Lecture 12. Aperture and Noise Analysis of Clocked Comparators

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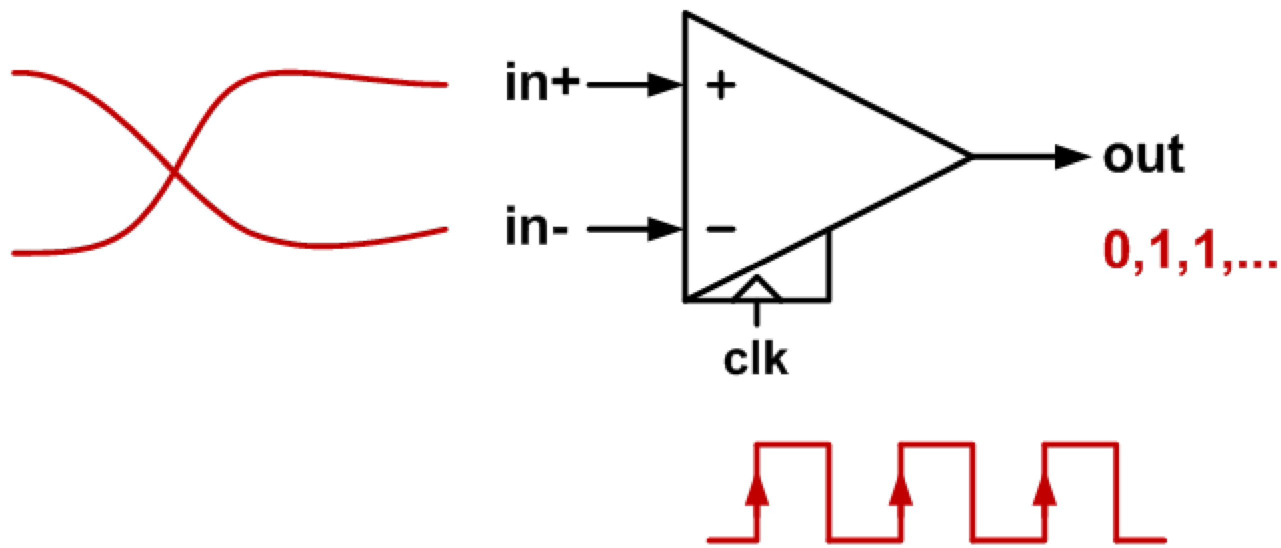
Clocked Comparators

- a.k.a. regenerative amplifier, sense-amplifier, flip-flop, latch, etc.
- At every clock edge, sample the input (continuous) and decide whether it is 0 or 1 (binary)
  - Therefore, it's inherently nonlinear operation

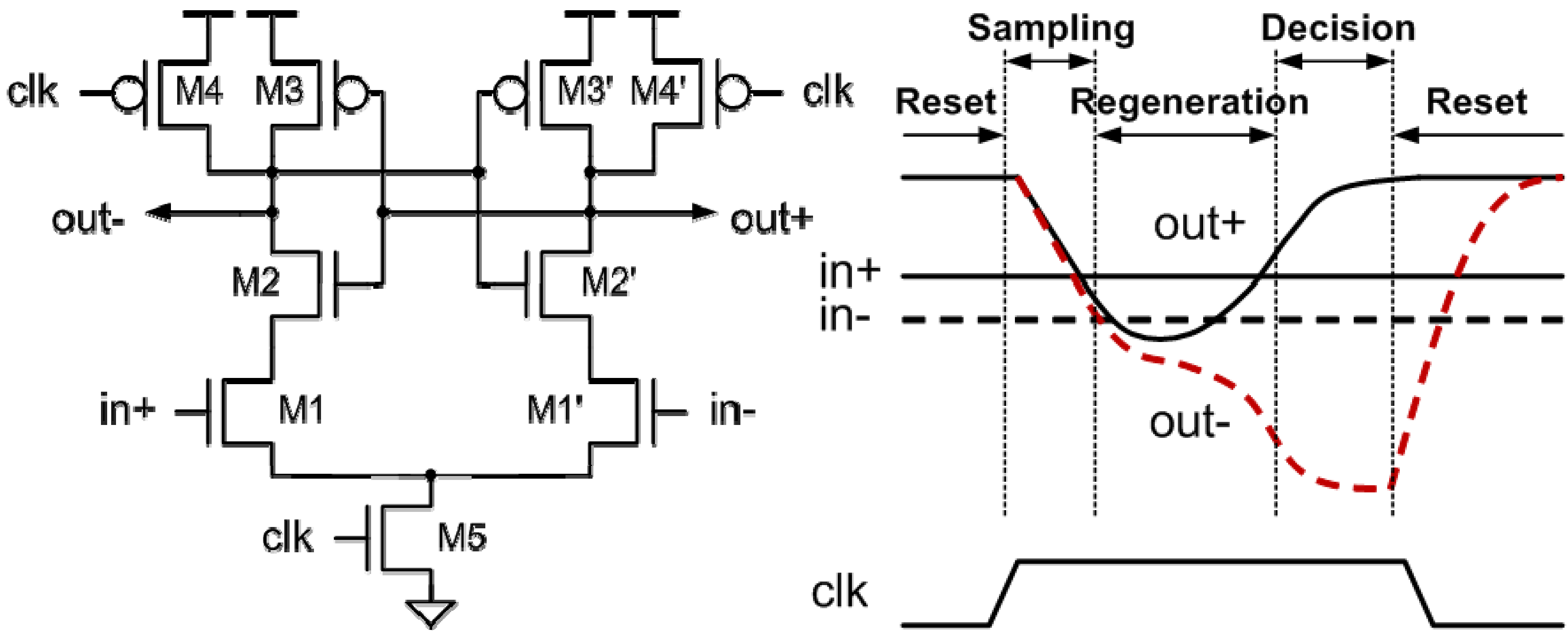


Comparator Characteristics

- Offset and hysteresis
- Sampling aperture, timing resolution, uncertainty window
- Regeneration gain, voltage sensitivity, metastability
- Random decision errors, input-referred noise

