Introduction to Nuclear Fusion (409.308A, 3 Credits)

Prof. Dr. Yong-Su Na

(32-206, Tel. 880-7204)

Contents

Contents

Contents

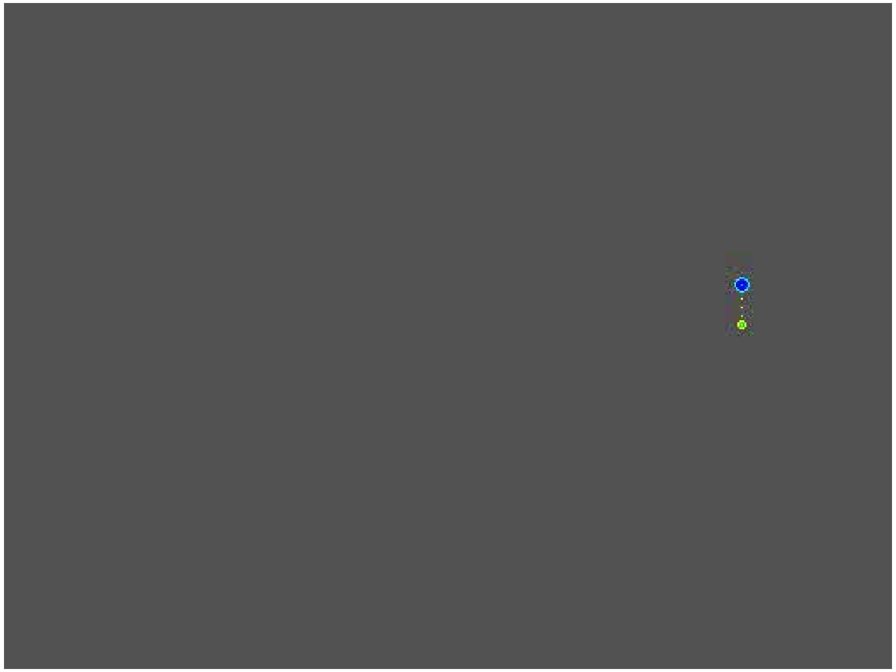
Individual Charge Trajectories

- **Equation of Motion**

- Basic relation determining the motion of an individual charged particle of mass *m* and charge *q* in a combined electric (**E**) and magnetic (**B**) field

- Neglecting electomagnetic fields generated by movement of the charge itself

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$



Individual Charge Trajectories

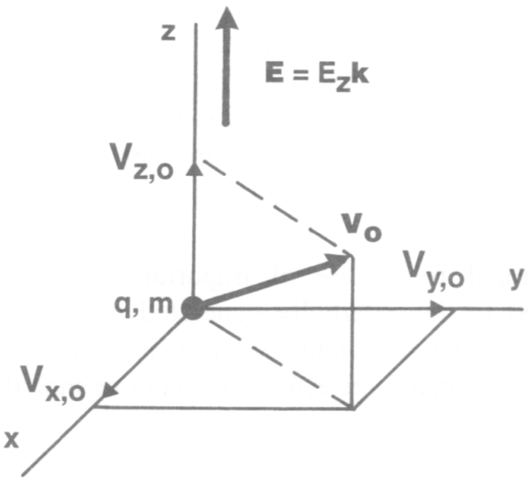
- **Homogeneous Electric Field**

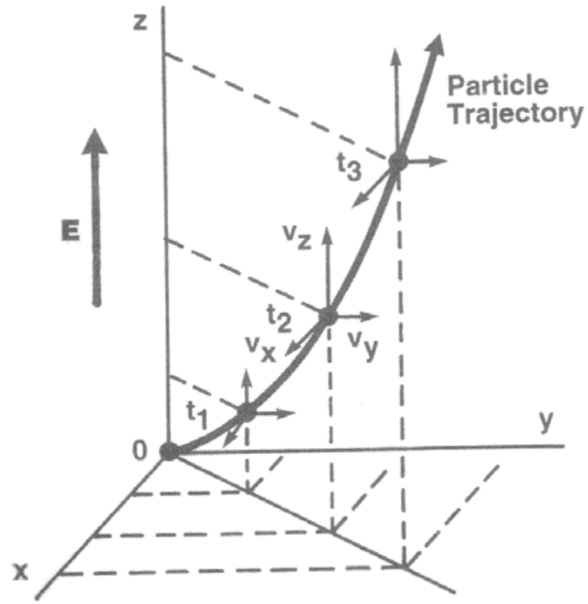
$$m\frac{d\mathbf{v}}{dt} = qE_z\mathbf{k}$$

