

**Introduction to  
Nuclear Fusion  
(409.308A, 3 Credits)**

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- Present Status and Future Prospect

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- Fusion Reactions

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Week 4. Review of Plasma Physics

- Plasma Confinement, Transport,  
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Week 5. Inertial Confinement

Week 6. Magnetic Confinement

- Mirror, Pinches, and Stellarator

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Week 6. Magnetic Confinement

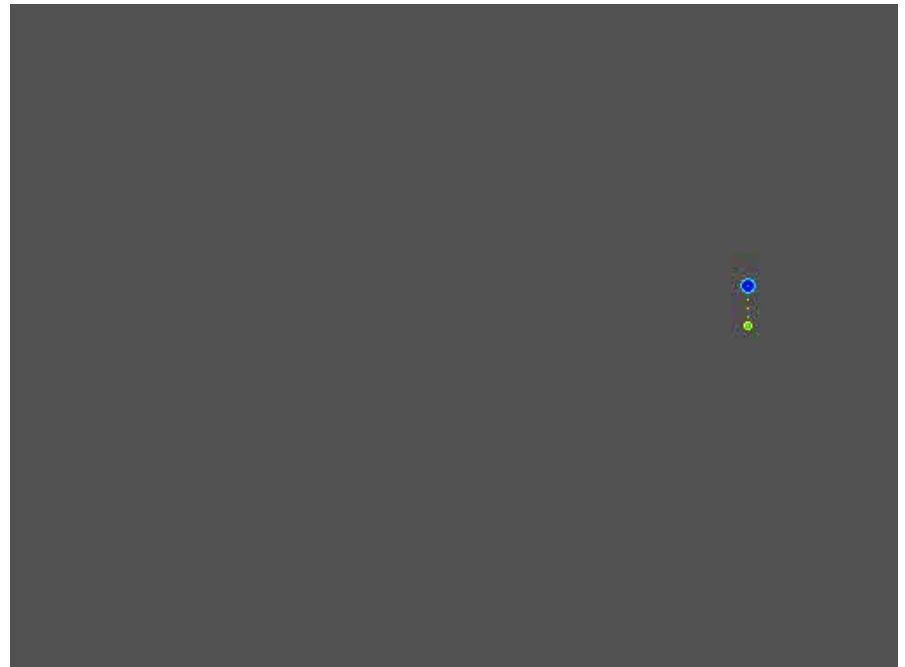
- Mirror, Pinches, and Stellarator

# Individual Charge Trajectories

- **Equation of Motion**

- Basic relation determining the motion of an individual charged particle of mass  $m$  and charge  $q$  in a combined electric ( $\mathbf{E}$ ) and magnetic ( $\mathbf{B}$ ) field
- Neglecting electromagnetic fields generated by movement of the charge itself

$$m \frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$



# Individual Charge Trajectories

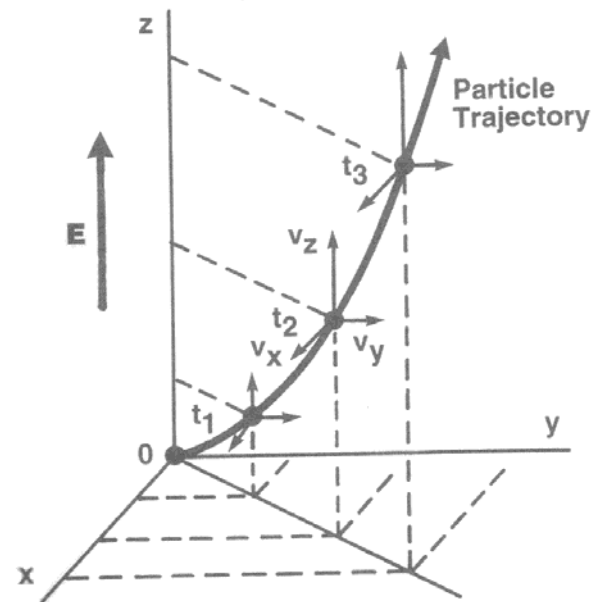
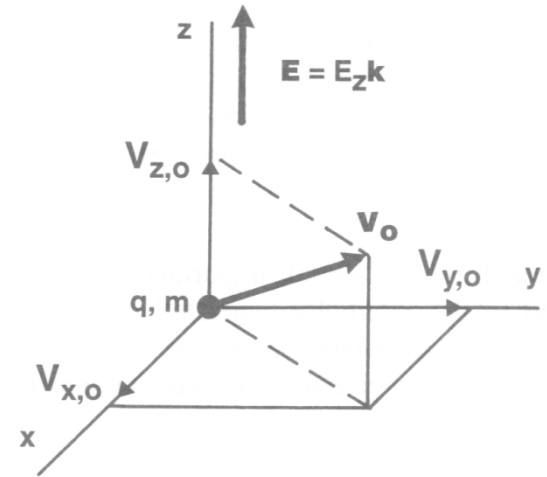
- Homogeneous Electric Field

