Deformation of Concrete

Fall 2010

Dept of Architecture
Seoul National University

7th week

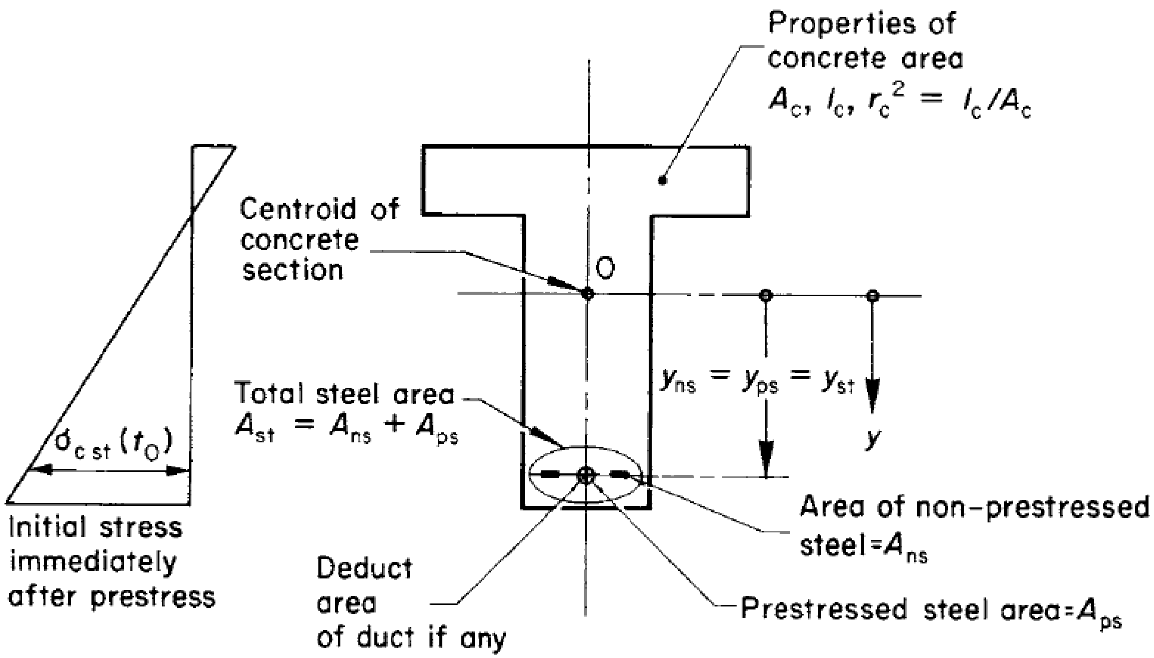
Special cases of uncracked sections

Chap. 3. Special Cases of uncracked sections and calculation of displacement

1. Introduction
2. Prestress loss
3. Effect of presence of non-prestressed steel
4. Reinforced concrete section w/o prestress
5. Approximate equations
6. Graphs
7. Multi-stage prestressing
8. Displacement

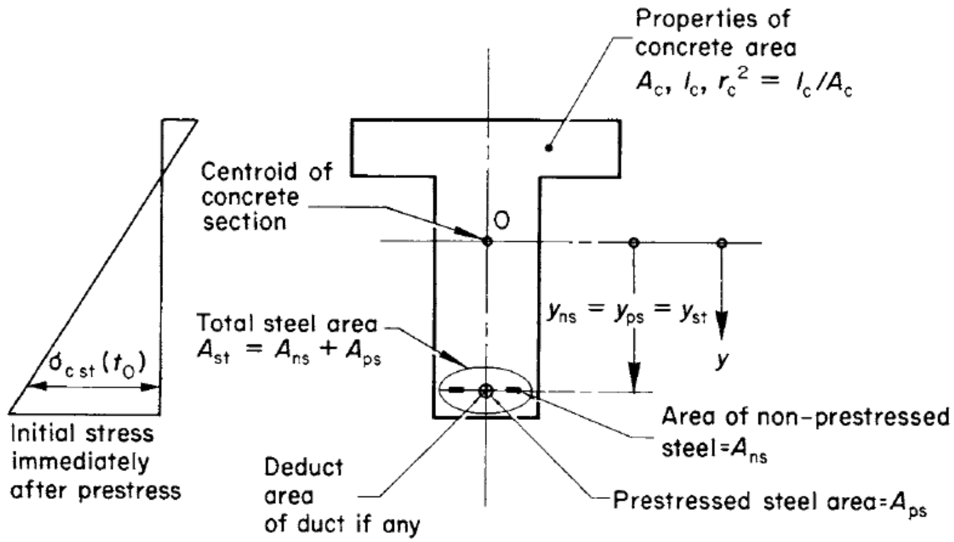
3.1 introduction

1. One type of concrete
2. Reinforcing bars and prestressing steel are located in one layer



Α	α	alpha	a	“f**a**ther”
Β	β	beta	b	
Γ	γ	gamma	g	
Δ	δ	delta	d	
Ε	ϵ	epsilon	e	“**e**nd”
Ζ	ζ	zêta	z	
Η	η	êta	ê	“h**ey**”
Θ	θ	thêta	th	“**th**ick”
Ι	ι	iota	i	“**i**t”
Κ	κ	kappa	k	
Λ	λ	lamda	l	
Μ	μ	mu	m	
Ν	ν	nu	n	
Ξ	ξ	xi	ks	“bo**x**”
Ο	ο	omikron	o	“**o**ff”
Π	π	pi	p	
Ρ	ρ	rho	r	
Σ	σ, ς	sigma	s	“**s**ay”
Τ	τ	tau	t	
Υ	υ	upsilon	u	“p**u**t”
Φ	ϕ	phi	f	
Χ	χ	chi	ch	“Ba**ch**”
Ψ	ψ	psi	ps	
Ω	ω	omega	ô	“gr**ow**”

3.2 prestress loss



Total reinforcement area

$$A_{st} = A_{ns} + A_{ps}$$

Modulus of elasticity

$$E_{st} = E_{ns} = E_{ps}$$

Equilibrium

$$\Delta P_c = -\Delta P_{ns} - \Delta P_{ps}$$

$$\Delta P_{ps} = A_{ps} \Delta \sigma_{ps}$$

$$\Delta P_{ns} = A_{ns} \Delta \sigma_{ns}$$

Compatibility condition

$$\Delta \varepsilon_s = \frac{\Delta \sigma_{ns}}{E_{st}} =$$ Creep + Shrinkage + due to ΔP_c

$$\frac{\Delta \sigma_{ns}}{E_{st}} = \frac{\Delta \sigma_{ps} - \Delta \bar{\sigma}_{pr}}{E_{st}} = \frac{\sigma_{cst}(t_0)\varphi(t,t_0)}{E_c(t_0)} + \varepsilon_{cs}(t,t_0) + \frac{1}{\bar{E}_c(t,t_0)}\left(\frac{\Delta P_c}{A_c} + \frac{\Delta P_c y_{st}^2}{I_c}\right)$$

$$\Delta P_c = -\frac{\varphi(t,t_0)\sigma_{cst}(t_0)A_{st}\left[E_{st}/E_c(t_0)\right] + \varepsilon_{cs}(t,t_0)E_{st}A_{st} + \Delta\bar{\sigma}_{pr}A_{ps}}{1 + \frac{A_{st}}{A_c}\frac{E_{st}}{E(t,t_0)}\left(1 + \frac{y_{st}^2}{r_c^2}\right)}$$

$$\Delta\bar{\sigma}_{pr} = \chi_r \Delta\sigma_{pr}$$

The value χ_r depends upon the magnitude of the total loss $\Delta\sigma_{ps}$ which is generally not known. Thus for calculation of the total loss due to creep, shrinkage and relaxation, an assumed value of $\Delta\bar{\sigma}_{pr}$ is substituted in Equation (3.4) to give a first estimate of ΔP_c. This answer is used to obtain an improved reduced relaxation value and Equation (3.4) is used again to calculate a better estimate of ΔP_c. In most cases, a first estimate of $\chi_r = 0.7$ followed by one iteration gives sufficient accuracy.

3.2.1 $\Delta\varepsilon$ and $\Delta\sigma$

$$\Delta\varepsilon_o = \varepsilon_{cs}(t,t_0) + \varphi(t,t_0)\,\varepsilon_o(t_0) + \frac{\Delta P_c}{\bar{E}_c(t,t_0)\,A_c}$$

$$\Delta\psi = \varphi(t,t_0)\,\psi(t_0) + \frac{\Delta P_c y_{st}}{\bar{E}_c(t,t_0)\,I_c}$$

