7. Excavation design in massive elastic rock

7.1 General principles of excavation design

- Mining excavation

  1) Service openings

  - Mine access, ore haulage drive, airway, crusher chambers…
  - Duty life: mining life of the orebody

  2) Production openings

  - Ore sources, stopes, drill headings, stope access…
  - Duty life: life of stope (as short as a few months)
- Excavation design in massive elastic rock
  - The simplest design problem in mining rock mechanics

7.1 General principles of excavation design

- Two points in mine design
  - Existence of an extensive damage zone or failure rock near the boundary of the opening is common even in successful mining practice
  - Basic mining design objective is to avoid large, uncontrolled displacement of rock in the excavation boundary.

- Logical framework for mine excavation design in massive rock
  - Refer to Fig 7.2

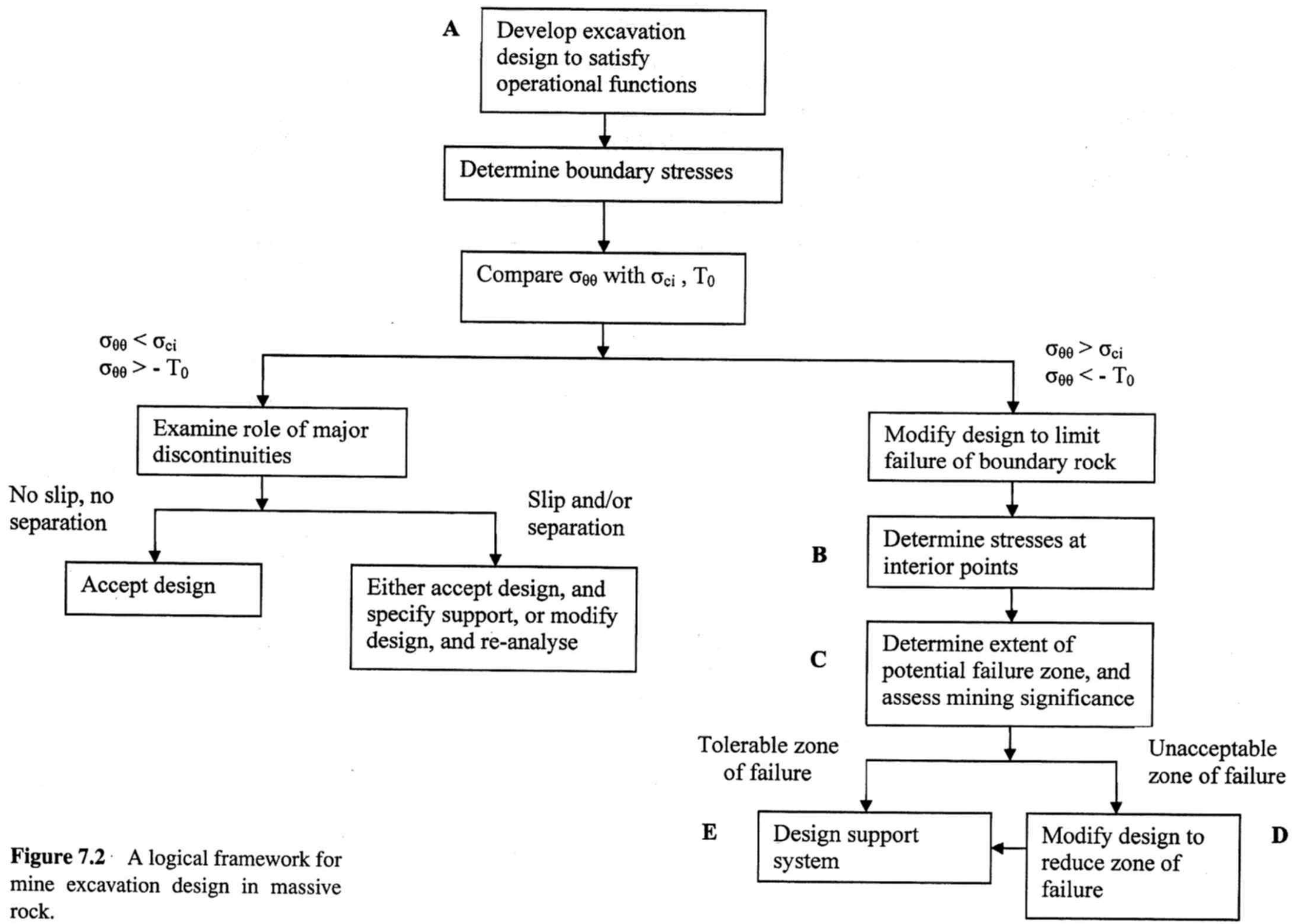


Figure 7.2 A logical framework for mine excavation design in massive rock.

7.2 Zone of influence of an excavation

- Zone of influence

  - A domain of significant disturbance of the pre-mining stress field by an excavation

  - Depends on excavation shape and pre-mining stresses

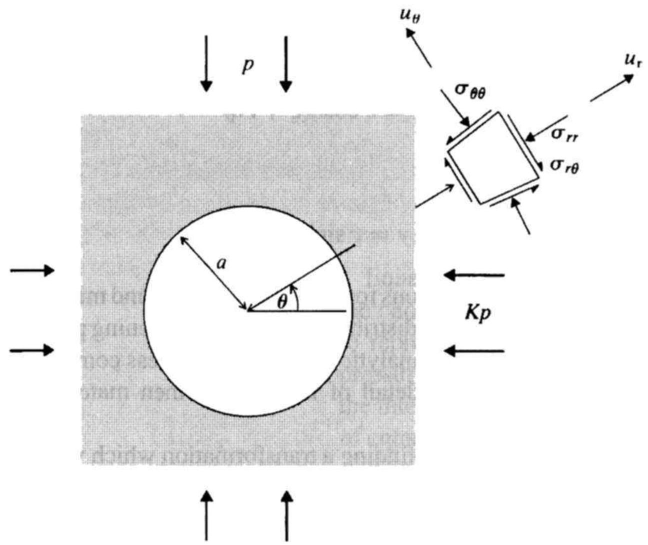
- Stress distribution around a circular hole (Kirsch equations)

  - General case:

$$\sigma_{rr} = \frac{p}{2}\left[(1+K)\left(1-\frac{a^2}{r^2}\right)-(1-K)\left(1-4\frac{a^2}{r^2}+3\frac{a^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_{\theta\theta} = \frac{p}{2}\left[(1+K)\left(1+\frac{a^2}{r^2}\right)+(1-K)\left(1+3\frac{a^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_{r\theta} = \frac{p}{2}(1-K)\left(1+2\frac{a^2}{r^2}-3\frac{a^4}{r^4}\right)\sin 2\theta$$