$$m \frac{d\mathbf{v}}{dt} = qE_z \mathbf{k}$$

$$v_x(0) = v_{x,0}, \quad v_y(0) = v_{y,0}, \quad v_z(0) = v_{z,0}$$

$$x(t) = v_{x,0}t, \quad y(t) = v_{y,0}t,$$

$$z(t) = v_{z,0}t + \left( \frac{q}{m} E_z \right) t^2$$

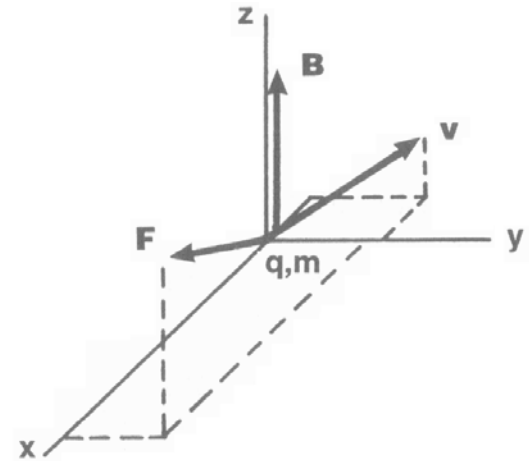


# Individual Charge Trajectories

- Homogeneous Magnetic Field

$$m \frac{d\mathbf{v}}{dt} = q(\mathbf{v} \times B_z \mathbf{k})$$

$$v_z(0) = v_{z,0} = v_{\parallel}$$



$$v_x(t) = v_{\perp} \cos(\omega_c t + \phi), \quad v_y(t) = \mp v_{\perp} \sin(\omega_c t + \phi), \quad v_z(t) = v_{z,0} = v_{\parallel}$$

$$x(t) = x_0 + \frac{v_{\perp}}{\omega_c} \sin(\omega_c t + \phi), \quad y(t) = y_0 \pm \frac{v_{\perp}}{\omega_c} \cos(\omega_c t + \phi), \quad z(t) = v_{\parallel} t + z_0$$

$$v_{\perp} = \sqrt{v_x^2 + v_y^2}, \quad \tan(\phi) = \mp \frac{v_y(0)}{v_x(0)}, \quad \omega_c = \frac{|q|B_z}{m} \text{ cyclotron frequency}$$

