Deformation of Concrete

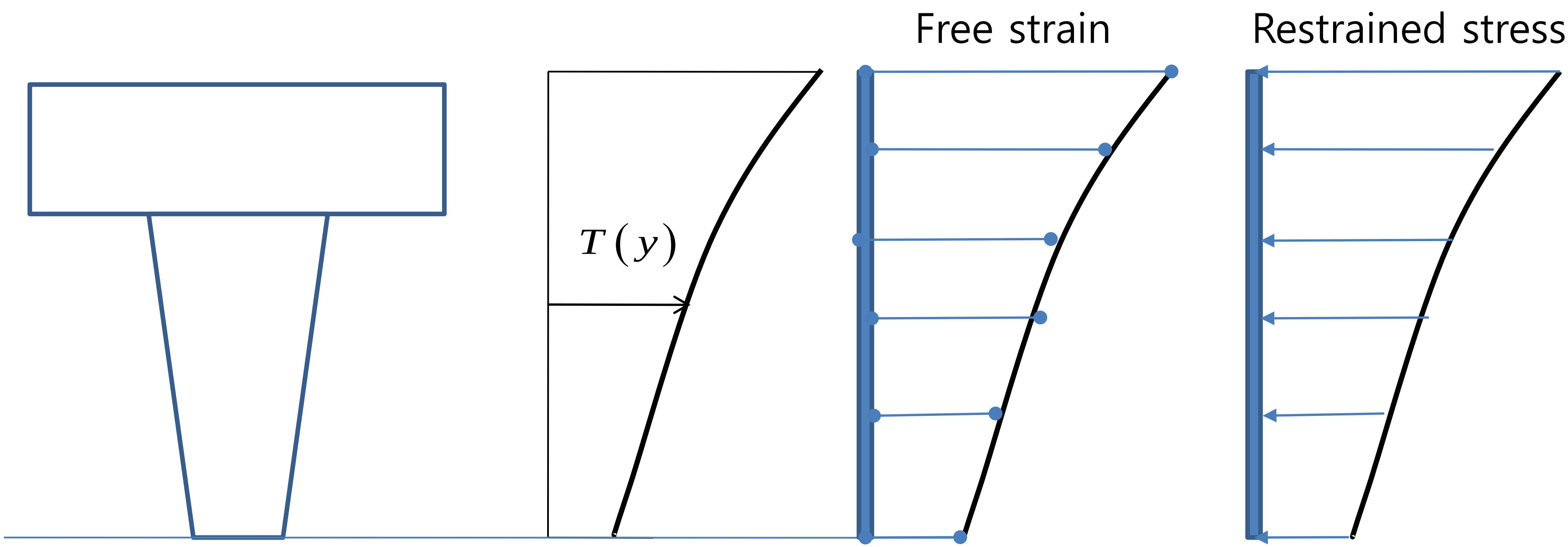
Fall 2010

Dept of Architecture
Seoul National University

4^{th} week

Thermal loading

2.4 Non-linear temperature variation



$$\varepsilon_f = \alpha_t T$$

$$\sigma = -E\varepsilon_f = -E\alpha_t T$$

Restrained stress resultant

$$\Delta N = -\int E\varepsilon_f dA$$

$$\Delta M = -\int E\varepsilon_f y dA$$

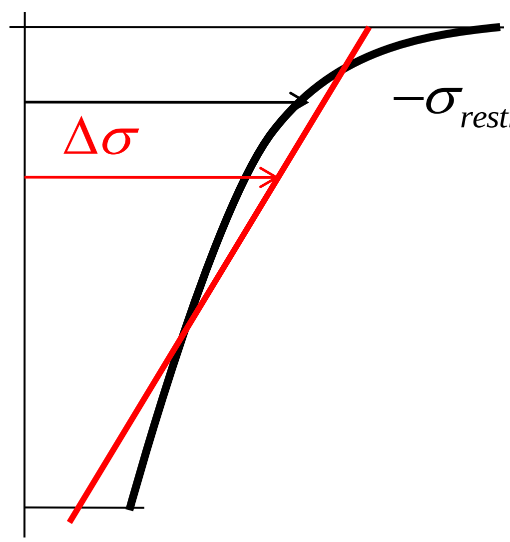
Strain due to restrained stress resultant

$$\begin{Bmatrix} \Delta\varepsilon_o \\ \Delta\psi \end{Bmatrix} = \frac{1}{E_{ref}\left(AI - B^2\right)} \begin{bmatrix} I & -B \\ -B & A \end{bmatrix} \begin{Bmatrix} -\Delta N \\ -\Delta M \end{Bmatrix}$$