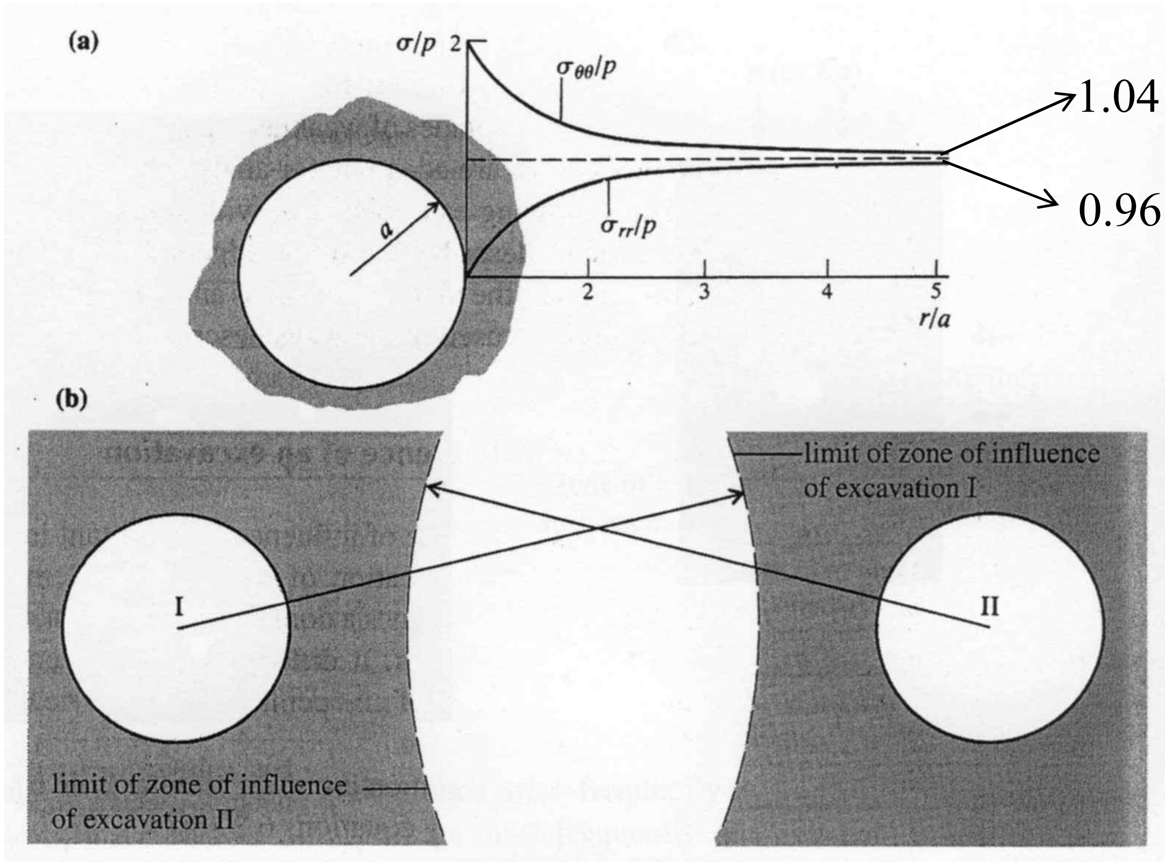
7.2 Zone of influence of an excavation

- Hydrostatic stress case:

$$\sigma_{rr} = p\left(1-\frac{a^2}{r^2}\right)$$

$$\sigma_{\theta\theta} = p\left(1+\frac{a^2}{r^2}\right)$$

$$\sigma_{r\theta} = 0$$



1) Openings of the same radius

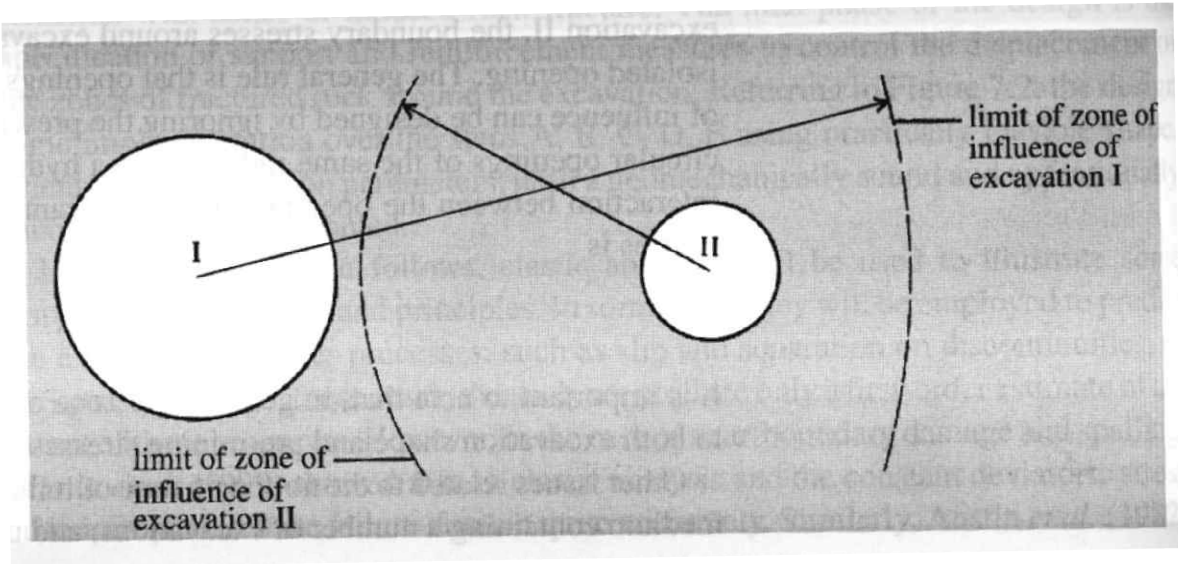
- $D_{I,II} \geq 6a$

(within ±5% from the field stresses)

7.2 Zone of influence of an excavation

2) Openings of the different radius

- Genera rule: openings lying outside one another's zones of influence can be designed by ignoring the presence of all others
- Boundary stresses around II can be obtained by calculating the state of stress at the center of II which is adopted as the far-field stresses in the Kirsch equations, prior to its excavation.



7.2 Zone of influence of an excavation

3) Elliptic opening

- General shapes of openings can be represented by ellipses inscribed in the opening cross sections.
- Zone of influence of an elliptic excavation:

$$W_I = H\left[A\alpha\left|q(q+2) - K(3+2q)\right|\right]^{1/2}$$

or

$$W_I = H\left[\alpha\left(A\left(K+q^2\right)+Kq^2\right)\right]^{1/2}$$

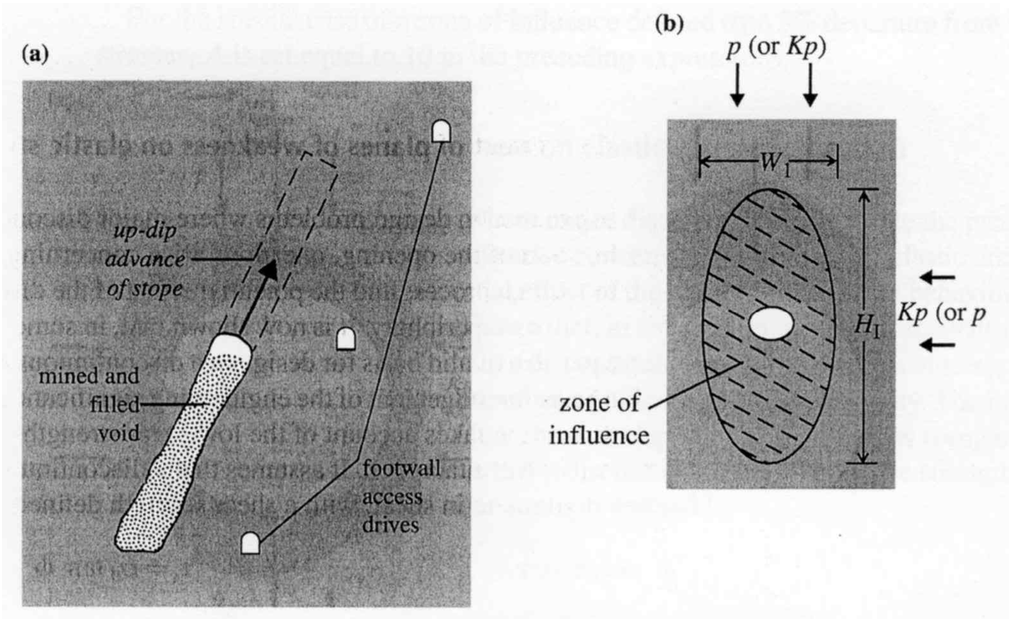
$$H_I = H\left[A\alpha\left|K(1+2q) - q(3q+2)\right|\right]^{1/2}$$

or

$$H_I = H\left[\alpha\left(A\left(K+q^2\right)+1\right)\right]^{1/2}$$

$q = W/H, \quad A = 100/2c,$

$\alpha = 1$, if $K < 1$, and $\alpha = 1/K$, if $K > 1$



7.3 Effect of planes of weakness on elastic stress distribution

- Elastic analysis for the excavations with discontinuities
  - In some cases, provides a perfectly valid basis for design
  - or a basis for judgment of engineering significance of a discontinuity.
- Basic assumption of discontinuities
  - Zero tensile strength
  - Non-dilatant in shear
  - Shear strength follows

$$\tau = \sigma_n \tan \phi$$

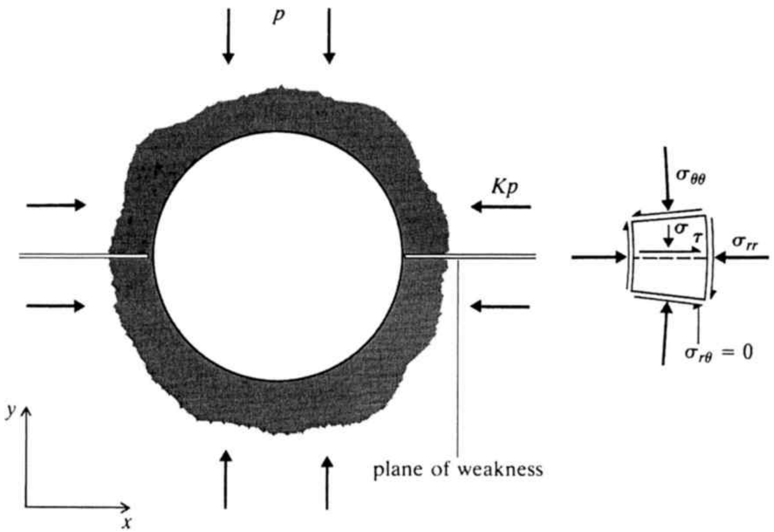
7.3 Effect of planes of weakness on elastic stress distribution

- Case 1: Horizontal discontinuity passing through the opening center
  - $\sigma_{r\theta} = 0$ for all r at $\theta = 0$: no slip, σ_{rr} and $\sigma_{\theta\theta}$ are principal stresses.
  - The plane of weakness (discontinuity) has no effect on the elastic stress distribution

$$\sigma_{rr} = \frac{p}{2}\left[(1+K)\left(1-\frac{a^2}{r^2}\right)-(1-K)\left(1-4\frac{a^2}{r^2}+3\frac{a^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_{\theta\theta} = \frac{p}{2}\left[(1+K)\left(1+\frac{a^2}{r^2}\right)+(1-K)\left(1+3\frac{a^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_{r\theta} = \frac{p}{2}(1-K)\left(1+2\frac{a^2}{r^2}-3\frac{a^4}{r^4}\right)\sin 2\theta$$



7.3 Effect of planes of weakness on elastic stress distribution

- Case 2: Vertical discontinuity passing through the opening center

  1) At $K \geq 1/3$

  - $\sigma_{r\theta} = 0$ for all r at $\theta = 90^{\circ}$: no slip, σ_{rr} and $\sigma_{\theta\theta}$ are principal stresses.

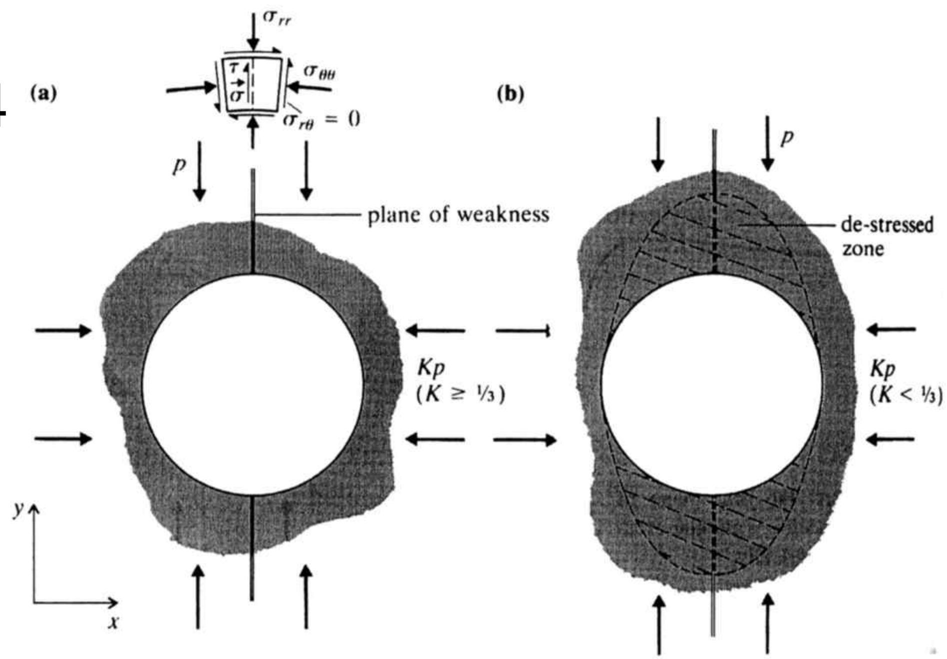
  2) At $K < 1/3$

  - De-stressed zone from Eq.6.24

$$\sigma_B = p\left(K - 1 + \frac{2K}{q}\right) = 0$$

$$\text{or } q = \frac{2K}{1-K} \quad \left(q = \frac{W}{H} = \frac{2a}{H}\right)$$

