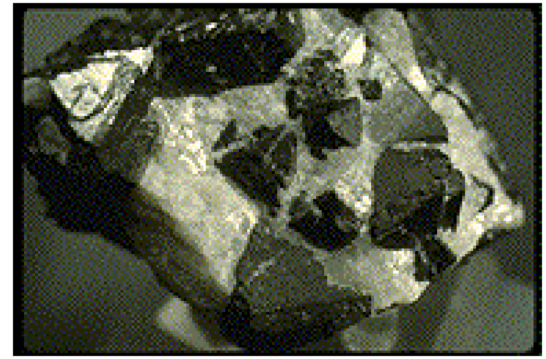
11

Foundations of Magnetism

11.1 Introduction



The magnetic rocks is lodestone which is the naturally
occurring mineral magnetite, Fe_3O_4

How do lodestones become magnetic?
Lodestones are rich in magnetite, an iron oxide mineral. Lightning with a typical current of 1,000,000 Amps creates a magnetic field large enough to saturate the magnetization of nearby lodestone outcrops. This is a rare event- on average lightning strikes that close to any point on the Earths surface happens once in 1-10 million years.

11.2 Basic Concepts in Magnetism

11.2 .1 Magnetic Poles

$$F = \frac{p_1 p_2}{d^2}$$

F = Force between two poles

p_1, p_2 = pole strength

d = distance between two poles

Propotionality constant = 1

$$F = pH$$

H = Field strength or field intensity

Propotionality constant = 1

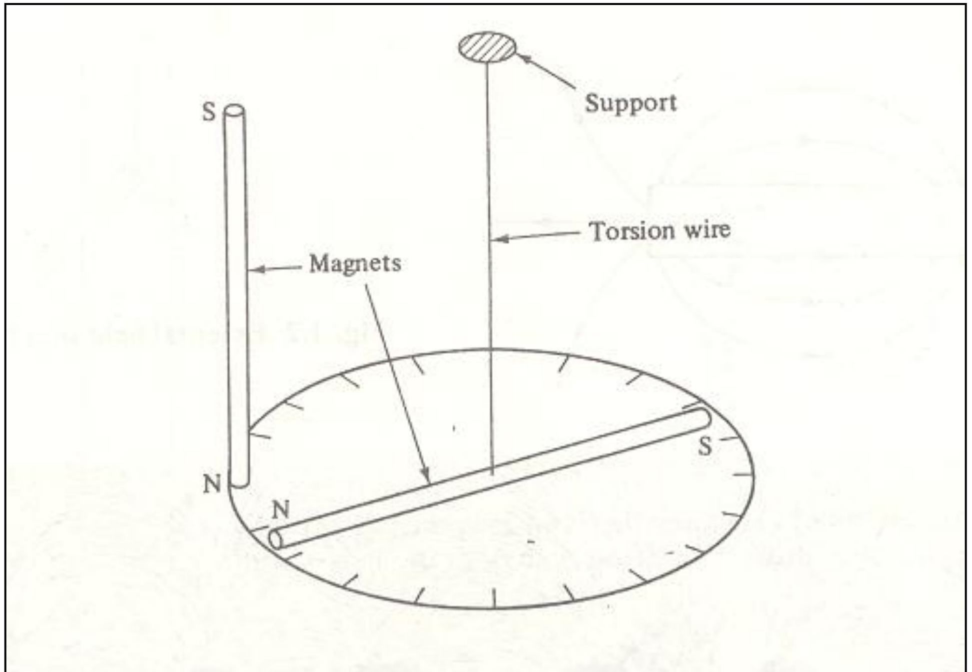


Fig. 1.1 Torsion balance for measuring the forces between poles.

Field of unit strength is one which exerts a force of one dyne on a unit pole.

$$H = \frac{p}{d^2}$$

1 Oe = 1 Line of force/cm^2 = 1 maxwell/cm^2

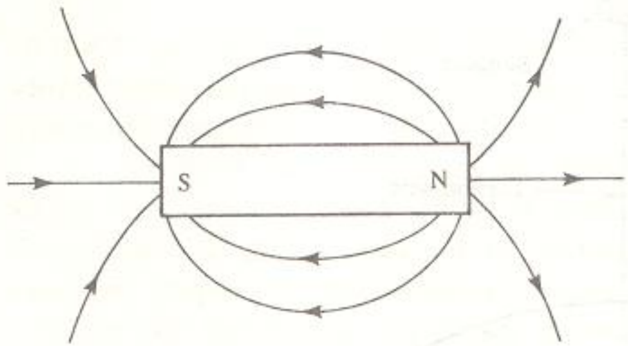


Fig. 1.2 External field of a bar magnet.