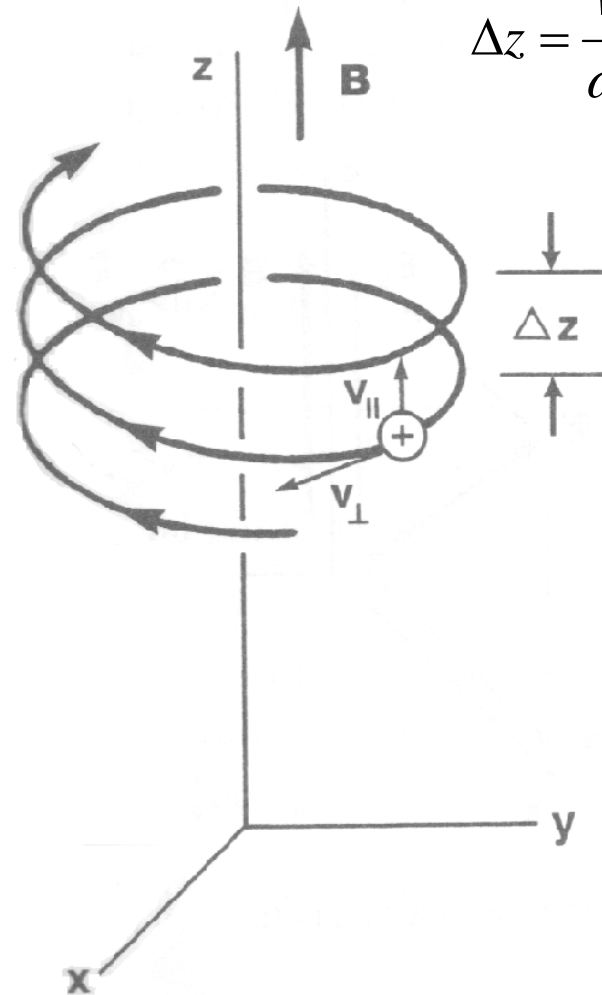
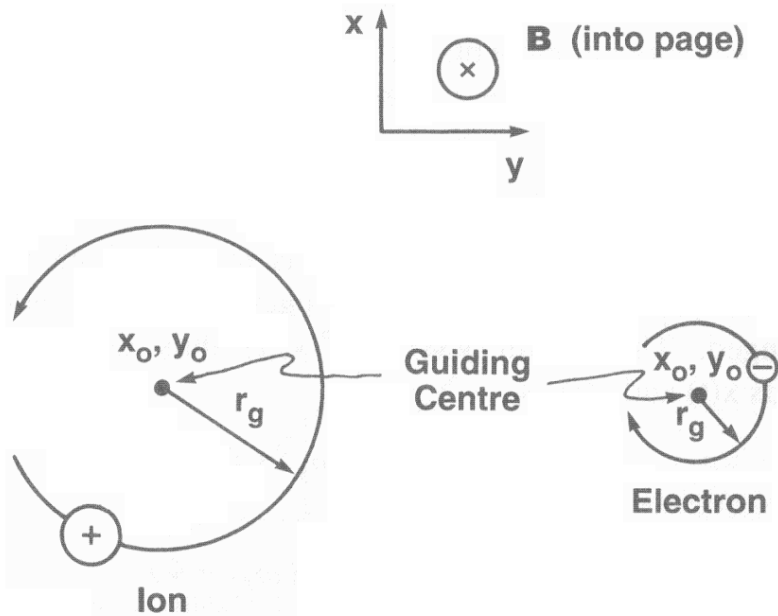
$$(x - x_0)^2 + (y - y_0)^2 = \left( \frac{v_{\perp}}{\omega_c} \right)^2 = \left( \frac{v_{\perp} m}{|q| B_z} \right)^2 = r_L^2 \text{ Larmor radius}$$

# Individual Charge Trajectories

- Homogeneous Magnetic Field

Pitch of the helix

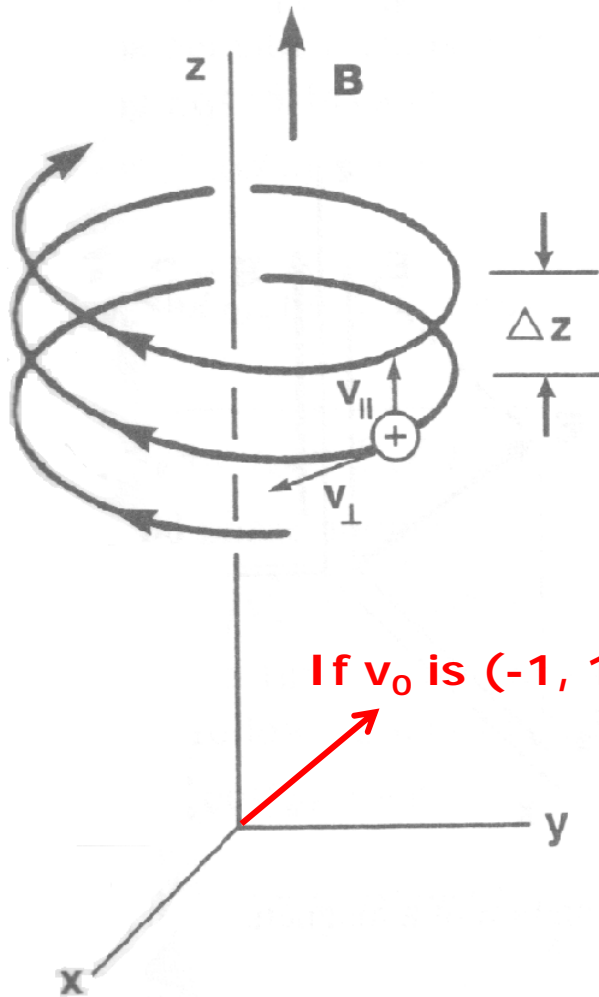
$$\Delta z = \frac{v_{\parallel}}{\omega_c}$$





# Individual Charge Trajectories

- Homogeneous Magnetic Field



If  $\mathbf{v}_0$  is  $(-1, 1, 1)$ ?