$$v_x(0) = v_{x,0}, \quad v_y(0) = v_{y,0}, \quad v_z(0) = v_{z,0}$$

$$x(t) = v_{x,0}t, \quad y(t) = v_{y,0}t,$$

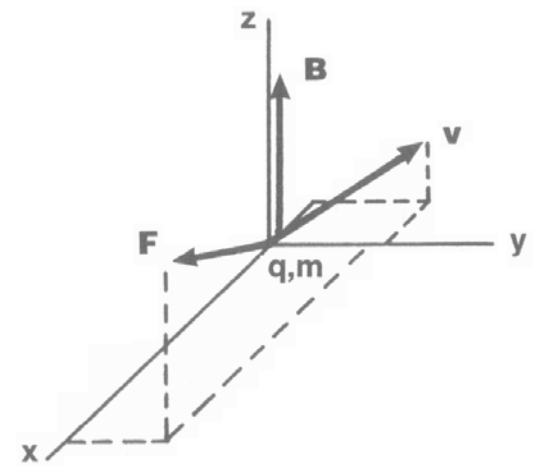
$$z(t) = v_{z,0}t + \left(\frac{q}{m}E_z\right)t^2$$





Individual Charge Trajectories

- **Homogeneous Magnetic Field**



$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{v}\times B_z\mathbf{k}\right)$$

$$v_z(0) = v_{z,0} = v_{\parallel}$$

$$v_x(t) = v_\perp \cos(\omega_c t + \phi), \quad v_y(t) = \mp v_\perp \sin(\omega_c t + \phi), \quad v_z(t) = v_{z,0} = v_{\parallel}$$

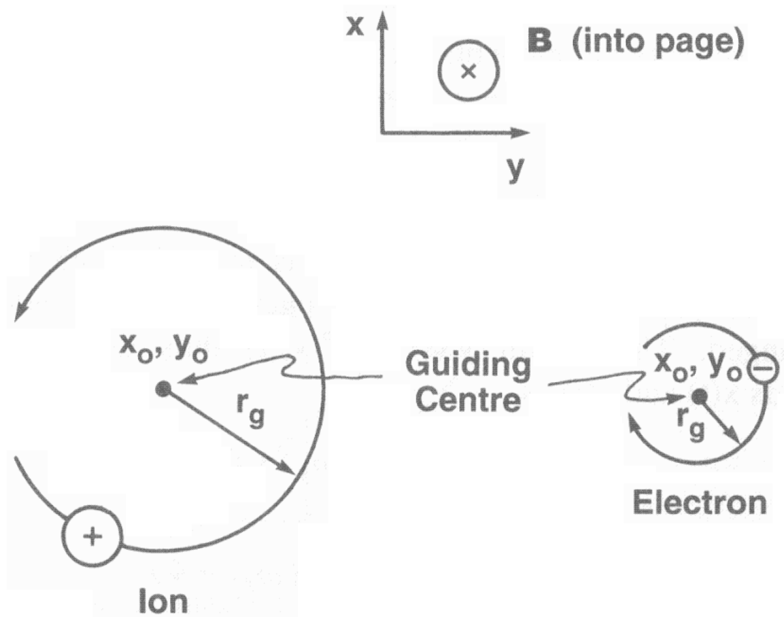
$$x(t) = x_0 + \frac{v_\perp}{\omega_c}\sin(\omega_c t + \phi), \quad y(t) = y_0 \pm \frac{v_\perp}{\omega_c}\cos(\omega_c t + \phi), \quad z(t) = v_{\parallel} t + z_0$$

$$v_\perp = \sqrt{v_x^2 + v_y^2}, \quad \tan(\phi) = \mp\frac{v_y(0)}{v_x(0)}, \quad \omega_c = \frac{|q|B_z}{m}$$ cyclotron frequency

$$(x - x_0)^2 + (y - y_0)^2 = \left(\frac{v_\perp}{\omega_c}\right)^2 = \left(\frac{v_\perp m}{|q|B_z}\right)^2 = r_L^2$$ Larmor radius

