0\,\text{deg}, \quad \mu=0.5$

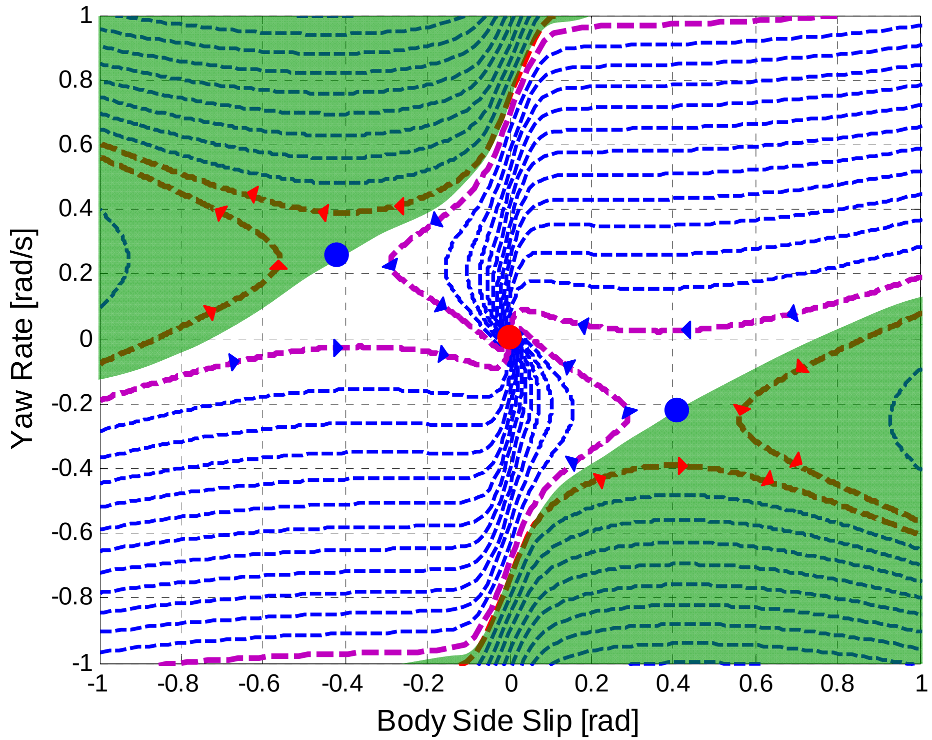


▼ $\beta - \gamma$ Phase Plane Trajectory



- $v_x = 70kph, \quad \delta_f = 0\deg, \quad \mu=0.85$
- $v_x = 70kph, \quad \delta_f = 0\deg, \quad \mu=0.5$





8. Vehicle Stability Control

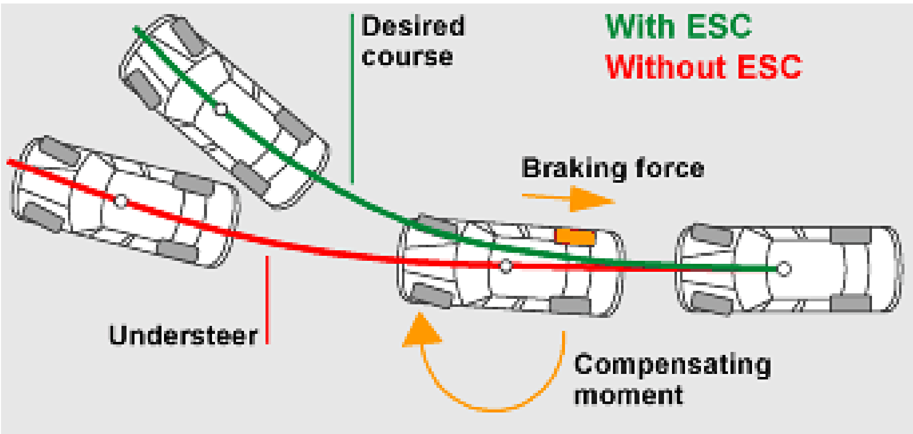
8.4 Electronic Stability Program (ESP)

- Why is ESP ?
  - An innovative safety system
  - Actively supporting the driver
  - Enhanced driving stability in situations with critical vehicle dynamics
  - Functions of ABS and TSC are integrated

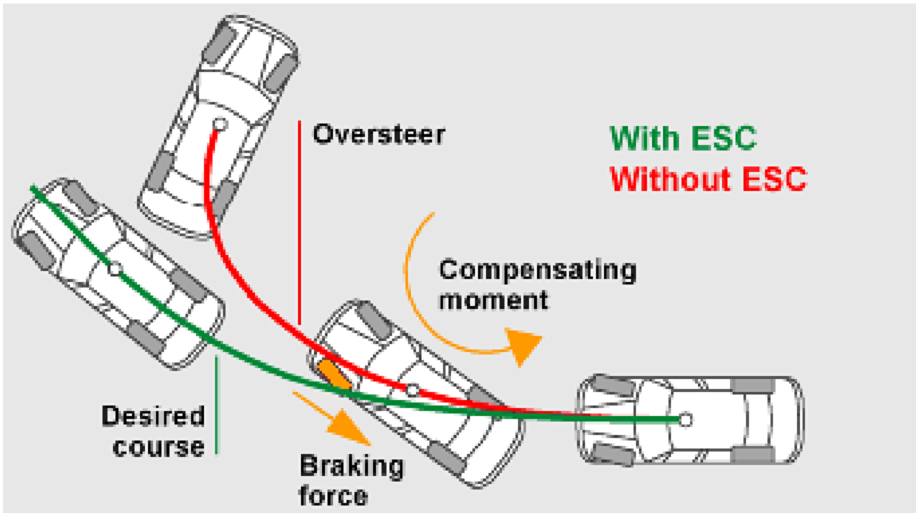
- What does ESP do ?

- ESP actively enhances vehicle stability
- (staying in lane and in direction)
  - Through interventions in the braking system or the motor management
  - To prevent critical situations, i.e., skidding, from leading to an accident
  - To minimize the risk of side crashs

▪ Under Steer



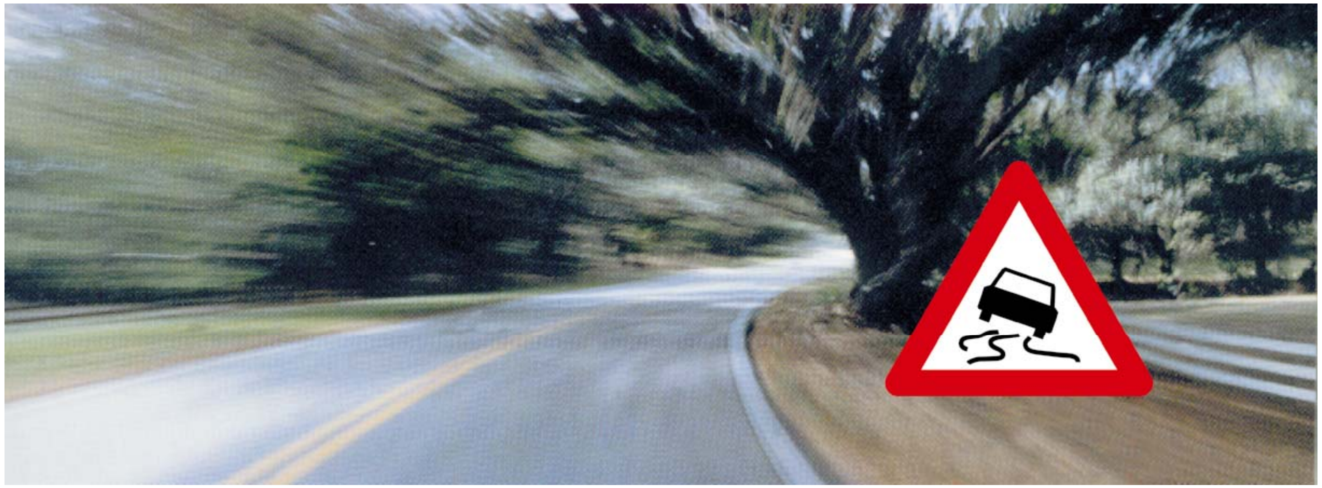
▪ Over Steer



- What is so special about ESP ? (1)

ESP watches out:

- surveys the vehicle's behavior
  (longitudinal and lateral dynamics)
- watches the vehicle-operator commands
  (Steering angle, brake pressure, accelerator-pedal travel)
- is continuously active
  in the background



- What is so special about ESP ? (2)

ESP knows:

- recognizes critical situations – in many cases before the driver does
- considers the possible ways of intervening:
  - Wheel-individual brake application
  - Intervention in the motor management

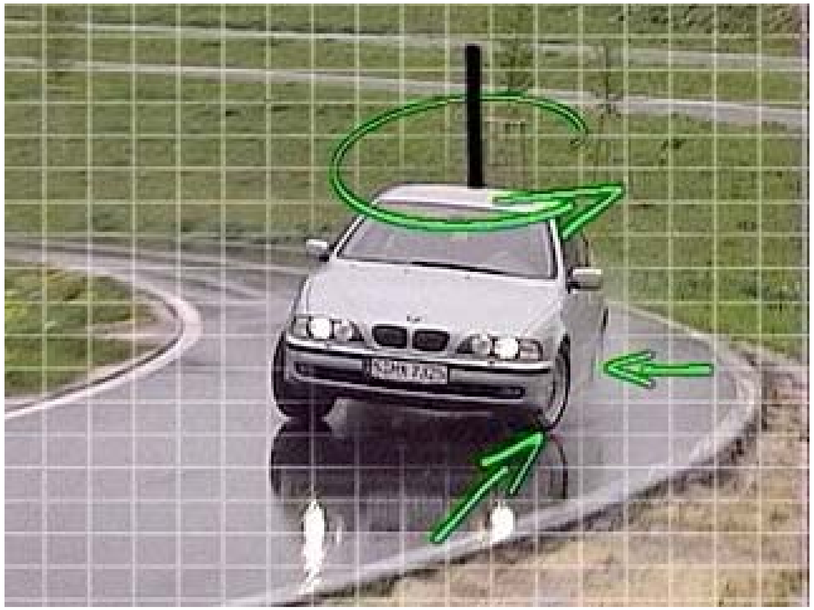
- What is so special about ESP ? (3)

ESP acts as quick as lightning:

– without reaction time

– calculated intervention in the brakes or the motor anagement

– reduces risk of skidding



Why is ESP so important ? (1)

Frequent cause for accidents:

The driver loses control of his vehicle.

I.e. Through

- speeding
- misinterpretation of the course or the road condition
- sudden swerving

ESP helps prevent accidents

Why is ESP so important ? (2)

28% of all accidents involving personal injury happen

- without prior conflict with another road user
- through loss of control of the vehicle followed by a collision with another car

60% of all accidents with fatal injuries are caused by side crashs

- These side crashes are mainly caused by skidding because of excessive speed, driving errors or excessive steering movements

(Source: RESIKO-Survey of GDV – Gesamtverband der deutschen Versicherungen – General Association of German Insurance Companies)

ESP helps prevent side crashes

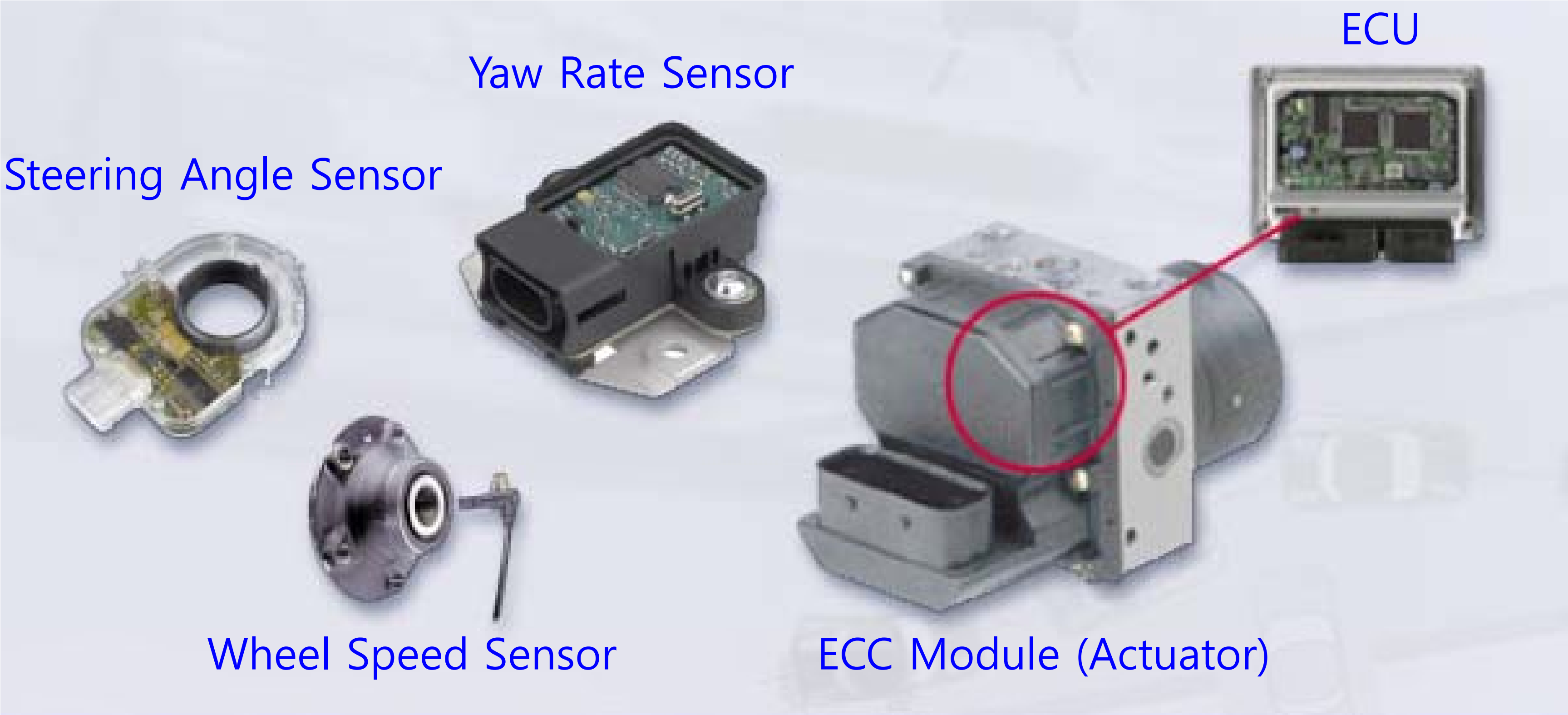
Why is ESP so important ? (3)

Recommendation of the General Association of German Insurance Companies

" Practice shows that vehicle dynamic control systems like ESP are capable of making skidding avoidable or at least increase control. With their widespread introduction a substantial decrease in the number of serious accidents could be expected."