$$\Delta h = a\left(\frac{1-3K}{2K}\right)$$



7.3 Effect of planes of weakness on elastic stress distribution

- Case 3: Horizontal discontinuity passing through the opening
  - Normal & shear stresses at the intersections on boundary

$$\sigma_n = \sigma_{\theta\theta} \cos^2 \theta$$

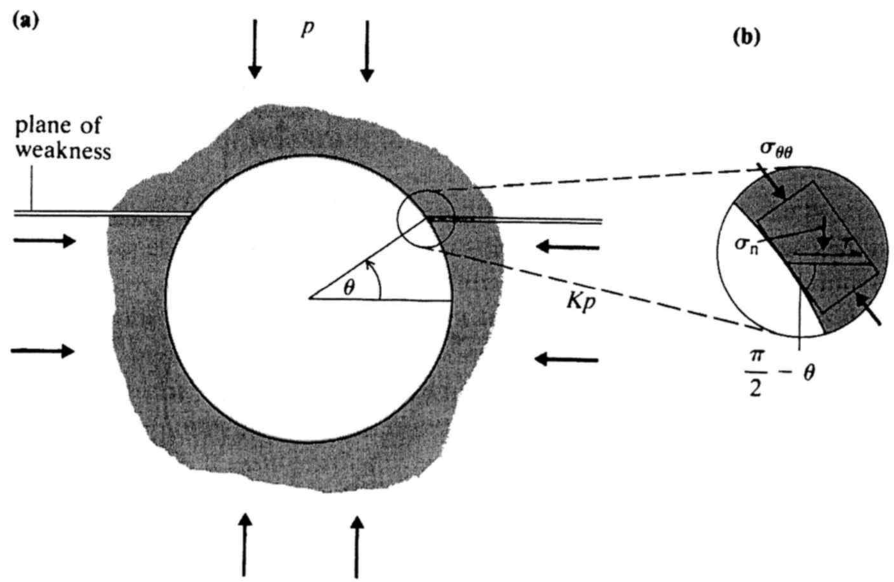
$$\tau = \sigma_{\theta\theta} \sin\theta \cos\theta$$

$$\tau = \sigma_n \tan\phi \quad (\text{slip occurrs})$$

$$\rightarrow \quad \sigma_{\theta\theta} \sin\theta \cos\theta = \sigma_{\theta\theta} \cos^2 \theta \tan\phi$$

$$\tan\theta = \tan\phi \quad \text{or} \quad \sigma_{\theta\theta} \frac{\sin(\theta - \phi)}{\cos\phi} = 0$$

- Slip occurs when $\theta = \phi$ or $\sigma_{\theta\theta} = 0$
- Intersection regions are either de-stressed or at low confining stress



7.3 Effect of planes of weakness on elastic stress distribution

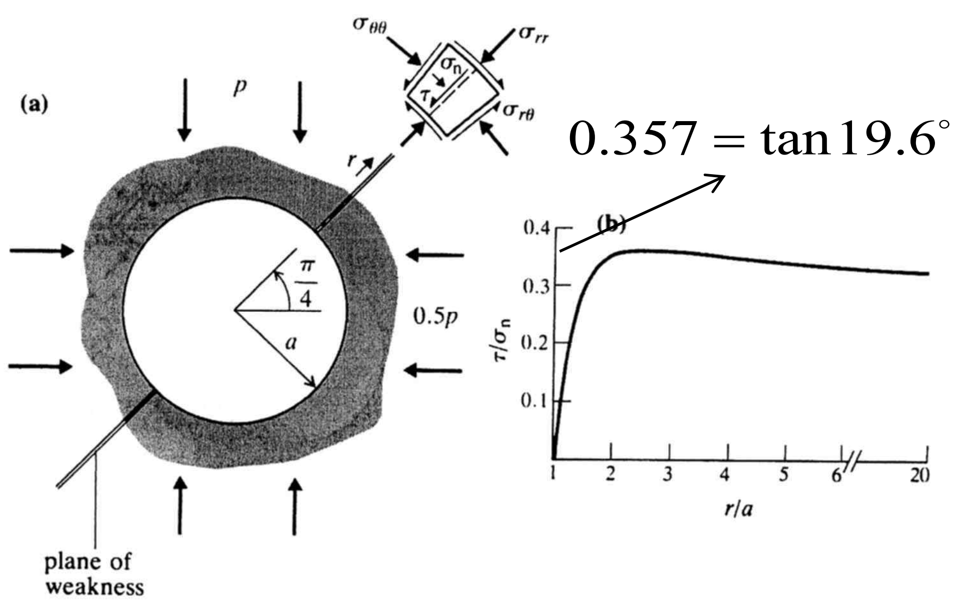
- Case 4: Arbitrary discontinuity passing through the opening center

  - Normal & shear stresses on the weak plane

$$\sigma_n = \sigma_{\theta\theta} = \frac{p}{2}1.5\left(1+\frac{a^2}{r^2}\right)$$

$$\tau = \sigma_{r\theta} = \frac{p}{2}0.5\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)$$

  - Slip does not occur if $\phi > 19.6^\circ$



7.3 Effect of planes of weakness on elastic stress distribution

- Case 5: Horizontal discontinuity not intersecting the opening
  - Normal & shear stresses on the weak plane

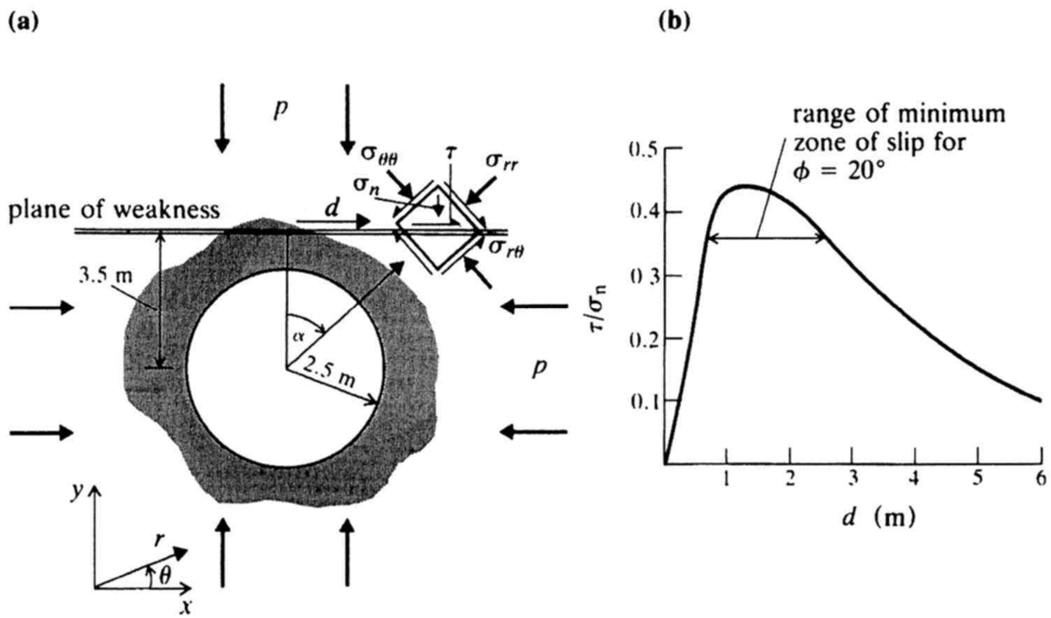
$$\sigma_n = \frac{1}{2}(\sigma_{rr} + \sigma_{\theta\theta}) + \frac{1}{2}(\sigma_{rr} - \sigma_{\theta\theta})\cos 2\alpha$$