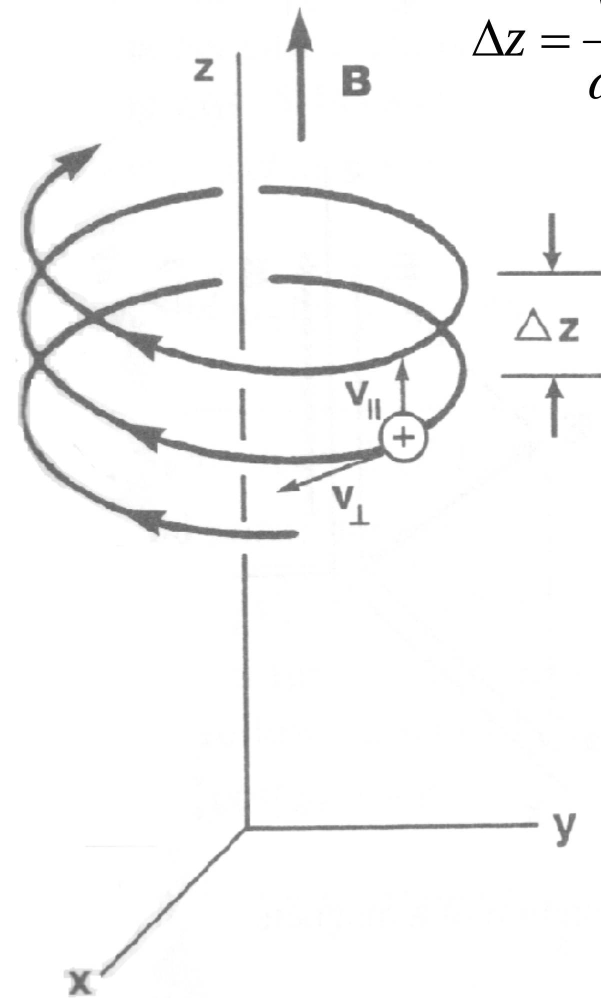
Individual Charge Trajectories

- **Homogeneous Magnetic Field**



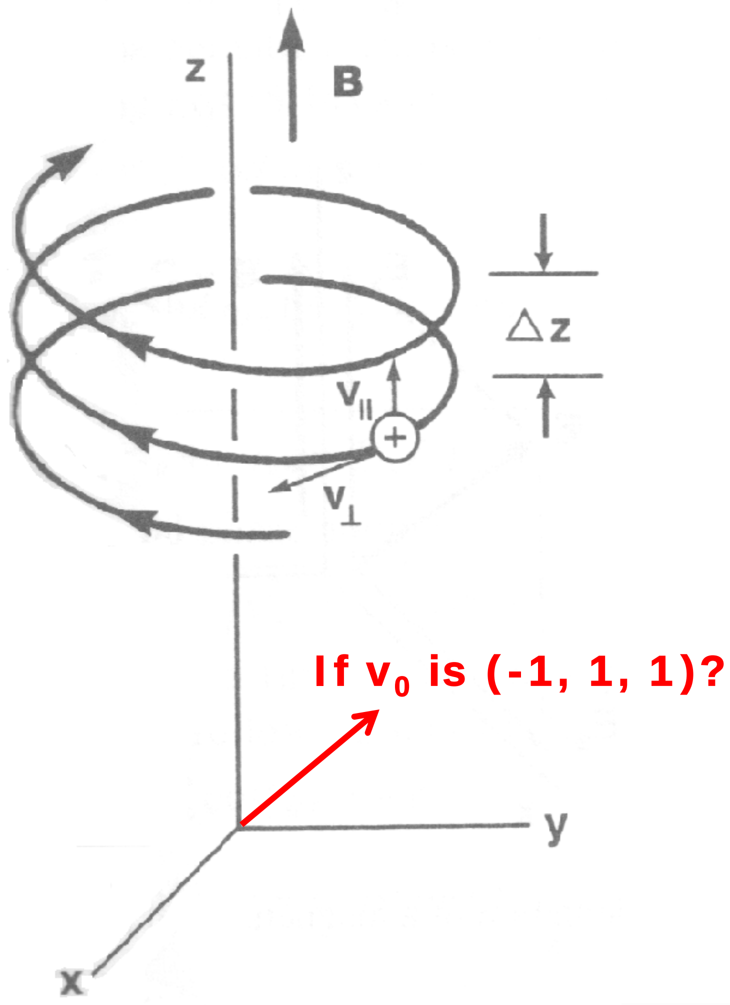
Pitch of the helix

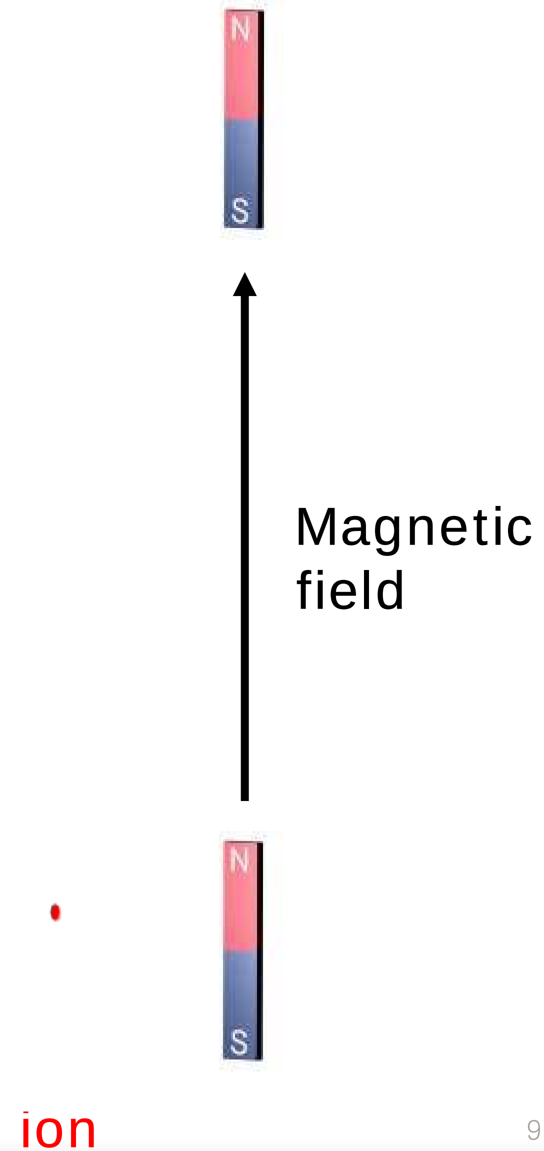
$$\Delta z = \frac{v_{\parallel}}{\omega_c}$$



Individual Charge Trajectories

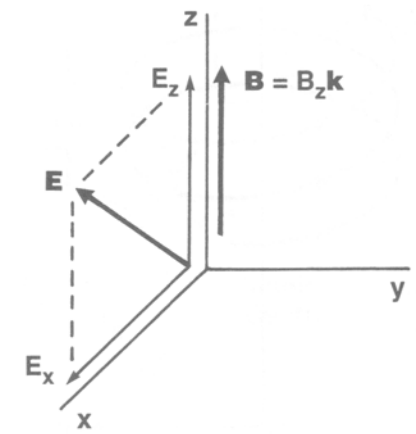
- Homogeneous Magnetic Field





Individual Charge Trajectories

- **Combined Homogeneous Electric and Magnetic Field**



$$m\frac{d\mathbf{v}}{dt} = q\left[\left(E_x\mathbf{i} + E_z\mathbf{k}\right) + \mathbf{v}\times B_z\mathbf{k}\right]$$

$$v_z(0) = v_{z,0} = v_{\parallel}$$