Magnetic field



ion

# Individual Charge Trajectories

- Combined Homogeneous Electric and Magnetic Field

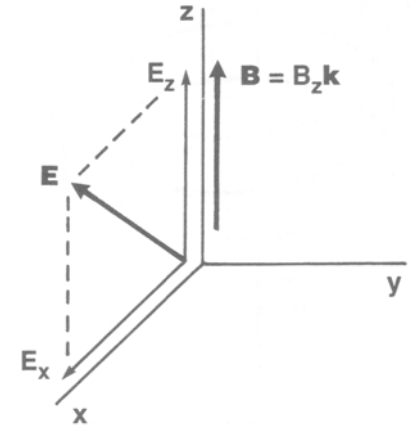
$$m \frac{d\mathbf{v}}{dt} = q \left[ (E_x \mathbf{i} + E_z \mathbf{k}) + \mathbf{v} \times B_z \mathbf{k} \right]$$

$$v_z(0) = v_{z,0} = v_{\parallel}$$

$$v_x(t) = v_0^* \cos(\omega_c t + \phi^*), \quad v_y(t) = \mp v_0^* \sin(\omega_c t + \phi^*) - \frac{E_x}{B_z},$$

$$v_z(t) = v_{\parallel} + \left( \frac{qE_z}{m} \right) t$$

$$v_0^* = \sqrt{v_x^2 + \left( v_y + \frac{E_x}{B_z} \right)^2} = \frac{v_0 \cos \phi}{\cos \phi^*}, \quad \tan(\phi^*) = \mp \frac{v_y(0) + \frac{E_x}{B_z}}{v_x(0)}$$



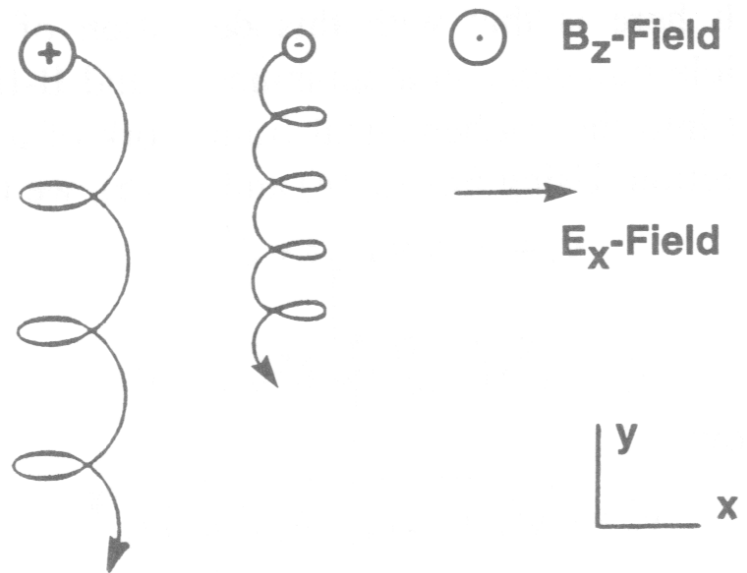
# Individual Charge Trajectories

- Combined Homogeneous Electric and Magnetic Field

$$x(t) = x_0 + \frac{v_0^*}{\omega_c} \sin(\omega_c t + \phi^*), \quad y(t) = y_0 \pm \frac{v_0^*}{\omega_c} \cos(\omega_c t + \phi^*) - \frac{E_x}{B_z} t,$$

$$z(t) = z_0 + v_{\parallel} t + \frac{1}{2} \left( \frac{qE_z}{m} \right) t^2$$

$$(x - x_0)^2 + \left( y - y_0 + \frac{E_x}{B_z} t \right)^2 = \left( \frac{v_0^*}{\omega_c} \right)^2$$