$$= p\left(1 - \frac{a^2}{r^2}\cos 2\alpha\right)$$

$$\tau = -\frac{1}{2}(\sigma_{rr} - \sigma_{\theta\theta})\sin 2\alpha$$

$$= p\frac{a^2}{r^2}\sin 2\alpha$$

  - Slip does not occur if $\phi > 24^\circ$



7.4 Excavation shape and boundary stresses

- Elliptic opening

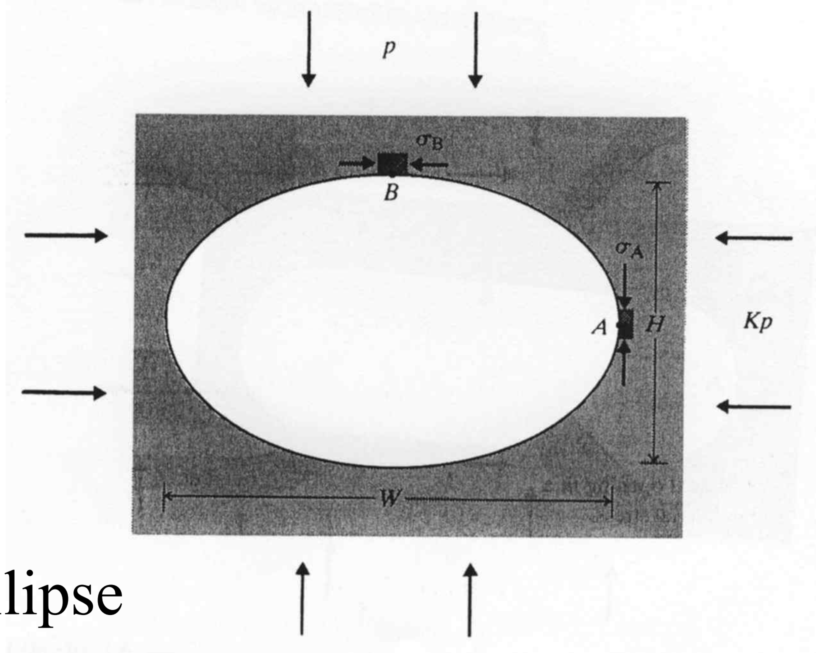
$$\sigma_A = p(1-K+2q) = p\left(1-K+\sqrt{\frac{2W}{\rho_A}}\right)$$

$$\sigma_B = p\left(K-1+\frac{2K}{q}\right) = p\left(K-1+K\sqrt{\frac{2W}{\rho_B}}\right)$$

- $q = W/H$

- ρ : radius of curvature

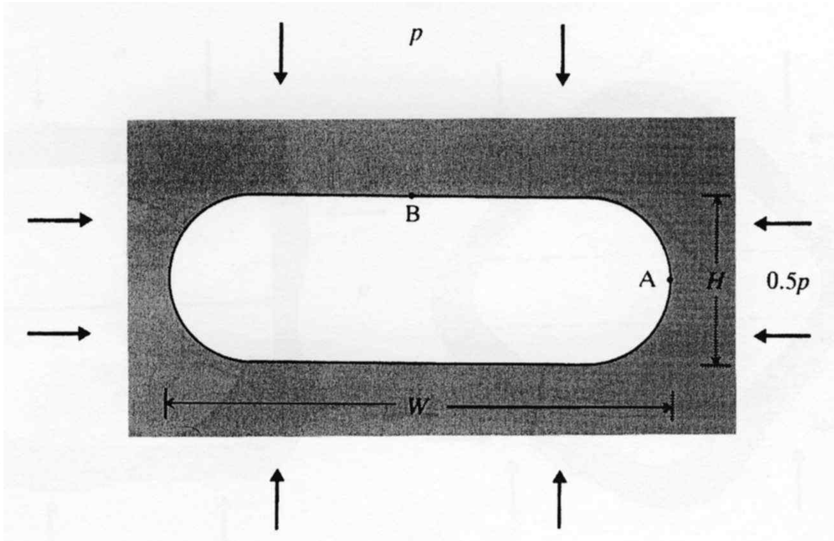
- $\frac{1}{\rho} = \frac{x'y'' - x''y'}{\left(x'^2 + y'^2\right)^{3/2}}$: curvature of an ellipse



- Larger curvature makes higher stress concentration

7.4 Excavation shape and boundary stresses

- Ovaloidal opening
  - Applying the boundary stress of an ellipse inscribed in the ovaloid



- $\sigma_A = p\left(1 - 0.5 + \sqrt{\dfrac{2 \times 3H}{H/2}}\right) = 3.96p$

  $\sigma_B = p\left(0.5 - 1 + 0.5\sqrt{\dfrac{2H}{(3H)^2/2H}}\right) = -0.17p$

- By B.E.M: $\sigma_A = 3.60p$, $\sigma_B = -0.15p$

7.4 Excavation shape and boundary stresses

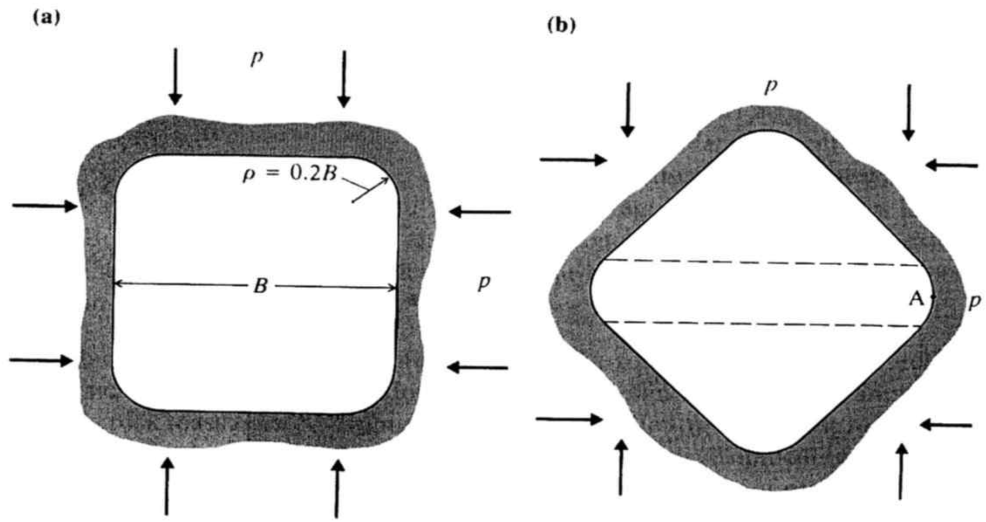
- Square opening with rounded corners

- Applying the boundary stress of an ellipse whose curvature is the same as those of the rounded corners

