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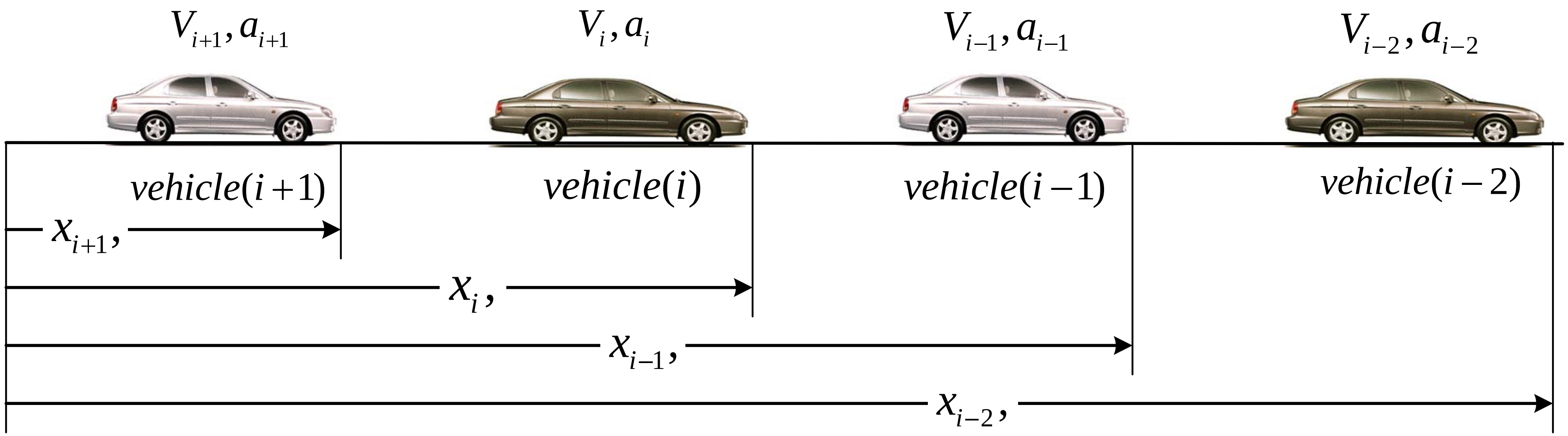
7.1 Motivation of String Stability for ACC Vehicles

• If the vehicle following control law ensures individual vehicle stability, the spacing error should converge to zero when the preceding vehicle moves at constant speed.

• However, the spacing error is expected to be non-zero during acceleration or deceleration of the preceding vehicle.



(1) Define the measured inter-vehicle spacing as

$$\varepsilon_i = x_i - \left(x_{i-1} - l_{i-1}\right) \qquad \text{Eq.1}$$

where, l_{i-1} is the length of the preceding vehicle.

7.1 Motivation of String Stability for ACC Vehicles

(2) Under the constant spacing policy, the spacing error of the i-*th* vehicle is then defined as

$$\delta_i = L_{des} + x_i - x_{i-1} \qquad \text{Eq.2}$$

where, L_{des} is the desired constant value of inter-vehicle spacing and includes the preceding vehicle length.

(3) Assuming that the acceleration of vehicle can be expressed as follows: the acceleration of vehicle can be instantaneously controlled, the ACC system can be expressed as follows:

$$a_{i\_des} = a_i = \ddot{x}_i - \underbrace{\ddot{x}_{i-1}}_{=0} = -k_p \cdot \delta_i - k_v \cdot \dot{\delta}_i \qquad \text{Assuming that: } a_p = \ddot{x}_{i-1} = 0$$

$$\ddot{x}_i = -k_p \cdot \delta_i - k_v \cdot \dot{\delta}_i \qquad \text{Eq.3}$$

where, k_p and k_v are positive.