Stress change in concrete

$$\Delta\sigma = \frac{\Delta P_c}{A_c} + \frac{\Delta P_c y_{st}}{I_c}\,y$$

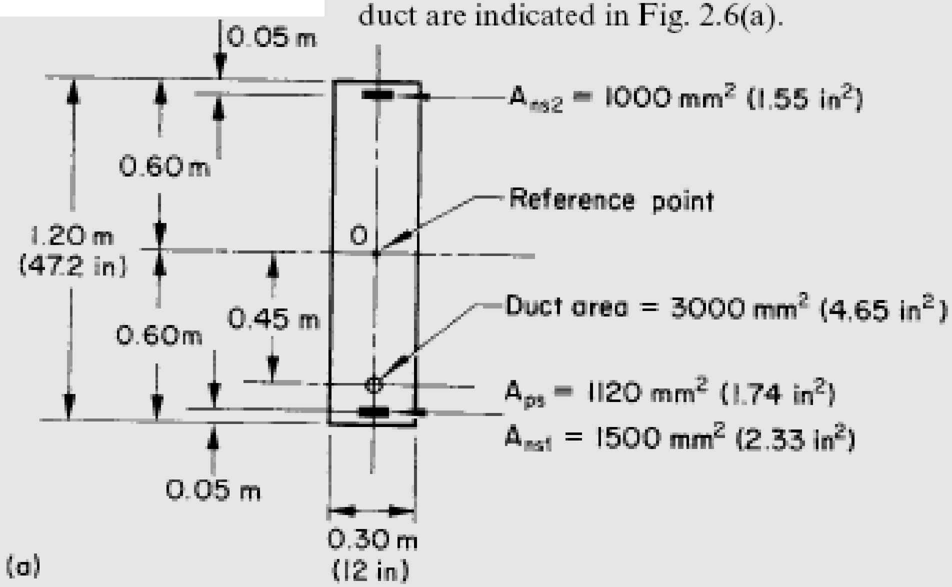
Since $I_c = A_c r_c^2$

$$\Delta\sigma_c = \frac{\Delta P_c}{A_c}\left(1 + \frac{y_{st}\,y}{r_c^2}\right)$$

$$\Delta\sigma_{ns} = E_{st}\left(\Delta\varepsilon_o + y_{st}\Delta\psi\right)$$

$$\Delta\sigma_{ps} = E_{st}\left(\Delta\varepsilon_o + y_{st}\Delta\psi\right) + \Delta\bar{\sigma}_{pr}$$

A prestress force $P = 1400 \times 10^3 N$ (315 kip) and a bending moment $M = 390 \times 10^3$ N-m (3450 kip-in) are applied at age t_0 on the rectangular post-tensioned concrete section shown in Fig. 2.6(a). Calculate the stresses, the axial strain and curvature at age t_0 and t given the following data: $E_c(t_0) = 30.0$ GPa (4350 ksi); $E_{ns} = E_{ps} = 200$ GPa (29×10^3 ksi); uniform free shrinkage value $\varepsilon_{cs}(t, t_0) = -240 \times 10^{-6}$; $\varphi(t, t_0) = 3$; $\chi = 0.8$; reduced relaxation, $\Delta\bar{\sigma}_{pr} = -80$ MPa (−12 ksi). The dimensions of the section and cross-section areas of the reinforcement and the prestress duct are indicated in Fig. 2.6(a).



$$I_c = 42.588 * 10^{-3} \text{ m}^4$$

$$A_c = 0.357 \text{ m}^2$$

$$r_c^2 = 0.1193 \text{ m}^2$$

$$P = 1400 \text{ kN}$$

$$M = 390 \text{ kN-m}$$

$$E_c(t_0) = 30 \text{ GPa}$$

Intrinsic relaxation $\Delta\sigma_{ps,\infty} = -115 \text{ MPa}$

Characteristic tensile strength $f_{ptk} = 1700 \text{ MPa}$

$$I_c = 42.588 * 10^{-3}\ \mathrm{m}^4$$

$$A_c = 0.357\ \mathrm{m}^2$$

$$r_c^2 = 0.1193\ \mathrm{m}^2$$

$$P = 1400\ \mathrm{kN}$$

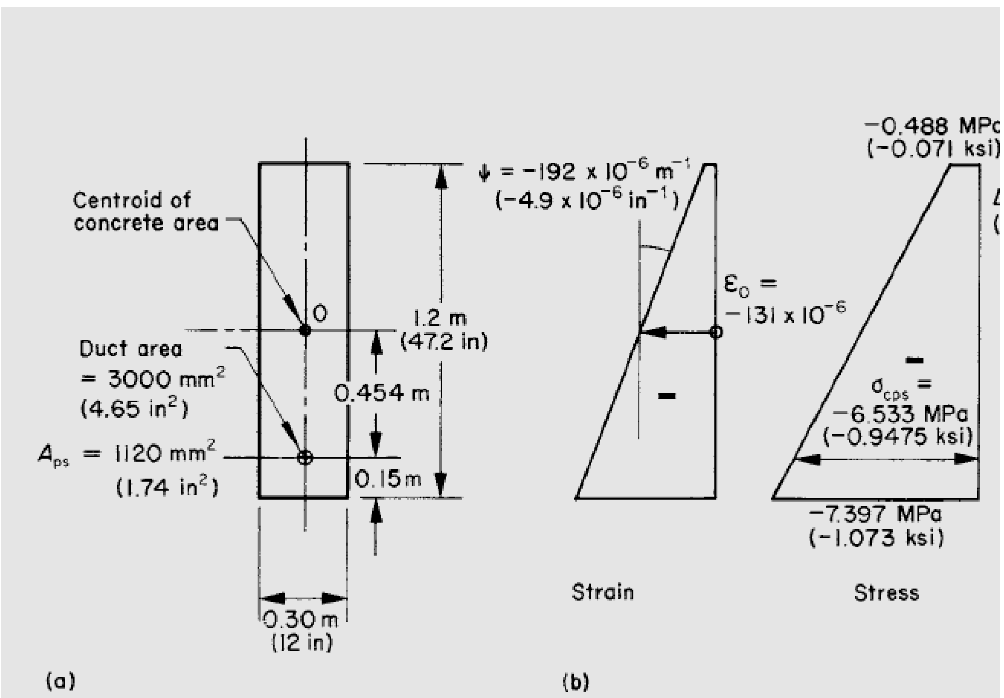
$$M = 390\ \mathrm{kN\text{-}m}$$

$$E_c(t_0) = 30\ \mathrm{GPa}$$

Strain distribution right after pressing

$$\begin{Bmatrix} \varepsilon_o \\ \psi \end{Bmatrix} = \frac{1}{E_{ref}} \begin{bmatrix} A & B \\ B & I \end{bmatrix}^{-1} \begin{Bmatrix} N \\ M \end{Bmatrix} = \frac{1}{E_{ref}\left(AI - B^2\right)} \begin{bmatrix} I & -B \\ -B & A \end{bmatrix} \begin{Bmatrix} N \\ M + Py_{st} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_o \\ \psi \end{Bmatrix}_{t=0} = \frac{1}{E_{ref}\left(AI - B^2\right)} \begin{bmatrix} I & -B \\ -B & A \end{bmatrix} \begin{Bmatrix} -1400 \\ 390 - 1400 * 0.454 \end{Bmatrix}$$



$$\sigma_{cps}(t_0) = -6.533\ \mathrm{MPa}$$

Assume the relaxation factor $\chi_r = 0.7$

$$\Delta\bar{\sigma}_{pr} = \chi_r \Delta\sigma_{pr} = 0.7(-115) = 80.5$$

$$\bar{E}_c\left(t,t_0\right) = 8.824\ \mathrm{GPa}$$

$$\varphi\left(t,t_0\right) = 3.0$$

$$A_c = 0.357\ \mathrm{m}^2$$

$$r_c^2 = 0.1193\ \mathrm{m}^2$$

$$E_c\left(t_0\right) = 30\ \mathrm{GPa}$$

$$\Delta P_c = -\frac{\varphi\left(t,t_0\right)\sigma_{cst}\left(t_0\right)A_{st}\left[E_{st}/E_c\left(t_0\right)\right] + \varepsilon_{cs}\left(t,t_0\right)E_{st}A_{st} + \Delta\bar{\sigma}_{pr}A_{ps}}{1+\dfrac{A_{st}}{A_c}\dfrac{E_{st}}{E\left(t,t_0\right)}\left(1+\dfrac{y_{st}^2}{r_c^2}\right)}$$

In the absence of non-prestressed steel $\Delta P_c = -\Delta P_{ps}$

$$\Delta \sigma_{ps} = \frac{\Delta P_{ps}}{A_{ps}} = -\frac{\Delta P_c}{A_{ps}} = -\frac{242.7*10^3}{1120*10^{-6}} \text{Pa} = 216.7\text{MPa}$$

To find an improved estimate the relaxation reduction factor

Relaxation reduction coefficient χ_r

$$\Delta \bar{\sigma}_{pr} = \chi_r \Delta \sigma_{pr}$$

Reduced relaxation

χ_r is a function of λ and Ω

$$\lambda = \frac{\sigma_{\mathrm{p0}}}{f_{\mathrm{ptk}}} = \frac{1250}{1770} = 0.706$$

$$\Omega = -\left(\frac{\Delta\sigma_{ps} - \Delta\sigma_{pr}}{\sigma_{p0}}\right) = \frac{216.7 - 115}{1250} = 0.081$$