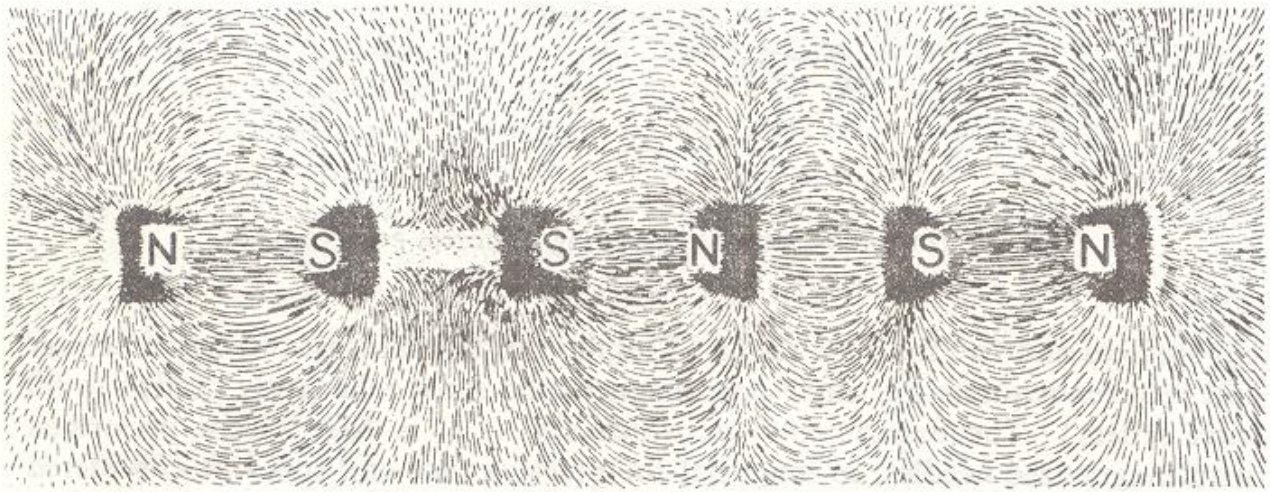
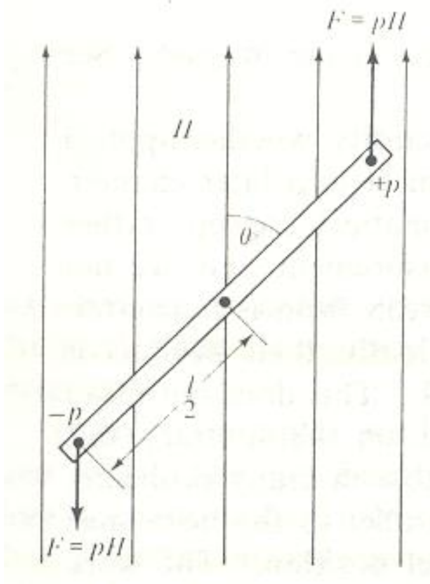


Fig. 1.3 Fields of bar magnets revealed by iron filings. (Courtesy Encyclopedia Brittanica).

11.2.2 Magnetic Moment



Moment of this couple, where p: pole strength

$$\frac{l}{2}(pH\sin\theta)+\frac{l}{2}(pH\sin\theta)=pHl\sin\theta$$

When H = 1 Oe and θ = 90°, the moment is given by

$$m = pl$$

m = magnetic moment of the magnet

$$dE_p = 2(\frac{l}{2})(pH\sin\theta)\,d\theta = mH\sin\theta\,d\theta$$ **(E_p = potential energy)**

$$E_p = \int_{90^\circ}^{\theta} mH\sin\theta\,d\theta = -mH\cos\theta$$

$$E_p = -\boldsymbol{m}\cdot\boldsymbol{H}$$ **(Vector Form)**

11.2.3 Intensity of magnetization

$$M = \frac{m}{V} = \frac{pl}{V} = \frac{p}{V/l} = \frac{p}{a}$$

M = intensity of magnetization or magnetization

V = volume of the magnet

a = cross-sectional area of the magnet

p = pole strength

l = interpolar distance

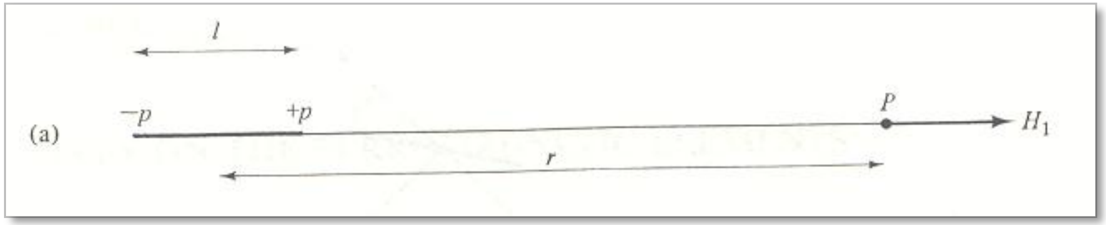
$$\sigma = \frac{m}{w} = \frac{m}{V\rho} = \frac{M}{\rho} \quad \text{emu/g}$$

