재료의 전자기적 성질

PART II.
OPTICAL PROPERTIES OF MATERIALS

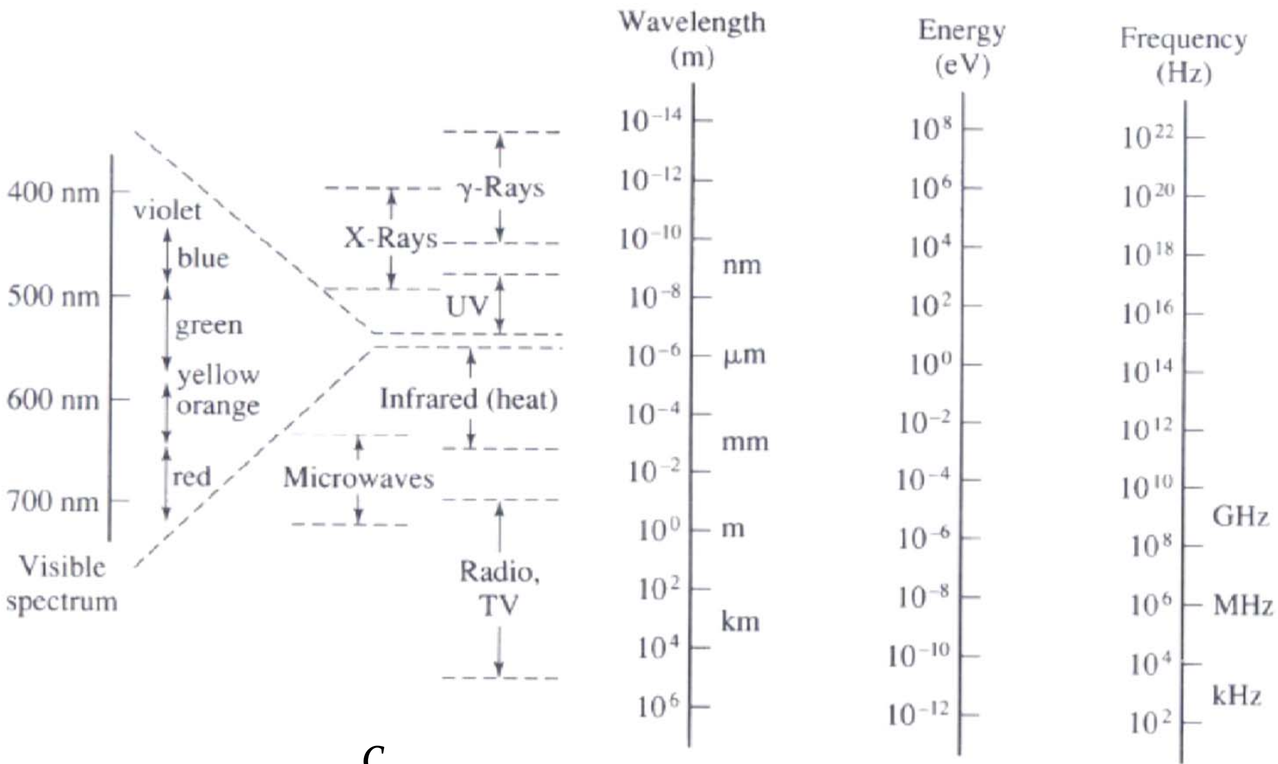
7 The Optical Constants

Goethe (1749-1832): **"Treatise on Color"**

Color is not an absolute property of matter (such as the resistivity), but requires a living being for its perception and description.

If the color blue is removed from the spectrum, then blue, violet, and green are missing and red and yellow remain." Thin gold films are bluish-green when viewed in transmission. These colors are missing in reflection. Consequently, gold appears reddish-yellow.