$$\chi_r = 0.8$$

$$\Delta\bar{\sigma}_{pr} = \chi_r \Delta\sigma_{pr}$$

$$= 0.8(-115) = -92 \text{ MPa}$$

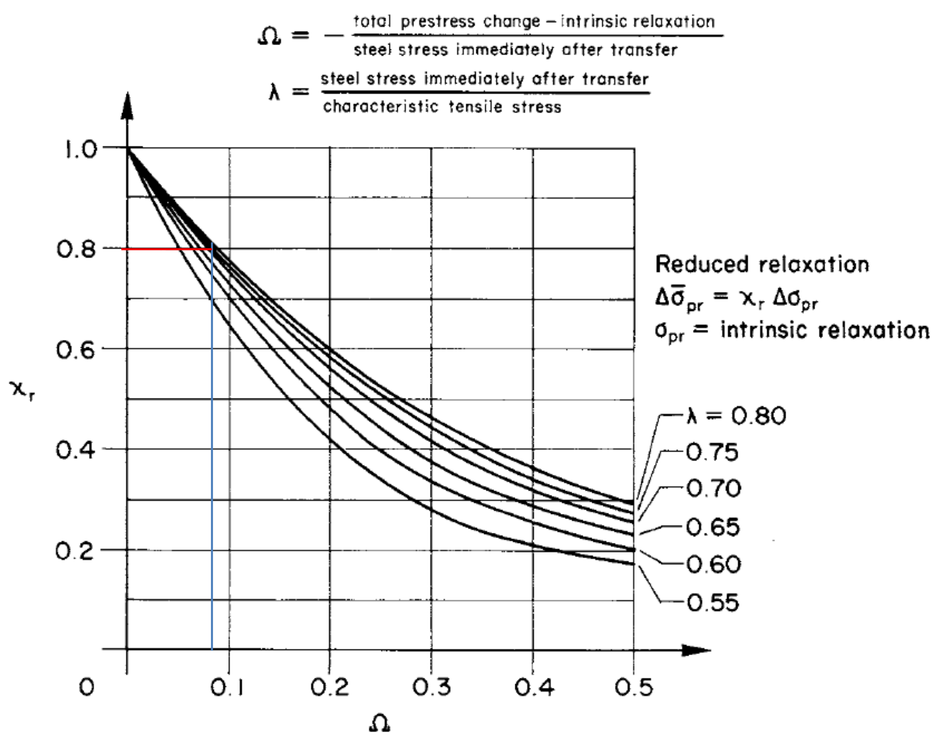


Figure 1.4 Relaxation reduction coefficient χ_r.

$$\Delta P_c = -\frac{\varphi(t,t_0)\sigma_{cst}(t_0)A_{st}\left[E_{st}/E_c(t_0)\right]+\varepsilon_{cs}(t,t_0)E_{st}A_{st}+\Delta\bar{\sigma}_{pr}A_{ps}}{1+\dfrac{A_{st}}{A_c}\dfrac{E_{st}}{E(t,t_0)}\left(1+\dfrac{y_{st}^2}{r_c^2}\right)}$$

$$\Delta\sigma_{ps} = \frac{\Delta P_{ps}}{A_{ps}} = -\frac{\Delta P_c}{A_{ps}}$$

$$\Delta\sigma_c = \frac{\Delta P_c}{A_c}\left(1+\frac{y_{st}y}{r_c^2}\right)$$

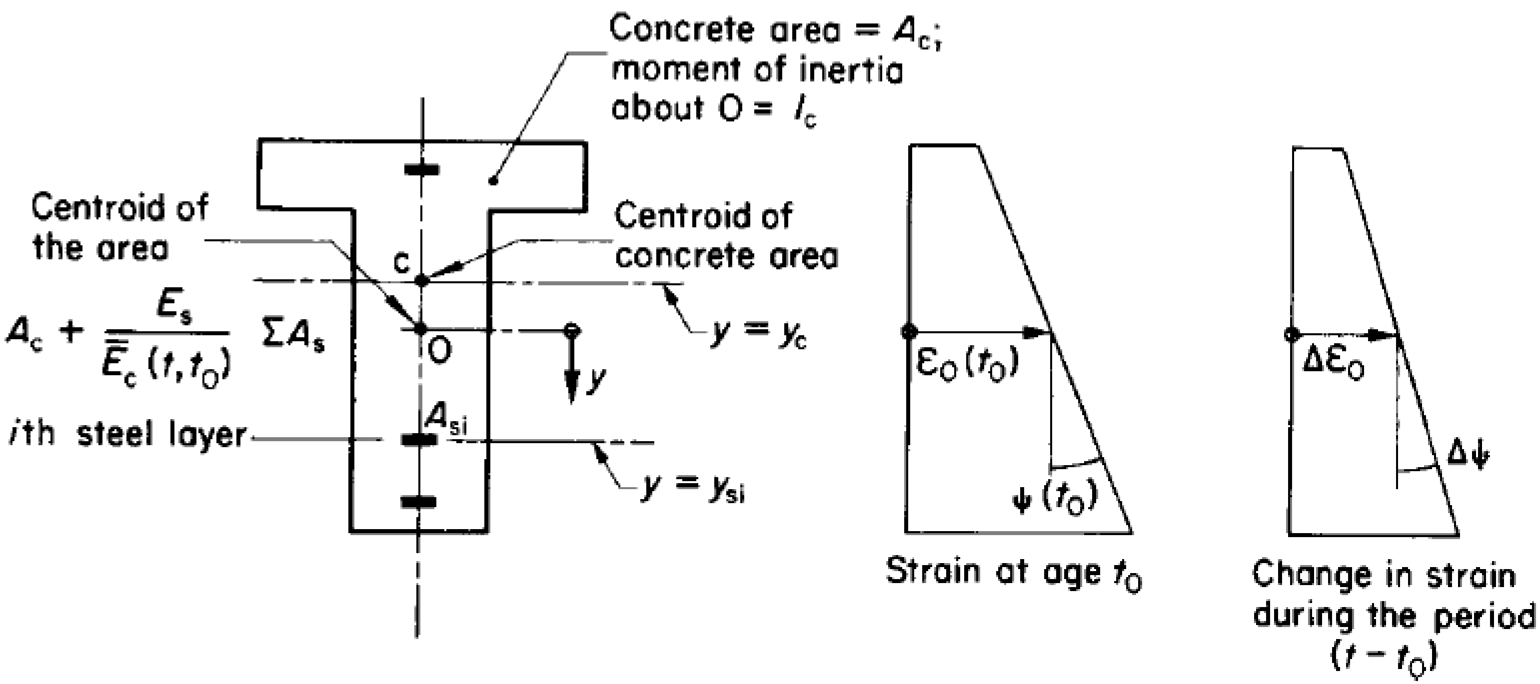
$$\Delta\varepsilon_o = \varepsilon_{cs}(t,t_0)+\varphi(t,t_0)\varepsilon_o(t_0)+\frac{\Delta P_c}{\bar{E}_c(t,t_0)A_c}$$

$$\Delta\psi = \varphi(t,t_0)\psi(t_0)+\frac{\Delta P_c y_{st}}{\bar{E}_c(t,t_0)I_c}$$

Table 3.1 Comparison of strains, curvatures and losses of prestress in two identical cross-sections with and without non-prestressed reinforcement (Examples 2.2 and 3.1).

	Symbol used	*Without non-prestressed reinforcement*	*With non-prestressed reinforcement*
Axial strain immediately after prestress	ε_O	-131×10^{-6}	-126×10^{-6}
Curvature immediately after prestress	ψ	$-192 \times 10^{-6}\ m^{-1}$	$-170 \times 10^{-6}\ m^{-1}$
Change in axial strain due to creep, shrinkage and relaxation	$\Delta\varepsilon_O$	-556×10^{-6}	-470×10^{-6}
Change in curvature due to creep, shrinkage and relaxation	$\Delta\psi$	$-283 \times 10^{-6}\ m^{-1}$	$-128 \times 10^{-6}\ m^{-1}$
Axial strain at time $t = \infty$	$\varepsilon_O + \Delta\varepsilon_O$	-687×10^{-6}	-596×10^{-6}
Curvature at time $t = \infty$	$\psi + \Delta\psi$	$-475 \times 10^{-6}\ m^{-1}$	$-298 \times 10^{-6}\ m^{-1}$
Change in force in prestressed steel (the loss)	$A_{ps}\Delta\sigma_{ps}$	−243 kN	−208 kN
Axial force on concrete immediately after prestress	$\int \sigma_c(t_0) dA_c$	−1400 kN	−1329 kN
Axial force on concrete at $t = \infty$	$\int \sigma_c(t) dA_c$	−1157 kN	−878 kN
Change in force on concrete, ΔP_c	$\int [\sigma_c(t_0) - \sigma_c(t)] dA_c$	243 kN	451 kN

3.4 RC w/o prestress



$$\Delta\varepsilon_O = \eta\{\varphi(t, t_0)[\varepsilon_O(t_0) + \psi(t_0)y_c] + \varepsilon_{cs}(t, t_0)\} \tag{3.15}$$

$$\Delta\psi = \kappa\left[\varphi(t, t_0)\left(\psi(t_0) + \varepsilon_O(t_0)\frac{y_c}{r_c^2}\right) + \varepsilon_{cs}(t, t_0)\,\frac{y_c}{r_c^2}\right] \tag{3.16}$$

$$\eta = A_c/\overline{A} \tag{3.17}$$

$$\kappa = I_c/\bar{I} \tag{3.18}$$

$$\Delta\sigma_c = \overline{E}_c(t, t_0)\{-[\varepsilon_O(t_0) + \psi(t_0)y]\varphi(t, t_0) - \varepsilon_{cs}(t, t_0) + \Delta\varepsilon_O + \Delta\psi \mathrm{y}\} \tag{3.19}$$

$$\begin{Bmatrix} \Delta N \\ \Delta M \end{Bmatrix}_{\text{creep}} = -\bar{E}_c(t, t_0)\varphi(t, t_0) \begin{bmatrix} A_c & A_c y_c \\ A_c y_c & I_c \end{bmatrix} \begin{Bmatrix} \varepsilon_O(t_0) \\ \psi(t_0) \end{Bmatrix} \tag{3.20}$$

$$\begin{Bmatrix} \Delta N \\ \Delta M \end{Bmatrix}_{\text{shrinkage}} = -\bar{E}_c(t, t_0)\varepsilon_{cs}(t, t_0) \begin{Bmatrix} A_c \\ A_c y_c \end{Bmatrix} \tag{3.21}$$