σ = specific magnetization

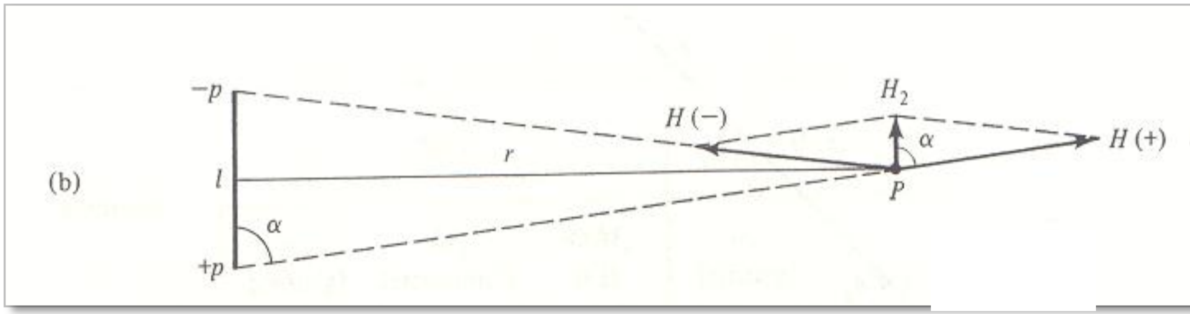
w = mass of magnet

ρ = density of magnet

11.2.4 Magnetic dipoles



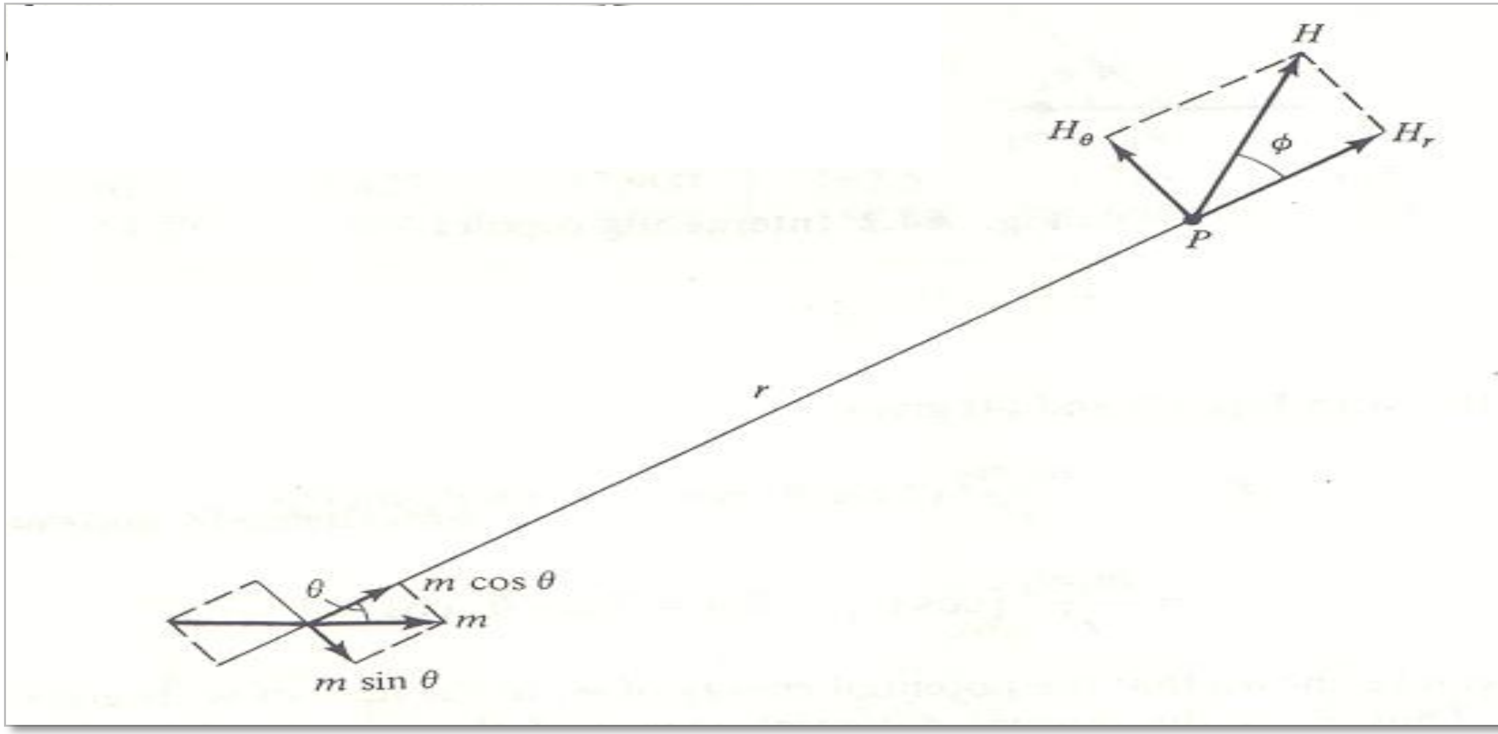
$$H_1 = \frac{p}{[r-(l/2)]^2} - \frac{p}{[r+(l/2)]^2} = \frac{2prl}{[r^2-(l^2/4)]^2}$$