- $$\sigma_A = p\left(1-1+\left[\frac{2B\left(\sqrt{2}-0.4\left(\sqrt{2}-1\right)\right)}{0.2B}\right]^{1/2}\right)$$

$$=3.53p$$

- By B.E.M: σ_A= 3.14p
- Boundary stress is dominated by the local geometry: St Venant's Principle

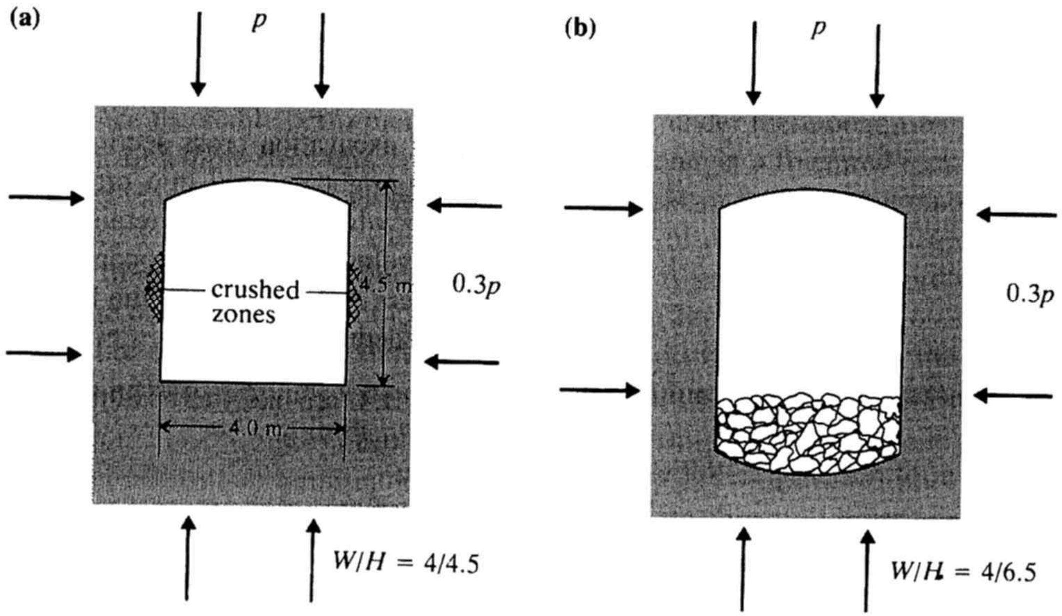


7.4 Excavation shape and boundary stresses

- Effect of changing the relative dimensions

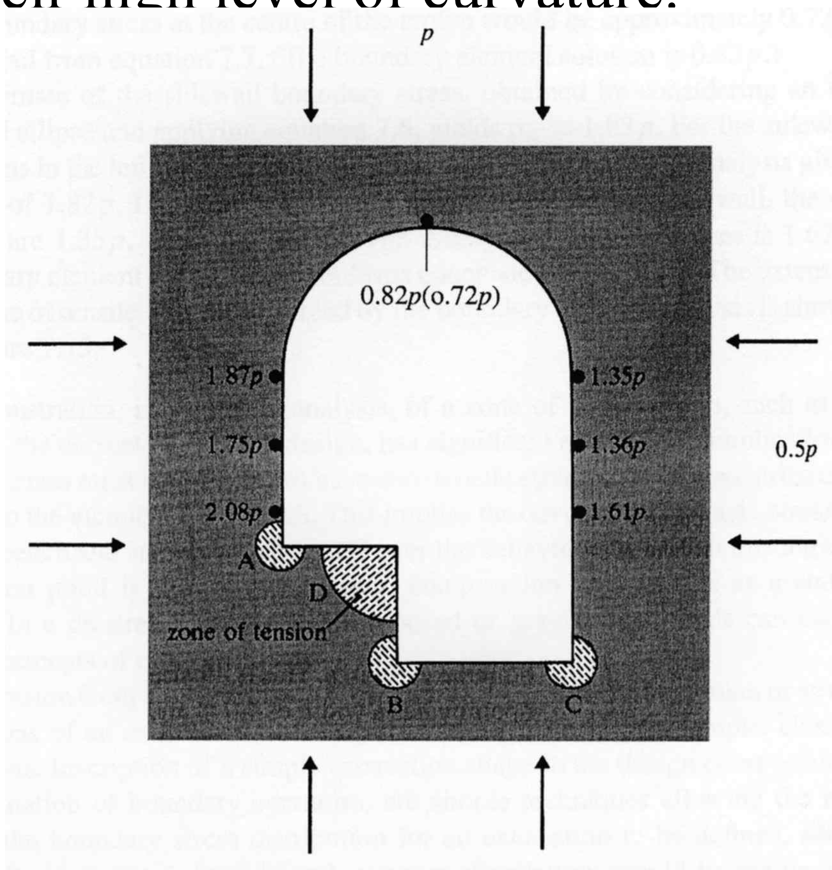
 - Sidewall stress 2.5p → 1.7p

 - The maximum boundary stress can be reduced if the opening dimension is increased in the direction of the major principal stress.



7.4 Excavation shape and boundary stresses

- Effect of local geometry of an opening
  - Width/height = 2/3
  - A, B, C are highly stressed due to their high level of curvature.
  - D is at low state of stress (tensile failure)
  - Rock mass in compression may behave as a stable continuum while in a de-stressed state, small loads can cause large displacement of rock units.



7.5 Delineation of zones of rock failure

- Estimation of the extent of fracture zones provides a basis for prediction of rock mass performance, modification of excavation design, or assessing support and reinforcement requirement.

- The solution procedure suggested here examines only the initial, linear component of the problem. For mining engineering purposes, the suggested procedure is usually adequate.

- Extent of boundary failure
  - Applicable compressive strength at boundary is σ_{ci}.
  - Tensile strength of rock mass is taken to be zero.