7.1 Introduction



$$E = h\nu = h\frac{c}{\lambda}$$

$$E\lambda = hc = (6.626\times10^{-34}(J\cdot\sec))\bullet(3\times10^{8}(m))$$

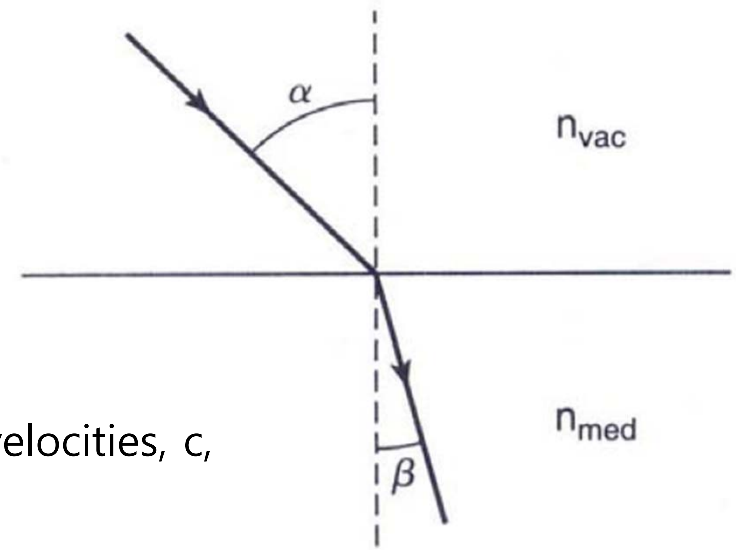
$$= (6.626\times10^{-34}/1.6\times10^{-19}(J/eV))\bullet(3\times10^{14}(\mu m))$$

$$= 1.24 eV\bullet\mu m$$

7.2 Index of Refraction, n

Snell's law

$$\frac{\sin\alpha}{\sin\beta} = \frac{n_{med}}{n_{vac}} = n.$$



Index of refraction:

the refraction is caused by the different velocities, c, of the light in the two media

$$\frac{\sin\alpha}{\sin\beta} = \frac{c_{vac}}{c_{med}} \quad \Rightarrow \quad n = \frac{c_{\mathrm{vac}}}{c_{\mathrm{med}}} = \frac{c}{\upsilon}$$

Dispersion:

A properties that the magnitude of the refractive index depending on the wavelength of the incident light

7.3 Damping Constant, *k*

Metals damp the intensity of light in a relatively short distance. Thus, an additional materials constant is needed to characterize the optical properties of metals.

Electromagnetic Wave Equation

$$c^2 \frac{\partial^2 E_x}{\partial z^2} = \varepsilon \frac{\partial^2 E_x}{\partial t^2} + \frac{\sigma}{\varepsilon_0} \frac{\partial E_x}{\partial t}$$

$$E_x = E_0 \exp\left[i\omega \left(t - \frac{zn}{c} \right) \right]$$

Trial Solution

$$E_x = E_0 \exp[i\omega(t - \frac{z \cdot n}{c})]$$

$$= E_0 \exp[i(\omega t - \frac{z \cdot n \cdot \omega}{c})]$$

$$= E_0 \exp[i(\omega t - \frac{z \cdot (\frac{c}{v}) \cdot 2\pi(\frac{v}{\lambda})}{c})]$$

$$= E_0 \exp[i(\omega t - z(\frac{2\pi}{\lambda})]$$

$$= E_0 \exp[i(\omega t - kz)]$$

$$\hat{n}^2 = \varepsilon - \frac{\sigma}{\varepsilon_0 \omega} i = \varepsilon - \frac{\sigma}{2\pi\varepsilon_0 \nu} i$$

Complex index of refraction,

Set $\hat{n} = n_1 - in_2$, $\hat{n} = n - ik$

$$\hat{n}^2 = n^2 - k^2 - 2nki = \varepsilon - \frac{\sigma}{2\pi\varepsilon_0 \nu} i$$

where, n_2 and k is called the **damping constant**

$$\varepsilon = n^2 - k^2 \qquad \sigma = 4\pi\varepsilon_0 nk\nu$$

$$\hat{n}^2 = n^2 - k^2 - 2nik \equiv \hat{\varepsilon} = \varepsilon_1 - i\varepsilon_2$$

$$\varepsilon_1 = n^2 - k^2 \qquad \varepsilon_2 = 2nk = \frac{\sigma}{2\pi\varepsilon_0\nu}$$

Absorption

Where, ε_1 and ε_2 are called the real and imaginary parts of the complex dielectric constant, $\hat{\varepsilon}$, respectively.

