4.3 Hyperstatic systems

- "Statically determinate" or "isostatic" ("the same") -- No. of eqns of equilibrium
  = total No. of unknown internal forces
  {reaction forces / forces acting in the members}
  - unknown forces can be determined from the eqns of equilibrium alone, without using the strain-displacement relation / constitutive laws
  - Example 4.1

- "Statically indeterminate" or "hyperstatic" ("larger") systems ... total No. of unknown forces > No. of equilibrium eqns

- "degree of redundancy" N_R ... No. of unknown internal forces − No. of eqns of equilibrium
  - Example 4.2 : $N_R = 4 - 3 = 1$ "a single degree of redundancy"
    "hyperstatic of order 1"
  - simultaneous solution of the 3 fundamental groups of equations

- Difference between iso- and hyperstatic systems
  i) solution procedure
    - iso- : eqns. of equilibrium are only needed
    - hyper- : equilibrium eqns cannot be solved independently of the other 2 sets of eqns of elasticity
    - 2 main approaches {the displacement method / " force "}

ii) nature of the sol. for the unknown internal forces

- iso - : internal forces can be expressed in terms of the externally applied forces → " is independent of "

- hyper - : internal forces depend on the applied loads, but also on the stiffness of the structure

→ internal force distribution depends on the stiffness characteristics of the structure

iii) - hyper - : "dual load paths"

- equilibrium eqns are not sufficient to determine how much of the load will be carried by load path 1, 2, ...
- according to their relative stiffness, the stiffer load path will carry more load than the more compliant one

- iso - : "single load path" ← • more damage tolerant

4.3.2 The displacement or stiffness method

• expressing the governing in terms of displacements

① equilibrium eqns of the system ··· free body diagrams

② constitutive laws ··· express internal forces in terms of member deformations or strains

③ strain-displacement eqns ··· express system deformation in terms of displacements

④ Introduce ③ → ② ··· find the internal forces in terms of displacements

⑤ " ④ → ① ··· yield the eqns of equilibrium in terms of displacement

⑥ Solve ⑤ ··· find the displacement of the system

⑦ Find system deformations ··· back-substitute the displacements into ③

⑧ " internal forces ··· " deformations " ②

4.3.3 The force or flexibility method

- focuses on the solution of the system internal forces. strains and displacements are then recovered

① equilibrium eqns of the system

② determine N_R

③ cut the system at N_R locations and define a single relative displacement for each of the cuts. → originally hyperstatic system is transformed into an isostatic system.

④ Apply N_R redundant forces, each along the relative displacements.
Express all internal forces in terms of { the applied loads, N_R redundant forces

⑤ constitutive laws ··· express system deformations in terms of N_R redundant forces

⑥ strain-displacement eqns ··· express the relative displacements at N_R cuts in terms of N_R redundant forces

⑦ impose vanishing of the relative displacements at N_R cuts

⑧ recover system deformations and system displacements

{ force method → a linear set of eqns of size N_R, can be applied effectively using good engineering judgement
{ displacement " → " N_D (unknown displacements)
more amenable to automated solution processes