7.5 Delineation of zones of rock failure

- Case of a circular excavation having σ_{ci} of 16 MPa:

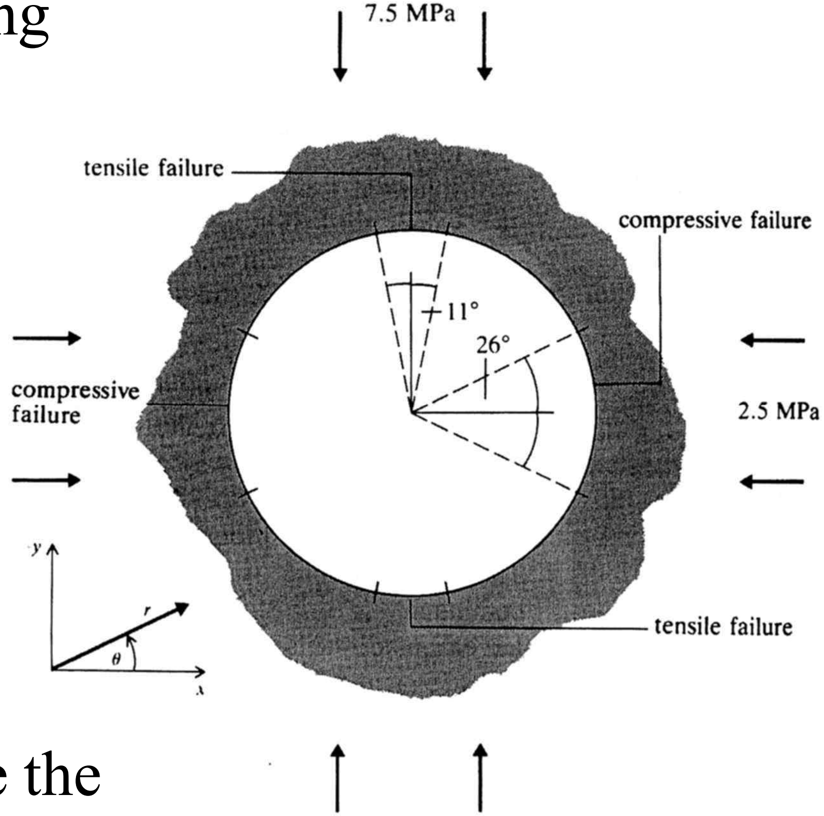
$$\sigma_{\theta\theta} = p\left[1 + K + 2(1-K)\cos 2\theta\right]$$

$$7.5\left[1.3 + 1.4\cos 2\theta\right] \geq 16 \quad \text{(compressive)}$$

$$\rightarrow \quad -26^\circ \leq \theta \leq 26^\circ \quad \text{or} \quad 154^\circ \leq \theta \leq 206^\circ$$

$$7.5\left[1.3 + 1.4\cos 2\theta\right] \leq 0 \quad \text{(tensile)}$$

$$\rightarrow \quad 79^\circ \leq \theta \leq 101^\circ \quad \text{or} \quad 259^\circ \leq \theta \leq 281^\circ$$

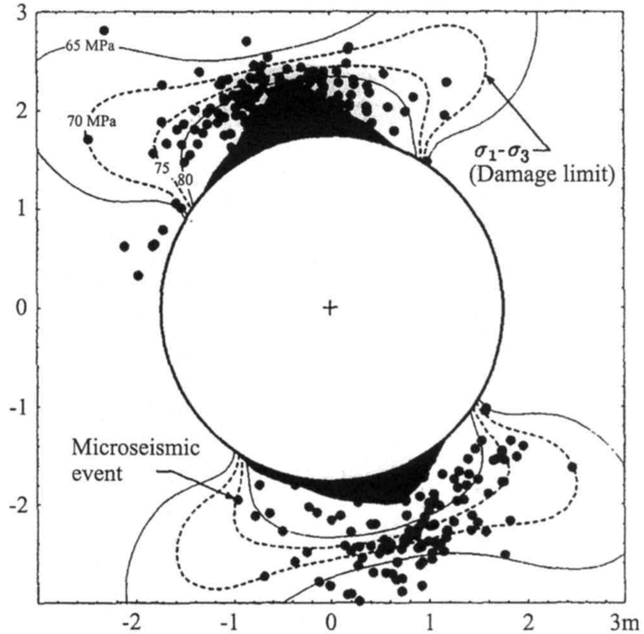


- Change in shape, installation of support/reinforcement, or increase the height of the opening can be used.

7.5 Delineation of zones of rock failure

- Extent of failure zones in rock mass
  - Close to the boundary (within a radius):
    the constant deviator stress criterion is useful.

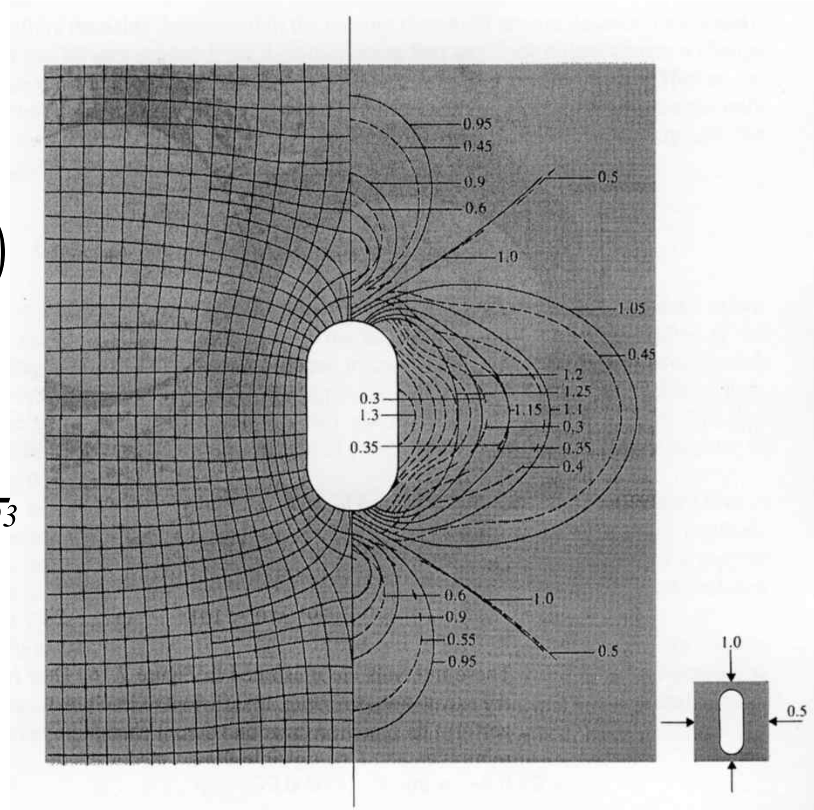
  - Example of an circular opening in
    Lac du Bonnet granite: the maximum
    deviator stress contour of 75 MPa
    predicted well the failure domain.



7.5 Delineation of zones of rock failure

- General cases including interior zones of rock mass:
  Hoek-Brown criterion with σ_{cd}

1) Principal stress contour method
   a. Calculate various values of $\left(\sigma_3, \sigma_1^f\right)$
   b. Contour plots of σ_1 and σ_3 are superimposed.
   c. Find the intersections of σ_1 and σ_3 isobars satisfying the failure criterion.



7.5 Delineation of zones of rock failure

2) Direct comparison with the failure criterion

a. Calculate the state of stress (principal stress)

b. Compare with the failure criterion.

c. Display failure locations throughout the rock mass.