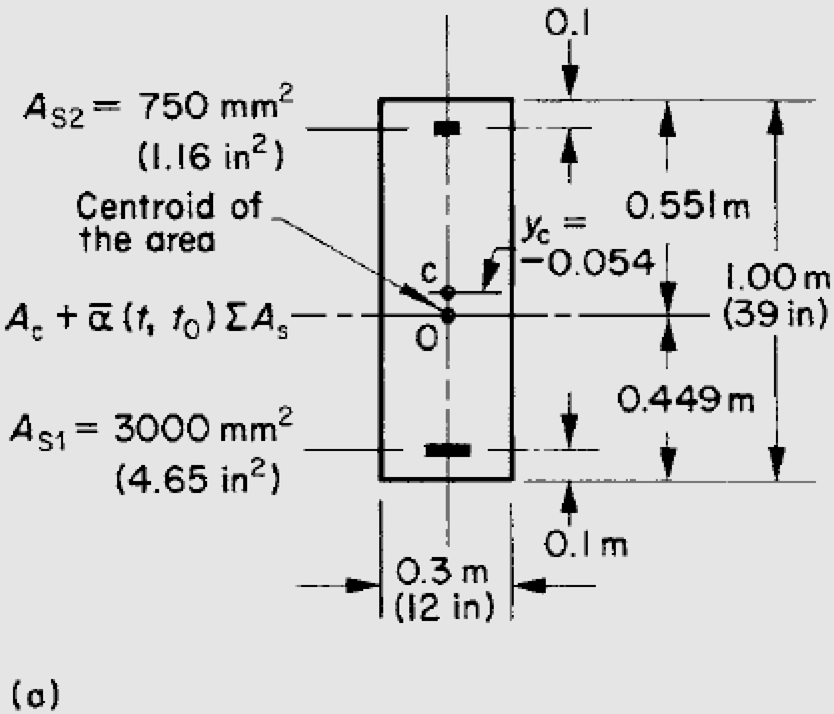
The sum of Equations (3.20) and (3.21) gives the forces necessary to restrain creep and shrinkage:

$$\begin{Bmatrix} \Delta N \\ \Delta M \end{Bmatrix} = -\bar{E}_c(t, t_0) \times \begin{Bmatrix} \{\varphi(t, t_0)[\varepsilon_O(t_0) + \psi(t_0) y_c] + \varepsilon_{cs}(t, t_0)\} A_c \\ \left[\varphi(t, t_0)\left(\psi(t_0) + \varepsilon_O(t_0)\dfrac{y_c}{r_c^2}\right) + \varepsilon_{cs}(t, t_0)\dfrac{y_c}{r_c^2}\right] A_c r_c^2 \end{Bmatrix} \tag{3.22}$$

$$\begin{Bmatrix} \Delta \varepsilon_O \\ \Delta \psi \end{Bmatrix} = \frac{1}{\bar{E}_c(t, t_0)} \begin{Bmatrix} -\Delta N/\bar{A} \\ -\Delta M/\bar{I} \end{Bmatrix} \tag{3.23}$$

Example 3.2 Section subjected to uniform shrinkage

Find the stress and strain distribution in the cross-section in Fig. 3.4(a) due to uniform free shrinkage $\varepsilon_{cs}(t, t_0) = -300 \times 10^{-6}$, using the following data: $E_c(t_0) = 30\,\text{GPa}$ (4350 ksi); $E_s = 200\,\text{GPa}$ (29 000 ksi); $\varphi(t, t_0) = 3$; $\chi = 0.8$. The section dimensions and reinforcement areas are given in Fig. 3.4(a).



$$\bar{E}_c(t, t_0) = \frac{30 \times 10^9}{1 + 0.8 \times 3} = 8.824\,\text{GPa}\ (1280\,\text{ksi})$$

$$\bar{\alpha}(t, t_0) = \frac{200}{8.824} = 22.665.$$

$$A_c = 0.2963\,\text{m}^2 \quad I_c = 25.26 \times 10^{-3}\,\text{m}^4 \quad r_c^2 = I_c/A_c = 84.75 \times 10^{-3}\,\text{m}^2.$$

The area and moment of inertia of the age-adjusted transformed section about an axis through O are:

$$\bar{A} = 0.3811\,\text{m}^2 \qquad \bar{I} = 37.50 \times 10^{-3}\,\text{m}^4.$$

The axial strain and curvature reduction coefficient (Equations (3.17) and (3.18)) are:

$$\eta = \frac{0.2963}{0.3811} = 0.777 \qquad \kappa = \frac{25.26}{37.50} = 0.674.$$

Substitution in Equations (3.15) and (3.16) gives the changes in axial strain and in curvature due to shrinkage:

$$\Delta\varepsilon_O = 0.777(-300 \times 10^{-6}) = -233 \times 10^{-6}$$

$$\Delta\psi = 0.674\,(-300 \times 10^{-6})\,\frac{-0.054}{84.45 \times 10^{-3}}$$

$$= 129 \times 10^{-6}\,\mathrm{m}^{-1}\ (3.23\,\mathrm{in}^{-1}).$$

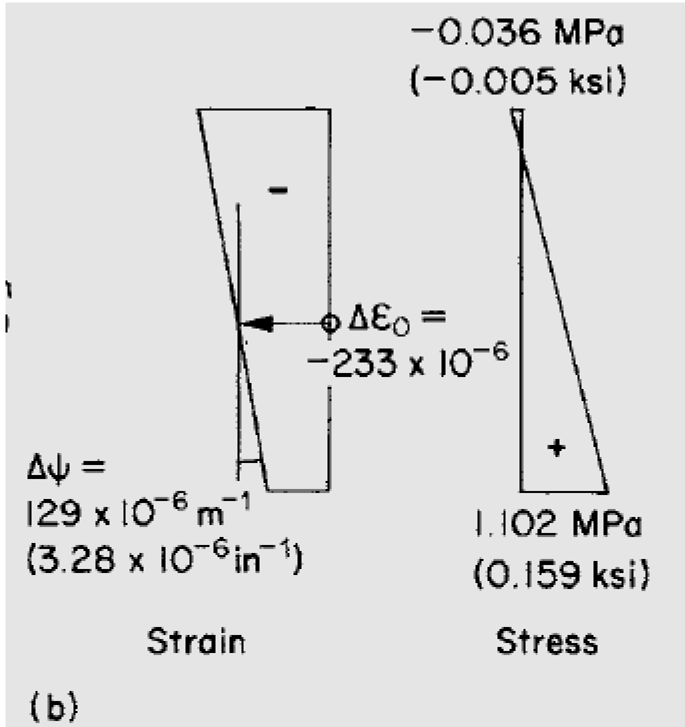
The changes in concrete stress due to shrinkage (Equation (3.19)) are:

$$(\Delta\sigma_c)_{top} = 8.824 \times 10^9[-(-300) + (-233) + 129(-0.551)]10^{-6}\ \mathrm{Pa}$$

$$= -0.036\,\mathrm{MPa}\ (-0.005\,\mathrm{ksi}).$$

$$(\Delta\sigma_c)_{bot} = 8.824 \times 10^9[-(-300) + (-233) + 129(0.449)]10^{-6}\ \mathrm{Pa}$$

$$= 1.102\,\mathrm{MPa}\ (0.159\,\mathrm{ksi}).$$



−0.036 MPa
(−0.005 ksi)
−
Δε₀ =
−233 x 10⁻⁶
+
Δψ =
129 x 10⁻⁶ m⁻¹
(3.28 x 10⁻⁶ in⁻¹)
1.102 MPa
(0.159 ksi)
Strain
Stress
(b)

Example 3.3 Section subjected to normal force and moment

The same cross-section of Example 3.2 (Fig. 3.4(a)) is subjected at age t_0 to an axial force = –1300 kN at mid-height and a bending moment of 350 kN-m. It is required to find the changes during the period $(t - t_0)$ in axial strain, curvature and in concrete stress due to creep. Use the same data as in Example 3.2 but do not consider shrinkage. Assume no cracking.

The applied forces are a bending moment of 350 kN-m and an axial force of – 1300 kN at mid-height. Replacing these by equivalent couple and axial force at the reference point O, gives (see Fig. 3.4(a)):

$$N = -1300\,\text{kN} \quad (-292\,\text{kip}) \qquad M = 350 + 1300(0.051)$$

$$= 416.3\,\text{kN-m}\ (3685\,\text{kip-in}).$$

These two values are substituted in Equation (2.32) to give the instantaneous axial strain and curvature:

$$\varepsilon_O(t_0) = -120 \times 10^{-6} \qquad \psi(t_0) = 428 \times 10^{-6}\,\mathrm{m}^{-1} \quad (10.9\,\mathrm{in}^{-1}).$$

The stress and strain distributions at age t_0 are shown in Fig. 3.4(c). The modulus of elasticity of concrete used for calculating the values of this figure is $E_c(t_0) = 30\,\mathrm{GPa}$.

The values $\bar{E}_c(t, t_0)$, $\bar{\alpha}(t, t_0)$, η and κ are the same as in Example 3.2.