# Individual Charge Trajectories

- Combined Homogeneous Magnetic Field and an Arbitrary Force

$$\mathbf{v} = \mathbf{v}_{gc} + \mathbf{v}_g$$

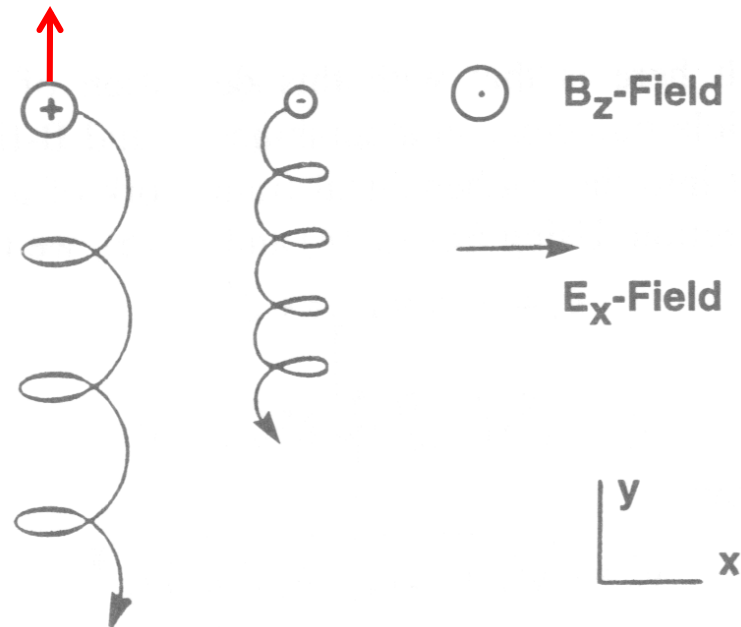
$$m \frac{d\mathbf{v}_{gc}}{dt} + m \frac{d\mathbf{v}_g}{dt} = F + q(\mathbf{v}_{gc} \times \mathbf{B}) + q(\mathbf{v}_g \times \mathbf{B})$$

$$v_{gc,\parallel}(t) = v_{gc,\parallel}(0) + \frac{1}{m} \int F_{\parallel} dt$$

$$\bar{\mathbf{v}}_{gc,\perp} = \frac{\bar{\mathbf{F}} \times \mathbf{B}}{qB^2} = \mathbf{v}_{DF}$$

$$\mathbf{v}_{D,E} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

If  $\mathbf{v}_0$  is (0, 1, -1)?

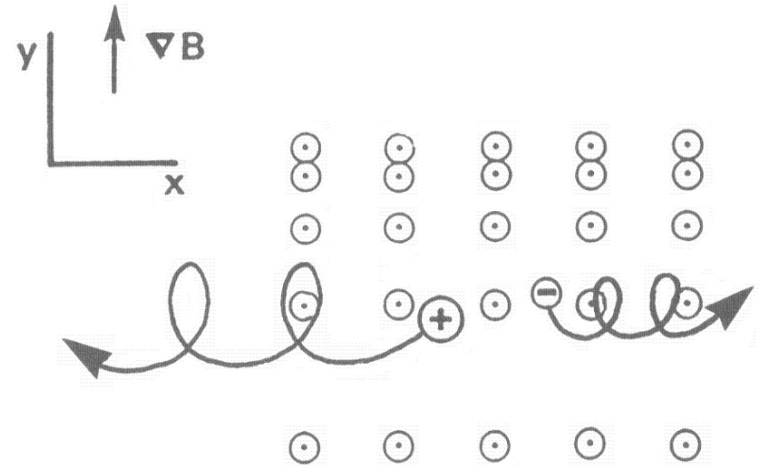
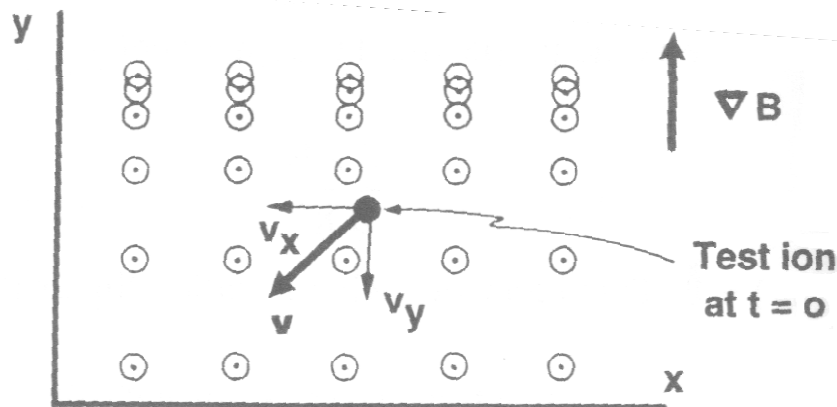


# Individual Charge Trajectories

- Spatially Varying Magnetic Field

$$m \frac{d\mathbf{v}}{dt} = q[\mathbf{v} \times \mathbf{B}(r)]$$

$$\nabla B = \frac{\partial B_z}{\partial y} \mathbf{j}, \quad v_{\parallel} = 0$$