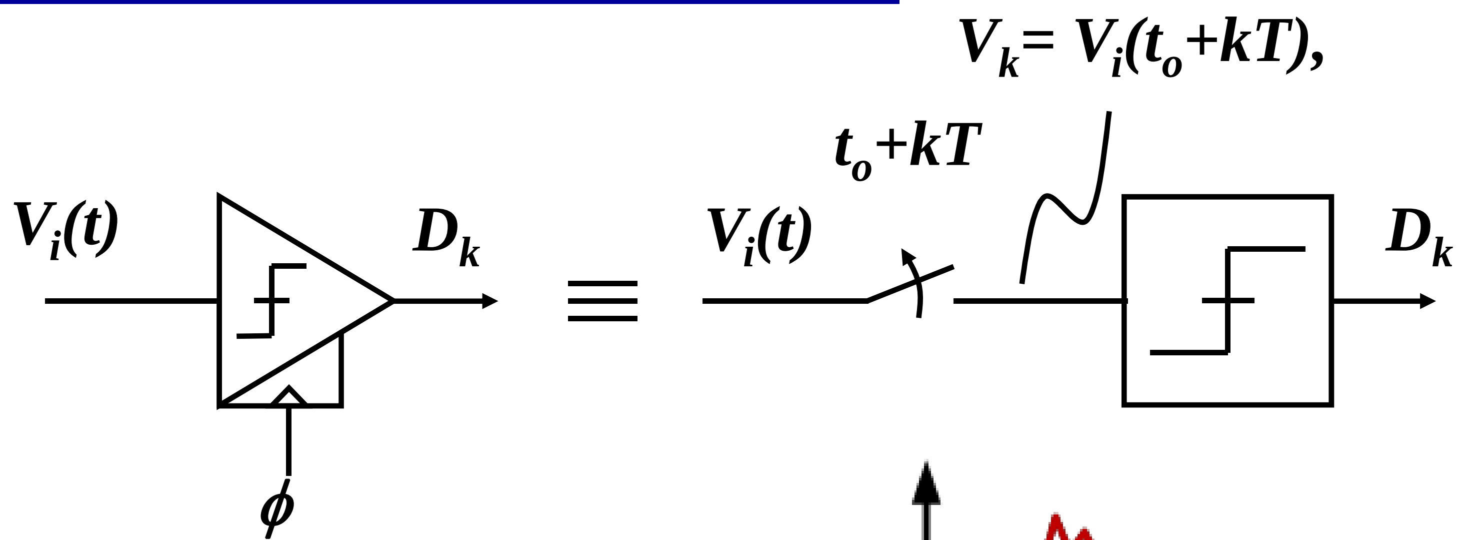
⇨ *Can be analyzed and simulated based on a linear, time-varying (LTV) model of the comparator*

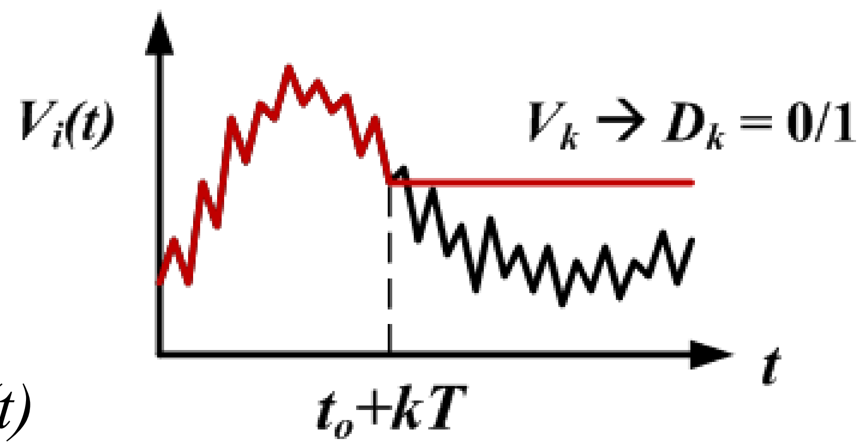
Clocked Comparator Operation



- 4 operating phases: reset, sample, regeneration & decision
- Sampling & regeneration phases can be modeled as LTV

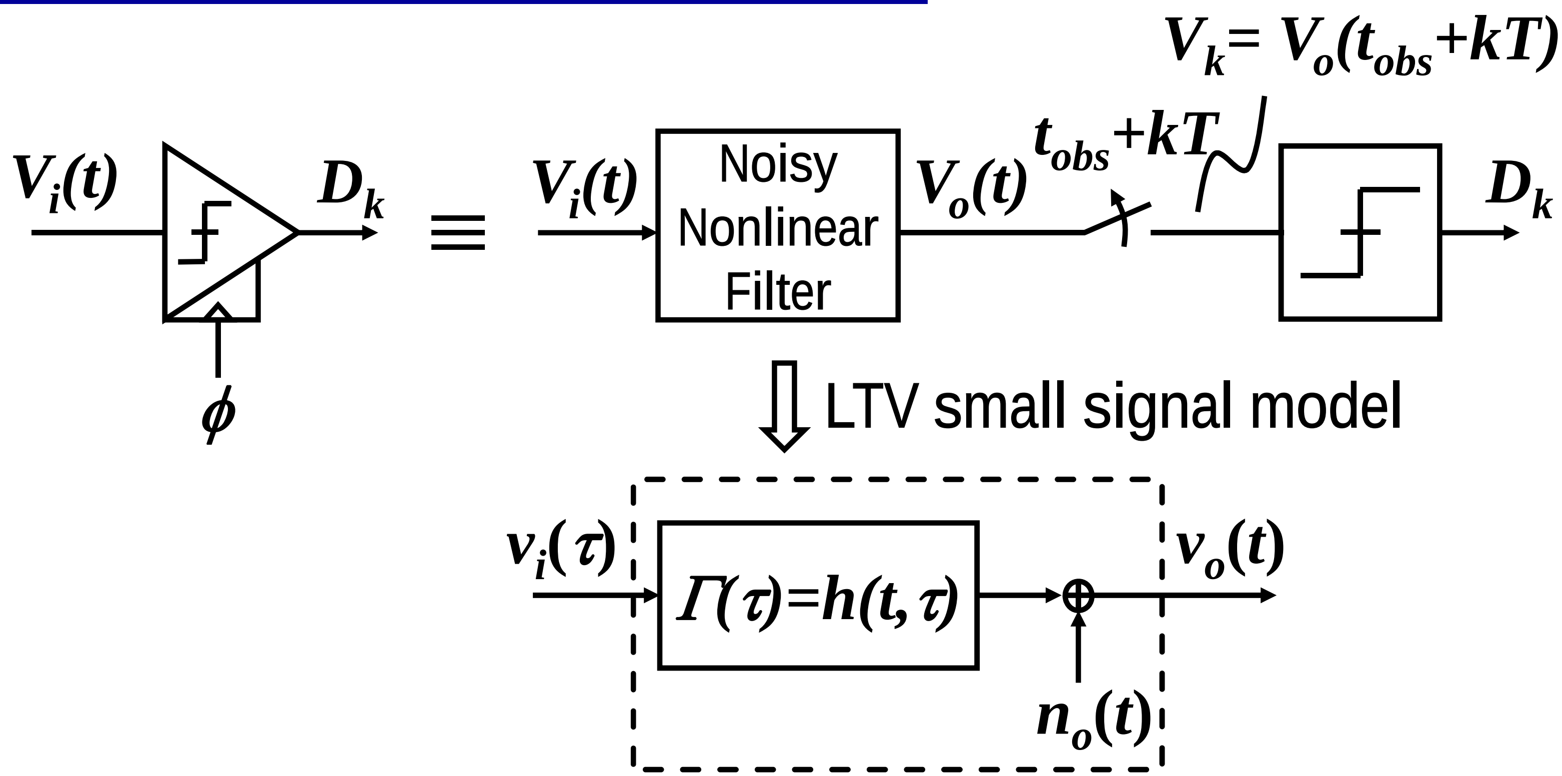
An Ideal Comparator Model





- Sampling and decision
  - Infinitely-fast tracking of $V_i(t)$
- A realistic comparator acts on a filtered version of $V_i(t)$

