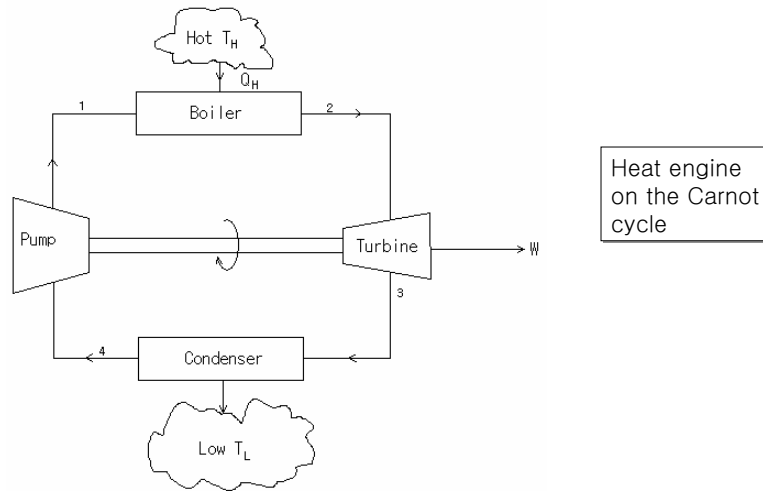


Chapter 6(계속) Entropy – Changes in reversible processes



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- 1→2 Isothermal + Reversible

$$s_2 - s_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{REV} = \frac{1}{T_H} Q_{1,2}$$

- 2→3 Adiabatic + Reversible = Isentropic

$$s_2 = s_3$$

- 3→4 Isothermal + Reversible

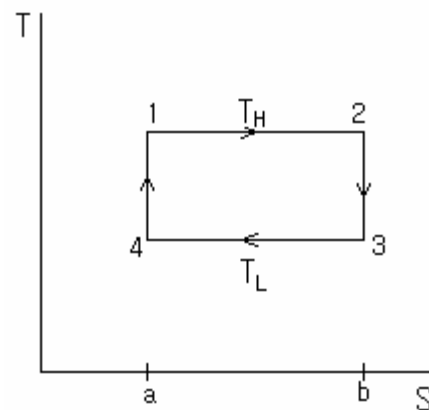
$$s_4 - s_3 = \int_3^4 \left(\frac{\delta Q}{T} \right)_{REV} = \frac{1}{T_L} Q_{3,4}$$

- 4→1 Isentropic

$$s_4 = s_1$$

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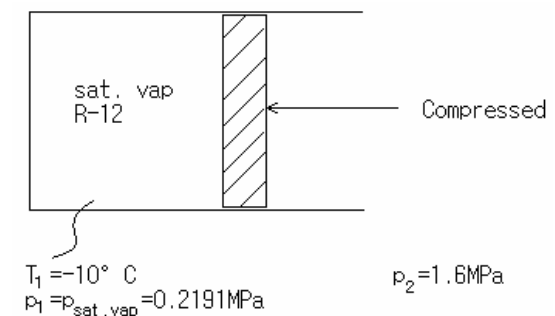


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- Example

Consider a cylinder fitted with a piston that contains saturated R-12 vapor at -10°C . Let this vapor be compressed in a **reversible & adiabatic** process until the pressure is 1.6MPa. What is the work per kilogram of R-12 for this process? Find W?



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$$m_2 - m_1 + m_e - m_i = 0 \rightarrow m_2 = m_1$$

$$q_{1,2} = u_2 - u_1 + w_{1,2} = 0 \quad \text{Adiabatic}$$

$$\rightarrow w_{1,2} = u_2 - u_1$$

From R-12 saturated vapor table,

$$h_g = 183.058 \text{ kJ/kg}$$

$$s_g = 0.7014 \text{ kJ/kg} = s_1$$

Since reversible and adiabatic = isentropic,

$$s_1 = s_2 = 0.7014 \text{ kJ/kg}$$

$$p_2 = 1.6 \text{ MPa}$$

Using p_2, s_2 find u_2 from table,

$$w_{1,2} = u_1 - u_2 = 166.4 - 200.57 = -34.17 \text{ kJ/kg}$$

• Thermodynamic Relations

Consider a simple compressible substance.