7.1 Motivation of String Stability for ACC Vehicles

(4) Acceleration of the spacing error of the i-*th* vehicle

$$\ddot{\delta}_i = \frac{d\left(L_{des} + x_i - x_{i-1}\right)}{dt} = \ddot{x}_i - \ddot{x}_{i-1} \qquad \text{Eq.4}$$

(5) Substituting Eq.3 into Eq.4,

$$\begin{aligned} \ddot{\delta}_i &= \ddot{x}_i - \ddot{x}_{i-1} = \left(-k_p \cdot \delta_i - k_v \cdot \dot{\delta}_i\right) - \left(-k_p \cdot \delta_{i-1} - k_v \cdot \dot{\delta}_{i-1}\right) \\ &= -k_p \cdot \delta_i - k_v \cdot \dot{\delta}_i + k_p \cdot \delta_{i-1} + k_v \cdot \dot{\delta}_{i-1} \end{aligned} \qquad \text{Eq.5}$$

(6) Laplace Transform

$$s^2 \cdot \delta_i(s) = -k_p \cdot \delta_i(s) - k_v \cdot s \cdot \delta_i(s) + k_p \cdot \delta_{i-1}(s) + k_v \cdot s \cdot \delta_{i-1}(s)$$

$$\Rightarrow$$

$$\left(s^2 + k_v \cdot s + k_p\right) \cdot \delta_i(s) = \left(k_v \cdot s + k_p\right) \cdot \delta_{i-1}(s)$$

$$\Downarrow$$

$$G_i(s) = \frac{\delta_i}{\delta_{i-1}}(s) = \frac{k_v \cdot s + k_p}{s^2 + k_v \cdot s + k_p} \qquad \text{Eq.6}$$

7.1 Motivation of String Stability for ACC Vehicles

(7) Eq. 6 shows that the spacing error of i-th vehicle (δ_i) converges to zero when δ_{i-1} is zero. Thus individual vehicle stability is ensured.

$$\lim_{t\to\infty}\delta_i(t) = \lim_{s\to 0} s\cdot\left\{\frac{k_v\cdot s + k_p}{s^2 + k_v\cdot s + k_p}\cdot\delta_{i-1}(s)\right\} = \lim_{s\to 0}\frac{k_v\cdot s^2 + k_p\cdot s}{s^2 + k_v\cdot s + k_p}\cdot\delta_{i-1}(s)$$

$$= \lim_{s\to 0}\frac{k_v\cdot s^2 + k_p\cdot s}{s^2 + k_v\cdot s + k_p}\cdot\delta_{i-1}(s)$$

$$= 0$$

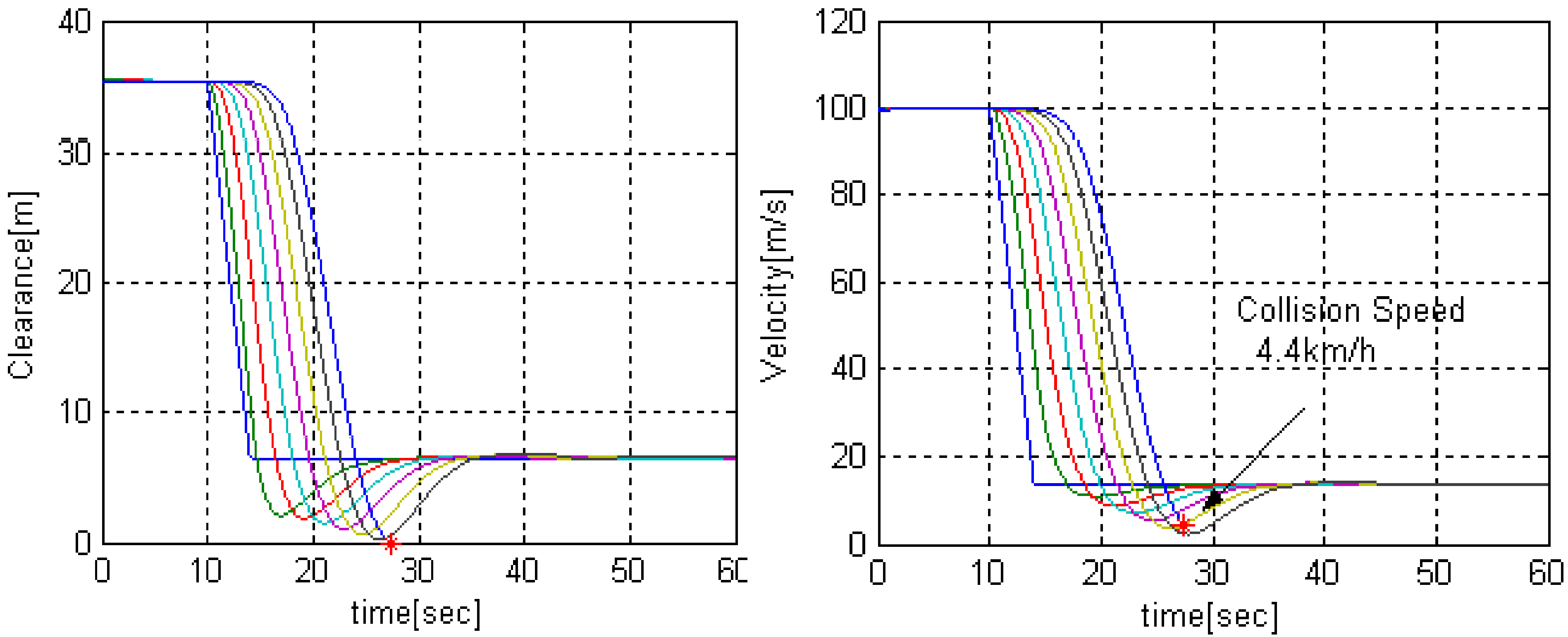
(8) However, the collision at i+N *th* vehicles can be occued as follows:

$$\frac{\delta_{i+N}}{\delta_{i-1}}(s) = \underbrace{G_i(s)\cdot G_i(s)\cdots G_i(s)}_{N} = \frac{\left(k_v\cdot s + k_p\right)^N}{\left(s^2 + k_v\cdot s + k_p\right)^N}$$

7.1 Motivation of String Stability for ACC Vehicles

(8) Step Response

$$\frac{\delta_{i+N}}{\delta_{i-1}}(s) = \underbrace{G_i(s)\cdot G_i(s)\cdots G_i(s)}_{N} = \frac{\left(k_v\cdot s + k_p\right)^N}{\left(s^2 + k_v\cdot s + k_p\right)^N}$$



7.2 Condition for string stability

Condition.1: The transfer function of the spacing error at i-*th* vehicle should satisfy

$$\left\| G_i(s) \right\|_\infty = \left\| \frac{\delta_i}{\delta_{i-1}}(s) \right\|_\infty \le 1$$

➔ The energy in the spacing error decreases as the spacing error propagates toward the tail of the string.

Condition.2: The impulse response function $g(t)$ corresponding to $G_i(s)$ should not change sign.

$$g(t) > 0 \qquad \forall t \ge 0$$

➔ The steady state spacing errors of the vehicles in the string have same sign.

Ref: Swaroop, D., 1995, "String Stability of Interconnected Systems: An Application to Platooning in Automated Highway Systems", Ph. D Dissertation, University of California, Berkely, 1995