LTV Model for Clocked Comparator



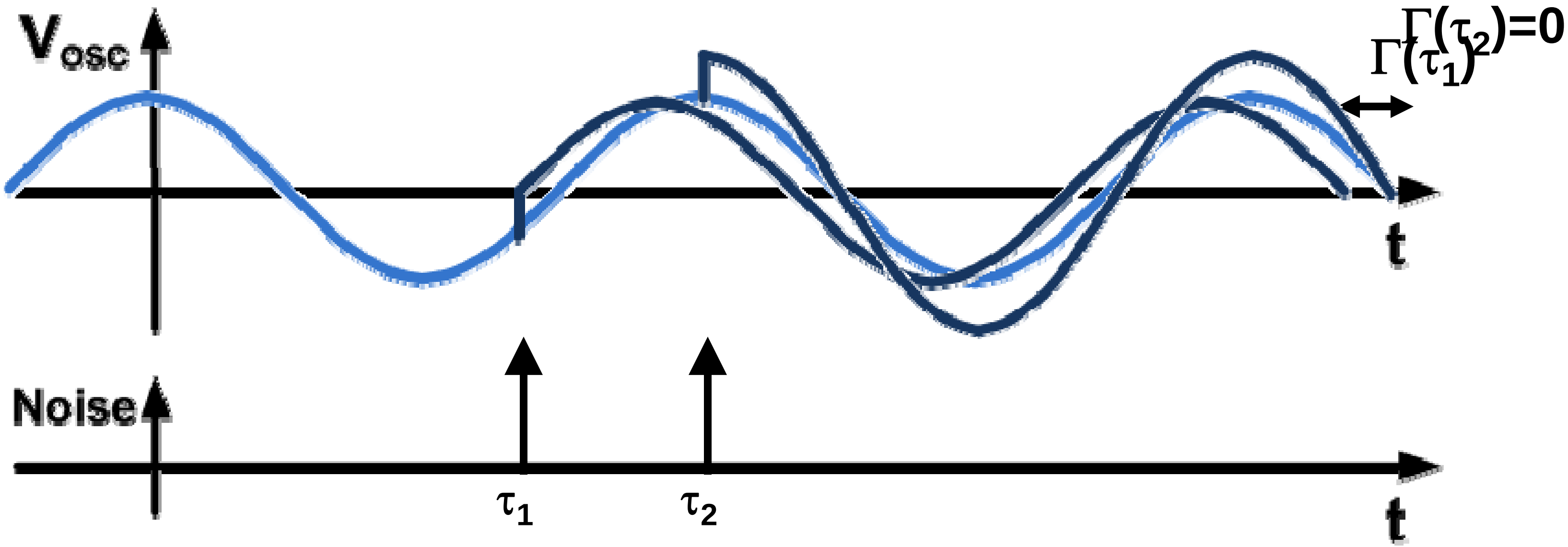
- Assumes a noisy, nonlinear filter before the sampling
- The filter's small-signal response is modeled with ISF $\Gamma(\tau)$

* J. Kim, et al., "Simulation and Analysis of Random Decision Errors in Clocked Comparators," IEEE TCAS-I, 08/2009.

ISF for Oscillators

- Impulse sensitivity function (ISF) $\Gamma(\tau)$ is defined as:

  $\Gamma(\tau)$ = the final shift in the oscillator phase due to a unit impulse arriving at time τ



* A. Hajimiri and T. H. Lee, “A General Theory of Phase Noise in Electrical Oscillators,” IEEE JSSC, Feb. 1998.

ISF for Oscillators (2)

- ISF describes the time-varying response of a oscillator
  - Responses to each impulse add up via superposition
  - For arbitrary noise input *n(t)*, the resulting phase shift $\Delta\phi$ is:

$$\Delta\phi = \int_{-\infty}^{\infty} \Gamma(\tau) \cdot n(\tau)\, d\tau$$

- ISF led to some key oscillator design idioms:
  - Sharpen the clock edge to lower ISF (i.e. minimize Γ_{RMS})
  - Align noise events within low-ISF period
  - Balance ISF (i.e. Γ_{DC}=0) to prevent 1/f-noise up-conversion

ISF for Samplers and Comparators

- For sample-and-hold circuits, the sampled voltage V_s can be expressed via a "sampling function" $f(t)$:

$$V_s = \int_{-\infty}^{\infty} f(\tau) \cdot V_i(\tau)\, d\tau$$

* H. O. Johansson, C. Svensson, "Time Resolution of NMOS Sampling Switches Used on Low-Swing Signals," JSSC, Feb. 1998.

- For clocked comparators, we simply add the "decision":

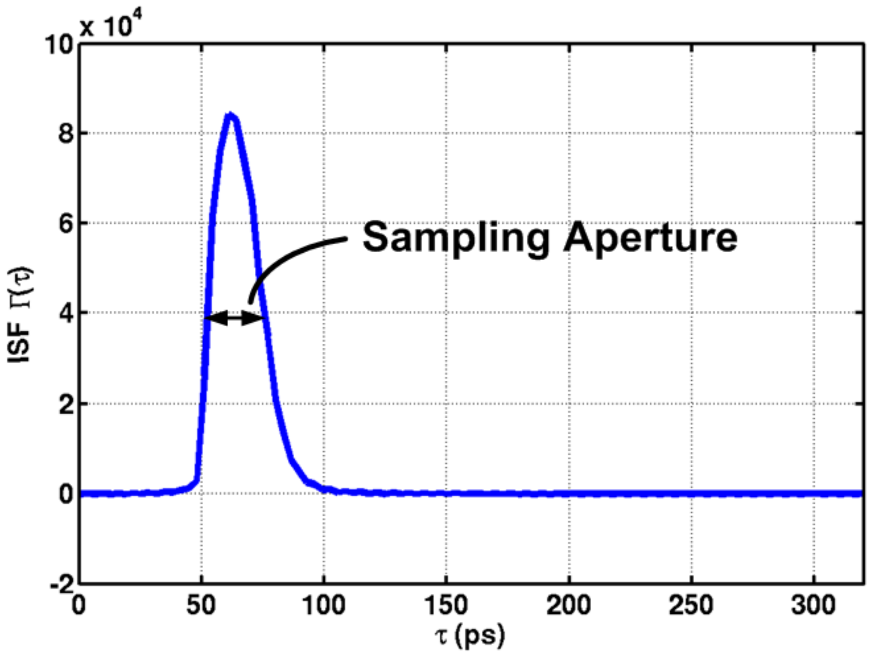
$$D_k = \mathrm{sgn}(V_k) = \mathrm{sgn}\left(\int_{-\infty}^{\infty} \Gamma(\tau) \cdot V_i(\tau)\, d\tau \right)$$

* P. Nuzzo, et al., "Noise Analysis of Regenerative Comparators for Reconfigurable ADC Architectures," TCAS-I, July 2008.