Substitution in Equations (3.15) and (3.16) gives the changes in axial strain and curvature due to creep (Fig. 3.4(d)):

$$\Delta\varepsilon_O = 0.777\{3[-120 + 428(-0.054)]10^{-6}\} = -334 \times 10^{-6}$$

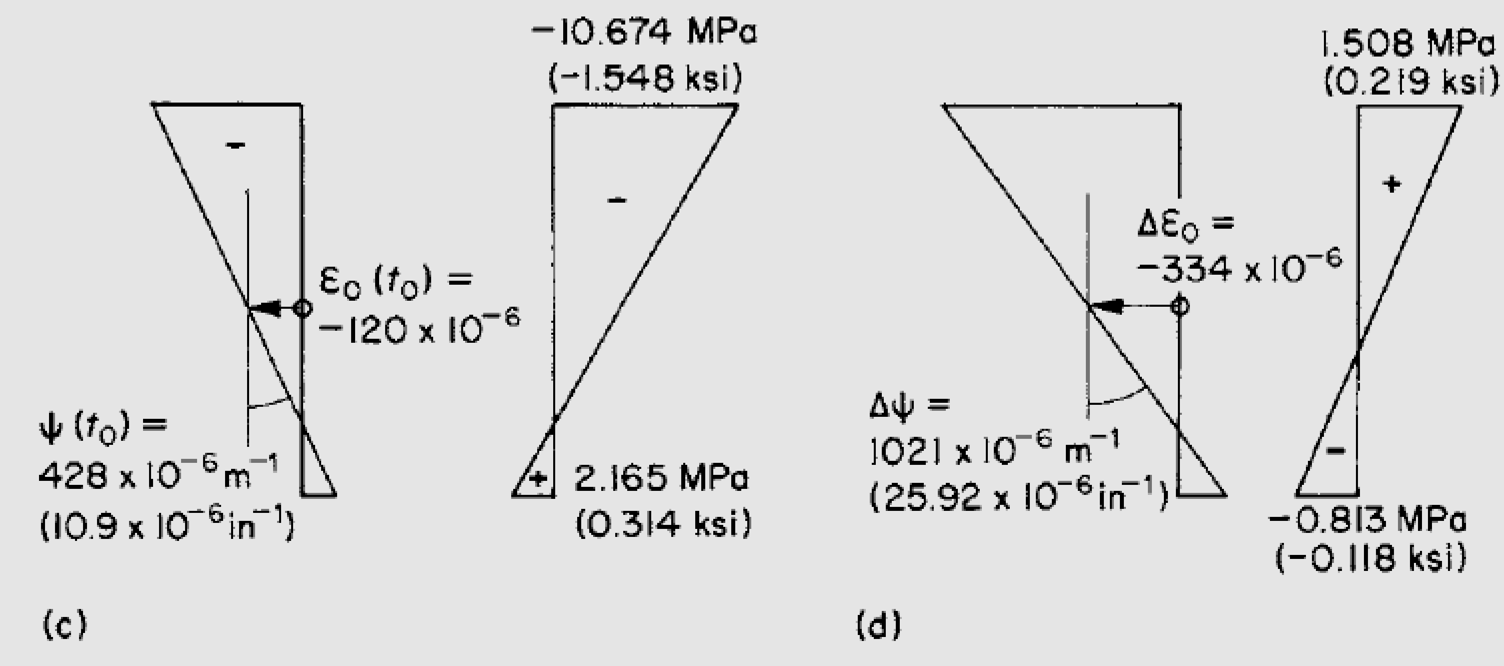
$$\Delta\psi = 0.674\left[3\left(428 + (-120)\frac{-0.054}{84.45 \times 10^{-3}}\right)10^{-6}\right]$$

$$= 1021 \times 10^{-6}\,\mathrm{m}^{-1} \quad (25.92 \times 10^{-6}\,\mathrm{in}^{-1}).$$

The corresponding changes in concrete stress (Equation (3.19)) are

$$(\Delta\sigma_c)_{top} = 8.824 \times 10^9\{-[-120 + 428(-0.551)]3$$
$$+ (-334) + 1021(-0.551)\}$$
$$= 1.508\,\text{MPa}\ (0.219\,\text{ksi})$$

$$(\Delta\sigma_c)_{bot} = 8.824 \times 10^9\{-[-120 + 428(0.449)]3 + (-334)$$
$$+ 1021(0.449)\} = -0.813\,\text{MPa}\ (-0.118\,\text{ksi}).$$



3.5 Approximate equation

$$\Delta\varepsilon_O = \eta\{\varphi(t, t_0)[\varepsilon_O(t_0) + \psi(t_0)y_c])\} \tag{3.24}$$

$$\Delta\psi = \kappa\left[\varphi(t, t_0)\left(\psi(t_0) + \varepsilon_O(t_0)\frac{y_c}{r_c^2}\right)\right] \tag{3.25}$$

$$\Delta\varepsilon_O = \eta\{\varphi(t, t_0)[\varepsilon_O(t_0) + \psi(t_0)y_c] + \varepsilon_{cs}(t, t_0)\} \tag{3.15}$$

$$\Delta\psi = \kappa\left[\varphi(t, t_0)\left(\psi(t_0) + \varepsilon_O(t_0)\frac{y_c}{r_c^2}\right) + \varepsilon_{cs}(t, t_0)\frac{y_c}{r_c^2}\right] \tag{3.16}$$

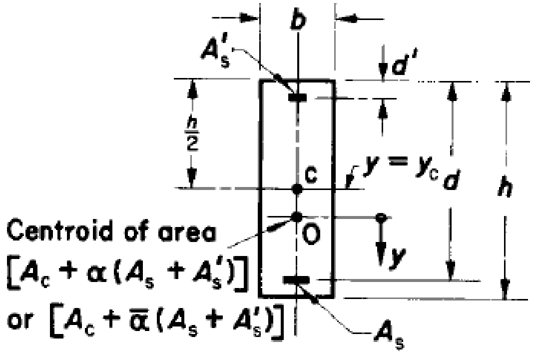
(*a*) *Creep due to axial force*: The change in axial strain due to creep in a reinforced section subjected to axial force

$$\Delta\varepsilon_O \simeq \eta\varepsilon_O(t_0)\varphi(t, t_0). \tag{3.26}$$

(*b*) *Creep due to bending moment*: The change in curvature due to creep in a reinforced concrete section subjected to bending moment

$$\Delta\psi \simeq \kappa\psi(t_0)\varphi(t, t_0). \tag{3.27}$$

3.6 Graph

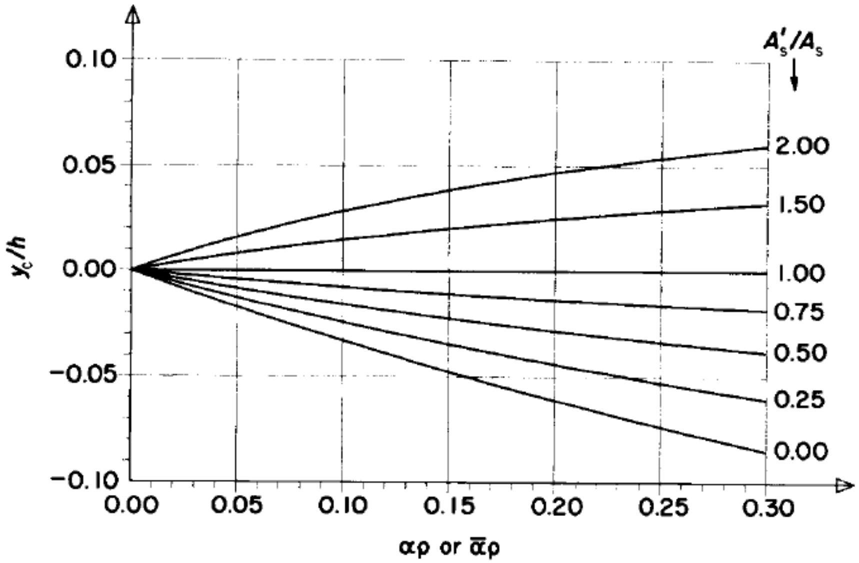


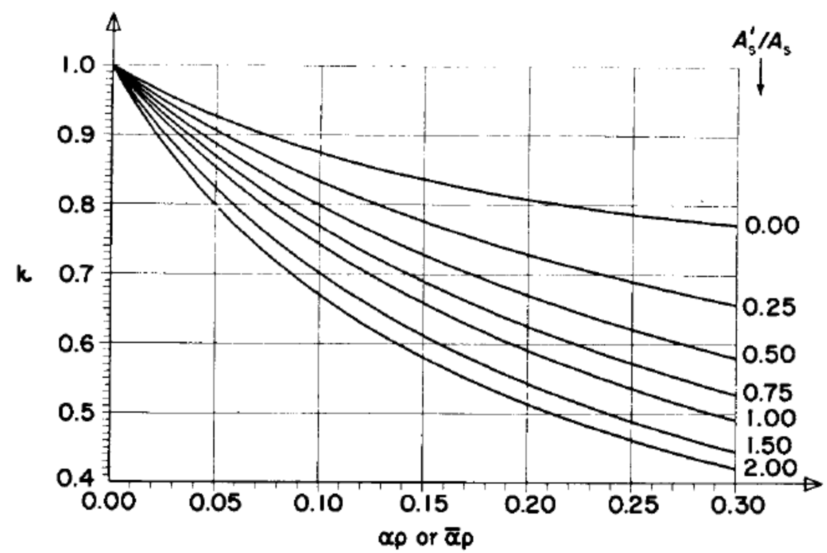
$$\alpha(t) = \frac{E_s}{E_c(t)}$$

$$\bar{\alpha}(t, t_0) = \frac{E_s}{\bar{E}_c(t, t_0)}$$

$$\rho = \frac{A_s}{bd}\ ;\ \rho' = \frac{A'_s}{bd}$$

$$I_c \cong \frac{bh^3}{12}\left[1 + 12\left(\frac{y_c}{h}\right)^2\right];\ \kappa = \frac{I_c}{I\ (\text{or}\ \bar{I})}$$





$$\alpha = \alpha(t_0) = E_s / E_c(t_0) \tag{3.28}$$

$$\bar{\alpha} = \bar{\alpha}(t, t_0) = E_s[1 + \chi\varphi(t, t_0)] / E_c(t_0). \tag{3.29}$$

b and h are breadth and height of the section.