$$H_2 = 2H(+)\cos\alpha = 2\left[\frac{p}{r^2+(l^2/4)}\right]\left[\frac{l/2}{\{r^2+(l^2/4)\}^{1/2}}\right] = \frac{pl}{[r^2+(l^2/4)]^{3/2}}$$

If r is large compared to l, this expression becomes

$$H_1 \cong \frac{2pl}{r^3} = \frac{2m}{r^3} \qquad H_2 \cong \frac{pl}{r^3} = \frac{m}{r^3}$$



$$H_r = \frac{2(m\cos\theta)}{r^3} \qquad H_\theta = \frac{m\sin\theta}{r^3}$$

$$H = (H_r^2 + H_\theta^2)^{1/2} = \frac{m}{r^3}(3\cos^2\theta + 1)^{1/2} \qquad \tan\phi = \frac{H_\theta}{H_r} = \frac{\tan\theta}{2}$$

$$H_p = H_r \cos\theta_2 - H_\theta \cos(90^\circ - \theta_2)$$

$$E_p = -m_2(H_r \cos\theta_2 - H_\theta \sin\theta_2)$$

$$H_r = \frac{2(m_1 \cos\theta_1)}{r^3}$$

$$H_\theta = \frac{m_1 \sin\theta_1}{r^3}$$

$$E_p = -\frac{m_1 m_2}{r^3}(2\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2)$$

$$= \frac{m_1 m_2}{r^3}[\cos(\theta_1 - \theta_2) - 3\cos\theta_1 \cos\theta_2]$$

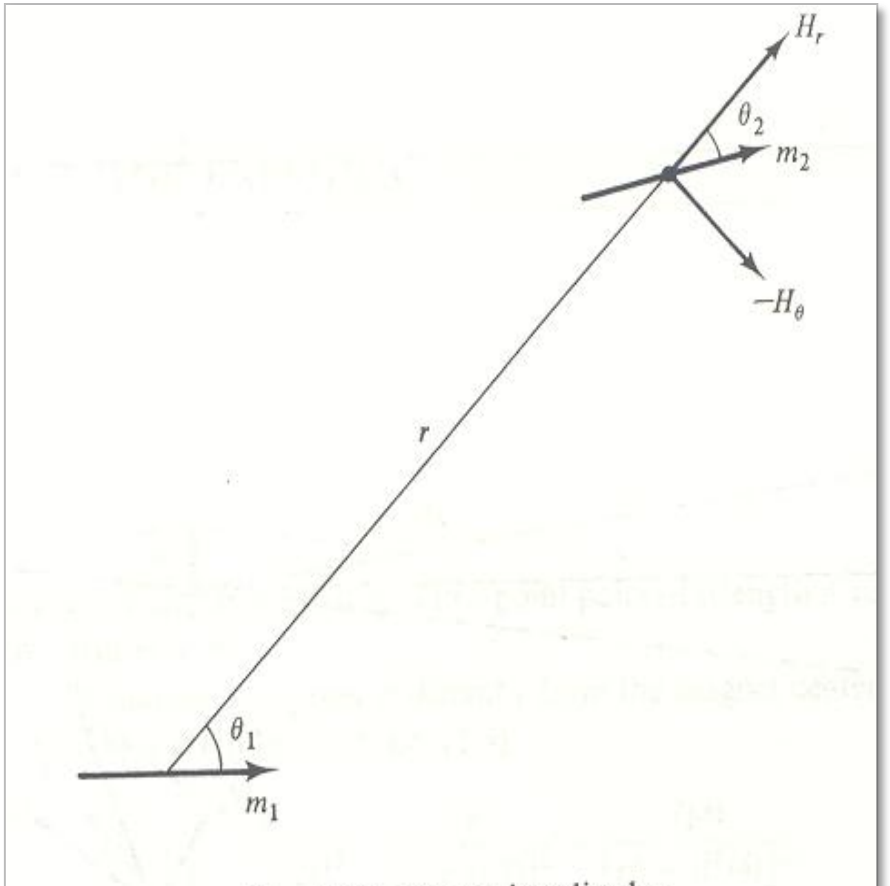
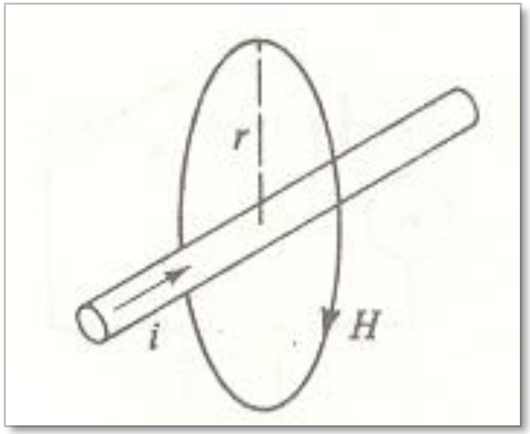


Fig. A4.2 Interacting dipoles.