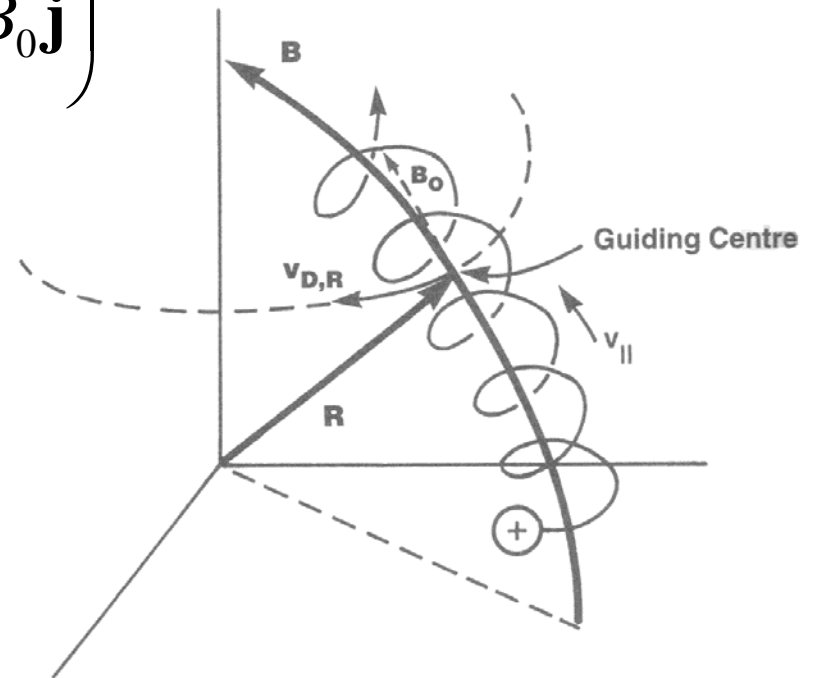
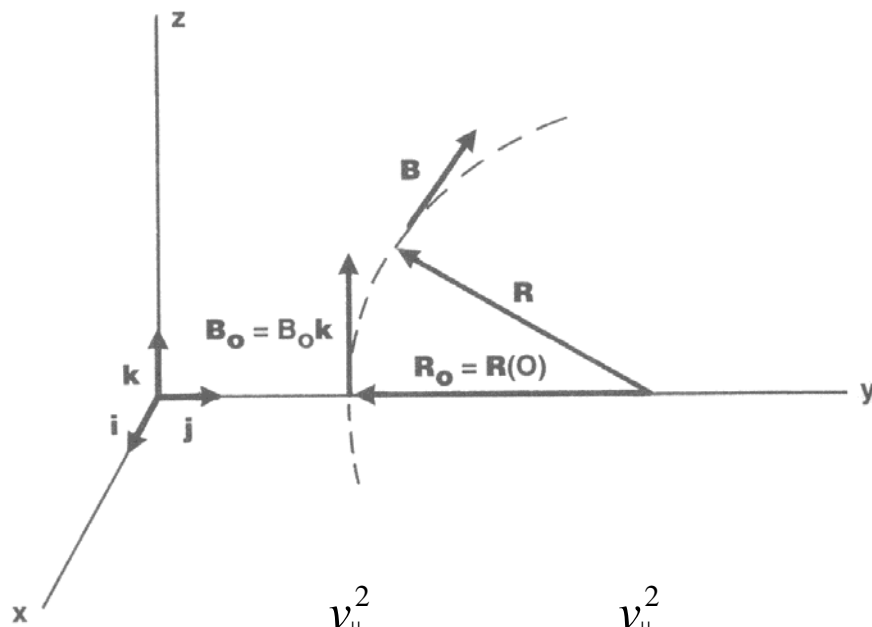
$$\begin{aligned} \bar{\mathbf{F}} &\approx -|q| \left( \frac{dB}{dy} \right) \left( \frac{v_{\perp}^2}{2\omega_c} \right) \mathbf{j} = -|q| \frac{v_{\perp}^2}{2\omega_c} \nabla B \\ &= -\frac{|q|}{2} v_{\perp} r_L \nabla B = -\frac{mv_{\perp}^2 / 2}{B} \nabla B \end{aligned}$$

$$\mathbf{v}_{D, \nabla B} = \pm \frac{v_{\perp}^2}{2\omega_c} \frac{\mathbf{B} \times \nabla B}{B^2} = \pm \frac{1}{2} v_{\perp} r_L \frac{\mathbf{B} \times \nabla B}{B^2}$$