The values of I (or $\bar{I}$) may be used in the calculations for the instantaneous curvature by Equation (2.16) or the change in curvature due to creep and shrinkage by Equation (2.40) (setting $\bar{B} = 0$).

The top graph in Fig. 3.5 gives the coordinate y_c of the centroid of the concrete area (mid-height of the section) with respect to point O. It is to be noted that in the common case when A_s is larger than A_s', y_c has a negative value. A_s and A_s' are the cross-section areas of the bottom and top reinforcement (Fig. 3.5).

The bottom graph in Fig. 3.5 gives the curvature reduction coefficient:

$$\kappa = \frac{I_c}{I \text{ (or } \bar{I})} \tag{3.30}$$

where I_c is the moment of inertia of the concrete area, A_c about an axis through O. I_c is given by

$$I_c \simeq \frac{bh^3}{12}\left(1 + 12\frac{y_c^2}{h^2}\right). \tag{3.31}$$

In this equation, the area A_c is considered equal to bh and its centroid at mid-height. In other words, the space occupied by the reinforcement is ignored. The graphs in Fig. 3.5 are calculated assuming that the distance between the centroid of the top or bottom reinforcement and the nearby extreme fibre is equal to 0.1 h. A small error results when the graphs are used with this distance between 0.05 h and 0.15 h.

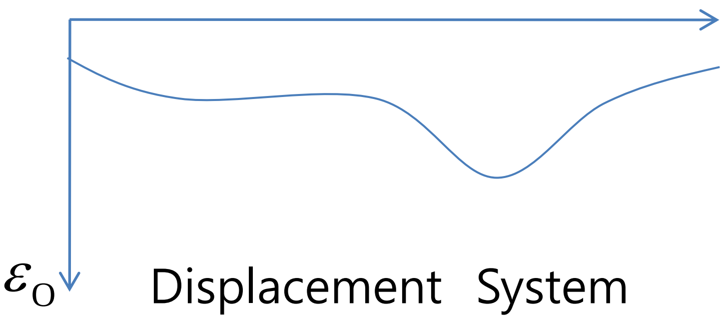
Assume that the prestress is applied at age t_0 and t_1 and we are interested in the stress and strain at these two ages and at a later age t_2. The analysis is to be done in four steps to calculate the following:

1 $\varepsilon_O(t_0)$ and $\psi(t_0)$ are the instantaneous strain at reference point O and the curvature immediately after application of the first prestress.
2 $\Delta\varepsilon_O(t_1, t_0)$ and $\Delta\psi(t_1, t_0)$ are the changes in strain at reference point O and in curvature during the period t_0 to t_1.
3 $\Delta\varepsilon_O(t_1)$ and $\Delta\psi(t_1)$ are the additional instantaneous strain at reference point O and curvature immediately after second prestress.
4 $\Delta\varepsilon_O(t_2, t_1)$ and $\Delta\psi(t_2, t_1)$ are the additional change in strain at reference point O and curvature during the period t_1 to t_2.