$$v_x(t) = v_0^* \cos(\omega_c t + \phi^*), \quad v_y(t) = \mp v_0^* \sin(\omega_c t + \phi^*) - \frac{E_x}{B_z},$$

$$v_z(t) = v_{\parallel} + \left(\frac{qE_z}{m}\right)t$$

$$v_0^* = \sqrt{v_x^2 + \left(v_y + \frac{E_x}{B_z}\right)^2} = \frac{v_0\cos\phi}{\cos\phi^*}, \quad \tan(\phi^*) = \mp\frac{v_y(0) + \dfrac{E_x}{B_z}}{v_x(0)}$$

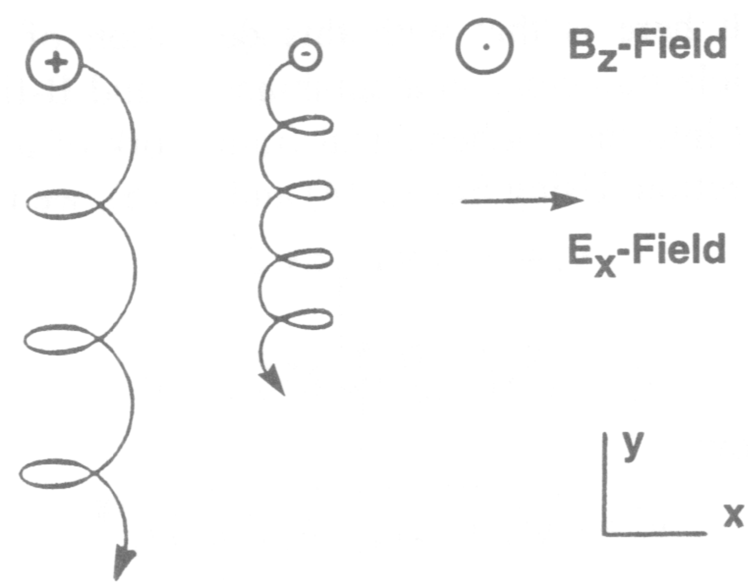
Individual Charge Trajectories

- **Combined Homogeneous Electric and Magnetic Field**

$$x(t) = x_0 + \frac{v_0^*}{\omega_c}\sin(\omega_c t + \phi^*), \quad y(t) = y_0 \pm \frac{v_0^*}{\omega_c}\cos(\omega_c t + \phi^*) - \frac{E_x}{B_z}t,$$