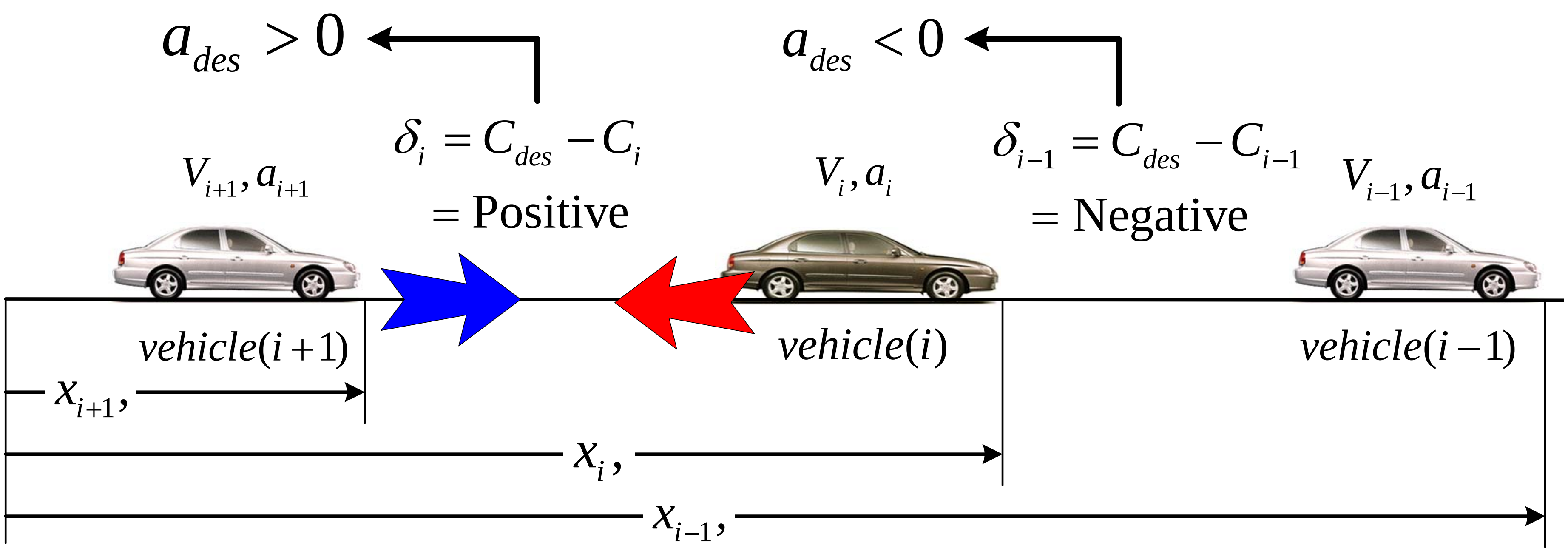
7.2 Condition for string stability

Condition.2: The impulse response function $g(t)$ corresponding to $G_i(s)$ should not change sign.

$$g(t) > 0 \qquad \forall t \geq 0$$

➔ The steady state spacing errors of the vehicles in the string have same sign.

Example:



7.2 Condition for string stability

Condition.1: The transfer function of the spacing error at i-*th* vehicle should satisfy

$$\left\|G_i(s)\right\|_\infty = \left\|\frac{\delta_i}{\delta_{i-1}}(s)\right\|_\infty \le 1$$

Example: $G(s) = \dfrac{\delta_i}{\delta_{i-1}}(s) = \dfrac{k_v \cdot s + k_p}{s^2 + k_v \cdot s + k_p} = \dfrac{k_p}{s^2 + k_v \cdot s + k_p} \cdot \left(\dfrac{k_v}{k_p} \cdot s + 1\right) = G_1(s) \cdot G_2(s)$

(1) For the magnitude of $G_1(s)$ to be less than 1

$$G_1(s) = \frac{k_p}{s^2 + k_v \cdot s + k_p} = \frac{1}{s^2 + 2 \cdot \varsigma \cdot \omega_n \cdot s + \omega_n^2}$$

$$\omega_n = \sqrt{k_p}$$

$$\varsigma = \frac{k_v}{2 \cdot \sqrt{k_p}}$$

$$\varsigma \ge \frac{1}{\sqrt{2}}$$

$$\frac{k_v}{2 \cdot \sqrt{k_p}} \ge \frac{1}{\sqrt{2}}$$

$$\Rightarrow$$

$$(i)\ k_v \ge \sqrt{2 \cdot k_p}$$

7.2 Condition for string stability

Condition.1: The transfer function of the spacing error at i-*th* vehicle should satisfy

$$\left\| G_i(s) \right\|_\infty = \left\| \frac{\delta_i}{\delta_{i-1}}(s) \right\|_\infty \leq 1$$

Example: $G(s) = \dfrac{\delta_i}{\delta_{i-1}}(s) = \dfrac{k_v \cdot s + k_p}{s^2 + k_v \cdot s + k_p} = \dfrac{k_p}{s^2 + k_v \cdot s + k_p} \cdot \left(\dfrac{k_v}{k_p} \cdot s + 1 \right) = G_1(s) \cdot G_2(s)$