(Source: RESIKO-Survey of GDV –
Gesamtverband der deutschen Versicherungen –
General Association of German Insurance Companies)

- What are the components of ESP ?
  - Sensors for monitoring vehicle-state and driver-inputs
  - ESP-ECU with micro processor
  - Hydraulic unit for stabilizing brake-application



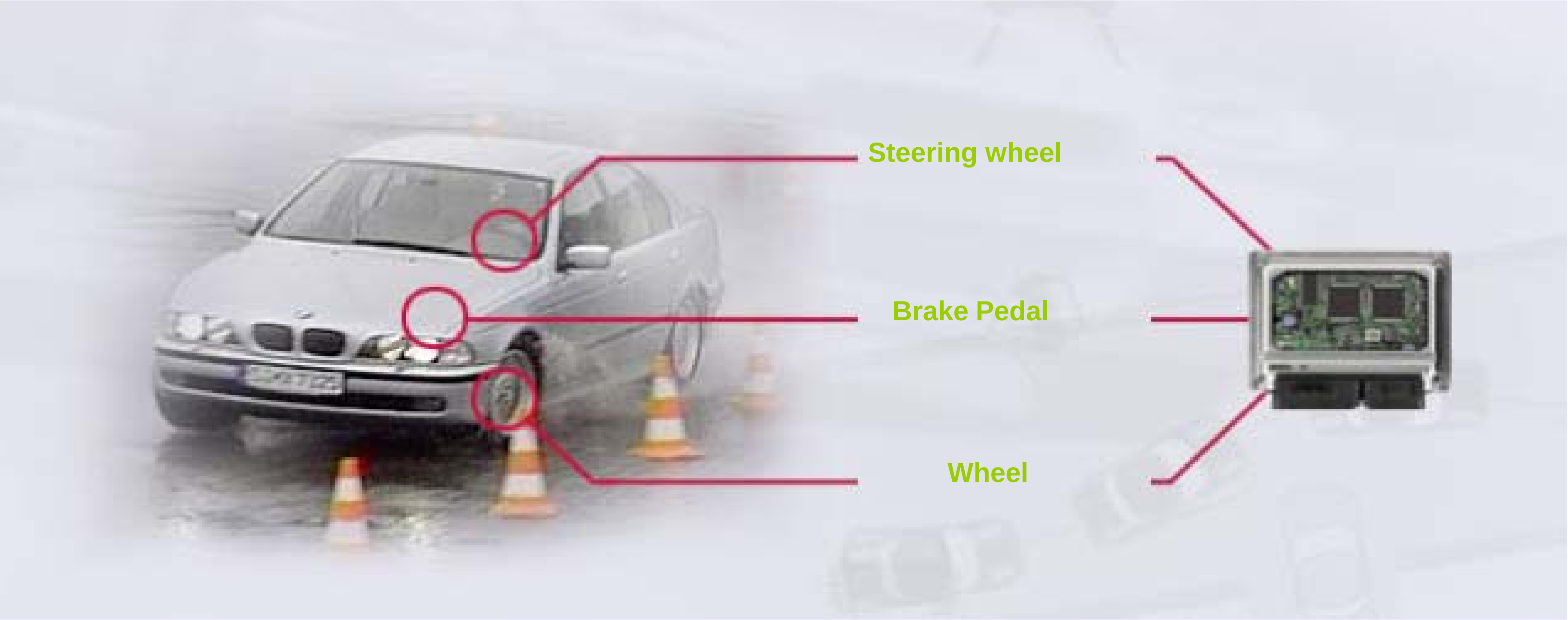
How does ESP work ? (1)

ESP analyzes:

What is the driver's intention?

Position of the steering wheel
+ wheel speed
+ position of the accelerator
+ brake pressure

= ECU recognizes driver's intention



How does ESP work ? (2)

ESP examines:
How does the vehicle behave?

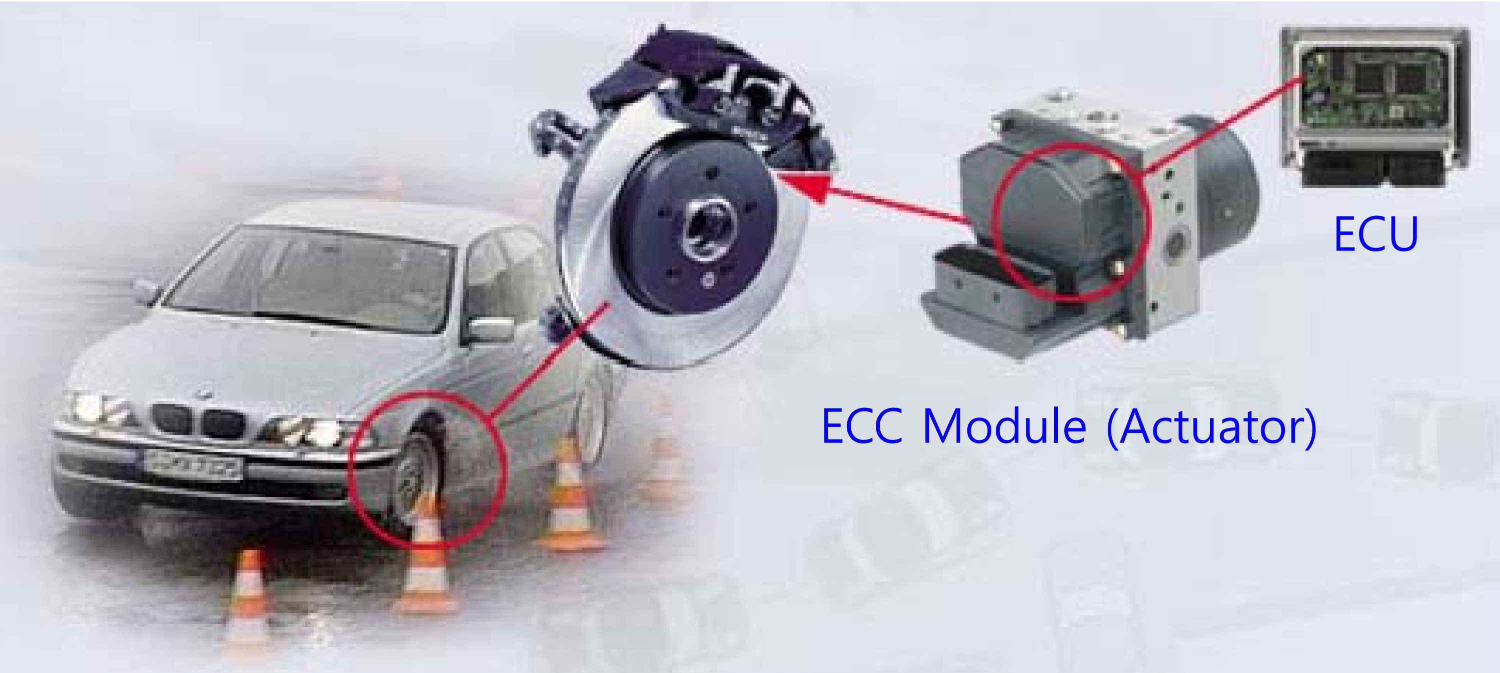
Yaw speed + lateral forces
= ECU calculates the vehicle's behavior

ESP's ECU compares vehicle behavior and driver's intention :
It recognizes any deviation from the set course

How does ESP work ? (3)

ESP acts: It "steers" through brake-application

- The ECU calculates the required measures
- The hydraulic unit quickly and individually supplies the brake pressure for each wheel
- In addition, ESP can reduce the engine torque via connection to the motor management



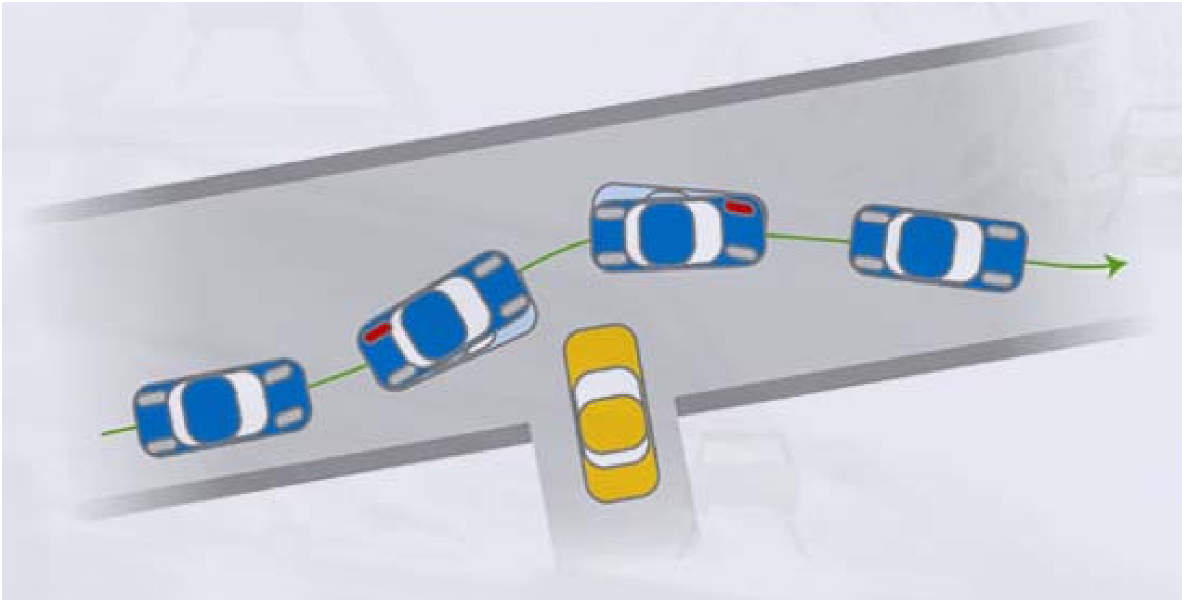
In what situations is ESP needed? (1)

Examples:

- Avoiding an obstacle
- Sudden wrenching of the steering wheel
- Driving on varying road surfaces
  (Longitudinal and/or lateral changes)

In what situations is ESP needed? (2)

Avoiding an obstacle



1) Hit the brakes, wrench the steering wheel:
   Vehicle tends to understeer

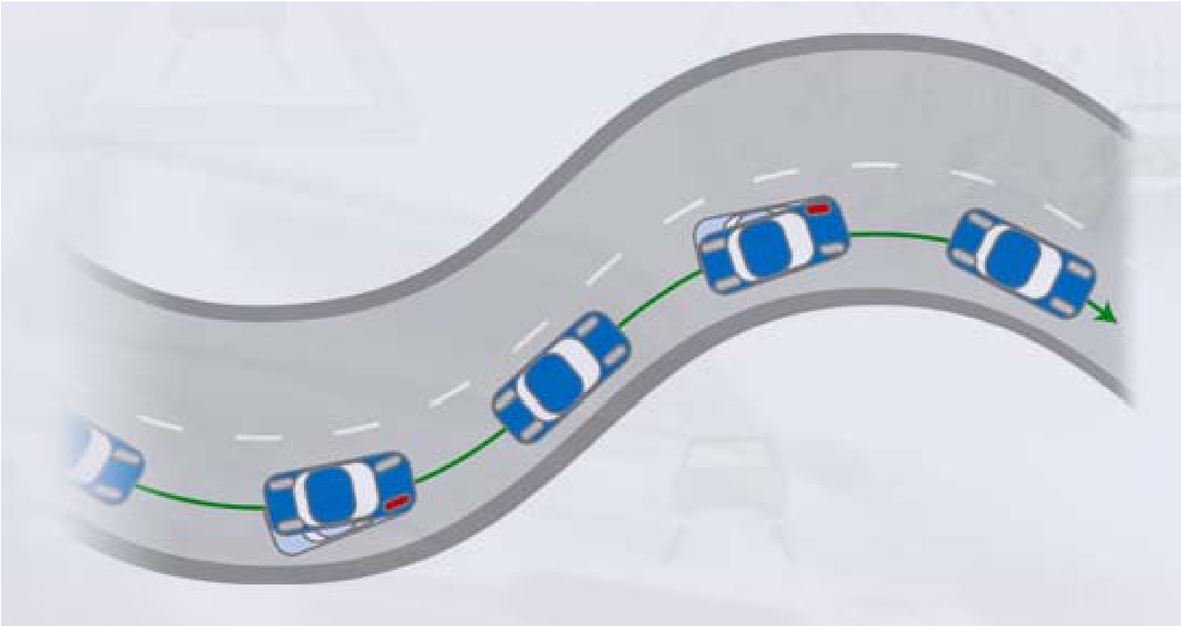
2) ESP brakes the left rear wheel, vehicle obeys steering-wheel input