11.2.5 Magnetic effects of currents



Outside the wire :

$$H = \frac{2i}{10r}\,\text{Oe}$$

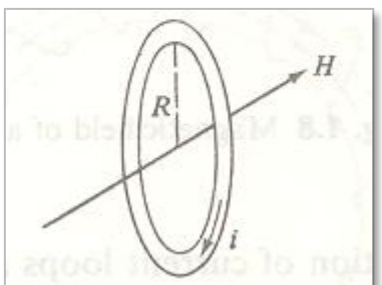
i = current in amperes

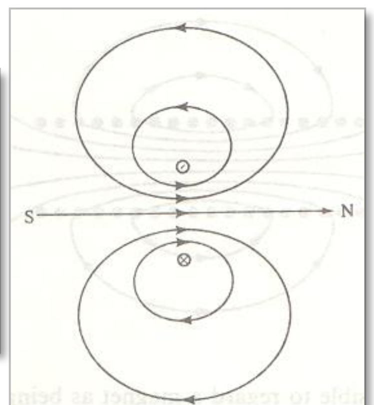
Inside the wire :

r = distance

$$H = \frac{2ir}{10r_0^2}\,\text{Oe}$$

r_0 = wire radius





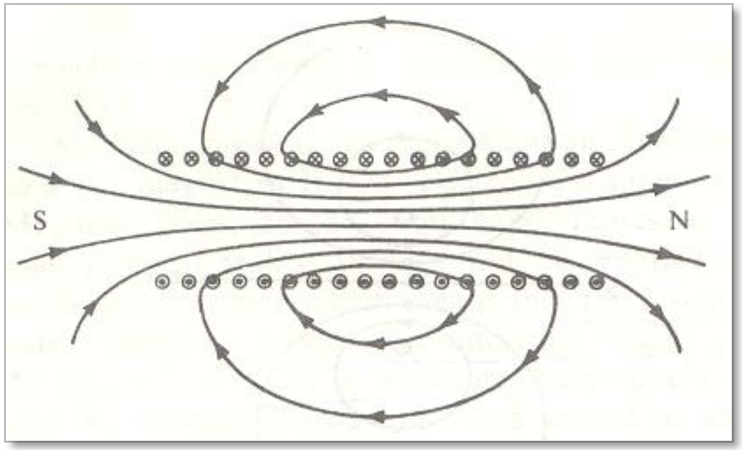
Field at the center along the axis :

$$H = \frac{2\pi i}{10R}\,\text{Oe}$$

Magnetic moment :

$$m\,(\text{loop}) = \frac{\pi R^2 i}{10} = \frac{Ai}{10}\,\text{erg/Oe}$$

A = area of the loop in cm^2



Field at the center along the axis :

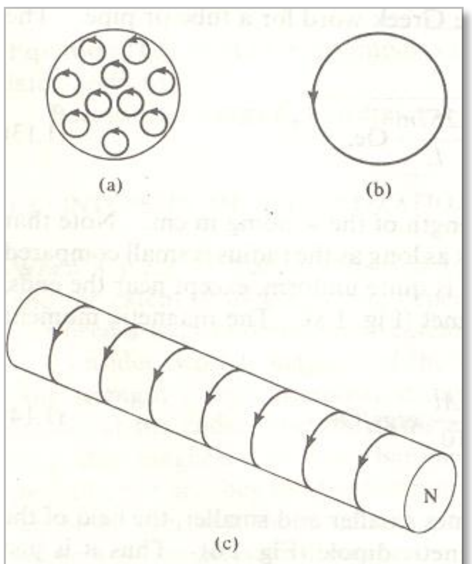
$$H = \frac{4\pi ni}{10L} = \frac{1.257ni}{L}\ \text{Oe}$$