(2) For the magnitude of $G_2(s)$ to not exceed 1 at frequencies up to the resonant frequency ,$\sqrt{k_p}$,

$$\frac{k_p}{k_v} > \sqrt{k_p} \quad \Rightarrow \quad (ii)\ \sqrt{k_p} > k_v$$

(3) Summary

$(i)\ k_v \geq \sqrt{2 \cdot k_p}$ It is not possible to find gains that satisfy $\left\| G_i(s) \right\|_\infty \leq 1$

$(ii)\ \sqrt{k_p} > k_v$

7.3 Controller Design for string stability

Controller is designed based on sliding control method in order to ensure string stability

(1) Define sliding surface

$$s_i = \dot{\varepsilon}_i + \frac{1}{1-\rho} \cdot \frac{\omega_n}{\varsigma + \sqrt{\varsigma^2 - 1}} \cdot \varepsilon_i + \frac{\rho}{1-\rho} \cdot \left(V_i - V_0\right) \qquad \text{Eq.1}$$

Where, V_i and V_0 denote the longitudinal velocity of the i-th vehicle and lead vehicle.

$$\varepsilon_i = x_i - x_{i-1} + l_{i-1}$$

(2) Tuning Parameters

ρ : Weighting of the lead vehicle's speed and acceleration. $0 < \rho < 1$

ς : Damping ratio $\varsigma \geq 1$

ω_n : Bandwidth of the controller

7.3 Controller Design for string stability

(3) Lyapunov Function

$$V = \frac{1}{2} s_i^2 \qquad \text{Eq.2}$$

(4) Differentiating the Lyapunov function,

$$\dot{V} = s_i \cdot \dot{s}_i < 0 \qquad \text{Eq.3}$$

(5) For stable system, the derivative of the Lyapunov function should be negative.

$$\dot{V} = s_i \cdot \dot{s}_i = -K \cdot s^2{}_i < 0$$

Eq.4

$$\dot{s}_i = -K \cdot s_i$$

Where, $K = \omega_n \cdot \left(\varsigma + \sqrt{\varsigma^2 - 1} \right)$

7.3 Controller Design for string stability

(6) Sliding Mode Control Method

$$\dot{s}_i = \ddot{\varepsilon}_i + \frac{1}{1-\rho} \cdot \frac{\omega_n}{\varsigma + \sqrt{\varsigma^2 - 1}} \cdot \dot{\varepsilon}_i + \frac{\rho}{1-\rho} \cdot \left(\dot{V}_i - \dot{V}_0 \right)$$

$$= \underbrace{\ddot{x}_i}_{=\ddot{x}_{i\_des}} - \ddot{x}_{i-1} + \frac{1}{1-\rho} \cdot \frac{\omega_n}{\varsigma + \sqrt{\varsigma^2 - 1}} \cdot \dot{\varepsilon}_i + \frac{\rho}{1-\rho} \cdot \left(\underbrace{\ddot{x}_i}_{=\ddot{x}_{i\_des}} - \ddot{x}_0 \right) \qquad \text{Eq.5}$$

$$= -\underbrace{\omega_n \cdot \left(\varsigma + \sqrt{\varsigma^2 - 1} \right)}_{K} \cdot \underbrace{\left\{ \dot{\varepsilon}_i + \frac{1}{1-\rho} \cdot \frac{\omega_n}{\varsigma + \sqrt{\varsigma^2 - 1}} \cdot \varepsilon_i + \frac{\rho}{1-\rho} \cdot \left(V_i - V_0 \right) \right\}}_{s_i}$$

