Deformation of Concrete

Fall 2010

Dept of Architecture
Seoul National University

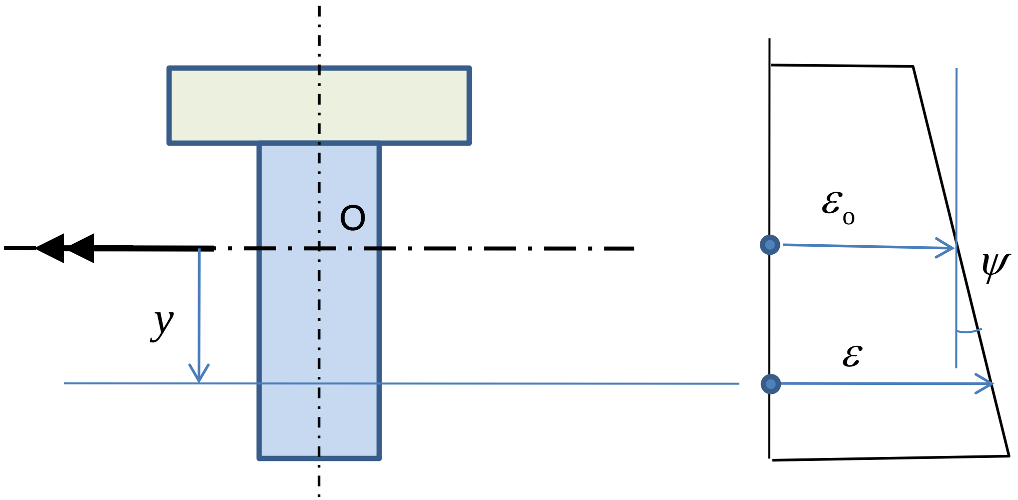
3rd week

Stress and Strain of uncracked sections

Chap 2. Stress and Strain of uncracked sections

1. Introduction
2. Sign convention
3. Curvature
4. Nonlinear temperature
5. Time-dependent s-s in composite section
6. General

2.2 sign convention



Section

Strain distribution

Compatibility condition

Plane sections remain after bending

$$\varepsilon = \varepsilon_o + \psi y$$

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Hookes Law

$$\sigma = E\left(\varepsilon_o + \psi y\right)$$

Integration of stress over the section

$$N = \int \sigma dA$$

$$M = \int \sigma y dA$$

Integration in terms of strain and E

$$N = \varepsilon_o \sum_i^m E_i \int dA + \psi \sum_i^m E_i \int y dA$$

$$M = \varepsilon_o \sum_i^m E_i \int y dA + \psi \sum_i^m E_i \int y^2 dA$$

Transformed section w.r.t. reference E

$$N = E_{ref}\left(A\varepsilon_o + B\psi\right)$$

$$M = E_{ref}\left(B\varepsilon_o + I\psi\right)$$

$$A = \sum_i^m \left(\frac{E_i}{E_{ref}} A_i\right)$$

$$B = \sum_i^m \left(\frac{E_i}{E_{ref}} B_i\right)$$

$$I = \sum_i^m \left(\frac{E_i}{E_{ref}} I_i\right)$$

In a matrix form

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = E_{ref} \begin{bmatrix} A & B \\ B & I \end{bmatrix} \begin{Bmatrix} \varepsilon_o \\ \psi \end{Bmatrix}$$

Strain distribution in a matrix form

$$\begin{Bmatrix} \varepsilon_o \\ \psi \end{Bmatrix} = \frac{1}{E_{ref}} \begin{bmatrix} A & B \\ B & I \end{bmatrix}^{-1} \begin{Bmatrix} N \\ M \end{Bmatrix} = \frac{1}{E_{ref}\left(AI - B^2\right)} \begin{bmatrix} I & -B \\ -B & A \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix}$$

If the reference axis is located through the neutral axis, B=0

$$\begin{Bmatrix} \varepsilon_o \\ \psi \end{Bmatrix} = \frac{1}{E_{ref}} \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix}^{-1} \begin{Bmatrix} N \\ M \end{Bmatrix} = \frac{1}{E_{ref}\left(AI\right)} \begin{bmatrix} I & 0 \\ 0 & A \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} = \frac{1}{E_{ref}} \begin{Bmatrix} N/A \\ M/I \end{Bmatrix}$$

$$\varepsilon_o = \frac{IN - BM}{E\left(AI - B^2\right)} \qquad \psi = \frac{-BN + AM}{E\left(AI - B^2\right)}$$

$$\sigma_o = \frac{IN - BM}{\left(AI - B^2\right)} \qquad \gamma = \frac{-BN + AM}{\left(AI - B^2\right)}$$