Magnetic moment :

$$m\,(\text{solenoid}) = \frac{nAi}{10}\ \text{erg/Oe}$$

n = number of turns

L = length of the winding in cm



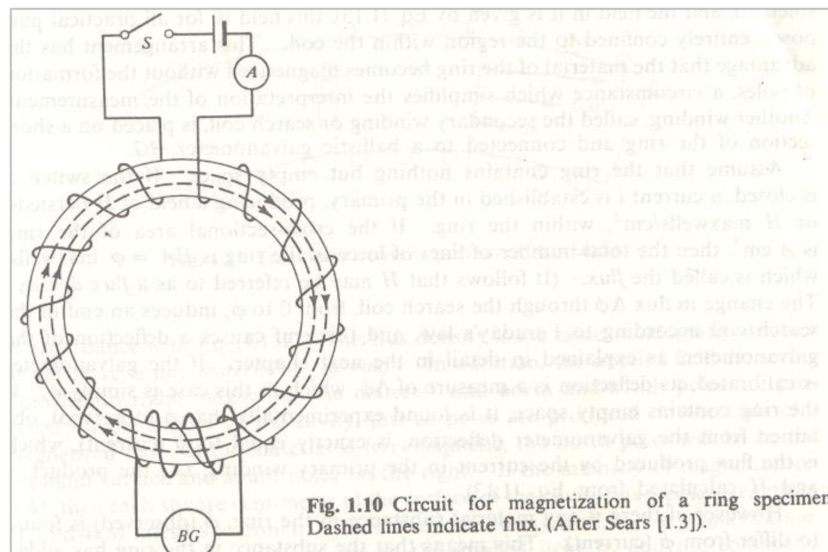


Fig. 1.10 Circuit for magnetization of a ring specimen. Dashed lines indicate flux. (After Sears [1.3]).

11.2.6 Varieties of magnetism

$$\phi = HA$$

ϕ = flux (concept of total flux)

H = field (maxwell/cm^2)

A = cross-sectional area of the ring (cm^2)

ϕ (observed) < ϕ (current), diamagnetic (for example, Cu, He)

ϕ (observed) > ϕ (current), paramagnetic (for example, Na Al)

or antiferromagnetic (for example, MnO, FeO)

ϕ (observed) ≫ ϕ (current), ferromagnetic (for example, Fe, Co, Ni)

or ferrimagnetic (for example, Fe_3O_4)

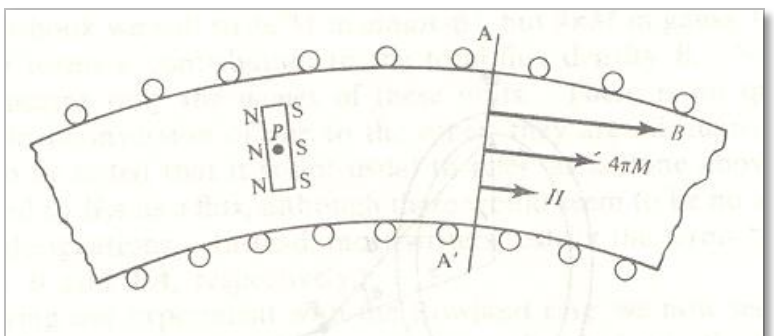


Fig. 1.11 Transverse cavity in portion of Rowland ring.

$$B = H + 4\pi M$$

The magnetic properties of a material are characterized not only by the magnitude and sign of M but also by the way in which M varies with H. The ratio of these two quantities is called the

susceptibility : $\kappa = \dfrac{M}{H}$ emu/cm^3 Oe

$$\chi = \kappa / \rho = \text{mass susceptibility (emu/g Oe), where } \rho = \text{density}$$

$$\chi_A = \chi A = \text{stomic susceptibility (emu/g atom Oe), where A = atomic weight}$$

$$\chi_M = \chi M' = \text{molecular susceptibility (emu/g mol Oe), where } M' = \text{molecular weight}$$

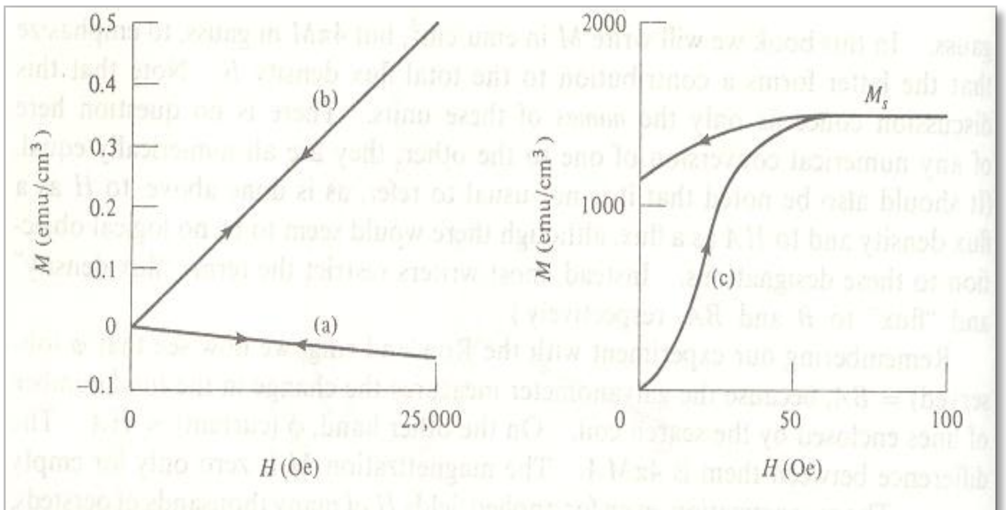


Fig. 1.13 Typical magnetization curves of (a) a diamagnetic, (b) a para- or antiferromagnetic, and (c) a ferro- or ferrimagnetic. (After Schoenberg [1.4]).