3) Reverse steering input:
   Vehicle tends to oversteer, ESP brakes the front right wheel

4) Vehicle becomes stable again

In what situations is ESP needed? (3)

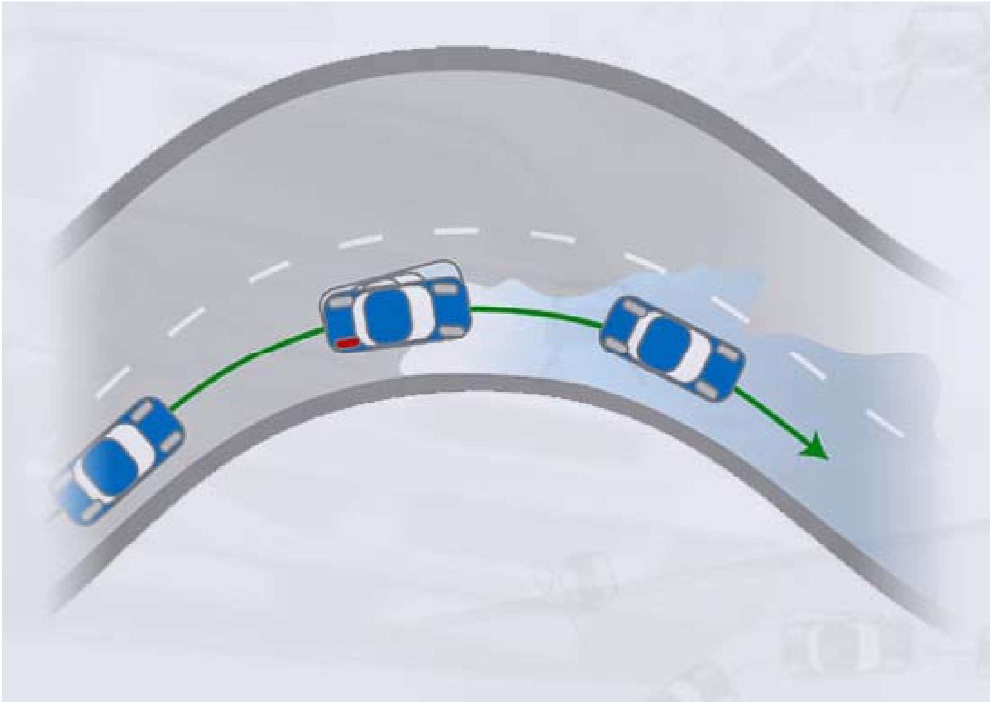
Sudden wrenching of the steering wheel



1) Vehicle tends to break away.
   Automatic breaking-pressure rise at the front right wheel
2) Vehicle is stable
3) Vehicle tends to break away.
   Automatic breaking-pressure rise at the front left wheel
4) Vehicle is stable

In what situations is ESP needed? (4)

Driving on varying road surfaces



Vehicle tends to break away (understeer):

– ESP intervenes and brakes the right rear wheel while at the same time reducing engine torque

8. Vehicle Stability Control

8.1 Bicycle model: Nonlinear and Linear models

8.2 Phase Plane Analysis

8.3 Phase Plane Analysis of Bicycle model

8.4 Electronic Stability Program (ESP)

8.5 Vehicle Stability Control Algorithm

8.5 Vehicle Stability Control (VSC) Algorithm

8.5.1 Direct Yaw Moment Control (Desired Yaw Rate Tracking)

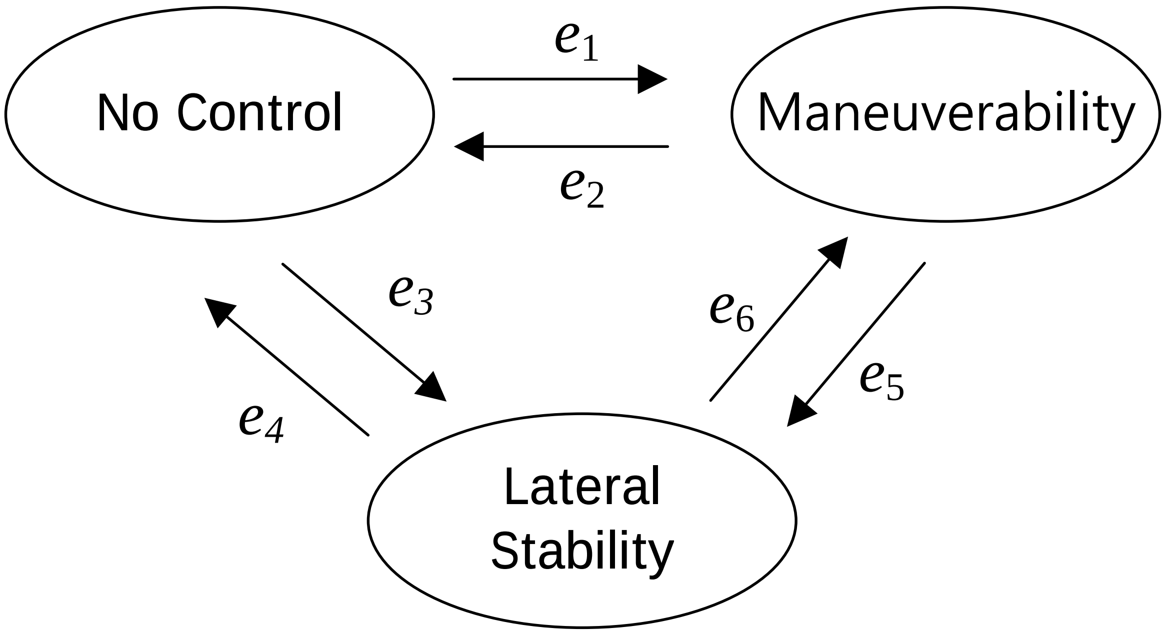
8.5.2 Direct Yaw Moment Control ($\beta - \dot{\beta}$ Phase Plane Analysis)

8.5.3 Simulation Results

8.5.1Direct Yaw Moment Control (Desired Yaw Rate Tracking)

1) Determination of Control Mode

- State-transition diagram for the control mode switching



- Desired Yaw Rate

$$\gamma_{desired} = \begin{cases} 0 & \textit{if} \text{ (No Control)} \\ \gamma_{des\_yaw} & \textit{if} \text{ (Manuverability)} \\ \gamma_{des\_lateral} & \textit{if} \text{ (Lateral Stability)} \end{cases}$$

8.5.1Direct Yaw Moment Control (Desired Yaw Rate Tracking)

1) Determination of Control Mode

- Events and conditions of activation

Event	Activation Condition
e_1	$\lvert\beta\rvert < \beta_{threshold}$, $\lvert\gamma_e\rvert \geq \gamma_{e\_threshold}$
e_2	$\lvert\beta\rvert < \beta_{threshold}$, $\lvert\gamma_e\rvert < \gamma_{e\_threshold}$
e_3	$\lvert\beta\rvert \geq \beta_{threshold}$
e_4	$\lvert\beta\rvert < \beta_{threshold}$, $\lvert\gamma_e\rvert < \gamma_{e\_threshold}$
e_5	$\lvert\beta\rvert \geq \beta_{threshold}$, $\lvert\gamma_e\rvert \geq \gamma_{e\_threshold}$
e_6	$\lvert\beta\rvert < \beta_{threshold}$, $\lvert\gamma_e\rvert \geq \gamma_{e\_threshold}$

Where, $\beta_{threshold} = 8\deg$ & $\gamma_{e\_threshold} = 5\deg/s$ are Control Threshold Values.

$\gamma_e = \gamma - \gamma_{desired}$ is yaw rate error.

8.5.1Direct Yaw Moment Control (Desired Yaw Rate Tracking)

2) Desired Yaw Rate for Maneuverability

- 2-DOF Bicycle Model based steady state value

$$x = \begin{bmatrix} \beta \\ \gamma \end{bmatrix} \qquad \dot{x} = A \cdot x + B \cdot \delta_f$$

$$A = \begin{bmatrix} -\dfrac{2(C_f + C_r)}{m \cdot v_x} & -1 - \dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{m \cdot v_x^2} \\ -\dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} & -\dfrac{2(l_f^2 \cdot C_f + l_r^2 \cdot C_r)}{I_z \cdot v_x} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} \dfrac{2 \cdot C_f}{m v_x} \\ \dfrac{2 \cdot l_f \cdot C_f}{I_z} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$x_{ss} = \begin{bmatrix} \beta_{ss} \\ \gamma_{ss} \end{bmatrix} = -A^{-1} \cdot B \cdot \delta_f$$

- Steady state yaw rate according to front steering angle

$$\gamma_{ss} = \frac{a_{21} \cdot b_1 - a_{11} \cdot b_2}{a_{11} \cdot a_{22} - a_{12} \cdot a_{21}} \cdot \delta_f$$

$$= \frac{v_x \cdot C_f \cdot C_r \left(l_f + l_r \right)}{C_f \cdot C_r \cdot (l_f + l_r)^2 + m \cdot v_x^2 (-l_f \cdot C_f + l_r \cdot C_r)} \cdot \delta_f$$

8.5.1Direct Yaw Moment Control (Desired Yaw Rate Tracking)

2) Desired Yaw Rate for Maneuverability

- Steady state yaw rate according to front steering angle

$$\gamma_{ss} = \frac{v_x \cdot C_f \cdot C_r \left(l_f + l_r\right)}{C_f \cdot C_r \cdot (l_f + l_r)^2 + m \cdot v_x{}^2(-l_f \cdot C_f + l_r \cdot C_r)} \cdot \delta_f$$

$$= \frac{v_x}{(l_f + l_r) + \frac{v_x{}^2}{g} \cdot \frac{mg}{(l_f + l_r)} \cdot (\frac{l_r}{C_f} - \frac{l_f}{C_r})} \cdot \delta_f \quad \Leftarrow \quad K_{us} = \frac{mg}{(l_f + l_r)} \cdot (\frac{l_r}{C_f} - \frac{l_f}{C_r})$$

$$= \frac{v_x}{(l_f + l_r) + \frac{v_x{}^2}{g} \cdot K_{us}} \cdot \delta_f = \frac{v_x}{(l_f + l_r) \cdot \left\{1 + \frac{v_x{}^2}{V_{char}^2}\right\}} \cdot \delta_f = \gamma_{ss}(v_x,\ \delta_f) \quad \Leftarrow \quad V_{char} = \sqrt{\frac{Lg}{K_{us}}}$$

- Considering tire/road friction

$$\gamma_{des\_yaw} = \begin{cases} \frac{1}{1 + \tau_e \cdot s} \gamma_{ss} & \text{if} \left(\left|\gamma_{ss}\right| < \frac{\mu g}{v_x}\right) \\ \frac{\mu g}{v_x} \cdot \mathrm{sgn}(\gamma_{ss}) & \text{elsewhere} \end{cases} \quad \Leftarrow \quad m \cdot \left|a_y\right| \simeq m \cdot \left|v_x \cdot \gamma_{ss}\right| \le \mu \cdot m \cdot g$$

Where, τ_e is a time constant.

8.5.1Direct Yaw Moment Control (Desired Yaw Rate Tracking)

3) Desired Yaw Rate for Lateral Stability

- Through 2-DOF bicycle model, the lateral dynamics are expressed as follows:

$$m \cdot v_x \cdot \left(\dot{\beta} + \gamma\right) = 2F_{yf} \cos\delta_f + 2F_{yr}$$

- Assuming that $\dot{v}_x \simeq 0$, the side slip angle dynamics can be expressed as follows:

$$\dot{\beta} = -\gamma + \frac{2 \cdot F_{yf} \cdot \cos\delta_f + 2 \cdot F_{yr}}{m \cdot v_x}$$

- desired yaw rate for lateral stability

$$\gamma_{des\_lateral} = K_1 \cdot \beta + \frac{2 \cdot F_{yf} \cdot \cos\delta_f + 2 \cdot F_{yr}}{m \cdot v_x}$$

- Then, the dynamics of the body side slip angle are stable, which implies that the body sideslip angle asymptotically converges to zero.

$$\dot{\beta} = -K_1 \cdot \beta$$

8.5.1Direct Yaw Moment Control (Desired Yaw Rate Tracking)