$$z(t) = z_0 + v_{\|}t + \frac{1}{2}\left(\frac{qE_z}{m}\right)t^2$$

$$(x - x_0)^2 + \left(y - y_0 + \frac{E_x}{B_z}t\right)^2 = \left(\frac{v_0^*}{\omega_c}\right)^2$$



Individual Charge Trajectories

- **Combined Homogeneous Magnetic Field and an Arbitrary Force**

$$\mathbf{v} = \mathbf{v}_{gc} + \mathbf{v}_g$$

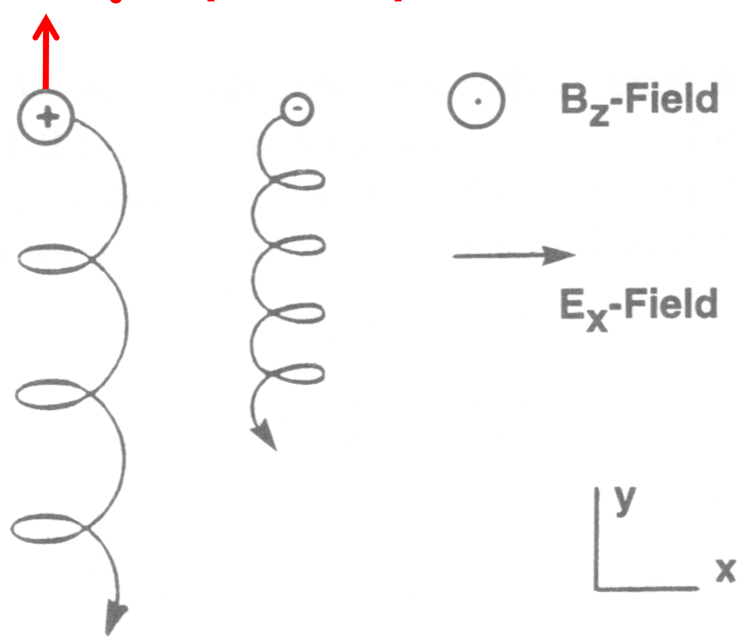
$$m\frac{d\mathbf{v}_{gc}}{dt} + m\frac{d\mathbf{v}_{gc}}{dt} = F + q\left(\mathbf{v}_{gc} \times \mathbf{B}\right) + q\left(\mathbf{v}_g \times \mathbf{B}\right)$$

If v_0 is (0, 1, -1)?

$$v_{gc,\|}(t) = v_{gc,\|}(0) + \frac{1}{m}\int F_{\|}dt$$

$$\overline{\mathbf{v}}_{gc,\perp} = \frac{\overline{\mathbf{F}} \times \mathbf{B}}{qB^2} = \mathbf{v}_{DF}$$

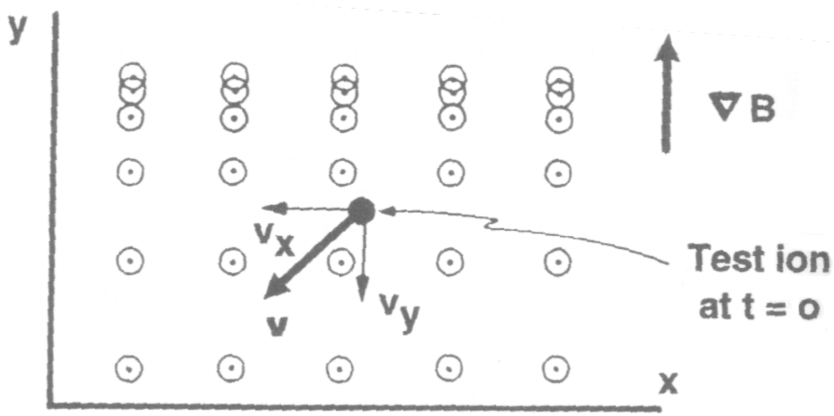
$$\mathbf{v}_{D,E} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