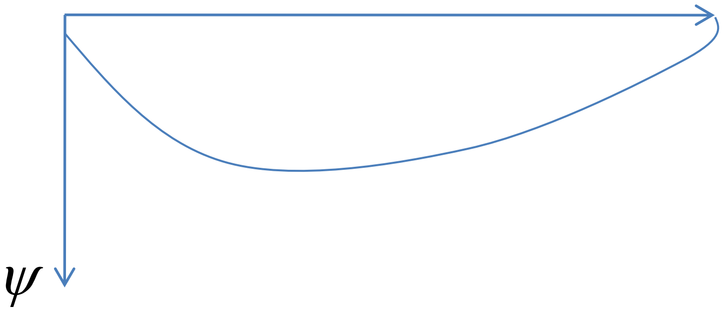
3.8 Displacement

- 3.8.1 unit load theory

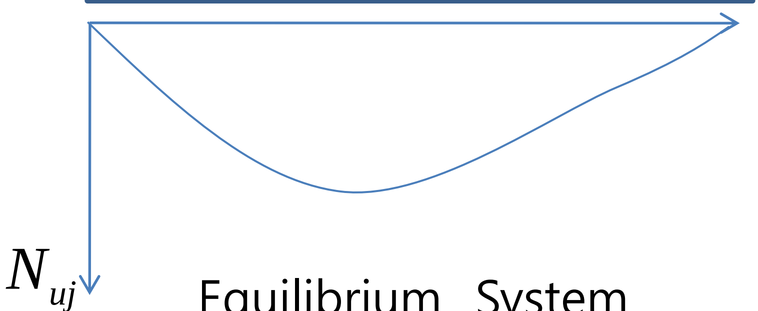
Real Strain and Curvature Distribution



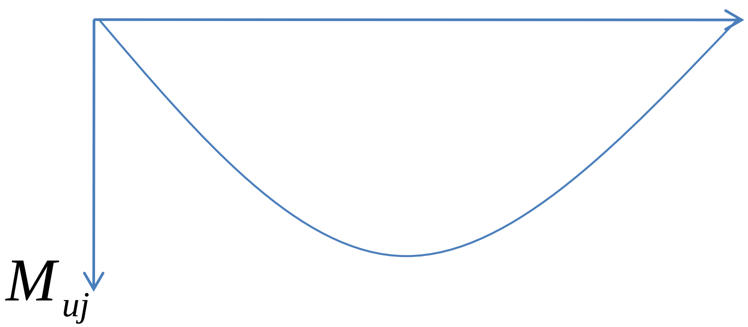
Displacement System



Virtual Axial and Moment Distribution due to unit virtual force at coordinate j

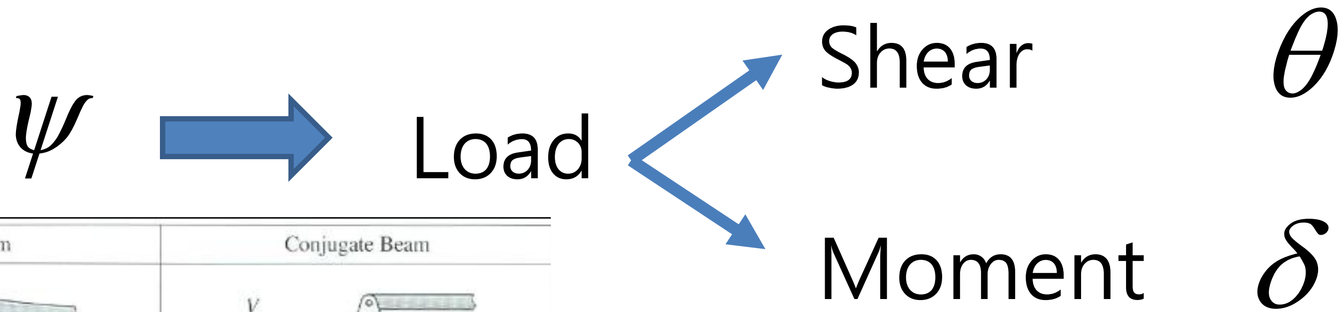


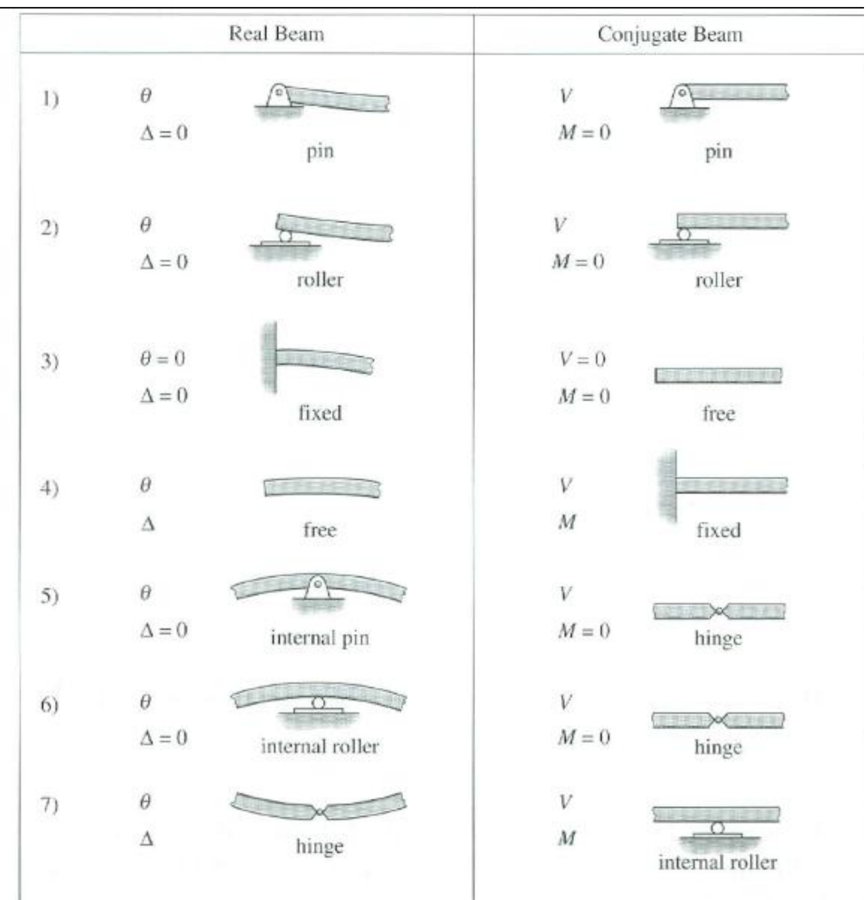
Equilibrium System

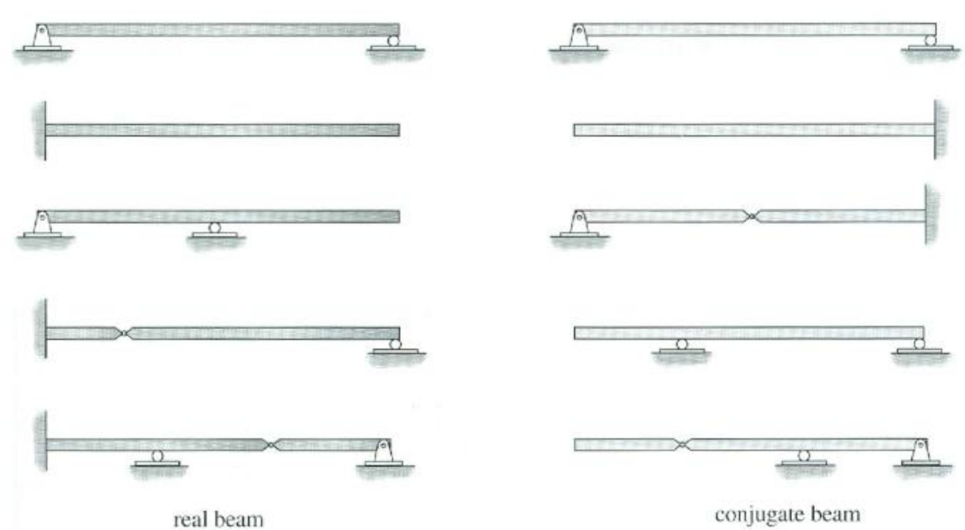


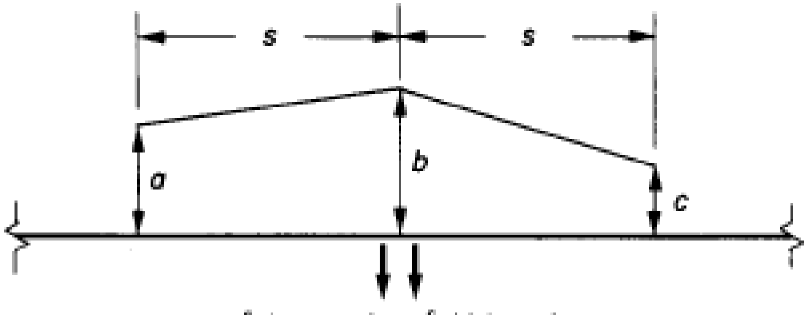
$$\Delta_j \times 1 = \int \varepsilon_o N_{uj} dx + \int \psi M_{uj} dx$$

3.8.2 Elastic weight: Conjugate Beam



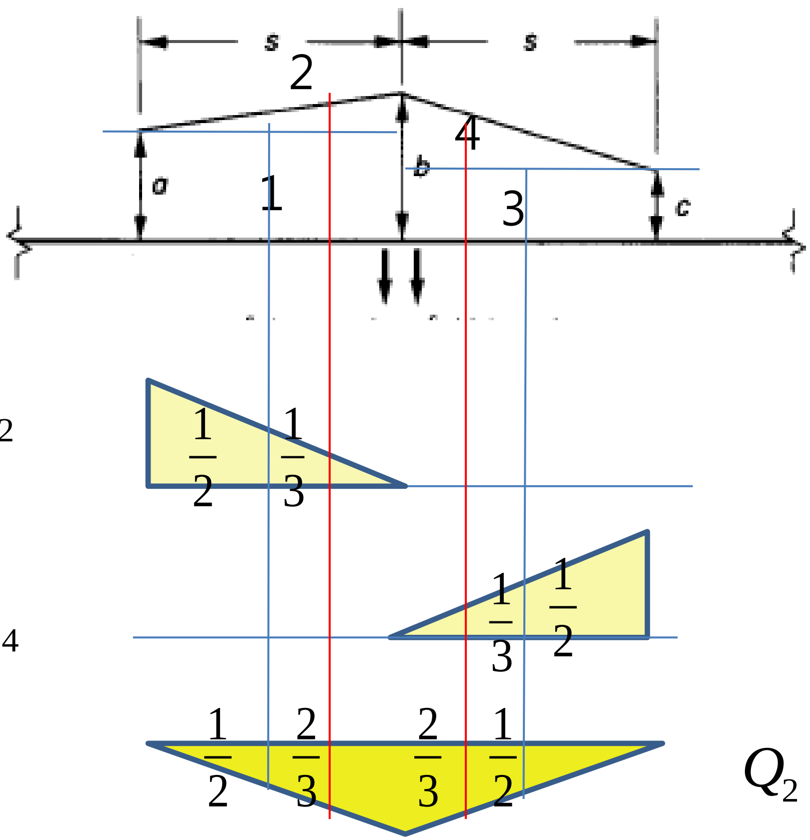






$$S_1 = \frac{1}{2} s\left(b - a\right)$$

$$S_2 = sa$$



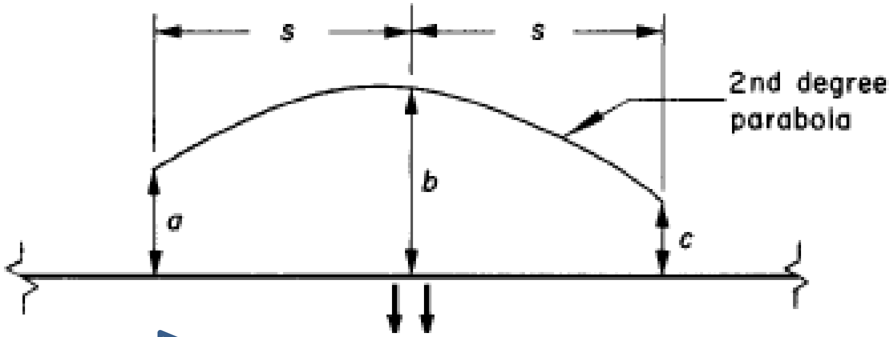
$$S_3 = \frac{1}{2} s\left(b - c\right)$$

$$S_4 = sc$$

$$Q_1 = \frac{1}{2} S_1 + \frac{1}{3} S_2$$

$$Q_3 = \frac{1}{2} S_3 + \frac{1}{3} S_4$$

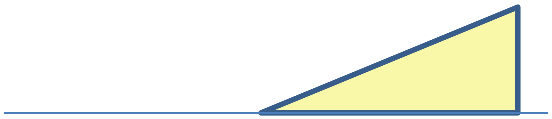
$$Q_2 = \frac{1}{2} S_1 + \frac{2}{3} S_2 + \frac{1}{2} S_3 + \frac{2}{3} S_4$$



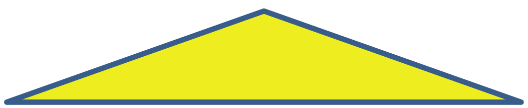
$$y = Ax^2 + Bx + C$$

$$A = \frac{a - 2b + c}{2s^2}, B = \frac{-3a + 3b - c}{2s}, C = a$$

$$Q_1 = \int_0^s \left(Ax^2 + Bx + C\right)\left(-\frac{x}{s} + 1\right)dx$$

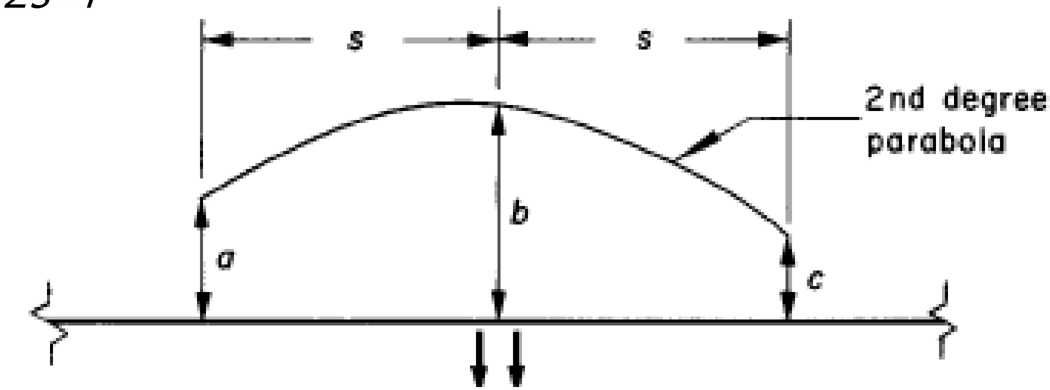


$$Q_3 = \int_s^{2s} \left(Ax^2 + Bx + C\right)\left(\frac{x}{s} - 1\right)dx$$

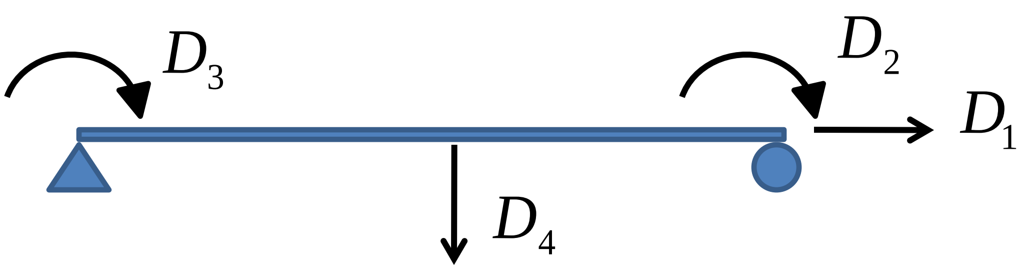


$$Q_2 = \int_0^s \left(Ax^2 + Bx + C\right)\left(\frac{x}{s}\right)dx + \int_s^{2s} \left(Ax^2 + Bx + C\right)\left(-\frac{x}{s} + 2\right)dx$$