4) Yaw Moment Input based on Sliding Control Method

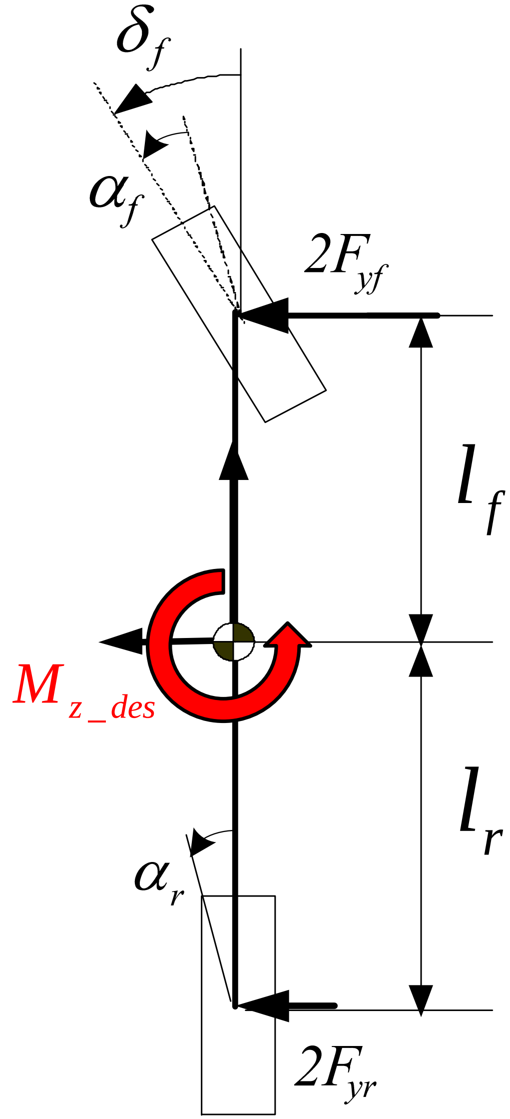
(1) 2 DOF Bicycle Model including Yaw Moment Input

$$x = \begin{bmatrix} \beta \\ \gamma \end{bmatrix} \qquad \dot{x} = A \cdot x + B \cdot \delta_f + F \cdot M_{z\_des} \quad \text{Eq.2}$$

Where,

$$A = \begin{bmatrix} -\dfrac{2(C_f + C_r)}{m \cdot v_x} & -1 - \dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{m \cdot v_x^2} \\ -\dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} & -\dfrac{2(l_f^2 \cdot C_f + l_r^2 \cdot C_r)}{I_z \cdot v_x} \end{bmatrix}$$

$$B = \begin{bmatrix} \dfrac{2 \cdot C_f}{m v_x} \\ \dfrac{2 \cdot l_f \cdot C_f}{I_z} \end{bmatrix} \qquad F = \begin{bmatrix} 0 \\ \dfrac{1}{I_z} \end{bmatrix}$$



8.5.1Direct Yaw Moment Control (Desired Yaw Rate Tracking)

4) Yaw Moment Input based on Sliding Control Method

(2) Define sliding surface

$$s = (\gamma - \gamma_{desired})$$ Eq.3

(3) Lyapunov Function

$$V = \frac{1}{2} s^2$$ Eq.4

(4) Differentiating the Lyapunov function,

$$\dot{V} = s \cdot \dot{s} < 0$$ Eq.5

(5) For stable system, the derivative of the Lyapunov function should be negative.

$$\dot{V} = s \cdot \dot{s} = -K_2 \cdot |s| < 0$$ Eq.6

8.5.1Direct Yaw Moment Control (Desired Yaw Rate Tracking)

4) Yaw Moment Input based on Sliding Control Method

(6) Substituting Eqns (2) and (3) into Eq.(5)

$$\dot{V} = s \cdot \dot{s} = -K_2 \cdot |s| \quad \Rightarrow \quad \dot{s} = -K_2 \cdot \mathrm{sgn}(s)$$

$$\dot{s} = (\dot{\gamma} - \dot{\gamma}_{desired})$$

Assuming that: $\dot{\gamma}_{desired} \simeq 0$

$$= -\frac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} \cdot \beta - \frac{2(l_f^{\ 2} \cdot C_f + l_r^{\ 2} \cdot C_r)}{I_z \cdot v_x} \cdot \dot{\psi} + \frac{2 \cdot l_f \cdot C_f}{I_z} \cdot \delta_f + \frac{M_{z\_des}}{I_z} \quad \text{Eq.7}$$

$$= -K_2 \cdot \mathrm{sgn}(s)$$

8.5.1Direct Yaw Moment Control (Desired Yaw Rate Tracking)

4) Yaw Moment Input based on Sliding Control Method

(7) Rewritten the above equation

$$M_{z\_des} = I_z \cdot \left(\frac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} \cdot \beta + \frac{2(l_f^{\,2} \cdot C_f + l_r^{\,2} \cdot C_r)}{I_z \cdot v_x} \cdot \dot{\psi} - \frac{2 \cdot l_f \cdot C_f}{I_z} \cdot \delta_f \right) - I_z \cdot K_2 \cdot \mathrm{sgn}(s)$$

(8) Finally, the saturation function with Φ is used to cope with the chattering phenomenon as follows:

$$M_{z\_des} = M_{z\_eq} - I_z \cdot K_2 \cdot sat(\frac{\dot{\gamma} - \dot{\gamma}_{desired}}{\Phi})$$

Where, $M_{z\_eq} = I_z \cdot \left(\frac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} \cdot \beta + \frac{2(l_f^{\,2} \cdot C_f + l_r^{\,2} \cdot C_r)}{I_z \cdot v_x} \cdot \dot{\psi} - \frac{2 \cdot l_f \cdot C_f}{I_z} \cdot \delta_f \right)$

8.5 Vehicle Stability Control (VSC) Algorithm

8.5.1 Direct Yaw Moment Control (Desired Yaw Rate Tracking)

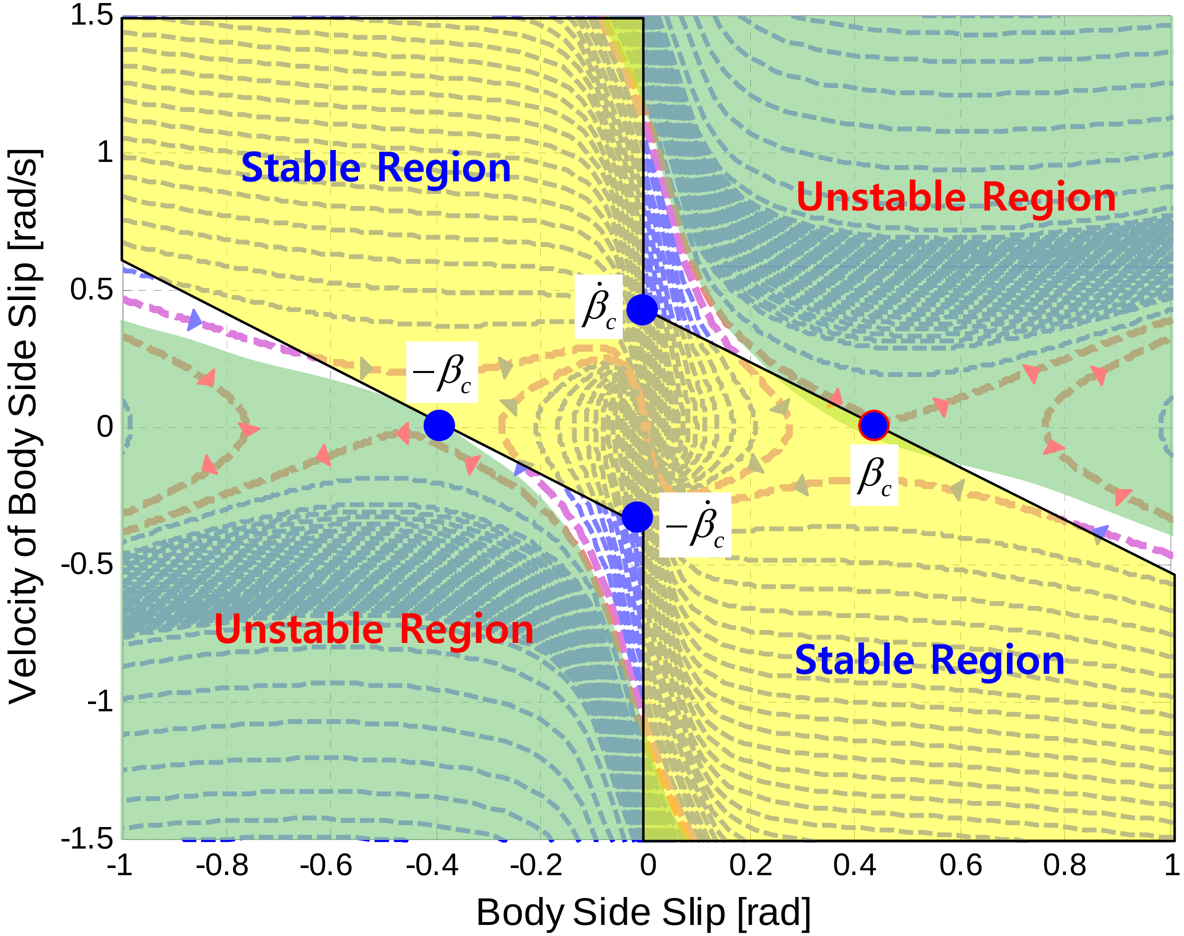
8.5.2 Direct Yaw Moment Control ($\beta - \dot{\beta}$ Phase Plane Analysis)

8.5.3 Simulation Results

8.5.2 Direct Yaw Moment Control ($\beta - \dot{\beta}$ Phase Plane Analysis)

1) Control Threshold

- Using Phase plane analysis
  - Predefined control threshold on phase plane axis
  - 'Look-up table' w.r.t. vehicle velocity & SWA



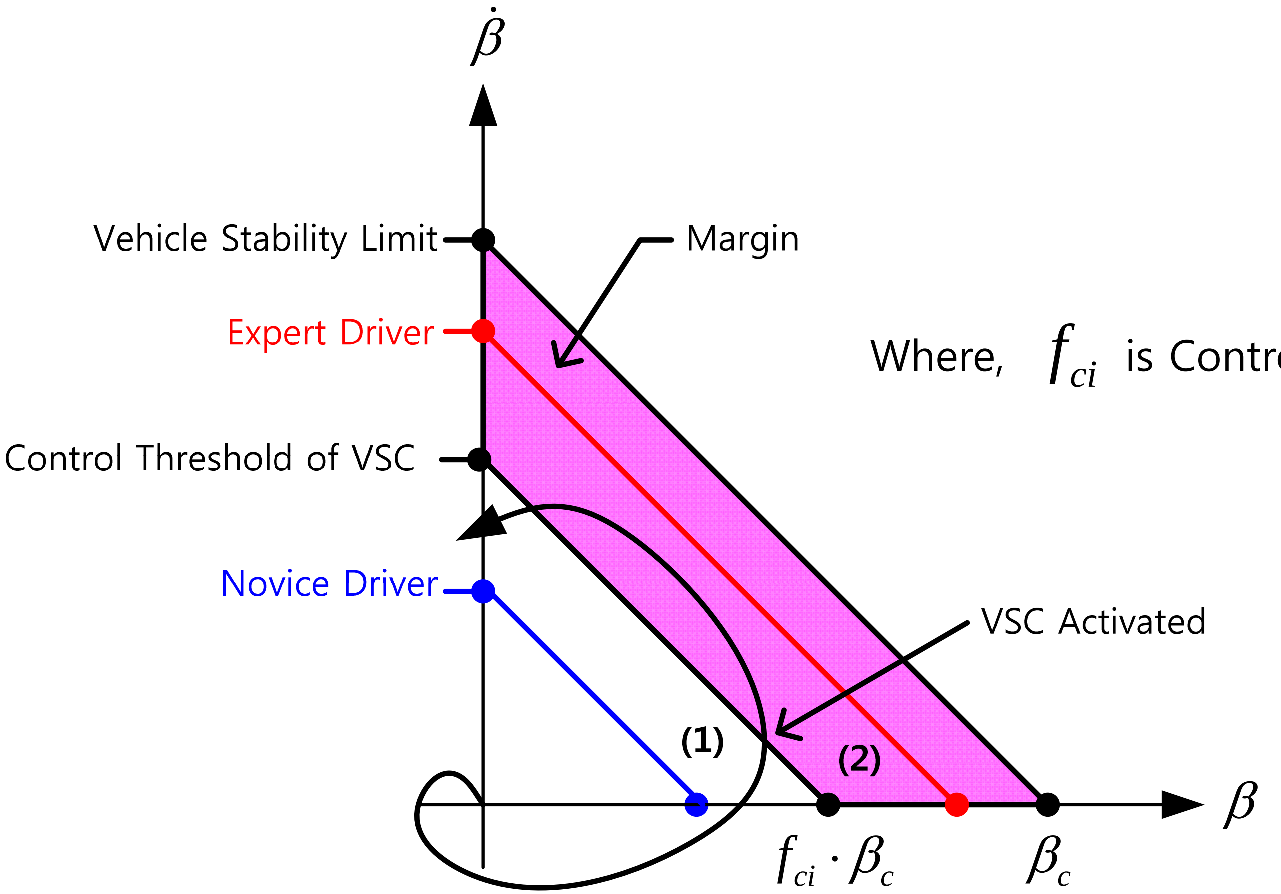
8.5.2 Direct Yaw Moment Control ($\beta - \dot{\beta}$ Phase Plane Analysis)