Saturation :

At large enough values of H, the Magnetization M becomes constant at saturation value of M_s.

Hysteresis or irreversibility :

After saturation, a decrease in H to zero does not reduce M to zero. Ferro- and ferrimagnetic materials can thus be made into permanent magnets

The ratio of B and H is the **permeability** :

$$\mu = \frac{B}{H}$$

Since $$B = H + 4\pi M$$

$$\frac{B}{H} = 1 + 4\pi \frac{M}{H}$$

$$\therefore \mu = 1 + 4\pi\kappa$$

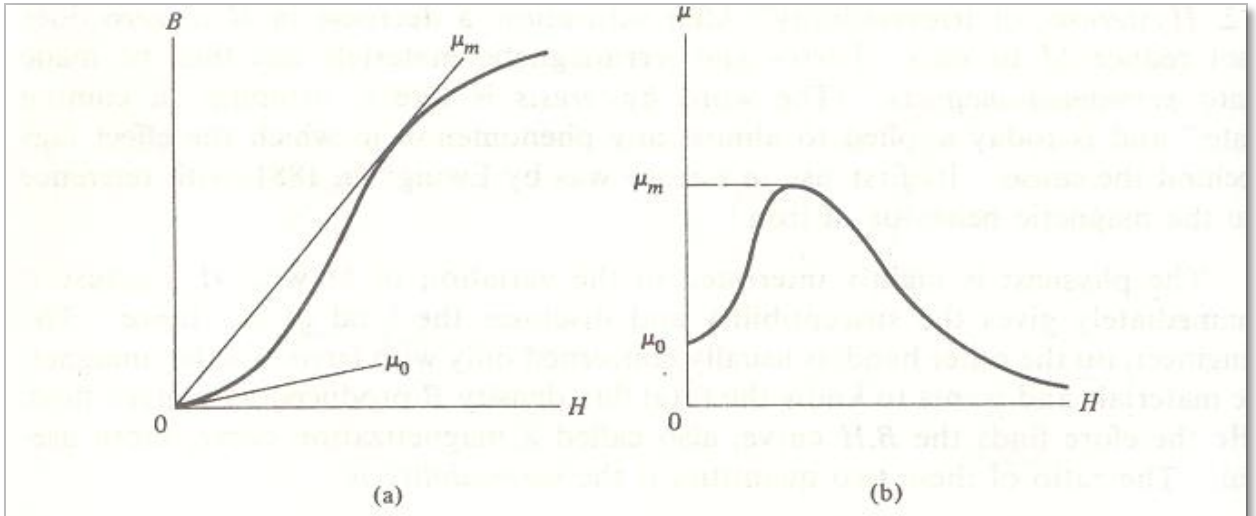
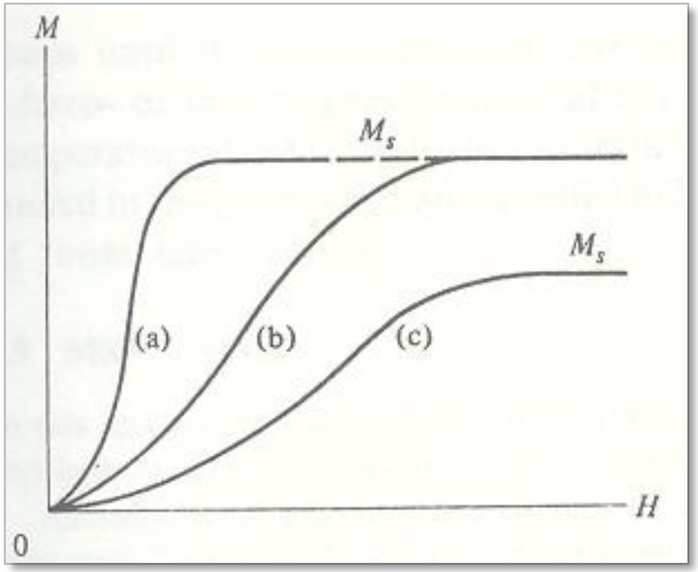


Fig. 1.14 (a) *B* versus *H* curve of a ferro- or ferrimagnetic, and (b) corresponding variation of μ with *H*.