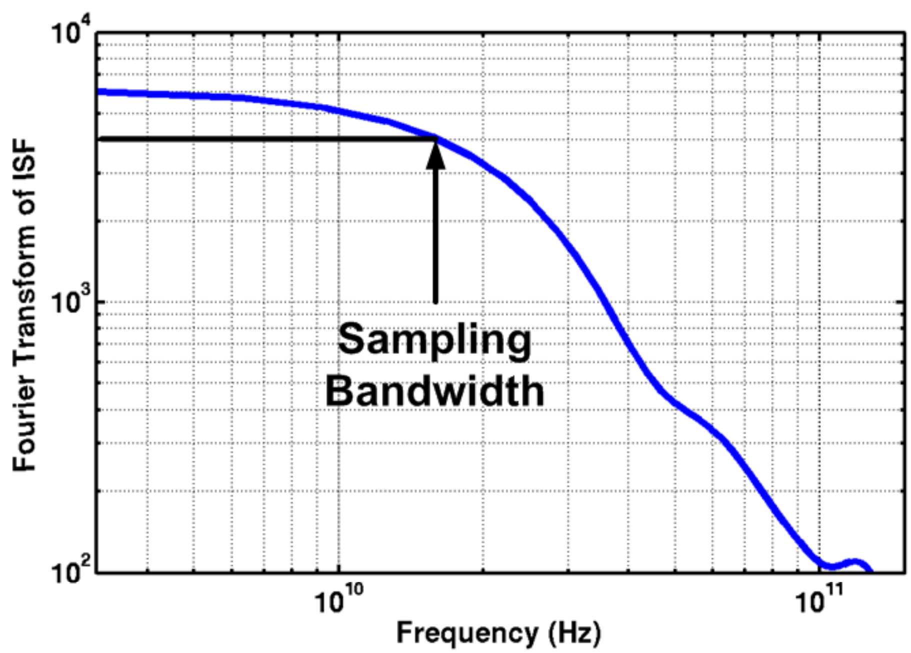
ISF for Clocked Comparators

- ISF shows sampling aperture, i.e. timing resolution
- In frequency domain, it shows sampling gain and BW

ISF $\Gamma(\tau)$



F.T. { $\Gamma(-\tau)$ }



Generalized ISF

- In general, ISF is a subset of a so-called *time-varying impulse response* $h(t,\ \tau)$ for LTV systems*:

$$y(t) = \int_{-\infty}^{\infty} h(t,\tau) \cdot x(\tau)\, d\tau$$

  - $h(t,\ \tau)$: the system response at t to a unit impulse arriving at τ
  - For LTI systems, $h(t,\ \tau) = h(t-\tau) \rightarrow$ convolution
- ISF $\Gamma(\tau) = h(t_0,\ \tau)$
  - t_0: the time at which the system response is observed
  - For oscillators, $t_0 = +\infty$
  - For comparators, t_0 is before the decision is made (more later)

* L. Zadeh, "Frequency Analysis of Variable Networks," Proc. I.R.E. Mar. 1950.

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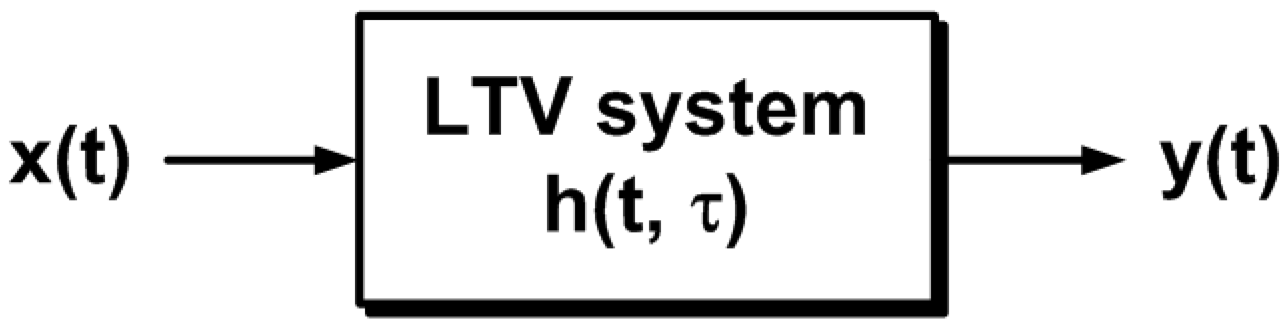
Noise in LTV Systems

- If the input $x(t)$ to an LTV system is a noise process, then the output $y(t)$ is a <u>*time-varying*</u> noise in general
  - Expressions become very complex (cyclo-stationary at best)
- We can keep things simple if we are interested in the noise only at one time point (in our case: $t_0 = t_{obs} + kT$)



LTV Output Noise at $t = t_0$

$$\sigma_y^{\,2}(t_0) = \mathrm{E}\left[y^2(t_0)\right] = \mathrm{E}\left[y(t_0)\cdot y(t_0)\right]$$

$$= \mathrm{E}\left[\left(\int_{-\infty}^{\infty} h(t_0,u)\cdot x(u)du\right)\cdot\left(\int_{-\infty}^{\infty} h(t_0,v)\cdot x(v)dv\right)\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left[x(u)\cdot x(v)\right]\cdot h(t_0,u)\cdot h(t_0,v)\,du\,dv$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{xx}(u,v)\cdot h(t_0,u)\cdot h(t_0,v)\,du\,dv$$

- $R_{xx}(u, v)$ is the auto-correlation of the input noise $x(t)$

Response to White and 1/f Noises

- If the input $x(t)$ is white noise, i.e. $R_{xx}(u, v) = \sigma_x^{\ 2} \cdot \delta(u\text{-}v)$:

$$\sigma_y^{\ 2}(t_0) = \sigma_x^{\ 2} \cdot \int_{-\infty}^{\infty} h^2(t_0, u)\, du = \sigma_x^{\ 2} \cdot \int_{-\infty}^{\infty} \Gamma^2(\tau)\, d\tau$$

- If the input $x(t)$ is 1/f noise, i.e. $R_{xx}(u, v) = \sigma_x^{\ 2}$:

$$\sigma_y^{\ 2}(t_0) = \sigma_x^{\ 2} \cdot \left[\int_{-\infty}^{\infty} h(t_0, u)\, du \right]^2 = \sigma_x^{\ 2} \cdot \left[\int_{-\infty}^{\infty} \Gamma(\tau)\, \mathrm{d}\tau \right]^2$$

- Agrees with Hajimiri/Lee's low-noise design idioms:
  - To minimize contribution of white noise, minimize Γ_{RMS}
  - To minimize contribution of 1/f noise, make $\Gamma_{DC} = 0$