1) Control Threshold

- VSC Control Threshold on $\beta - \dot{\beta}$ Phase Plane

Region (1),
VSC not activated,
Novice driver feels in danger

Region (2),
VSC activated,
Expert driver feels redundancy



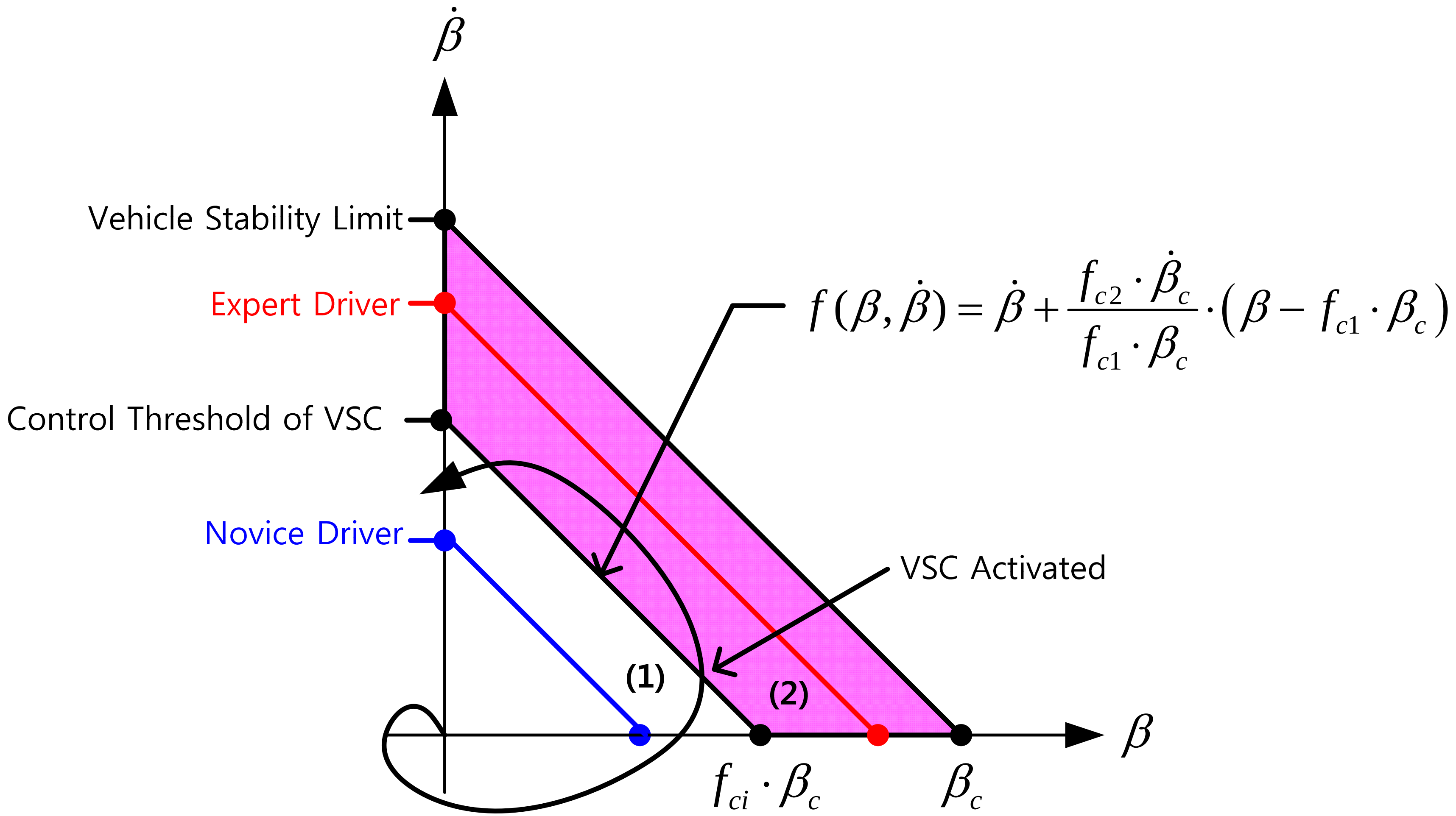
Where, f_{ci} is Control Threshold Factor

8.5.2 Direct Yaw Moment Control ($\beta - \dot{\beta}$ Phase Plane Analysis)

1) Control Threshold

VSC is not Activated $\quad if \left(f(\beta, \dot{\beta}) < 0 \right)$

VSC is Activated $\quad$ *elsewhere*



8.5.2 Direct Yaw Moment Control ($\beta - \dot{\beta}$ Phase Plane Analysis)

2) Desired Yaw Rate

- 2-DOF Bicycle Model based steady state value

$$x = \begin{bmatrix} \beta \\ \gamma \end{bmatrix} \qquad \dot{x} = A \cdot x + B \cdot \delta_f \Rightarrow \quad x_{ss} = \begin{bmatrix} \beta_{ss} \\ \gamma_{ss} \end{bmatrix} = -A^{-1} \cdot B \cdot \delta_f$$

$$\gamma_{ss} = \frac{v_x \cdot C_f \cdot C_r \left(l_f + l_r \right)}{C_f \cdot C_r \cdot (l_f + l_r)^2 + m \cdot v_x{}^2 (-l_f \cdot C_f + l_r \cdot C_r)} \cdot \delta_f$$

$$= \frac{v_x}{(l_f + l_r) \cdot \left\{ 1 + \frac{v_x{}^2}{V_{char}^2} \right\}} \cdot \delta_f = \gamma_{ss}(v_x,\ \delta_f)$$

- Considering tire/road friction

$$\gamma_{des\_yaw} = \begin{cases} \dfrac{1}{1 + \tau_e \cdot s} \gamma_{ss} & if \left(\left| \gamma_{ss} \right| < \dfrac{\mu g}{v_x} \right) \\ \dfrac{\mu g}{v_x} \cdot \mathrm{sgn}(\gamma_{ss}) & elsewhere \end{cases}$$

$$\Leftarrow \quad m \cdot \left| a_y \right| \simeq m \cdot \left| v_x \cdot \dot{\psi}_{des} \right| \le \mu \cdot m \cdot g$$

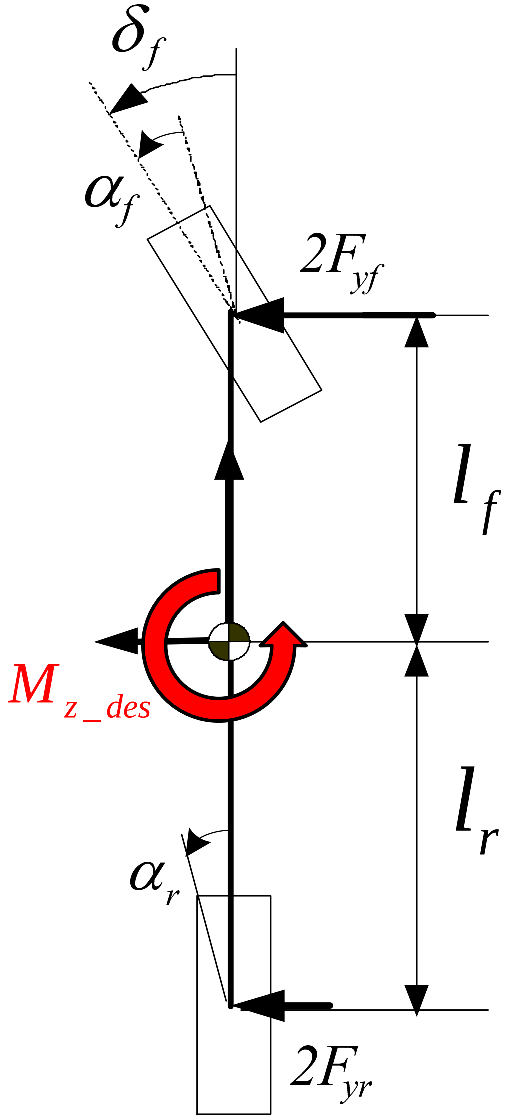
Where, τ_e is a time constant.

8.5.2 Direct Yaw Moment Control ($\beta - \dot{\beta}$ Phase Plane Analysis)

3) Yaw Moment Input based on Sliding Control Method

(1) 2 DOF Bicycle Model including Yaw Moment Input

$$x = \begin{bmatrix} \beta \\ \gamma \end{bmatrix} \qquad \dot{x} = A \cdot x + B \cdot \delta_f + F \cdot M_{z\_des} \quad \text{Eq.2}$$



Where,

$$A = \begin{bmatrix} -\dfrac{2(C_f + C_r)}{m \cdot v_x} & -1 - \dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{m \cdot v_x^2} \\ -\dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} & -\dfrac{2(l_f^2 \cdot C_f + l_r^2 \cdot C_r)}{I_z \cdot v_x} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$B = \begin{bmatrix} \dfrac{2 \cdot C_f}{m v_x} \\ \dfrac{2 \cdot l_f \cdot C_f}{I_z} \end{bmatrix} \qquad F = \begin{bmatrix} 0 \\ \dfrac{1}{I_z} \end{bmatrix}$$

8.5.2 Direct Yaw Moment Control ($\beta - \dot{\beta}$ Phase Plane Analysis)

3) Yaw Moment Input based on Sliding Control Method

(2) Define sliding surface

$$s = (\gamma - \gamma_{desired}) + \rho \cdot (\beta - \beta_{des}) \quad \text{Eq.3}$$

(3) Lyapunov Function

$$V = \frac{1}{2} s^2 \quad \text{Eq.4}$$

(4) Differentiating the Lyapunov function,

$$\dot{V} = s \cdot \dot{s} < 0 \quad \text{Eq.5}$$

(5) For stable system, the derivative of the Lyapunov function should be negative.

$$\dot{V} = s \cdot \dot{s} = -K \cdot |s| < 0 \quad \text{Eq.6}$$

8.5.2 Direct Yaw Moment Control ($\beta - \dot{\beta}$ Phase Plane Analysis)

3) Yaw Moment Input based on Sliding Control Method

(6) Substituting Eqns (2) and (3) into Eq.(5)

$$\dot{V} = s \cdot \dot{s} = -K \cdot |s| \quad \Rightarrow \quad \dot{s} = -K \cdot \mathrm{sgn}(s)$$

$$\dot{s} = (\dot{\gamma} - \dot{\gamma}_{desired}) + \rho \cdot (\dot{\beta} - \dot{\beta}_{des})$$

Assuming that: $\dot{\gamma}_{desired}$ and $\dot{\beta}_{des} \simeq 0$

$$= \left(a_{21} \cdot \beta + a_{22} \cdot \dot{\psi} + \frac{2 \cdot l_f \cdot C_f}{I_z} \cdot \delta_f + \frac{M_{z\_des}}{I_z} \right) + \rho \cdot \left(a_{11} \cdot \beta + a_{12} \cdot \dot{\psi} + \frac{2 \cdot C_f}{m v_x} \cdot \delta_f \right) \quad \text{Eq.7}$$

$$= -K \cdot \mathrm{sgn}(s)$$

3) Yaw Moment Input based on Sliding Control Method

(7) Rewritten the above equation

$$M_{z\_des} = -I_z \cdot \rho \cdot \left(a_{11} \cdot \beta + a_{12} \cdot \dot{\psi} + \frac{2 \cdot C_f}{m v_x} \cdot \delta_f \right) - I_z \cdot \left(a_{21} \cdot \beta + a_{22} \cdot \dot{\psi} + \frac{2 \cdot l_f \cdot C_f}{I_z} \cdot \delta_f \right) - I_z \cdot K \cdot \mathrm{sgn}(s)$$

$$= -I_z \cdot \left(\rho \cdot a_{11} + a_{21} \right) \cdot \beta - I_z \cdot \left(\rho \cdot a_{12} + a_{22} \right) \cdot \dot{\psi} - I_z \cdot \left(\rho \cdot \frac{2 \cdot C_f}{m v_x} + \frac{2 \cdot l_f \cdot C_f}{I_z} \right) \cdot \delta_f - I_z \cdot K \cdot \mathrm{sgn}(s)$$

Where,

$$A = \begin{bmatrix} -\dfrac{2(C_f + C_r)}{m \cdot v_x} & -1 - \dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{m \cdot {v_x}^2} \\ -\dfrac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} & -\dfrac{2({l_f}^2 \cdot C_f + {l_r}^2 \cdot C_r)}{I_z \cdot v_x} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

8.5.2 Direct Yaw Moment Control ($\beta - \dot{\beta}$ Phase Plane Analysis)

3) Yaw Moment Input based on Sliding Control Method

(8) Finally, the saturation function with Φ is used to cope with the chattering phenomenon as follows:

$$M_{z\_des} = M_{z\_eq} - I_z \cdot K \cdot sat\left\{ \frac{(\gamma - \gamma_{desired}) + \rho \cdot (\beta - \beta_{des})}{\Phi} \right\}$$

Where,

$$M_{z\_eq} = -I_z \cdot (\rho \cdot a_{11} + a_{21}) \cdot \beta - I_z \cdot (\rho \cdot a_{12} + a_{22}) \cdot \dot{\psi} - I_z \cdot \left(\rho \cdot \frac{2 \cdot C_f}{m v_x} + \frac{2 \cdot l_f \cdot C_f}{I_z} \right) \cdot \delta_f$$

8.5 Vehicle Stability Control (VSC) Algorithm

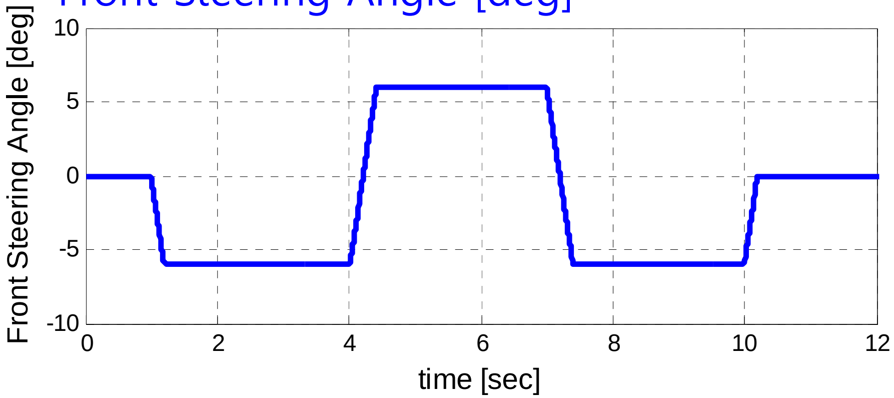
8.5.1 Direct Yaw Moment Control (Desired Yaw Rate Tracking)