For insulators (σ=0), $k \approx 0$, then $\varepsilon = n^2$ **Maxwell relation**

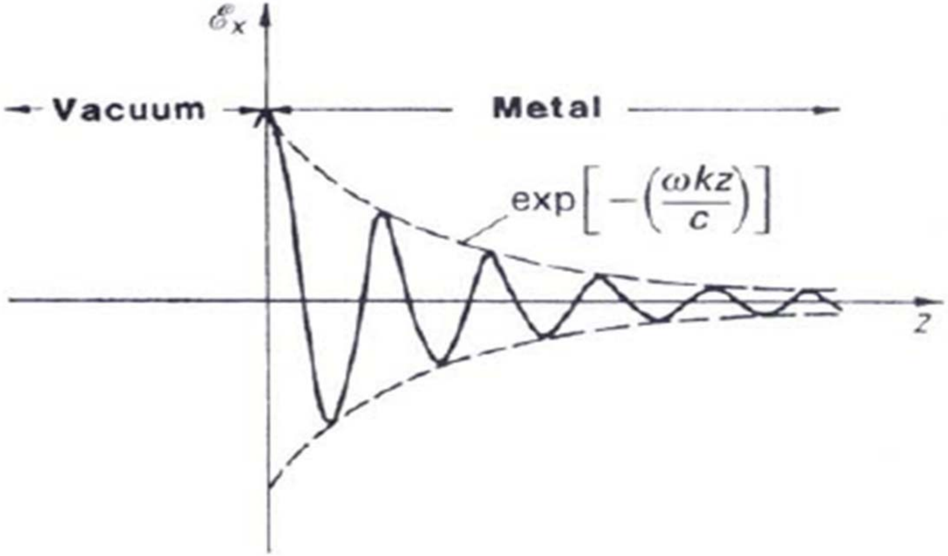
$$n^2 = \frac{1}{2}\left(\sqrt{\varepsilon^2 + \left(\frac{\sigma}{2\pi\varepsilon_0\nu}\right)^2} + \varepsilon\right) = \frac{1}{2}\left(\sqrt{\varepsilon_1^2 + \varepsilon_2^2} + \varepsilon_1\right)$$

$$k^2 = \frac{1}{2}\left(\sqrt{\varepsilon^2 + \left(\frac{\sigma}{2\pi\varepsilon_0\nu}\right)^2} - \varepsilon\right) = \frac{1}{2}\left(\sqrt{\varepsilon_1^2 + \varepsilon_2^2} - \varepsilon_1\right)$$

$$E_x = E_0 \exp\left[i\omega\left(t - \frac{z(n-ik)}{c}\right)\right]$$

$$E_x = E_0 \exp\left[-\frac{\omega k}{c} z\right] \cdot \exp\left[i\omega\left(t - \frac{zn}{c}\right)\right]$$

Damped amplitude **Undamped wave**



7.4 Characteristic Penetration Depth, *W*, and Absorbance,

$$I = E^2 = I_0 \exp\left(-\frac{2\omega k}{c} z\right)$$ ➡ **Damping Term**

$$\frac{I}{I_0} = \frac{1}{e} = e^{-1} \qquad z = W = \frac{c}{2\omega k} = \frac{c}{4\pi \nu k} = \frac{\lambda}{4\pi k}$$

Characteristic penetration depth

The inversion of W is sometimes called the (exponential) attenuation or the absorbance, α.

