

Chapter 5 First law analysis for an Open System in control volume

- Review of Ch. 4: 1st Law of Thermodynamics in the Closed System

$$\Delta Q \equiv \Delta E + \Delta W$$

$$(\delta Q = dE + \delta W)$$

- Now consider an Open System
CV (control volume) : a volume in space that has interest for a particular study or analysis.

Recall the first law

$$Q_{1 \rightarrow 2} = E_2 - E_1 + W_{1 \rightarrow 2}$$

We also note that this may be written as an average rate equation over the time interval :

$$\frac{\delta Q}{\delta t} = \frac{E_2 - E_1}{\delta t} + \frac{\delta W}{\delta t}$$

Or $\lim_{\delta t \rightarrow 0} \frac{\delta Q}{\delta t} = \dot{Q}$: heat transfer rate

$\lim_{\delta t \rightarrow 0} \frac{\delta W}{\delta t} = \dot{W}$: power

$\lim_{\delta t \rightarrow 0} \frac{\Delta E}{\delta t} = \frac{dE}{dt}$

$$\dot{Q} = \frac{dE}{dt} + \dot{W}$$

$$= \frac{dU}{dt} + \frac{d(KE)}{dt} + \frac{d(PE)}{dt} + \dot{W}$$

Rate equation form of the First Law

- Continuous System

The state quantities are expressed in terms of space and time, for example,

$$p = p(x, y, z, t)$$

$$T = T(x, y, z, t)$$

The mass, m and energy E can be expressed using the density, $\rho(x, y, z, t)$

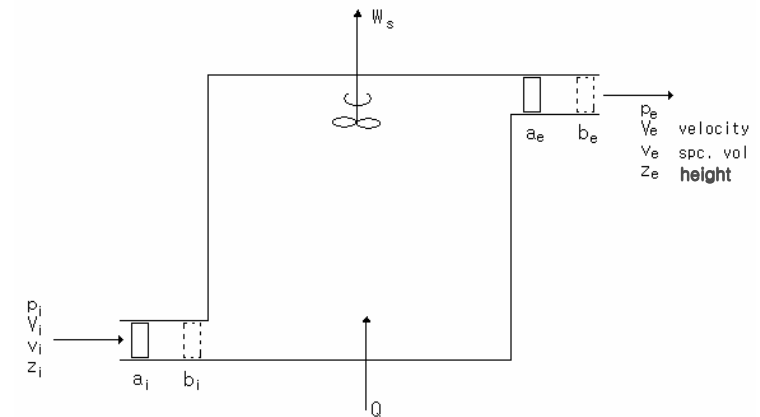
$$m(t) = \int \rho(x, y, z, t) dV$$

$$E(t) = \int \rho(x, y, z, t) e(x, y, z, t) dV$$

If the state of the system does not change with time, $\frac{\partial}{\partial t} \rightarrow 0$, it is called "steady state", and we note $\rho(x, y, z)$ and $e(x, y, z)$

- Consider an Open System (CV)

A typical Open System is depicted below.



Fluid enters through inlet - i (입구) and exits through - e (출구)

- In order to use the first law for a closed system (ie $\delta Q - \delta W = dE$), we need to modify the system.

- Then, during t and $t + \Delta t$, the total energy considering the inlet and the exit is,

$$E_1 = E_i + \Delta m_i (u_i + \frac{1}{2} V_i^2 + gz_i)$$

$$E_2 = E_{t+\Delta t} + \Delta m_e (u_e + \frac{1}{2} V_e^2 + gz_e)$$

Δt 사이의 열 전달 량: ΔQ

Δt 사이의 일: $\Delta W_s + \Delta W_v$

by Shaft

by Fluid

$$\Delta W = \Delta W_s - p_i v_i \Delta m_i + p_e v_e \Delta m_e$$

- Thus the first law as defined for a closed system may be written for an open system (or CV) as follows:

$$\Delta Q - (\Delta W_s - p_i v_i \Delta m_i + p_e v_e \Delta m_e) = \{E_{t+\Delta t} + \Delta m_e (u_e + \frac{1}{2} V_e^2 + gz_e)\} - \{E_t + \Delta m_i (u_i + \frac{1}{2} V_i^2 + gz_i)\} \quad \text{---(*)}$$

Heat

Work

Total Energy

- If Steady State,

$$\Delta m_i = \Delta m_e = \Delta m$$

$$E_{t+\Delta t} = E_t$$

The state of the mass at each point in the CV does not vary with time, $\frac{dm_{cv}}{dt} = 0$; $\frac{dE_{cv}}{dt} = 0$

- Then the first law for an open system (*) becomes

$$\Delta Q + \Delta m (p_i v_i - p_e v_e) = \Delta W_s + \Delta m (u_e - u_i + \frac{1}{2} (V_e^2 - V_i^2) + g(z_e - z_i))$$

using Enthalpy definition ($h=u+pv$)

$$\Delta Q + \Delta m (h_i + \frac{1}{2} V_i^2 + gz_i) = \Delta W_s + \Delta m (h_e + \frac{1}{2} V_e^2 + gz_e)$$

In terms of a rate equation,

$$\dot{Q} + \dot{m} (h_i + \frac{1}{2} V_i^2 + gz_i) = \dot{W}_s + \dot{m} (h_e + \frac{1}{2} V_e^2 + gz_e) \quad \text{-----(**)}$$

- Thus far, we considered only 1 inlet, 1 exit.
- What if there were more than 1?

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e$$

$$\dot{Q} + \sum_i \dot{m}_i (h_i + \frac{1}{2} V_i^2 + gz_i) = \dot{W}_s + \sum_e \dot{m}_e (h_e + \frac{1}{2} V_e^2 + gz_e)$$

We also note $dm = \rho V dA$ then

$$\dot{Q} + \int_{A_i} (h + \frac{1}{2} V^2 + gz) \rho V dA = \dot{W}_s + \int_{A_e} (h + \frac{1}{2} V^2 + gz) \rho V dA$$

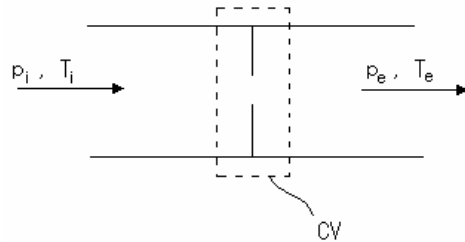
i.e. the cross sectional area of inlet/outlet vary.

- The Joule-Thomson Coefficient and the Throttling Process

Let $\mu = \left(\frac{\partial T}{\partial p}\right)_h$ J-T coefficient.

Consider a **throttling process**:

The flow through a partially opened valve or a restriction in the line. This usually occurs so rapidly and in such a small space that there is neither sufficient force, nor a large enough area for much heat transfer. So we assume such process to be **adiabatic**



$p_i > p_e$
Reduction in Pressure.

- Thus with steady state assumption, we note,

$$\delta W_s = 0, \Delta PE = 0, \Delta KE = 0, \delta Q = 0$$

Then the 1st Law for the CV becomes

$$h_i = h_e$$

For $\mu > 0$, ($\Delta p < 0$)
We note, $\Delta T < 0$ i.e. $T_e < T_i$

For $\mu < 0$ ($\Delta p < 0$ always)
We note, $\Delta T > 0$, i.e. $T_e > T_i$

For ideal gas, we know $h=h(T)$ only, thus fixing h means $T=\text{constant}$,

$$\left.\frac{\partial T}{\partial P}\right|_{h(T)} = 0 = \mu$$