# Individual Charge Trajectories

- Curvature Drift

$$m \frac{d\mathbf{v}}{dt} = q(\mathbf{v} \times B_0 \mathbf{k}) + q \left( \mathbf{v} \times \frac{z}{R} B_0 \mathbf{j} \right)$$

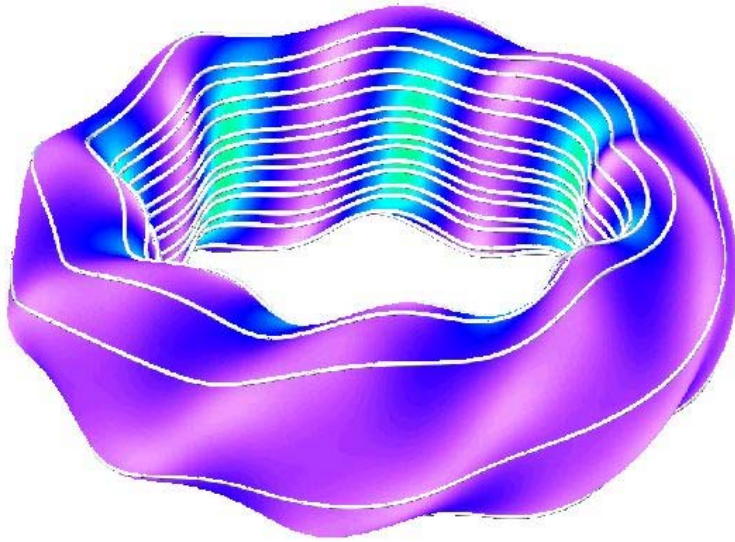


$$\bar{\mathbf{v}}_{gc,x} = \mp \frac{v_{||}^2}{\omega_c R} \mathbf{i}, \quad \bar{\mathbf{v}}_{gc,y} = \frac{v_{||}^2}{R} \bar{t} \mathbf{j} = \mathbf{a}_{cp} \bar{t}$$

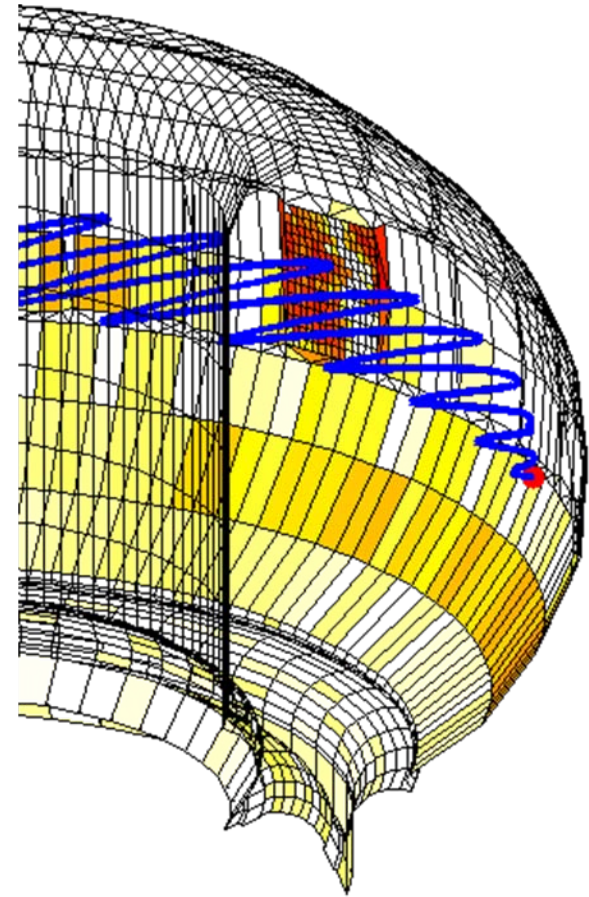
$$\mathbf{v}_{D,R} = \bar{\mathbf{v}}_{gc,x} = \frac{mv_{||}^2}{qB_0^2} \frac{\mathbf{R}_0 \times \mathbf{B}_0}{R^2}$$