7.6 Support and reinforcement of massive rock

- Explanation of the effect of support

(1) Elastic rock medium

- Stress before support:

$$\sigma_A = 170.2\ \text{MPa}, \quad \sigma_B = -8.0\ \text{MPa}$$

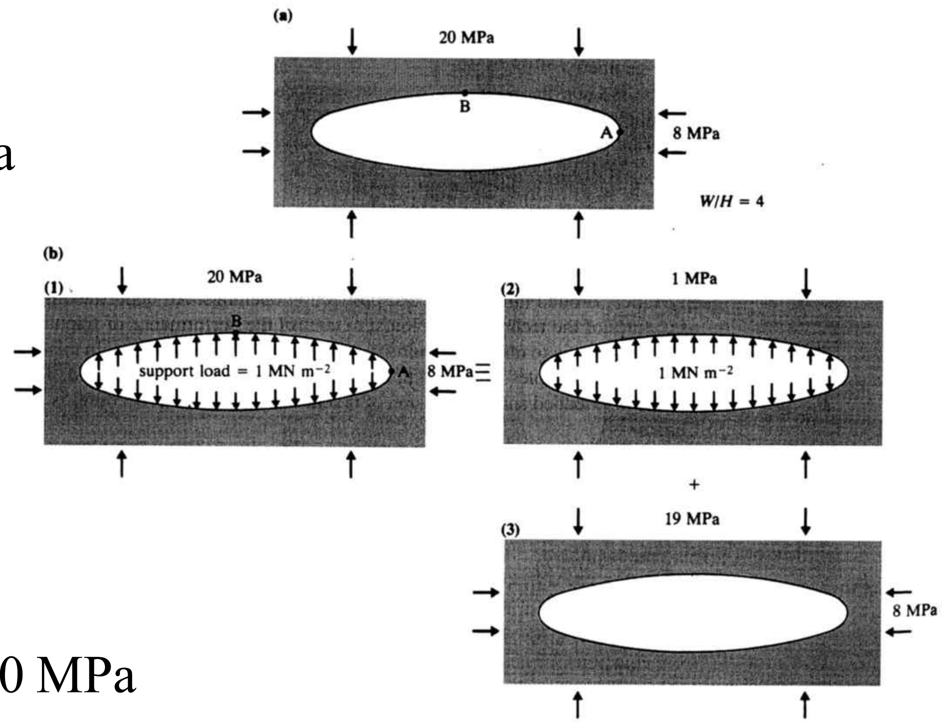
- Stress after support

$$\sigma_{A1} = \sigma_{A2} + \sigma_{A3}$$

$$= 1 + 19\left(1 - \frac{8}{19} + 8\right) = 161.0\ \text{MPa}$$

$$\sigma_{B1} = \sigma_{B2} + \sigma_{B3}$$

$$= 0 + 19\left(\frac{8}{19} - 1 + \frac{2\times 8}{19} \times \frac{1}{4}\right) = -7.0\ \text{MPa}$$



- Support pressure does not significantly modify the elastic distribution around an underground opening

7.6 Support and reinforcement of massive rock

(2) Elastic rock mass with a failed rock annulus

- Rock mass strength is assumed to follow Coulomb's criterion:

$$\sigma_1 = \sigma_3 \frac{1+\sin\phi}{1-\sin\phi} + \frac{2c\cos\phi}{1-\sin\phi} \;\rightarrow\; \sigma_1 = b\sigma_3 + C_0 : \text{Rock Mass}$$

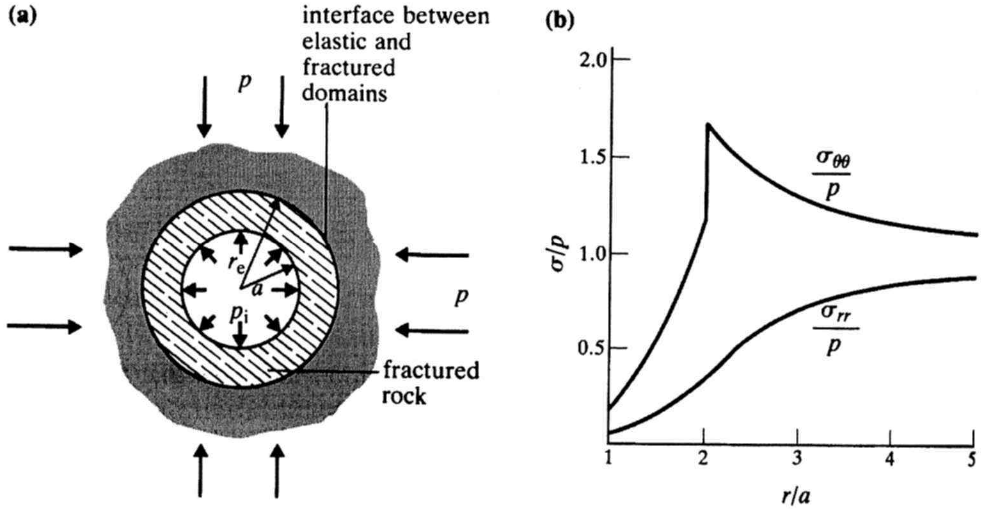
$$\sigma_1 = \sigma_3 \frac{1+\sin\phi^f}{1-\sin\phi^f} \;\rightarrow\; \sigma_1 = d\sigma_3 : \text{Fractured rock Mass}$$

- Equilibrium equations in fractured rock (c.f. Sec.2.11)

$$\frac{d\sigma_{rr}}{dr} = \frac{\sigma_{\theta\theta}-\sigma_{rr}}{r} = (d-1)\frac{\sigma_{rr}}{r}$$

$$\rightarrow \sigma_{rr} = p_i\left(\frac{r}{a}\right)^{d-1}, \sigma_{\theta\theta} = dp_i\left(\frac{r}{a}\right)^{d-1}$$

$$p_1 = p_i\left(\frac{r_e}{a}\right)^{d-1} \quad \text{or} \quad r_e = a\left(\frac{p_1}{p_i}\right)^{1/(d-1)}$$



7.6 Support and reinforcement of massive rock

- Stress in elastic zone (c.f. Eqn.6.8)

$$\sigma_{\theta\theta} = p\left(1+\frac{r_e^2}{r^2}\right) - p_1\frac{r_e^2}{r^2}, \quad \sigma_{rr} = p\left(1-\frac{r_e^2}{r^2}\right) + p_1\frac{r_e^2}{r^2}$$

- At the inner boundary of the elastic zone ($r = r_e$)

$$\sigma_{\theta\theta} = 2p - p_1, \quad \sigma_{rr} = p_1$$

- Applying to Coulomb's criterion ($\sigma_1 = b\sigma_3 + C_0$)

$$2p - p_1 = bp_1 + C_0 \quad \rightarrow \quad p_1 = \frac{2p - C_0}{1+b}$$

$$\rightarrow r_e = a\left[\frac{2p - C_0}{(1+b)p_i}\right]^{1/(d-1)}$$

- At $\phi = \phi^f = 35^\circ$, $p_i = 0.05p, \quad C_0 = 0.5p$

$$\rightarrow r_e = 1.99a$$