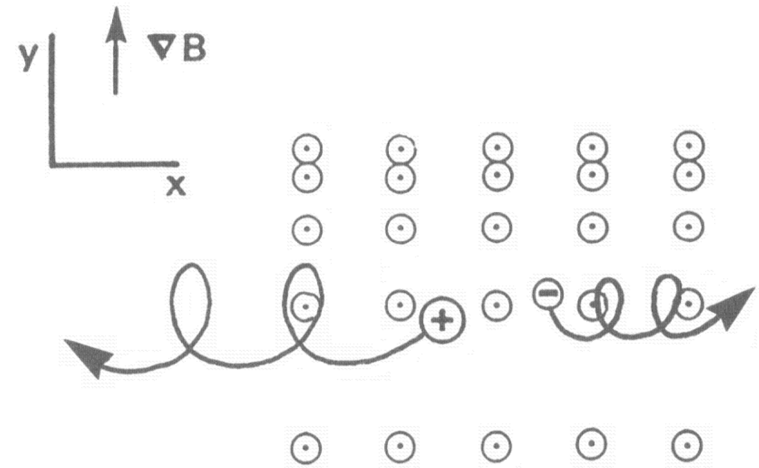


Individual Charge Trajectories

- **Spatially Varying Magnetic Field**

$$m\frac{d\mathbf{v}}{dt} = q\left[\mathbf{v}\times\mathbf{B}(r)\right] \qquad \nabla B = \frac{\partial B_z}{\partial y}\mathbf{j}, \quad v_{\parallel} = 0$$





$$\overline{\mathbf{F}} \approx -|q|\left(\frac{dB}{dy}\right)\left(\frac{v_\perp^2}{2\omega_c}\right)\mathbf{j} = -|q|\frac{v_\perp^2}{2\omega_c}\nabla B$$

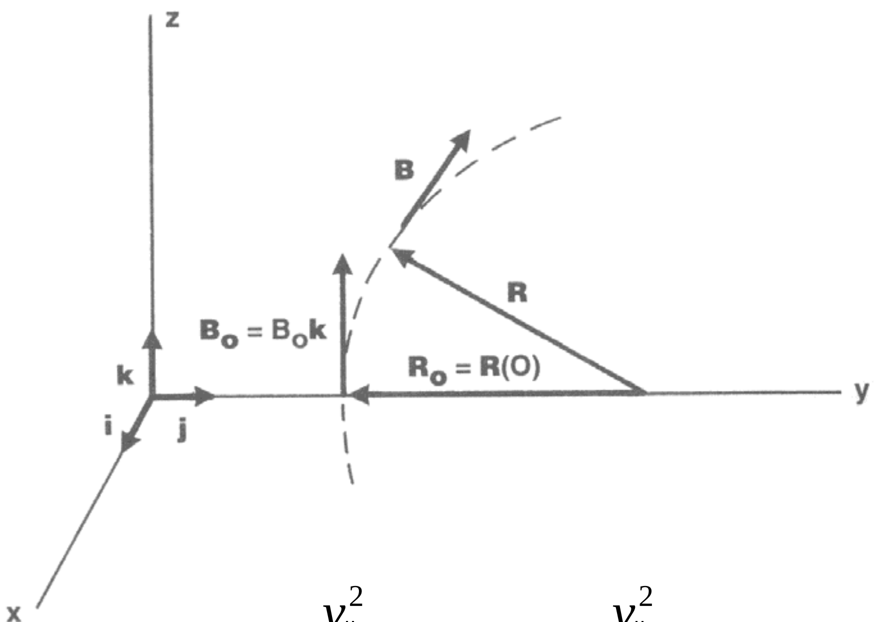
$$= -\frac{|q|}{2}v_\perp r_L \nabla B = -\frac{mv_\perp^2/2}{B}\nabla B$$