8.5.2 Direct Yaw Moment Control ($\beta - \dot{\beta}$ Phase Plane Analysis)

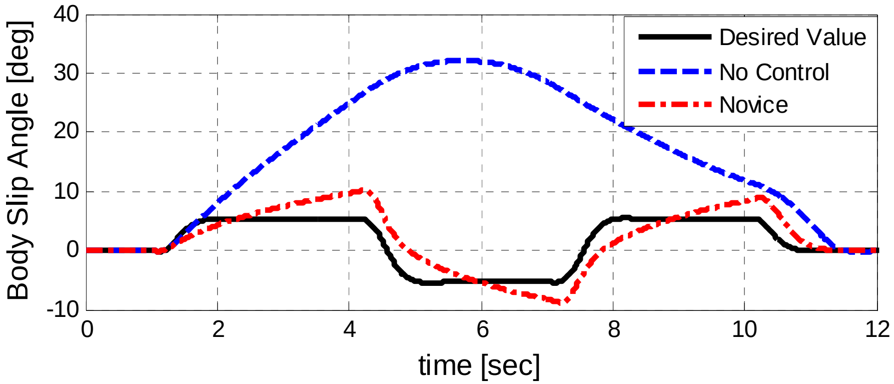
8.5.3 Simulation Results

8.5.3 Simulation Results: Vehicle Speed =100 kph (Novice Driver)

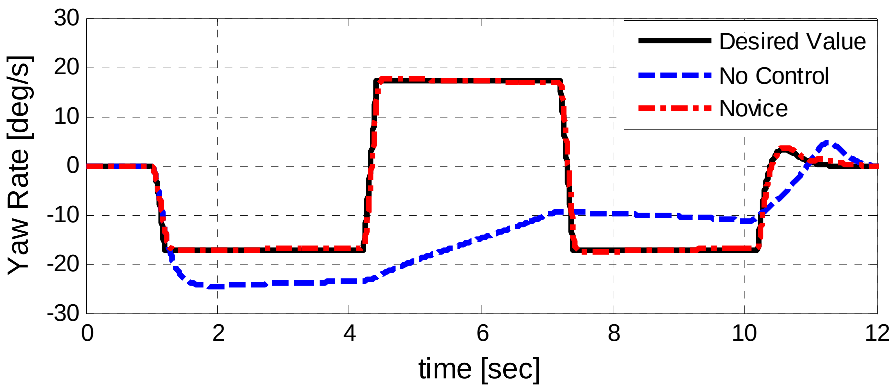
- Front Steering Angle [deg]



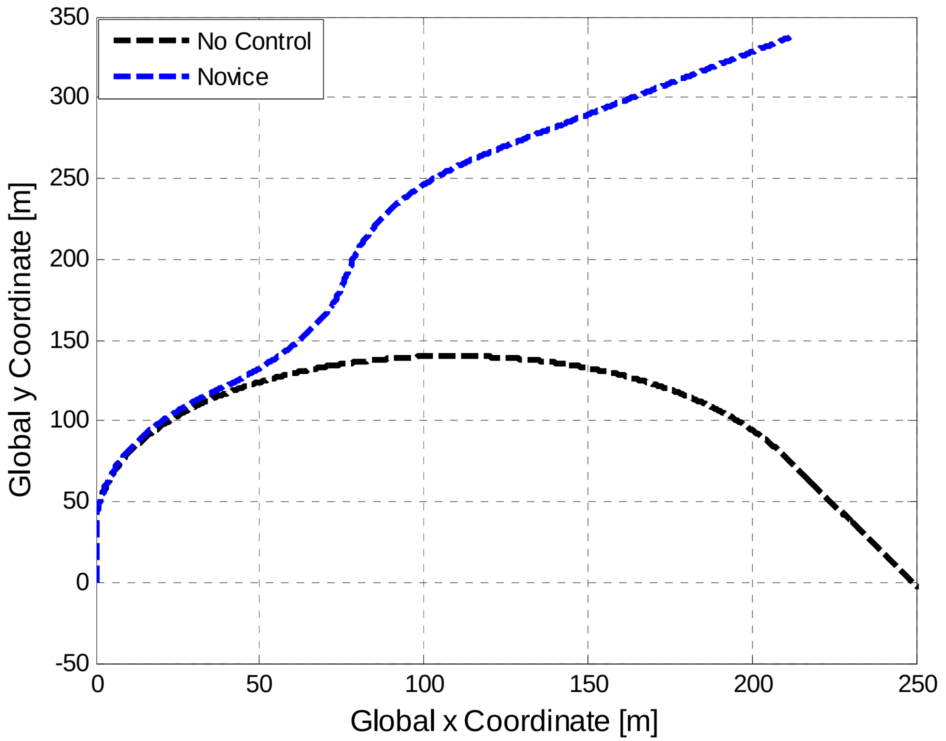
- Body Slip Angle [deg]



- Yaw Rate [deg/s]

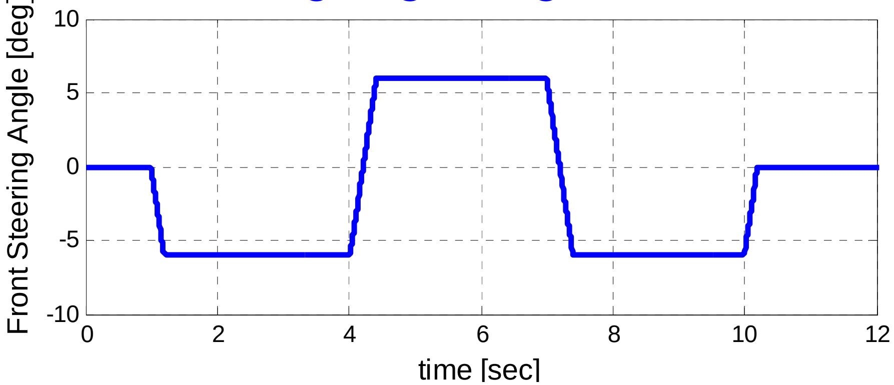


- Vehicle Trajectory

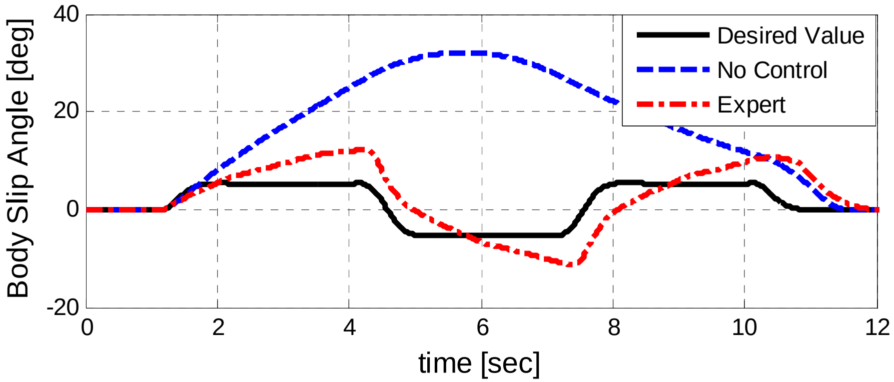


8.5.3 Simulation Results: Vehicle Speed =100 kph (Expert Driver)

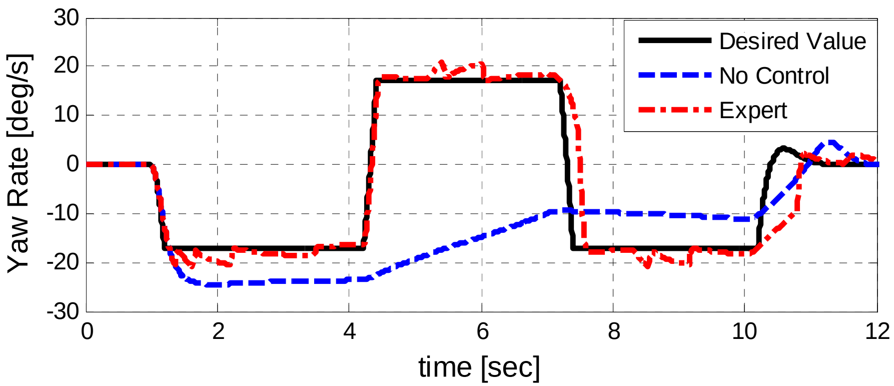
- Front Steering Angle [deg]



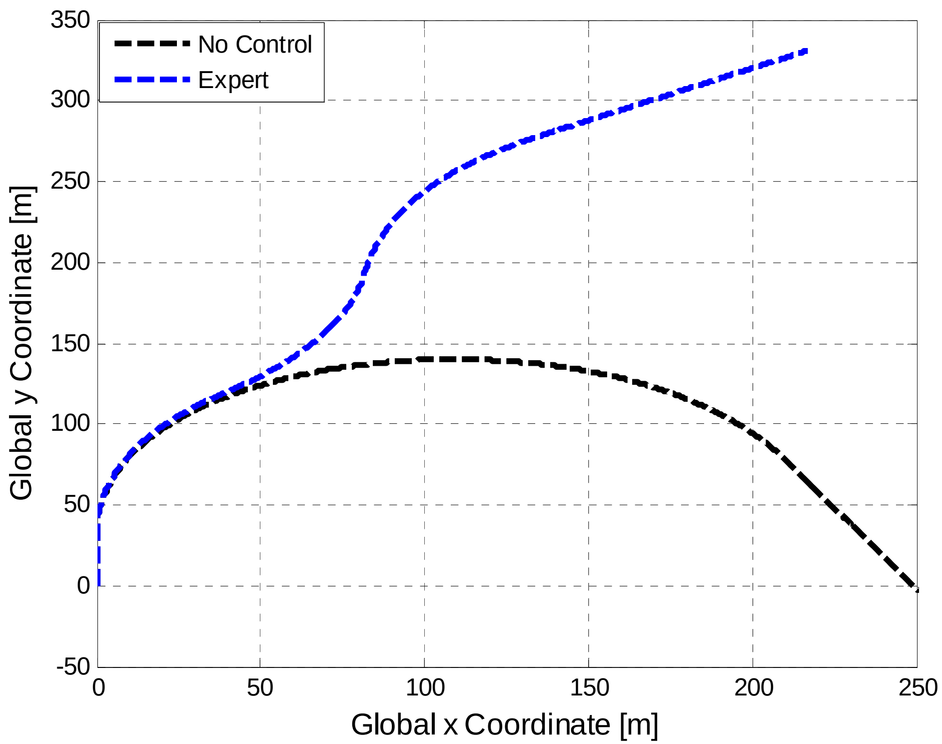
- Body Slip Angle [deg]



- Yaw Rate [deg/s]