$$T dS = dU + p dV \quad \text{(i)}$$

using

$$H \equiv U + pV$$

$$dH = dU + p dV + V dp$$

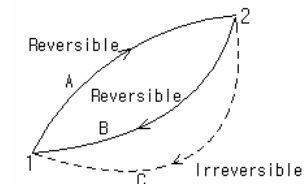
then

$$T dS = dH - V dp \quad \text{(ii)}$$

For reversible process, (i) becomes

$$\delta Q = T dS \quad \text{and} \quad \delta W = p dV$$

• Irreversible process



For reversible cycle, $\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T}\right)_A + \int_2^1 \left(\frac{\delta Q}{T}\right)_B = 0$

For irreversible cycle, $\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T}\right)_A + \int_2^1 \left(\frac{\delta Q}{T}\right)_C < 0$

Subtracting the two lines, we get

$$\int_2^1 \left(\frac{\delta Q}{T}\right)_B > \int_2^1 \left(\frac{\delta Q}{T}\right)_C$$

Since path B is reversible and since entropy is a property

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_B = \int_2^1 dS_B \quad \text{----- Reversible}$$

$$= \int_2^1 dS_C$$

$$\therefore \int_2^1 dS_C > \int_2^1 \left(\frac{\delta Q}{T}\right)_C \quad \text{----- Irreversible}$$

In general

$$s_2 - s_1 \geq \frac{\delta Q}{T} \quad (*) \text{ Most important inequality!}$$

- **Lost Work (LW)**

For a simple compressible substance,

$$pdV = \delta W + \delta LW$$

In other words,

$$TdS = dU + \delta W + \delta LW$$

That is, we can write **inequality** (*) as **equality**:

$$dS = \frac{\delta Q}{T} + \frac{\delta LW}{T} \quad \text{For irreversible process}$$

Ex) For reversible process, $\delta LW = 0$ i.e. no lost work.

- **Entropy change of an Ideal Gas**

We learned the thermodynamic relation,

$$TdS = du + pdv$$

For an ideal gas,

$$du = c_v dT, \quad p = \frac{RT}{v}$$

Therefore, $ds = c_v \frac{dT}{T} + R \ln \frac{v_2}{v_1}$

$$s_2 - s_1 = \int_1^2 c_v \frac{dT}{T} + R \ln \frac{v_2}{v_1} \quad \text{(iii)}$$

Similarly,

$$Tds = dh - vdp$$

For an ideal gas,

$$du = c_v dT, \quad \frac{v}{T} = \frac{R}{p}$$

Therefore, $ds = c_p \frac{dT}{T} - R \frac{dp}{p}$

$$s_2 - s_1 = \int_1^2 c_p \frac{dT}{T} - R \ln \frac{p_2}{p_1} \quad \text{(iv)}$$

If c_v, c_p are constants,

then (iii) becomes

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

(iv) Becomes

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

Now, we introduce,

$$\gamma = \frac{c_p}{c_v}$$

Recall,

$$R \equiv c_p - c_v$$

Then we have,

$$c_v = \frac{R}{\gamma-1}, c_p = \frac{\gamma R}{\gamma-1}$$

For a reversible adiabatic (isentropic) process, ($ds = 0$),

$$\begin{aligned} Tds &= du + pdv & \text{------(vi)} \\ &= c_v dT + pdv = 0 \end{aligned}$$

Now consider the ideal EOS

$$T = \frac{1}{R} pv \quad \text{------(vii)}$$

$$dT = \frac{1}{R} (pdv + vdp)$$

Substitute (vii) into (vi), we get

$$\frac{c_v}{R} (pdv + vdp) + pdv = 0$$

and using $c_v = \frac{R}{\gamma-1}$

$$\frac{1}{\gamma-1} (pdv + vdp) + pdv = 0$$

$$\frac{\gamma}{\gamma-1} (pdv) + \frac{1}{\gamma-1} (vdp) = 0$$

$$\gamma \frac{dv}{v} = -\frac{dp}{p}$$

$$\ln v^\gamma = -\ln p + c$$

$$\ln v^\gamma + \ln p = c$$

$$\ln(pv^\gamma) = c$$

Or

$$pv^\gamma = c \quad \text{Isentropic}$$

For all reversible adiabatic processes that involve an ideal gas with constant specific heat

Fourth useful relations...

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^\gamma = \left(\frac{V_1}{V_2}\right)^\gamma$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$