Assume parabolic variation of strain at O and curvature.
Note *2s=l*

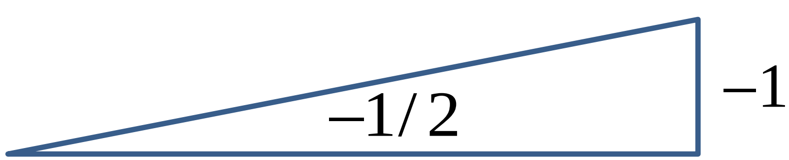


$$\{Q_1, Q_2, Q_3\} = \begin{bmatrix} 7l/48 & l/8 & -l/48 \\ l/24 & 5l/12 & l/24 \\ -l/48 & l/24 & 7l/48 \end{bmatrix} \begin{Bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{Bmatrix}$$

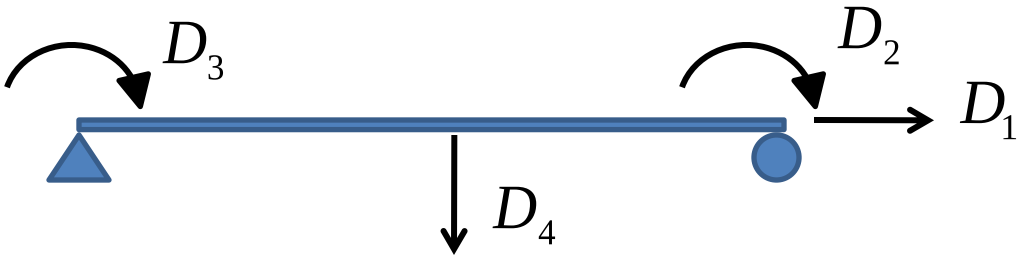


Apply a unit moment at the left support

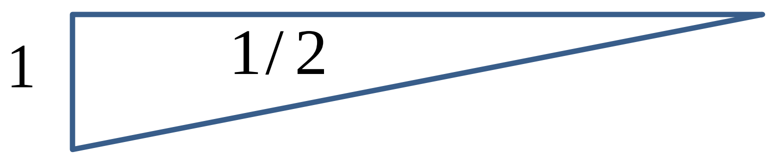
$$1 \times D_2 = [0, -0.5, -1] \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix}$$



Moment distribution by a unit load

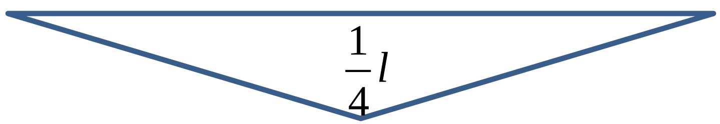


Apply a unit moment at the left support



$$1 \times D_3 = [1, 0.5, 0] \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix}$$

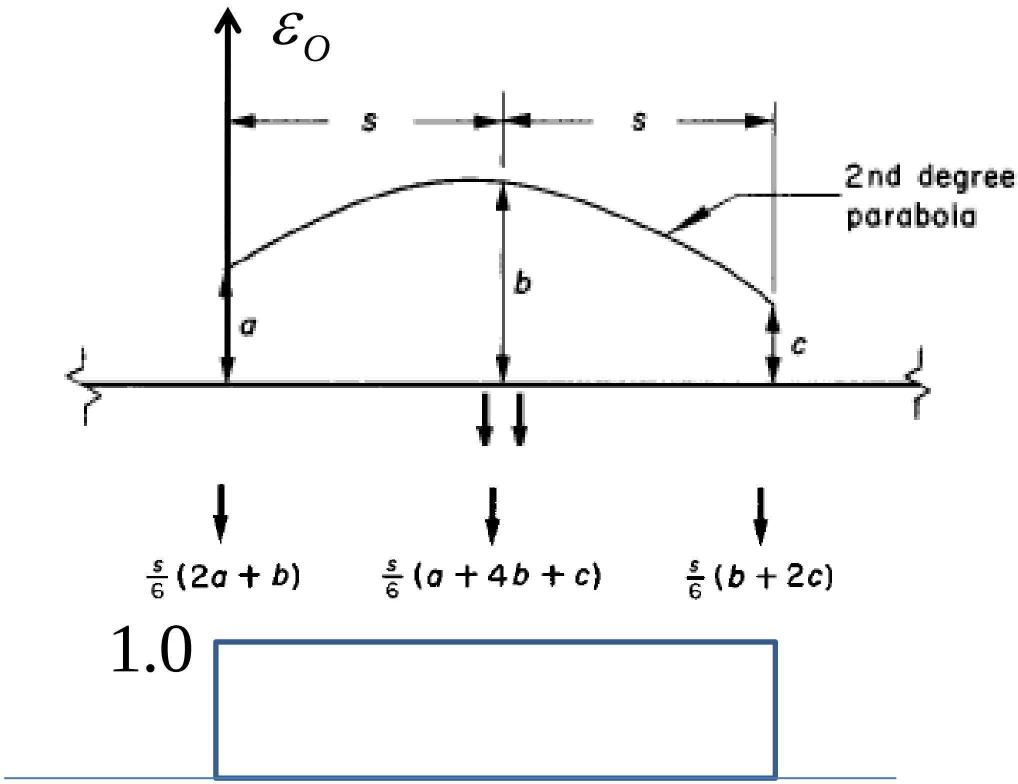
Apply a unit load at the center



$$1 \times D_4 = \left[1, \frac{l}{4}, 0\right] \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix}$$

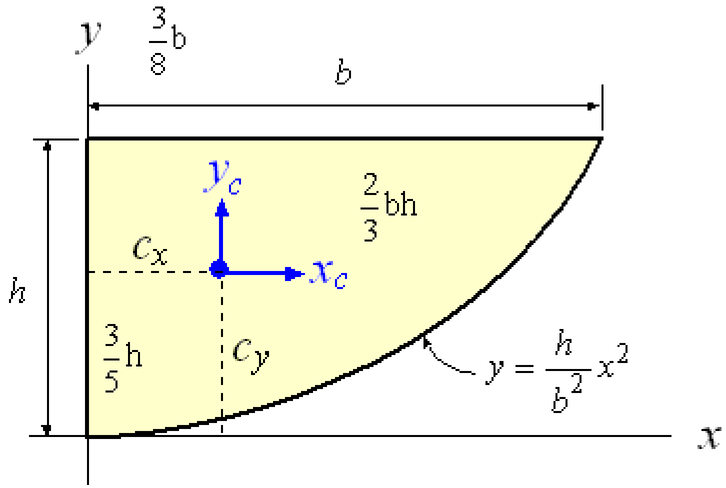
$$D_2 = \left[0, -\frac{1}{2}, -1\right] \begin{bmatrix} 7/48 & 1/8 & -1/48 \\ 1/24 & 5/12 & 1/24 \\ -1/48 & 1/8 & 7/48 \end{bmatrix} l \begin{Bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{Bmatrix} = l \left[0, -\frac{8}{24}, -\frac{8}{48}\right] \begin{Bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{Bmatrix}$$

Axial deformation

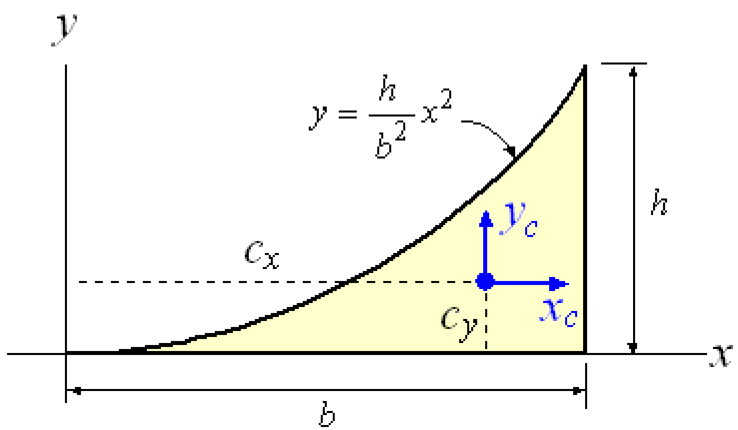


Axial force distribution due to a unit load

$$D_1 = [1,1,1]\begin{bmatrix} 7/48 & 1/8 & -1/48 \\ 1/24 & 5/12 & 1/24 \\ -1/48 & 1/8 & 7/48 \end{bmatrix} l \begin{Bmatrix} \varepsilon_{O1} \\ \varepsilon_{O2} \\ \varepsilon_{O3} \end{Bmatrix} = l\left[\frac{8}{48}, \frac{16}{24}, \frac{8}{48}\right]\begin{Bmatrix} \varepsilon_{O1} \\ \varepsilon_{O2} \\ \varepsilon_{O3} \end{Bmatrix}$$



y
3/8 b
b
h
y_c
2/3 bh
c_x
x_c
3/5 h
c_y
y = h/b^2 x^2
x



y
y = h/b^2 x^2
h
y_c
c_x
c_y
x_c
x
b