Random Decision Error Probability

- If we have multiple noise sources, their contributions add up via RMS sum assuming they are independent:

$$\sigma_{y,total}{}^2(t_o) = \sum_j \sigma_{y,j}{}^2(t_o)$$

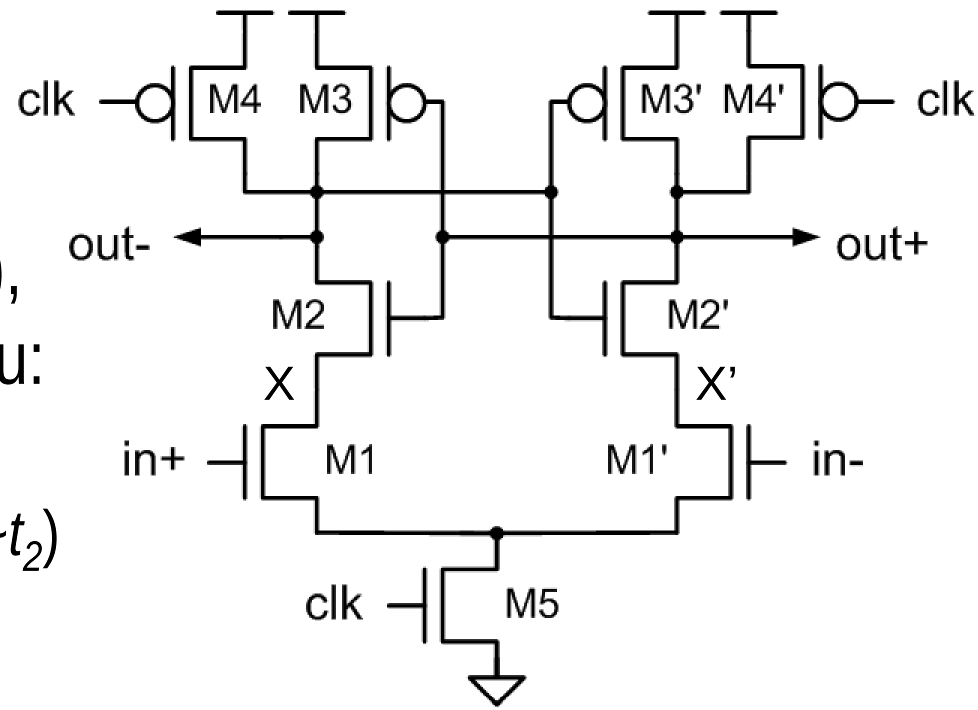
- If the comparator has signal V_o and noise $\sigma_{n,o}$ at t_{obs}, the decision error probability $P(error)$ can be estimated as:

$$\sqrt{SNR} = V_o(t_{obs}) / \sigma_{n,o}(t_{obs})$$

$$P(error) = Q\left(\sqrt{SNR}\right) = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{SNR}}^{\infty} \exp(-x^2/2)\, dx$$

Circuit Analysis Example

- A variant of StrongARM comparator
- When clk is low, the comparator is in reset
  - out+/- are at V_{dd}
  - X/X' are ~V_{dd}-V_{TN}



- When clk rises (say $t=t_0$), the comparator goes thru:
  - Sampling phase (t_0~t_1)
  - Regeneration phase (t_1~t_2)

1. Sampling Phase ($t = t_0 \sim t_1$)

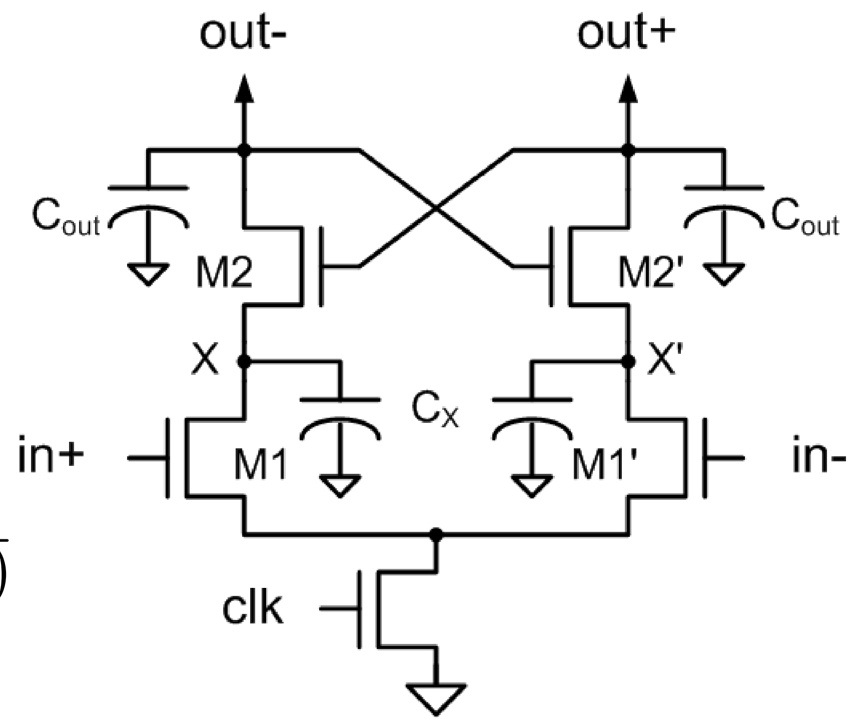
- While out+/- remain high:
  - M1-pair discharges X/X'
  - M2-pair discharges out+/-
- S.S. transfer from v_{in} to v_{out}:

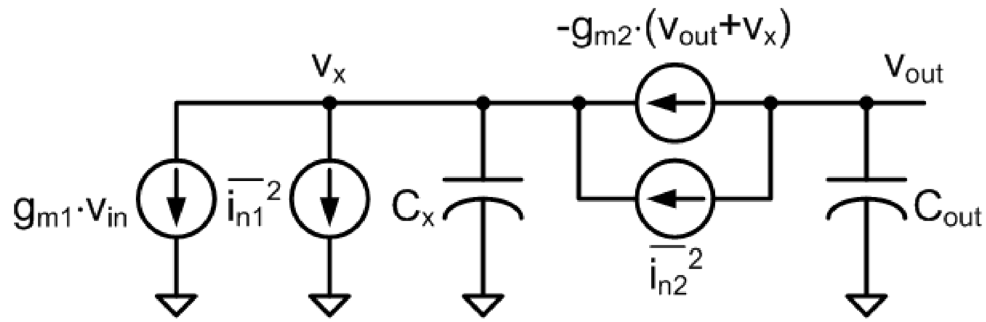
$$\frac{v_{out}(s)}{v_{in}(s)} = \frac{g_{m1} \cdot g_{m2}}{sC_{out}C_x\left(s + g_{m2}(C_{out} - C_x)/C_{out}C_x\right)}$$

$$\cong \frac{g_{m1} \cdot g_{m2}}{s^2 C_{out} C_x} = \frac{1}{s^2 \tau_{s1} \tau_{s2}}$$

- The ISF w.r.t. v_{in} is:

$$\Gamma(t) \cong \frac{t_1 - t}{\tau_{s1}\tau_{s2}} \cdot G_R$$





1. Sampling Phase (t = 0~t_1)

- S.S. response to M1 noise:

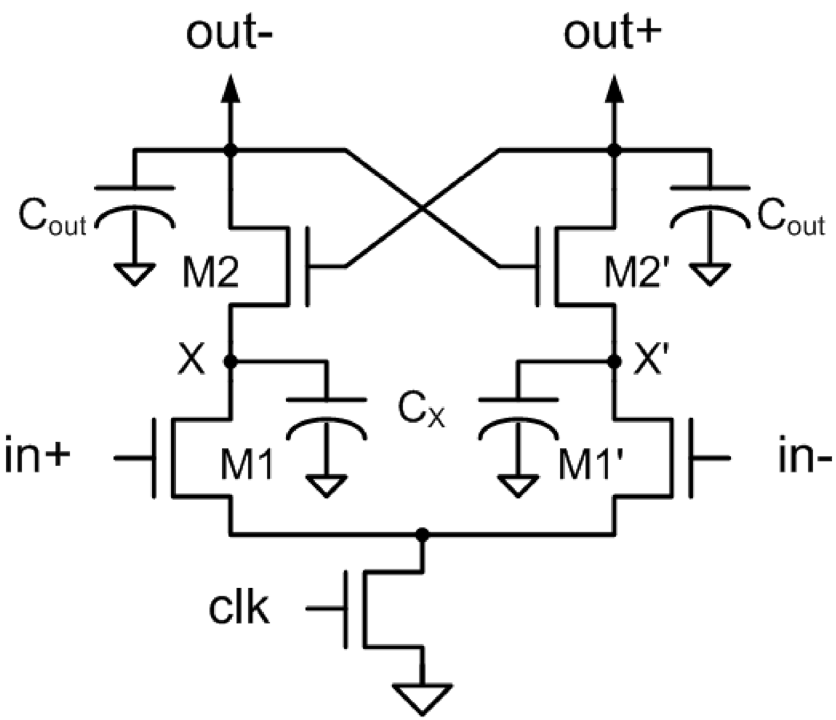
$$\frac{v_{out}(s)}{i_{n1}(s)} \cong \frac{g_{m2}}{s^2 C_{out} C_x}$$

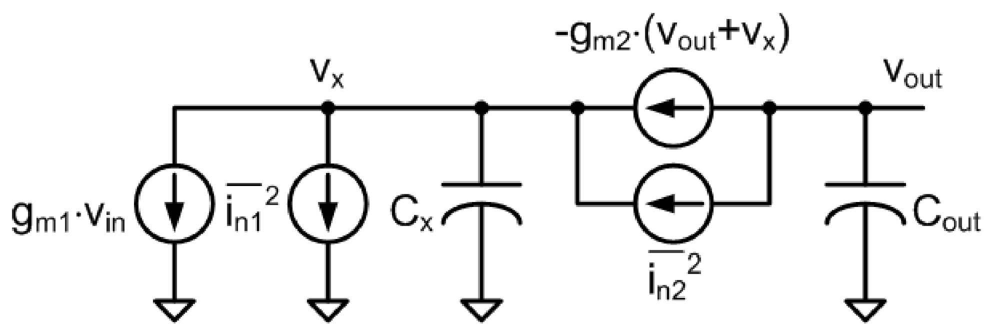
$$\Gamma_{n1}(t) \cong \frac{t_1 - t}{g_{m1}\tau_{s1}\tau_{s2}} \cdot G_R$$

- S.S. response to M2 noise:

$$\frac{v_{out}(s)}{i_{n2}(s)} \cong \frac{1}{sC_{out}}$$

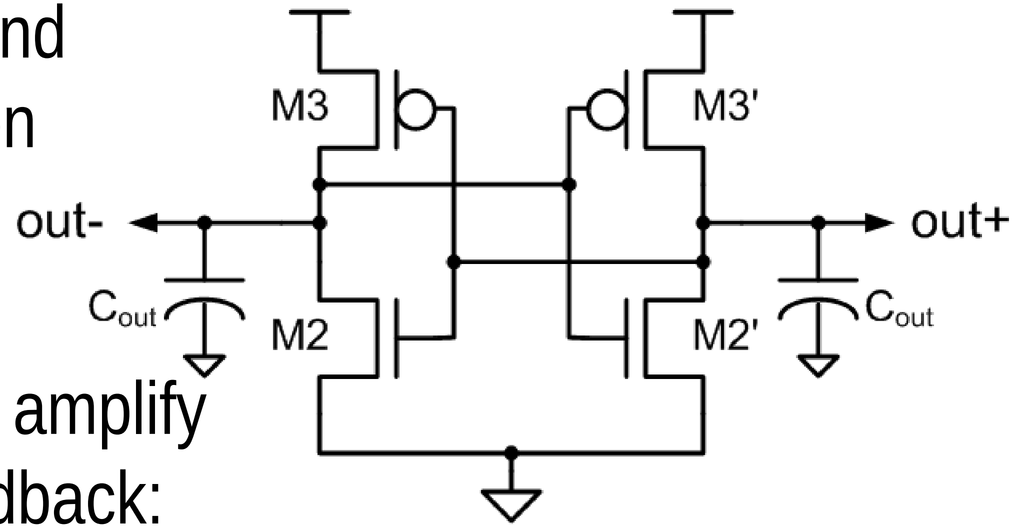
$$\Gamma_{n2}(t) \cong \frac{1}{C_{out}} \cdot G_R$$





2. Regeneration Phase ($t = t_1 \sim t_2$)

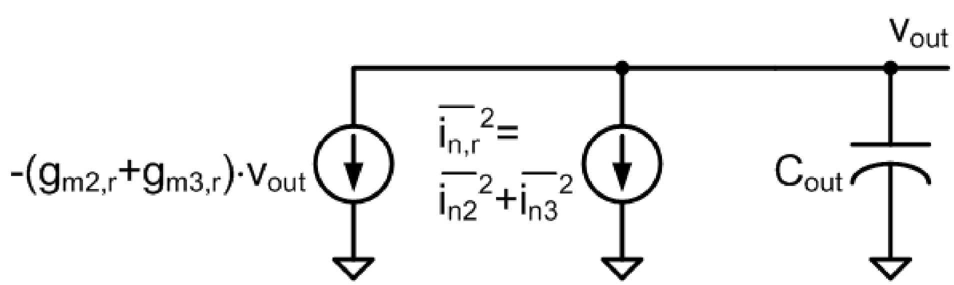
- We assume X/X' ~ 0V and M1-pair is in linear region
  - The circuit is no longer sensitive to v_{in} (ISF=0)



- Cross-coupled inverters amplify signals via positive-feedback:

$$G_R = \exp\left(\frac{t_2 - t_1}{\tau_R}\right)$$

$$\tau_R = C_{out} / (g_{m2,r} + g_{m3,r})$$



- The ISF w.r.t. noise is:

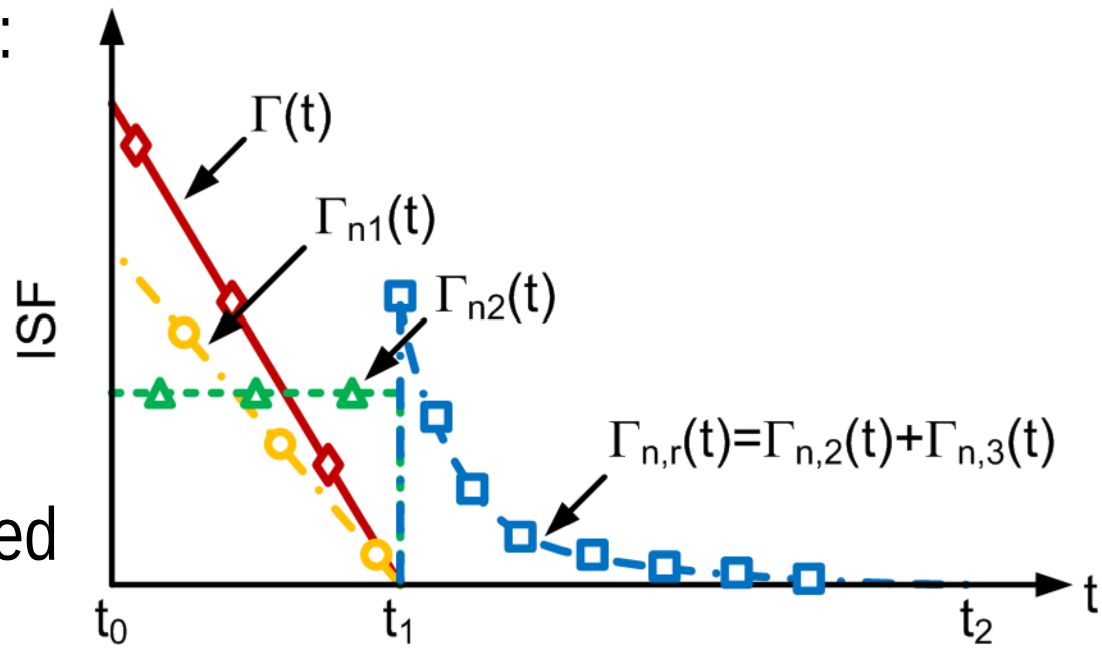
$$\Gamma_{n,r}(t) \cong \frac{1}{C_{out}} \cdot \exp\left(\frac{t_2 - t}{\tau_R}\right)$$

Putting It All Together

- The overall gain G is:

$$G = v_o(t_{obs}) / v_{i,dc} = \int_{-\infty}^{\infty} \Gamma(t)\,\mathrm{d}t$$

$$= \frac{(t_0 - t_1)^2}{2\tau_{s1}\tau_{s2}} \cdot \exp\left(\frac{t_2 - t_1}{\tau_R}\right)$$



- The total input-referred noise is:

$$\sigma_{n,i}^2 = \sigma_{n,o}^2 / G^2$$

$$\cong \frac{16kT\gamma}{3C_x} \cdot \frac{\tau_{s1}}{t_1 - t_0} + \frac{16kT\gamma}{C_{out}} \cdot \frac{\tau_{s1}{}^2 \cdot \tau_{s2}}{(t_1 - t_0)^3}$$

- Most of the noise is contributed by M1 and M2 pairs during the sampling phase

Design Trade-Offs

- The input-referred noise can be approximated as:

$$\sigma_{n,i}^2 \cong \frac{16kT\gamma}{3C_x} \cdot \frac{\tau_{s1}}{t_1 - t_0} + \frac{16kT\gamma}{C_{out}} \cdot \frac{{\tau_{s1}}^2 \cdot \tau_{s2}}{(t_1 - t_0)^3}$$

where

$$\frac{\tau_{s1}}{t_1 - t_0} \cong \frac{C_x}{C_{out}} \Big/ \left(\frac{g_{m1}}{I_{d1}} \cdot V_{Tp} \right), \quad \frac{\tau_{s2}}{t_1 - t_0} \cong 1 \Big/ \left(\frac{g_{m2}}{I_{d2}} \cdot V_{Tp} \right)$$

- Therefore, noise improves with larger g_m/I_d ratios and wider sampling aperture $(t_1\text{-}t_0)$
- However, sampling bandwidth and/or gain may degrade
  - Controlling the tail turn-on rate is a good way to keep high gain

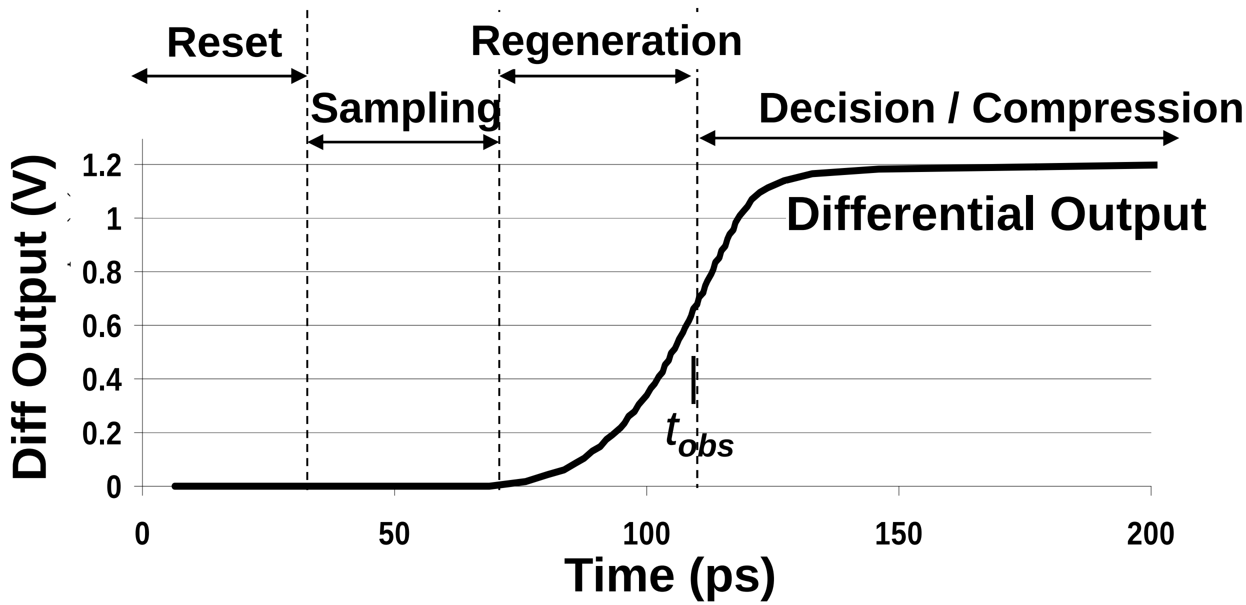
Simulating Aperture & Noise

- RF simulators (e.g. SpectreRF) can simulate small-signal LPTV response and noise efficiently:
  - Simulates linearized responses around a periodic steady-state
  - PAC analysis gives $H(j\omega;t)$ = Fourier transform of $h(t,\tau)$*
  - PNOISE analysis can give the noise PSD at one time point