7.3 Controller Design for string stability

(7) Arrange Eq.5

$$\left(1+\frac{\rho}{1-\rho}\right)\ddot{x}_{i\_des}=$$

$$\ddot{x}_{i-1}-\frac{1}{1-\rho}\cdot\frac{\omega_n}{\varsigma+\sqrt{\varsigma^2-1}}\cdot\dot{\varepsilon}_i+\frac{\rho}{1-\rho}\cdot\ddot{x}_0-\omega_n\cdot\left(\varsigma+\sqrt{\varsigma^2-1}\right)\cdot\dot{\varepsilon}_i$$

$$-\frac{\omega_n^2}{1-\rho}\cdot\varepsilon_i-\omega_n\cdot\left(\varsigma+\sqrt{\varsigma^2-1}\right)\cdot\frac{\rho}{1-\rho}\cdot\left(V_i-V_0\right)$$

$$\Rightarrow$$

$$\ddot{x}_{i\_des}=\left(1-\rho\right)\cdot\ddot{x}_{i-1}+\rho\cdot\ddot{x}_0-\left\{\frac{1}{\varsigma+\sqrt{\varsigma^2-1}}+\left(1-\rho\right)\cdot\left(\varsigma+\sqrt{\varsigma^2-1}\right)\right\}\cdot\omega_n\cdot\dot{\varepsilon}_i$$

$$-\omega_n^2\cdot\varepsilon_i-\omega_n\cdot\left(\varsigma+\sqrt{\varsigma^2-1}\right)\cdot\rho\cdot\left(V_i-V_0\right)$$

Eq.6

7.3 Controller Design for string stability

(8) Desired Acceleration

$$\ddot{x}_{i\_des} = (1-\rho)\cdot\ddot{x}_{i-1} + \rho\cdot\ddot{x}_0 - \left\{2\varsigma - \rho\cdot\left(\varsigma + \sqrt{\varsigma^2 - 1}\right)\right\}\cdot\omega_n\cdot\left(\underbrace{V_i - V_{i-1}}_{\dot{\varepsilon}_i}\right) - \omega_n^2\cdot\varepsilon_i$$

$$-\omega_n\cdot\left(\varsigma + \sqrt{\varsigma^2 - 1}\right)\cdot\rho\cdot(V_i - V_0)$$

Eq.7

$$= (1-\rho)\cdot\ddot{x}_{i-1} + \rho\cdot\ddot{x}_0 - k_v\cdot(V_i - V_{i-1}) - k_p\cdot(x_i - x_{i-1} + l_{i-1}) - k_0\cdot(V_i - V_0)$$

The longitudinal acceleration of the preceding vehicle and the lead vehicle is required in order to obtained the desired acceleration for string stability.

(9) Setting $\rho = 0$ for a two car platoon,

$$\ddot{x}_{i\_des} = \ddot{x}_{i-1} - 2\varsigma\cdot\omega_n\cdot\dot{\varepsilon}_i - \omega_n^2\cdot\varepsilon_i$$

We obtain the classical second-order system.

7.3 Controller Design for string stability

(10) Desired Acceleration in Eq.7 can ensure that the sliding surface converges to zero.

$$V = \frac{1}{2} s_i^2 > 0, \qquad \dot{V} = s_i \cdot \dot{s}_i = -K \cdot s^2{}_i < 0$$

(11) Verify the string stability

$$s_i - s_{i-1} = \frac{1}{1-\rho} \cdot \dot{\varepsilon}_i - \dot{\varepsilon}_{i-1} + \frac{1}{1-\rho} \cdot \frac{\omega_n}{\varsigma + \sqrt{\varsigma^2 - 1}} \cdot \left(\varepsilon_i - \varepsilon_{i-1} \right)$$

The sliding controller ensures that the left side of the above equation is zero.

If $i = 1$,

$$0 = \frac{1}{1-\rho} \cdot \dot{\varepsilon}_1 - \dot{\varepsilon}_0 + \frac{1}{1-\rho} \cdot \frac{\omega_n}{\varsigma + \sqrt{\varsigma^2 - 1}} \cdot \left(\varepsilon_1 - \varepsilon_0 \right)$$

Where, $\dot{\varepsilon}_0 = \varepsilon_0 = 0$ ◀ Clearance of the Lead vehicle in the platoon.

$$\dot{\varepsilon}_1 = -\frac{\omega_n}{\varsigma + \sqrt{\varsigma^2 - 1}} \varepsilon_1 \quad \Rightarrow \quad \varepsilon_1 \to 0$$