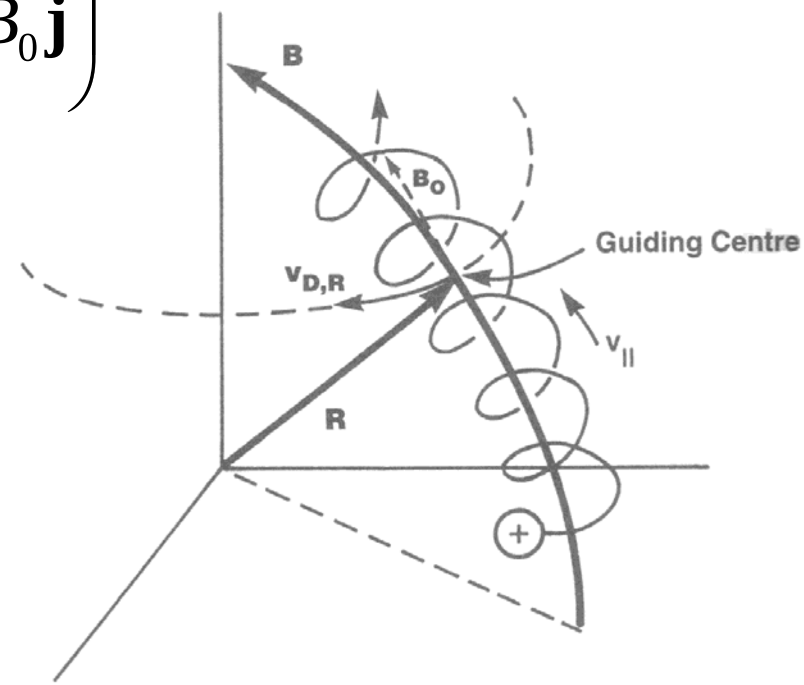
$$\mathbf{v}_{D,\nabla B} = \pm\frac{v_\perp^2}{2\omega_c}\frac{\mathbf{B}\times\nabla B}{B^2} = \pm\frac{1}{2}v_\perp r_L\frac{\mathbf{B}\times\nabla B}{B^2}$$

Individual Charge Trajectories

- **Curvature Drift**

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{v}\times B_0\mathbf{k}\right) + q\left(\mathbf{v}\times\frac{z}{R}B_0\mathbf{j}\right)$$



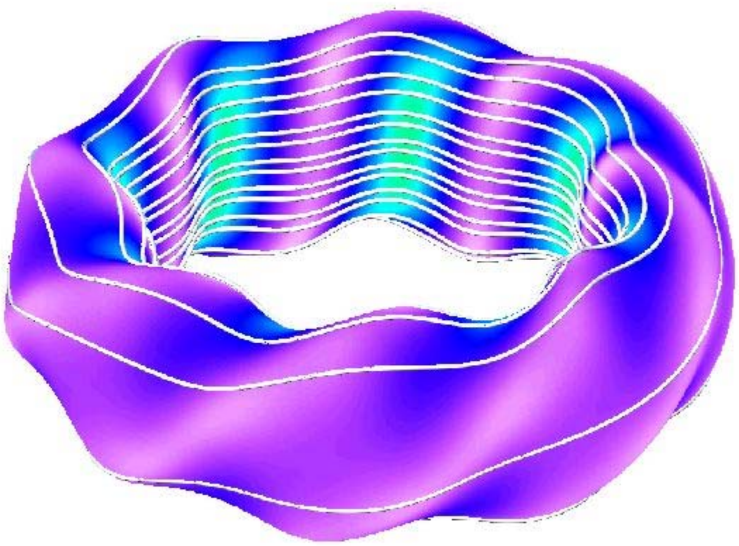


$$\bar{\mathbf{v}}_{gc,x} = \mp\frac{v_\parallel^2}{\omega_c R}\mathbf{i},\quad \bar{\mathbf{v}}_{gc,y} = \frac{v_\parallel^2}{R}\bar{t}\mathbf{j} = \mathbf{a}_{cp}\bar{t}$$

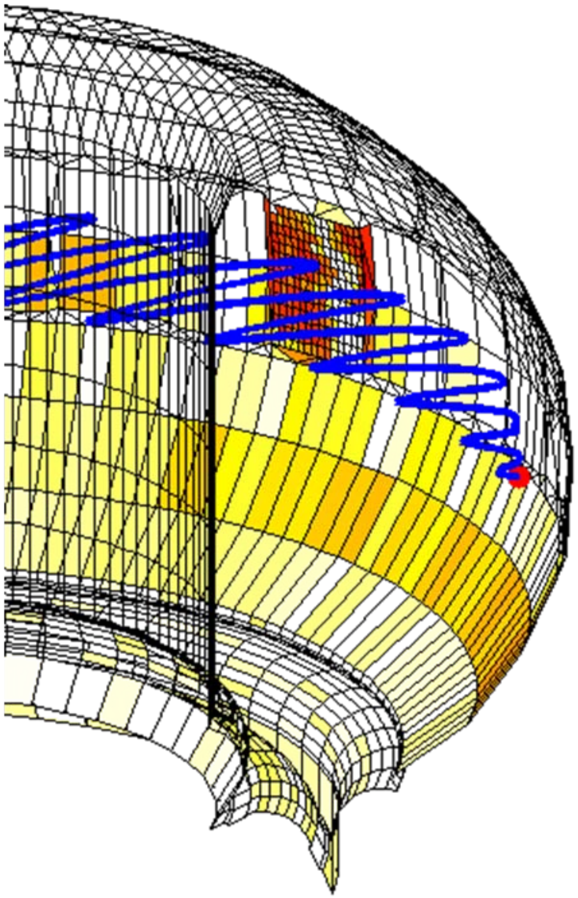
$$\mathbf{v}_{D,R} = \bar{\mathbf{v}}_{gc,x} = \frac{mv_\parallel^2}{qB_0^2}\frac{\mathbf{R}_0\times\mathbf{B}_0}{R^2}$$