# Individual Charge Trajectories

- Curvature Drift



<http://www.physics.ucla.edu/icnsp/Html/spong/spong.htm>



# Individual Charge Trajectories

- Axial Field Variations

$$m \frac{d\mathbf{v}}{dt} = q(\mathbf{v} \times B_z \mathbf{e}_z) + q(\mathbf{v} \times B_r \mathbf{e}_r)$$

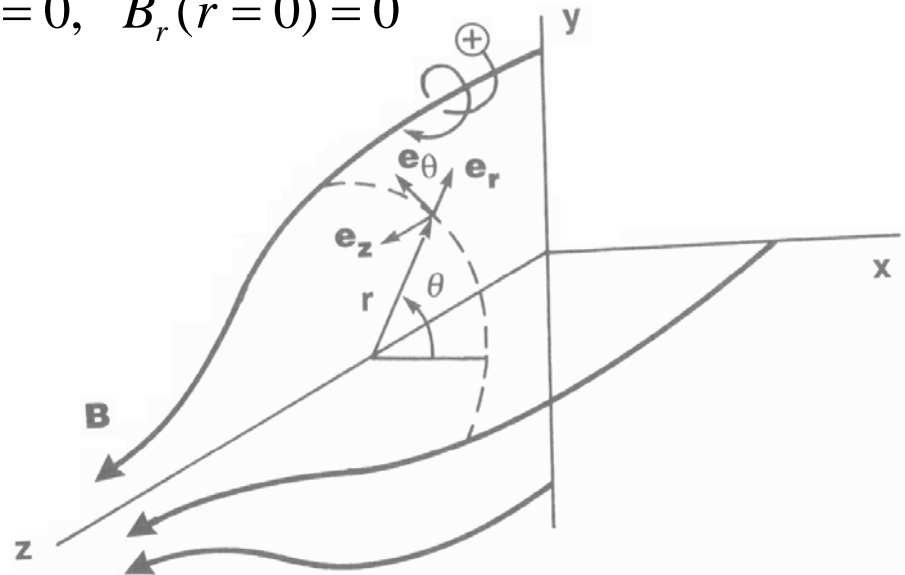
$$\nabla \cdot \mathbf{B} = 0 \rightarrow B_r \approx -\frac{r}{2} \frac{\partial B_z}{\partial z} \quad B_\theta = 0, \quad B_r(r=0) = 0$$

$$\mathbf{F}_{\parallel} = -\frac{1}{2} \frac{mv_{\perp}^2}{B} \nabla_{\parallel} B = -\mu \nabla_{\parallel} B$$

$\mu$ : magnetic moment of the gyrating particle

c.f.  $\mathbf{v}_{D,\nabla B} = \pm \frac{1}{2} v_{\perp} r_L \frac{\mathbf{B} \times \nabla B}{B^2}$

$$\mathbf{F}_{\perp} = \mp \frac{q}{2} v_{\perp} r_L \nabla_{\perp} B = -\frac{mv_{\perp}^2/2}{B} = -\mu \nabla_{\perp} B$$





# Individual Charge Trajectories

## • Invariant of Motion

$$\frac{d}{dt} E_0 = \frac{d}{dt} \left( \frac{1}{2} m v_{\perp}^2 + \frac{1}{2} m v_{\parallel}^2 \right) = 0 \quad \mu = \frac{m v_{\perp}^2 / 2}{B}$$

$$\mathbf{F}_{\parallel} = m \frac{d v_{\parallel}}{dt} = -\mu \nabla_{\parallel} \mathbf{B} = -\mu \frac{\partial \mathbf{B}}{\partial s} = -\mu \frac{\partial \mathbf{B}}{\partial s} \cdot \frac{ds}{dt} \cdot \frac{1}{v_{\parallel}} = -\frac{\mu}{v_{\parallel}} \frac{d\mathbf{B}}{dt} \rightarrow \frac{d}{dt} \left( \frac{1}{2} m v_{\parallel}^2 \right) = -\mu \frac{dB}{dt}$$

$$\rightarrow \frac{d}{dt} \left( \frac{1}{2} m v_{\perp}^2 + \frac{1}{2} m v_{\parallel}^2 \right) = \frac{d}{dt} (\mu B) + \left( -\mu \frac{dB}{dt} \right) = 0$$

$$\rightarrow \frac{d}{dt} (\mu) = 0 : \text{adiabatic invariant}$$

- If B is constant

$$- \frac{r_L}{B} \nabla_{\parallel} B \ll 1$$

$$- \frac{1}{\omega_c B} \frac{dB}{dt} \ll 1$$