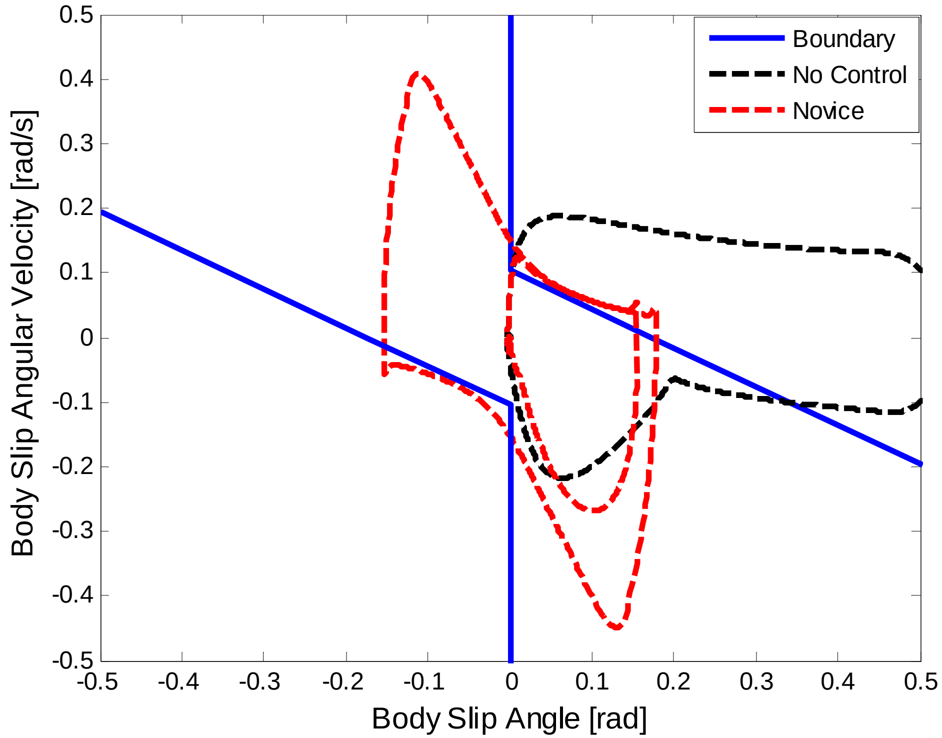


- Vehicle Trajectory

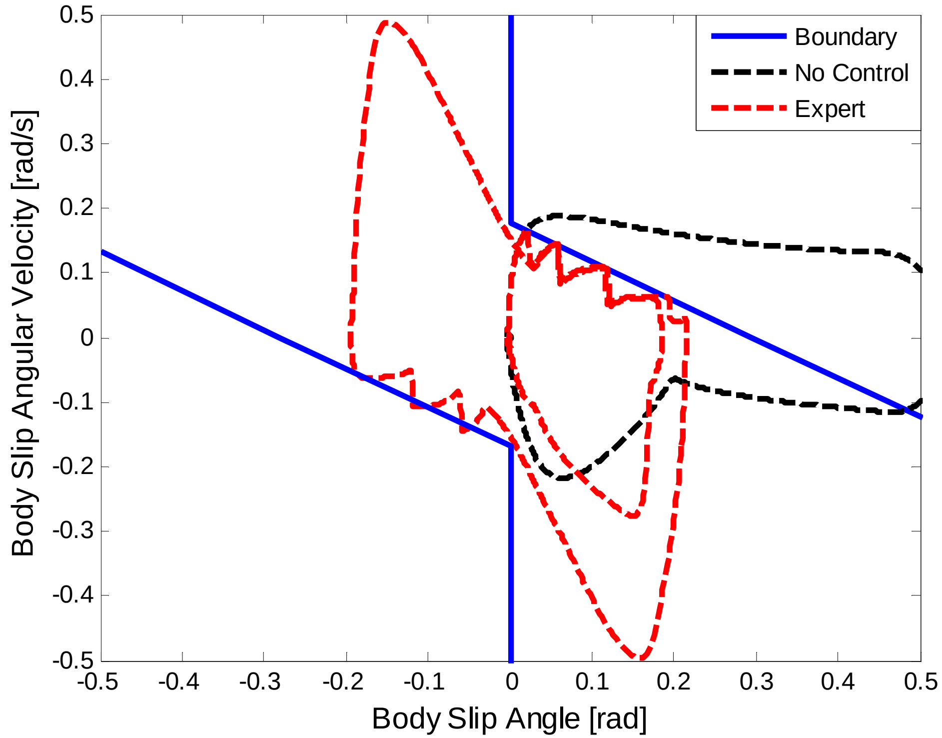


8.5.3 Simulation Results: Phase Plane

- Novice Driver

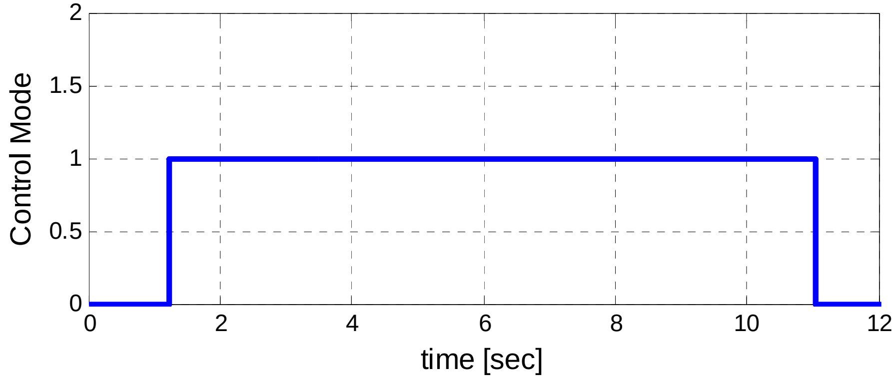


- Expert Driver

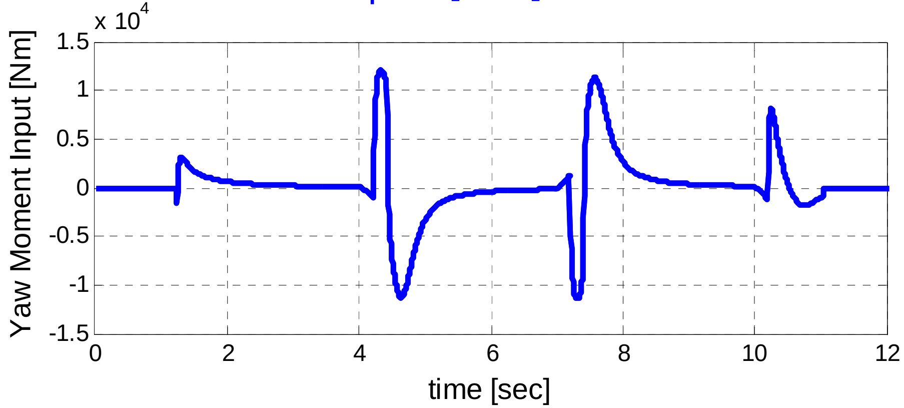


8.5.3 Simulation Results: Control Input

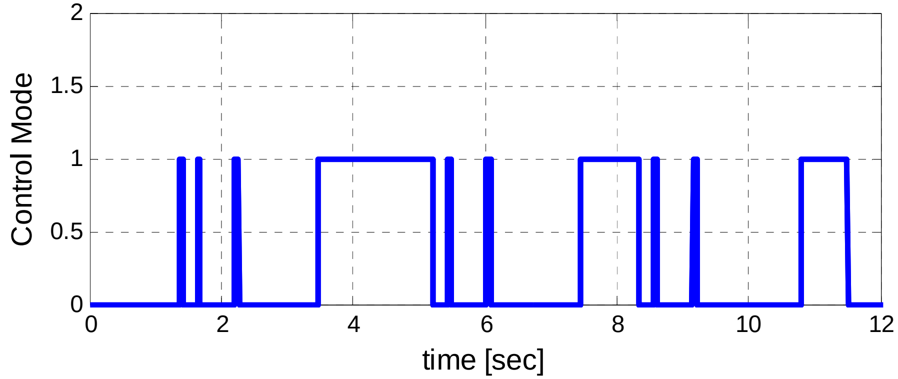
- Novice Driver
- Control Mode



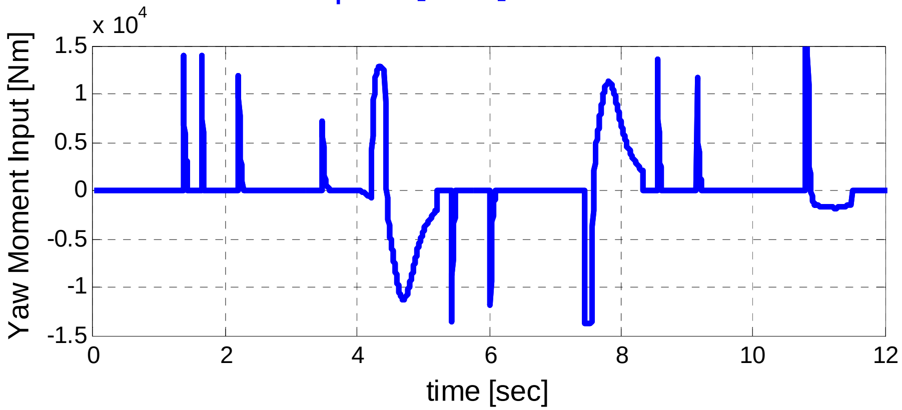
- Yaw Moment Input [Nm]



- Expert Driver
- Control Mode



- Yaw Moment Input [Nm]



8.6 Comparison of Control Performance according to Sliding Surface

8.6.1 Sliding Surface 1

8.6.2 Sliding Surface 2

8.6.3 Comparison of Simulation Results

8.6.1 Sliding Surface 1

(1) Sliding Surface using Yawrate Error

$$s_1 = \gamma - \gamma_{desired}$$

(2) Control Input should satisfy the following sliding conditions:

$$\dot{V} = s_1 \cdot \dot{s}_1 = -K \cdot |s_1| < 0$$

(3) Yaw Moment Input

$$M_{z\_des} = M_{z\_eq} - I_z \cdot K \cdot sat(\frac{\gamma - \gamma_{desired}}{\Phi})$$

Where, $M_{z\_eq} = I_z \cdot \left(\frac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} \cdot \beta + \frac{2(l_f{}^2 \cdot C_f + l_r{}^2 \cdot C_r)}{I_z \cdot v_x} \cdot \dot{\psi} - \frac{2 \cdot l_f \cdot C_f}{I_z} \cdot \delta_f \right)$

$$K = \text{sliding control gain}$$

8.6.1 Sliding Surface 1

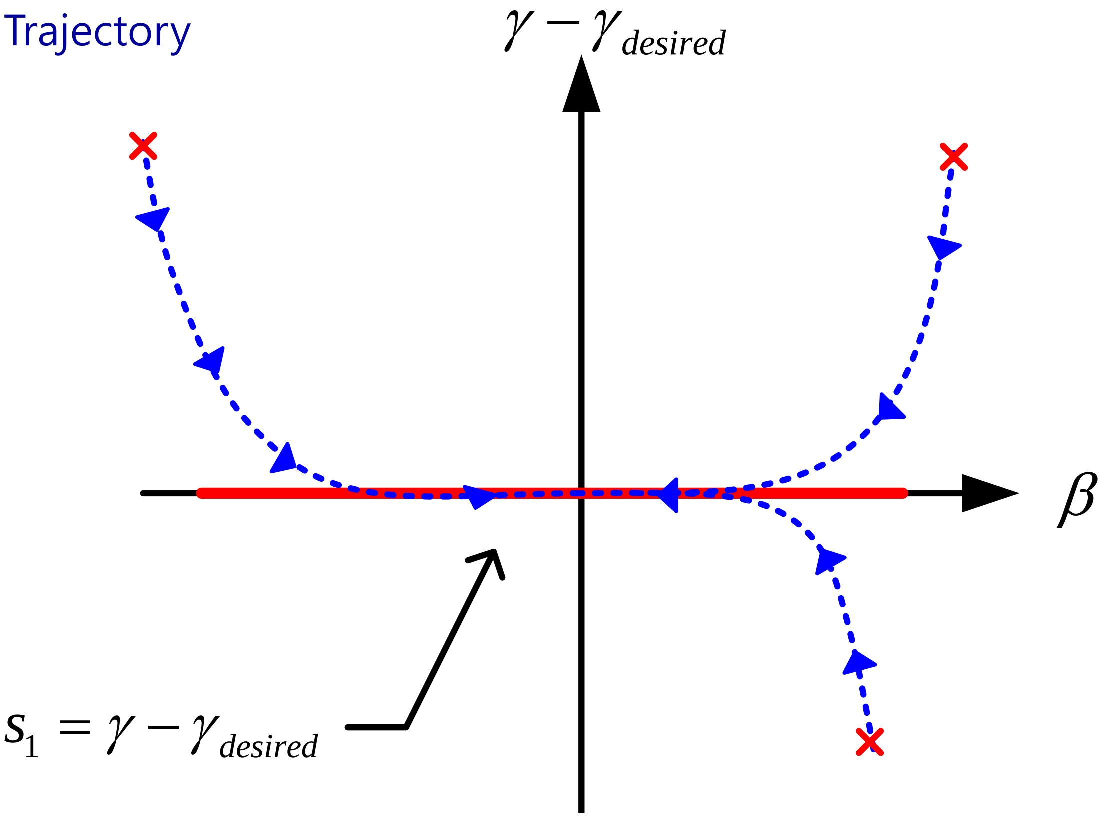
(1) Sliding Surface using Yawrate Error

$$s_1 = \gamma - \gamma_{desired}$$

(2) If sliding surface is zero,

$$s_1 = 0 \quad \Rightarrow \quad \gamma \to \gamma_{desired}$$

(3) State Trajectory



8.6.2 Sliding Surface 2

(1) Sliding Surface using Yawrate Error and Body Slip Angle

$$s_2 = (\gamma - \gamma_{desired}) + \rho \cdot \beta$$

(2) Control Input should satisfy the following sliding conditions:

$$\dot{V} = s_2 \cdot \dot{s}_2 = -K \cdot |s_2| < 0$$

(3) Yaw Moment Input

$$M_{z\_des} = M_{z\_eq} - I_z \cdot K \cdot sat\left\{ \frac{(\gamma - \gamma_{desired}) + \rho \cdot \beta}{\Phi} \right\}$$

Where, $M_{z\_eq} = -I_z \cdot (\rho \cdot a_{11} + a_{21}) \cdot \beta - I_z \cdot (\rho \cdot a_{12} + a_{22}) \cdot \dot{\psi} - I_z \cdot \left(\rho \cdot \frac{2 \cdot C_f}{mv_x} + \frac{2 \cdot l_f \cdot C_f}{I_z} \right) \cdot \delta_f$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} -\frac{2(C_f + C_r)}{m \cdot v_x} & -1 - \frac{2(l_f \cdot C_f - l_r \cdot C_r)}{m \cdot v_x^2} \\ -\frac{2(l_f \cdot C_f - l_r \cdot C_r)}{I_z} & -\frac{2(l_f^2 \cdot C_f + l_r^2 \cdot C_r)}{I_z \cdot v_x} \end{bmatrix}$$