Individual Charge Trajectories

- **Curvature Drift**



http://www.physics.ucla.edu/icnsp/Html/spong/spong.htm

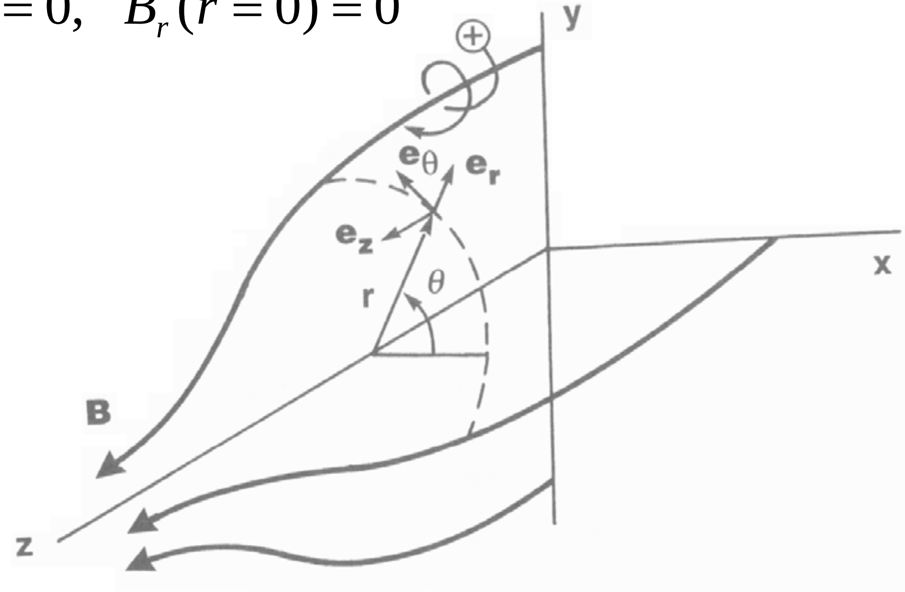


Individual Charge Trajectories

- **Axial Field Variations**

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{v}\times B_z\mathbf{e}_z\right) + q\left(\mathbf{v}\times B_r\mathbf{e}_r\right)$$

$$\nabla\cdot\mathbf{B} = 0 \rightarrow B_r \approx -\frac{r}{2}\frac{\partial B_z}{\partial z} \qquad B_\theta = 0,\quad B_r(r=0) = 0$$



$$\mathbf{F}_\parallel = -\frac{1}{2}\frac{mv_\perp^2}{B}\nabla_\parallel B = -\mu\nabla_\parallel B$$

μ : magnetic moment of the gyrating particle

c.f. $\mathbf{v}_{D,\nabla B} = \pm\frac{1}{2}v_\perp r_L\frac{\mathbf{B}\times\nabla B}{B^2}$

$$\mathbf{F}_\perp = \mp\frac{q}{2}v_\perp r_L\nabla_\perp B = -\frac{mv_\perp^2/2}{B} = -\mu\nabla_\perp B$$

Individual Charge Trajectories

- **Invariant of Motion**

$$\frac{d}{dt}E_0 = \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = 0 \qquad \mu = \frac{mv_\perp^2/2}{B}$$

$$\mathbf{F}_\parallel = m\frac{d\mathbf{v}_\parallel}{dt} = -\mu\nabla_\parallel\mathbf{B} = -\mu\frac{\partial\mathbf{B}}{\partial s} = -\mu\frac{\partial\mathbf{B}}{\partial s}\cdot\frac{ds}{dt}\cdot\frac{1}{v_\parallel} = -\frac{\mu}{v_\parallel}\frac{d\mathbf{B}}{dt} \longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right) = -\mu\frac{dB}{dt}$$

$$\longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = \frac{d}{dt}(\mu B) + \left(-\mu\frac{dB}{dt}\right) = 0$$

$$\longrightarrow \frac{d}{dt}(\mu) = 0$$: adiabatic invariant

- If B is constant
- $\frac{r_L}{B}\nabla_\parallel B << 1$
- $\frac{1}{\omega_c B}\frac{dB}{dt} << 1$