Thermal stress distribution

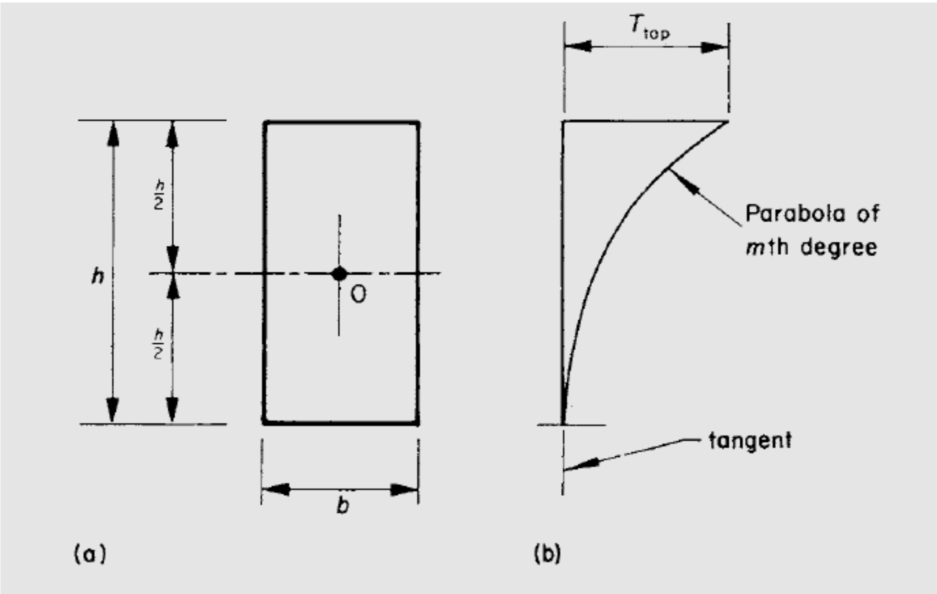
Stress due to restrained thermal strain $\Delta\sigma = E\left(\Delta\varepsilon_o + (\Delta\psi)y\right)$

Total stress $\sigma = \sigma_{restrained} + \Delta\sigma = E\left[-\varepsilon_f + \Delta\varepsilon_o + (\Delta\psi)y\right]$



$-\sigma_{restrained} = E\varepsilon_f = E\alpha_t T$

Ex. 2.1 Temperature varies over the depth in the form of a parabola of the *m*-th degree



$$T(y) = T_{top}\left(\frac{y+h/2}{h}\right)^m$$

$$\Delta N = -\int E\alpha_t T\, dA$$

$$= -\alpha_t T_{top} Eb\left[y\frac{1}{m+1}\left(\frac{h/2-y}{h}\right)^{m+1}\right]_{-h/2}^{h/2}$$

$$= -\alpha_t T_{top} Ebh\frac{1}{m+1}$$

$$\Delta M = -\int E\alpha_t T\, ydA$$

$$= -E\alpha_t T_{top}\frac{b}{h^m}\left[\left[\frac{(h/2-y)^{m+1}}{m+1}y\right]_{-h/2}^{h/2} - \int\frac{(h/2-y)^{m+1}}{m+1}dy\right]$$

$$= -E\alpha_t T_{top}\frac{b}{h^m}\left[\frac{h^{m+1}}{m+1}(-h/2) + \left[\frac{(h/2-y)^{m+2}}{(m+1)(m+2)}\right]_{-h/2}^{h/2}\right]$$

$$= E\alpha_t T_{top} bh^2\left[\frac{m}{2(m+1)(m+2)}\right]$$