$$\alpha = \frac{4\pi k}{\lambda} = \frac{2\pi \varepsilon_2}{\lambda n} = \frac{\sigma}{n c \varepsilon_0} = \frac{2\omega k}{c}$$ ➡ **Inversion of W :**
It is related to the energy loss

7.5 Reflectivity, *R*, and Transmittance, *T*

$$R = \frac{I_R}{I_0}$$: **Reflectivity**

$$T = \frac{I_T}{I_0}$$: **Transmissivity**

$$R = \frac{(n-1)^2}{(n+1)^2} \Rightarrow R = \left|\frac{\hat{n}-1}{\hat{n}+1}\right|^2$$

N is generally a complex quantity,
By definition R has to remain real.
Thus, the modulus of R becomes

$$R = \frac{(n-ik-1)}{(n-ik+1)} \cdot \frac{(n+ik-1)}{(n+ik+1)} = \frac{(n-1)^2+k^2}{(n+1)^2+k^2}$$

(Beer equation)

$$R = \frac{n^2+k^2+1-2n}{n^2+k^2+1+2n}$$

(1) $$n^2 + k^2 = \sqrt{(n^2 + k^2)^2} = \sqrt{n^4 + 2n^2k^2 + k^2}$$

$$= \sqrt{n^4 - 2n^2k^2 + k^4 + 4n^2k^2} = \sqrt{(n^2 - k^2)^2 + 4n^2k^2}$$

$$= \sqrt{\varepsilon_1^2 + \varepsilon_2^2}$$

(2) $$2n = \sqrt{4n^2} = \sqrt{2(n^2 + k^2 + n^2 - k^2)} = \sqrt{2(\sqrt{\varepsilon_1^2 + \varepsilon_2^2} + \varepsilon_1)}$$

$$\boxed{R = \frac{\sqrt{\varepsilon_1^2 + \varepsilon_2^2} + 1 - \sqrt{2(\sqrt{\varepsilon_1^2 + \varepsilon_2^2} + \varepsilon_1)}}{\sqrt{\varepsilon_1^2 + \varepsilon_2^2} + 1 + \sqrt{2(\sqrt{\varepsilon_1^2 + \varepsilon_2^2} + \varepsilon_1)}}}$$

7.6 Hangen-Rubens Relation

Find out the relationship between reflectivity and conductivity.

For small frequencies (i.e., $\upsilon < 10^{13}\ s^{-1}$), with $\varepsilon \sim 10$

$$\frac{\sigma}{2\pi\varepsilon_0 \nu} \approx \frac{10^{17}}{10^{13}} \gg \varepsilon \qquad n^2 \approx \frac{\sigma}{2\pi\varepsilon_0 \nu} \approx k^2$$

$$n^2 = \frac{1}{2}\left(\sqrt{\varepsilon^2 + \left(\frac{\sigma}{2\pi\varepsilon_0\nu}\right)^2} + \varepsilon\right) = \frac{1}{2}\left(\sqrt{\varepsilon_1^2 + \varepsilon_2^2} + \varepsilon_1\right)$$

$$k^2 = \frac{1}{2}\left(\sqrt{\varepsilon^2 + \left(\frac{\sigma}{2\pi\varepsilon_0\nu}\right)^2} - \varepsilon\right) = \frac{1}{2}\left(\sqrt{\varepsilon_1^2 + \varepsilon_2^2} - \varepsilon_1\right)$$

$$R = \frac{n^2 + 2n + k^2 + 1 - 4n}{n^2 + 2n + 1 + k^2} = 1 - \frac{4n}{2n^2 + 2n + 1}$$

(neglecting $2n+1 \ll 2n^2$)

$$R = 1 - \frac{2}{n} = 1 - 2\sqrt{\frac{\nu}{\sigma}\pi\varepsilon_0}$$

$$\boxed{R = 1 - 2\sqrt{\frac{\nu}{\sigma_0}\pi\varepsilon_0}}$$

States that metals with large electrical conductivity are good reflectors

Hagen-Rubens Equation

(works for IR region)