7.3 Controller Design for string stability

(11) Verify the string stability

If $i = 2$,
$$0 = \frac{1}{1-\rho}\cdot\dot{\varepsilon}_2 - \dot{\varepsilon}_1 + \frac{1}{1-\rho}\cdot\frac{\omega_n}{\varsigma + \sqrt{\varsigma^2 - 1}}\cdot\left(\varepsilon_2 - \varepsilon_1\right)$$

Where, $\dot{\varepsilon}_1 = -\dfrac{\omega_n}{\varsigma + \sqrt{\varsigma^2 - 1}}\varepsilon_1$

$$0 = \frac{1}{1-\rho}\cdot\dot{\varepsilon}_2 + \frac{1}{1-\rho}\cdot\frac{\omega_n}{\varsigma + \sqrt{\varsigma^2 - 1}}\cdot\left(\varepsilon_2 - \rho\cdot\varepsilon_1\right)$$

==>

$$\dot{\varepsilon}_2 = -\frac{\omega_n}{\varsigma + \sqrt{\varsigma^2 - 1}}\cdot\left(\varepsilon_2 - \rho\cdot\varepsilon_1\right)$$

$\Rightarrow$ $if\,(\varepsilon_1 \to 0) \quad \varepsilon_2 \to 0$

As a result,

$$s_i \to 0,\ \ s_{i-1} \to 0 \ \text{ and } \varepsilon_{i-1} \to 0 \quad \Rightarrow \quad \varepsilon_i \to 0$$

7.3 Controller Design for string stability

(12) Laplace Transform of Sliding Surface

$$s_i - s_{i-1} = 0 = \frac{1}{1-\rho}\cdot\dot{\varepsilon}_i - \dot{\varepsilon}_{i-1} + \frac{1}{1-\rho}\cdot\frac{\omega_n}{\varsigma+\sqrt{\varsigma^2-1}}\cdot\left(\varepsilon_i - \varepsilon_{i-1}\right)$$

==>

$$\frac{1}{1-\rho}\cdot s\cdot\varepsilon_i(s) - s\cdot\varepsilon_{i-1}(s) + \frac{1}{1-\rho}\cdot\frac{\omega_n}{\varsigma+\sqrt{\varsigma^2-1}}\cdot\left(\varepsilon_i(s)-\varepsilon_{i-1}(s)\right)=0$$

==>

$$\left(\frac{1}{1-\rho}\cdot s+\frac{1}{1-\rho}\cdot\frac{\omega_n}{\varsigma+\sqrt{\varsigma^2-1}}\right)\cdot\varepsilon_i(s) = \left(s+\frac{1}{1-\rho}\cdot\frac{\omega_n}{\varsigma+\sqrt{\varsigma^2-1}}\right)\varepsilon_{i-1}(s)$$

==>

$$\varepsilon_i(s) = \frac{s+\frac{1}{1-\rho}\cdot\frac{\omega_n}{\varsigma+\sqrt{\varsigma^2-1}}}{\frac{1}{1-\rho}\cdot s+\frac{1}{1-\rho}\cdot\frac{\omega_n}{\varsigma+\sqrt{\varsigma^2-1}}}\cdot\varepsilon_{i-1}(s) = \frac{s+A}{\frac{1}{1-\rho}\cdot s+A}\cdot\varepsilon_{i-1}(s)$$

7.3 Controller Design for string stability

(12) Laplace Transform of Sliding Surface

$$\varepsilon_i(s) = \frac{s + A}{\frac{1}{1-\rho} \cdot s + A} \cdot \varepsilon_{i-1}(s)$$

Where, ρ Weighting of the lead vehicle's speed and acceleration. $0 < \rho < 1$

$$\left| \frac{\varepsilon_i(j\omega)}{\varepsilon_{i-1}(j\omega)} \right| = \left| \frac{j\omega + A}{\frac{1}{1-\rho} \cdot j\omega + A} \right| = \sqrt{\frac{1 + A^2}{\left(\frac{1}{1-\rho} \right)^2 \omega^2 + A^2}} < 1$$

Also, $\varepsilon_{i-1} \to 0 \quad \Rightarrow \quad \varepsilon_i \to 0$