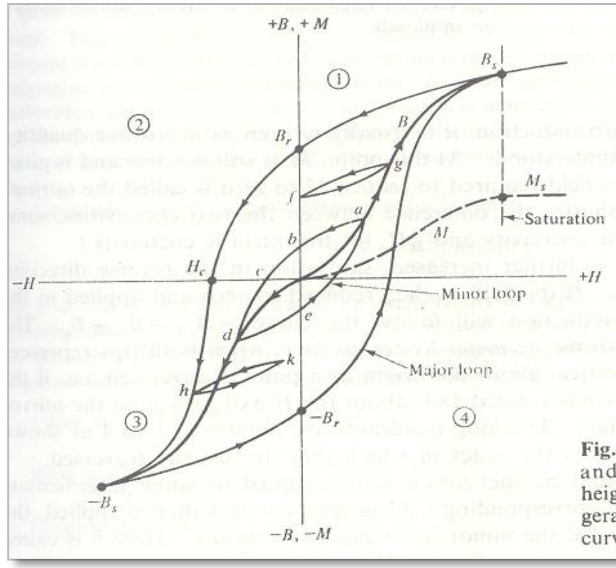
We can now characterize the magnetic behavior of various kinds of substances by their corresponding values of κ and μ :

1. **Empty Space :** K = 0, since there is no matter to magnetize, and μ = 1.
2. **Diamagnetic :** κ is small and negative, and μ slightly less than 1.
3. **Para- and antiferromagnetic :** κ s small and positive, and μ slightly greater than 1.
4. **Ferro- and ferrimagnetic :** κ and μ are large and positive, and both are functions of H.

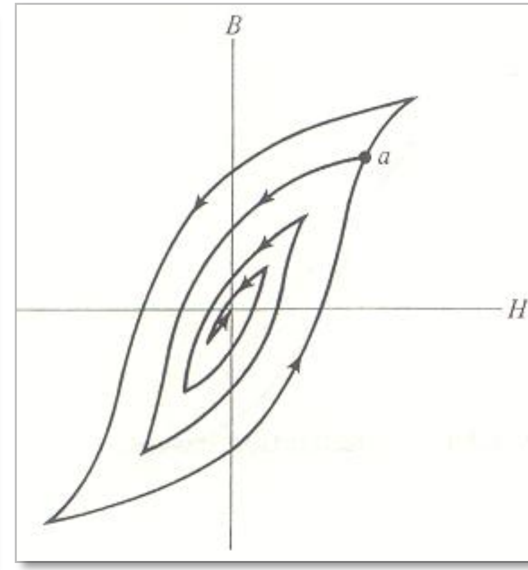
11.2.7 Magnetization curves and hysteresis loops



Magnetization curves



Magnetization curves and hysteresis loops. (The height of the M curve is exgerated relative to that of B curve.



Demagnetization by cycling with a decreasing amplitude.

11.3 Units

The scientific and technical literature on magnetism, particularly in the USA is still widely written in electromagnetic cgs(emu) units. In some European countries, and in many international scientific journals, the SI units are mandatory.

$$B = H + 4\pi M$$ (cgs units)

$$\frac{B}{H} = 1 + 4\pi \frac{M}{H} \quad \Leftarrow \quad \mu = \frac{B}{H}$$

$$\therefore \mu = 1 + 4\pi\kappa$$

11.3.1 MKS units

Coulomb's law of the force between poles: $F = \frac{p_1 p_2}{4\pi\mu_0 d^2}$ newtons

Force on a pole: $F = pH$ newtons

Field of a pole: $H = \frac{p}{4\pi\mu_0 d^2}$ ampere-turns/meter

Magnetic moment: $m = pl$ weber-meter

Potential energy: $E_{\mathrm{p}} = -mH\cos\theta$ joules

Magnetization: $M = \frac{m}{v} = \frac{\mathrm{p}}{\mathrm{a}}$ weber/meter^2

Field of straight wire: $H = \frac{i}{2\pi r}$ ampere/meter

Field of current loop: $H = \frac{i}{2R}$ ampere/meter

m (loop): $\mu_0 A i$ weber-meter

Field of solenoid: $H = \frac{ni}{L}$ ampere/meter

m (solenoid): $\mu_0 n A i$ weber-meter

Volume susceptibility: $\kappa = \frac{M}{H}$ weber/ampere meter

Absolute permeability: $\mu = \frac{B}{H}$ weber/ampere meter

Relative permeability: $\mu_r = \frac{\mu}{\mu_0} = \frac{B}{\mu_0 H}$

H: 1 ampere-turn/m $= 4\pi \times 10^{-3}$ oersted

B: 1 weber/meter^2 $= 10^4$ gauss $= 1$ tesla

M: 1 weber/meter^2 $= \frac{104}{4\pi}$ emu/cm^3

ϕ: 1 weber $= 10^8$ maxwells