8.6.2 Sliding Surface 2

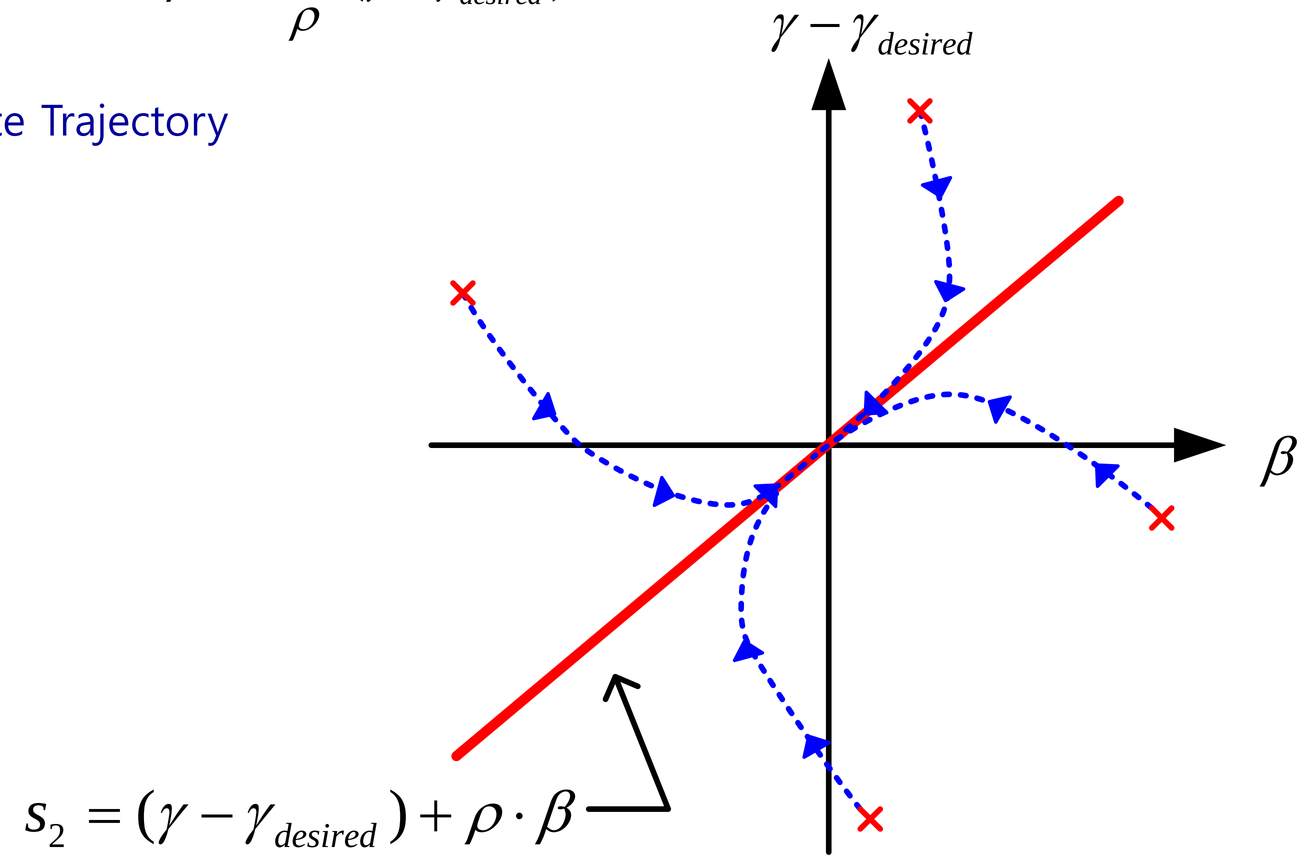
(1) Sliding Surface using Yawrate Error and Body Slip Angle

$$s_2 = (\gamma - \gamma_{desired}) + \rho \cdot \beta$$

(2) If sliding surface is zero,

$$s_2 = 0 \quad \Rightarrow \quad \beta = -\frac{1}{\rho} \cdot (\gamma - \gamma_{desired})$$

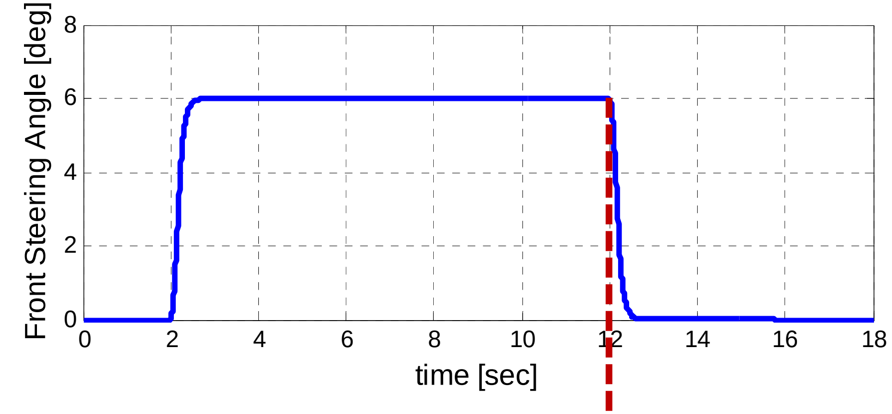
(3) State Trajectory



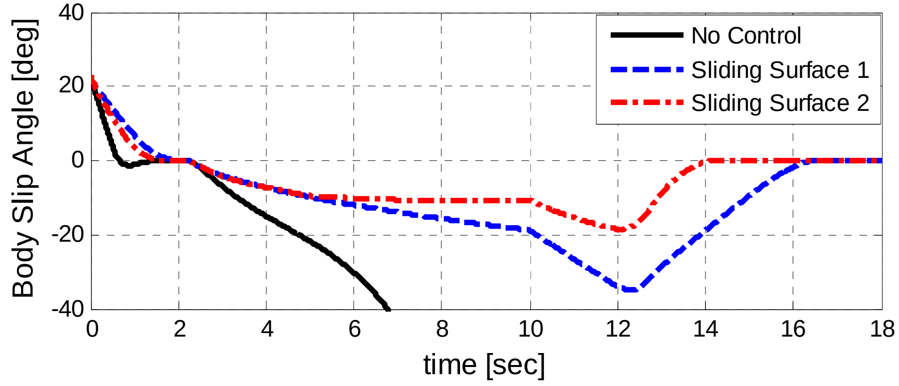
8.6.3 Comparison of Simulation Results

- Front Steering Angle = 6 deg (Unstable)
- Vehicle Speed : 100 km/h
- Initial Condition = $[\beta_{ini} \quad \gamma_{ini}]^T = [0.3\ rad \quad 0.3\ rad/s]^T$

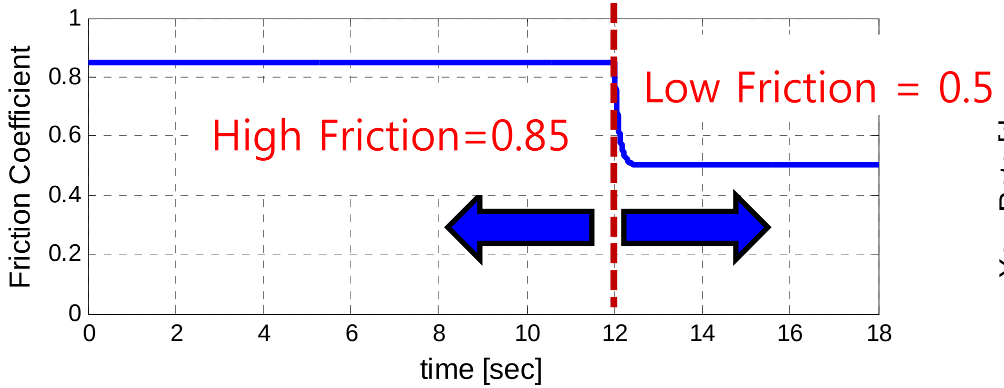
- Front Steering Angle [deg]



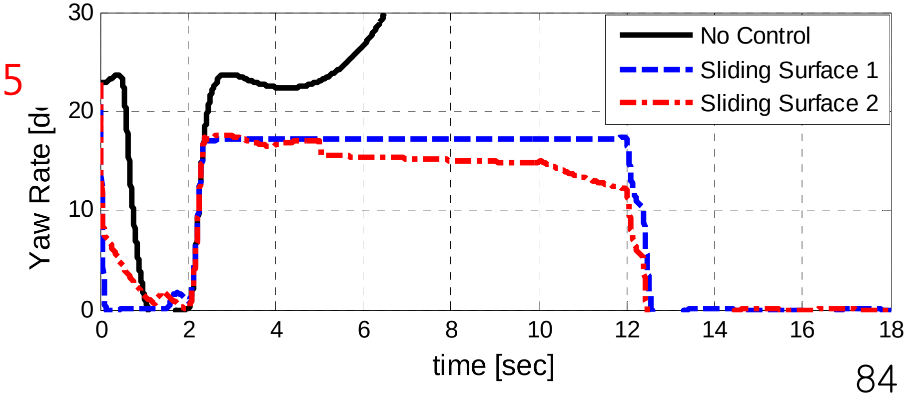
- Body Slip Angle [deg]



- Friction Coefficient



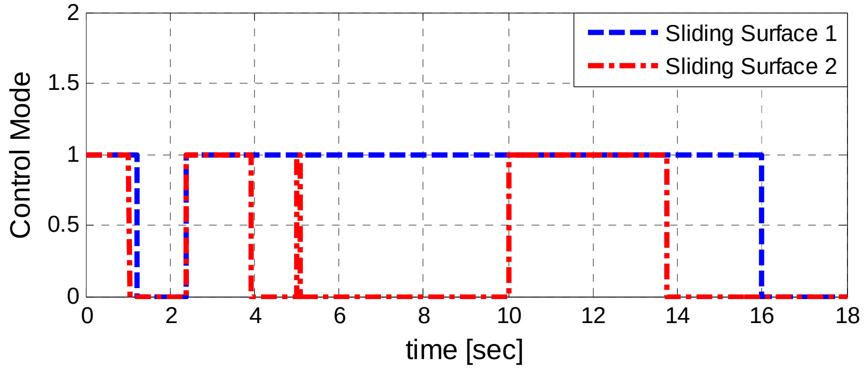
- Vehicle Yaw Rate [deg]



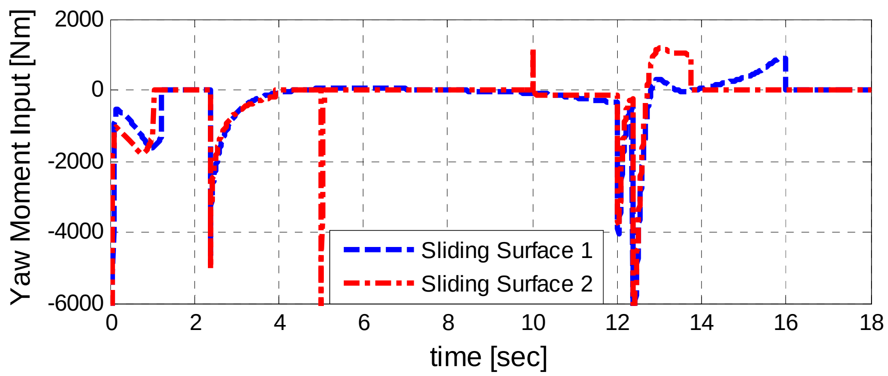
8.6.3 Comparison of Simulation Results

- Front Steering Angle = 6 deg (Unstable)
- Vehicle Speed : 100 km/h
- Initial Condition = $[\beta_{ini} \quad \gamma_{ini}]^T = [0.3\ rad \quad 0.3\ rad/s]^T$

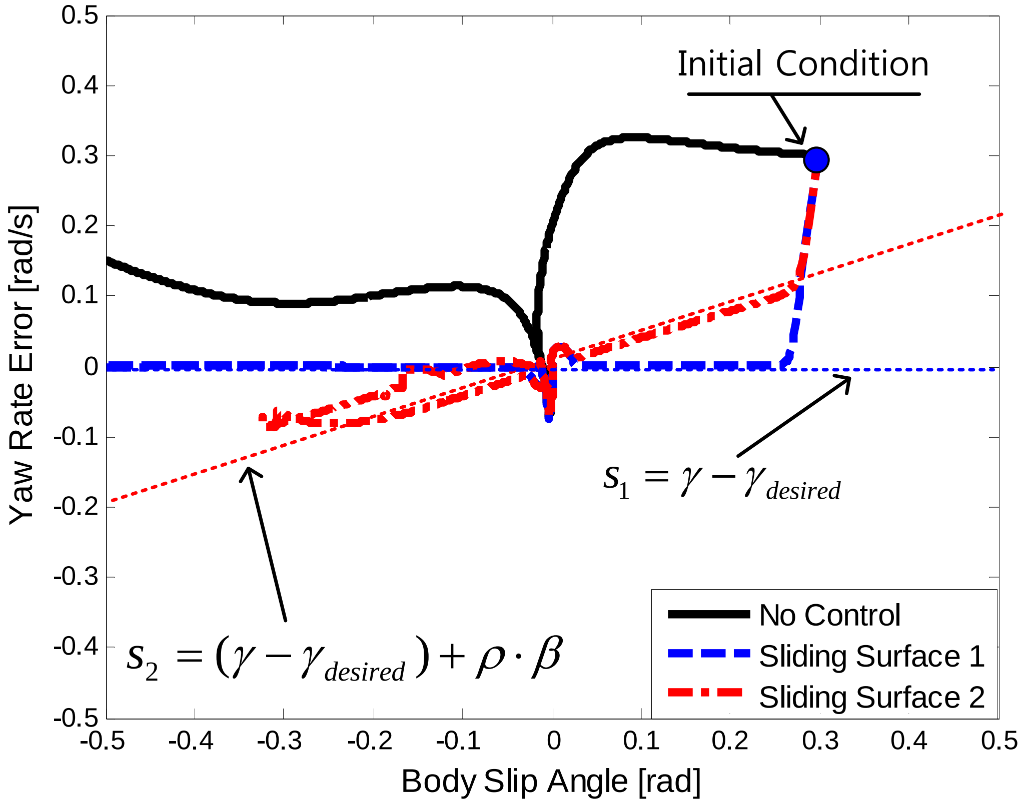
- Control Mode



- Yaw Moment Input [Nm]



- State Trajectory



8.6.3 Comparison of Phase Plane

- Front Steering Angle = 6 deg (Unstable)
- Vehicle Speed : 100 km/h

- $s_1 = \gamma - \gamma_{desired}$
- $s_2 = (\gamma - \gamma_{desired}) + \rho \cdot \beta$