- The remaining question is how to choose t_{obs}?
  - We'd like to choose it to mark the end of the regeneration
  - Since $\Gamma(\tau)$ in our LTV model captures sampling + regeneration

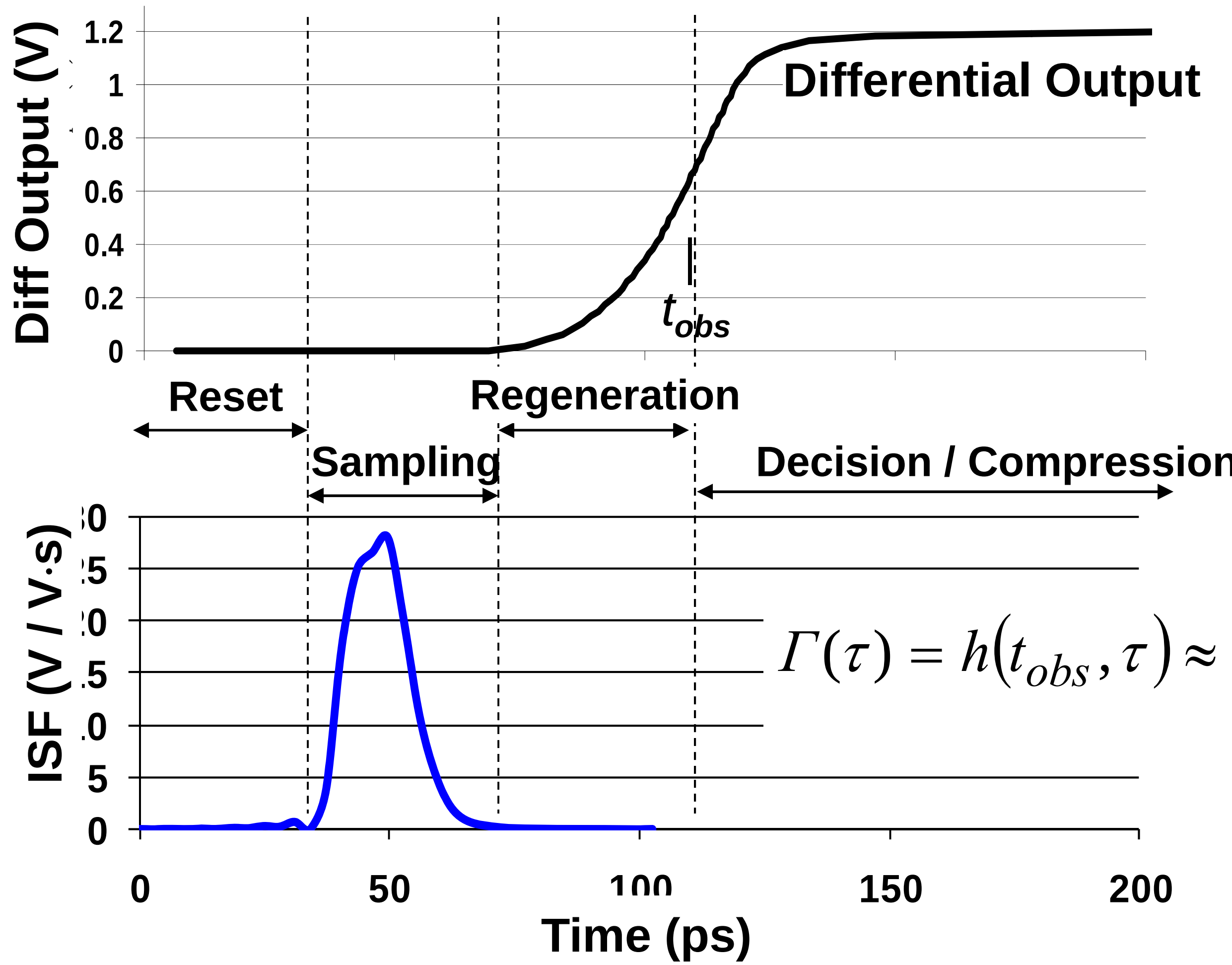
* J. Kim, et al., "Impulse Sensitivity Function Analysis of Periodic Circuits," ICCAD'08.

Comparator Periodic Steady-State (PSS)



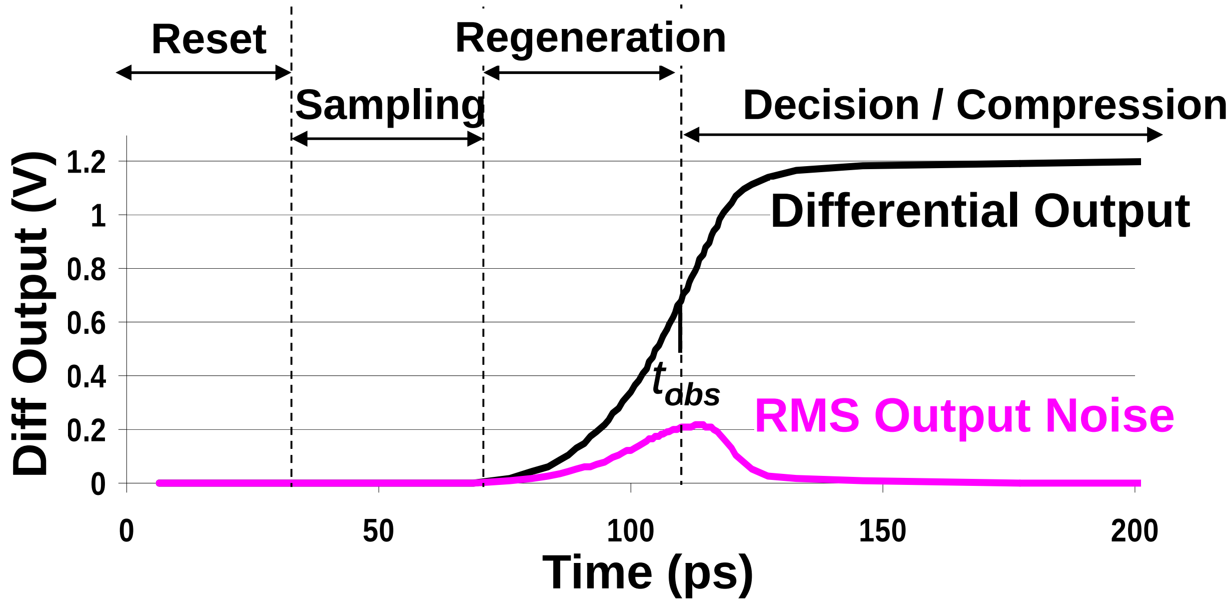
- PSS response of the comparator for a small DC input
  - Near the clock's rising edge; return to reset not shown

Comparator Sampling Aperture (PAC)



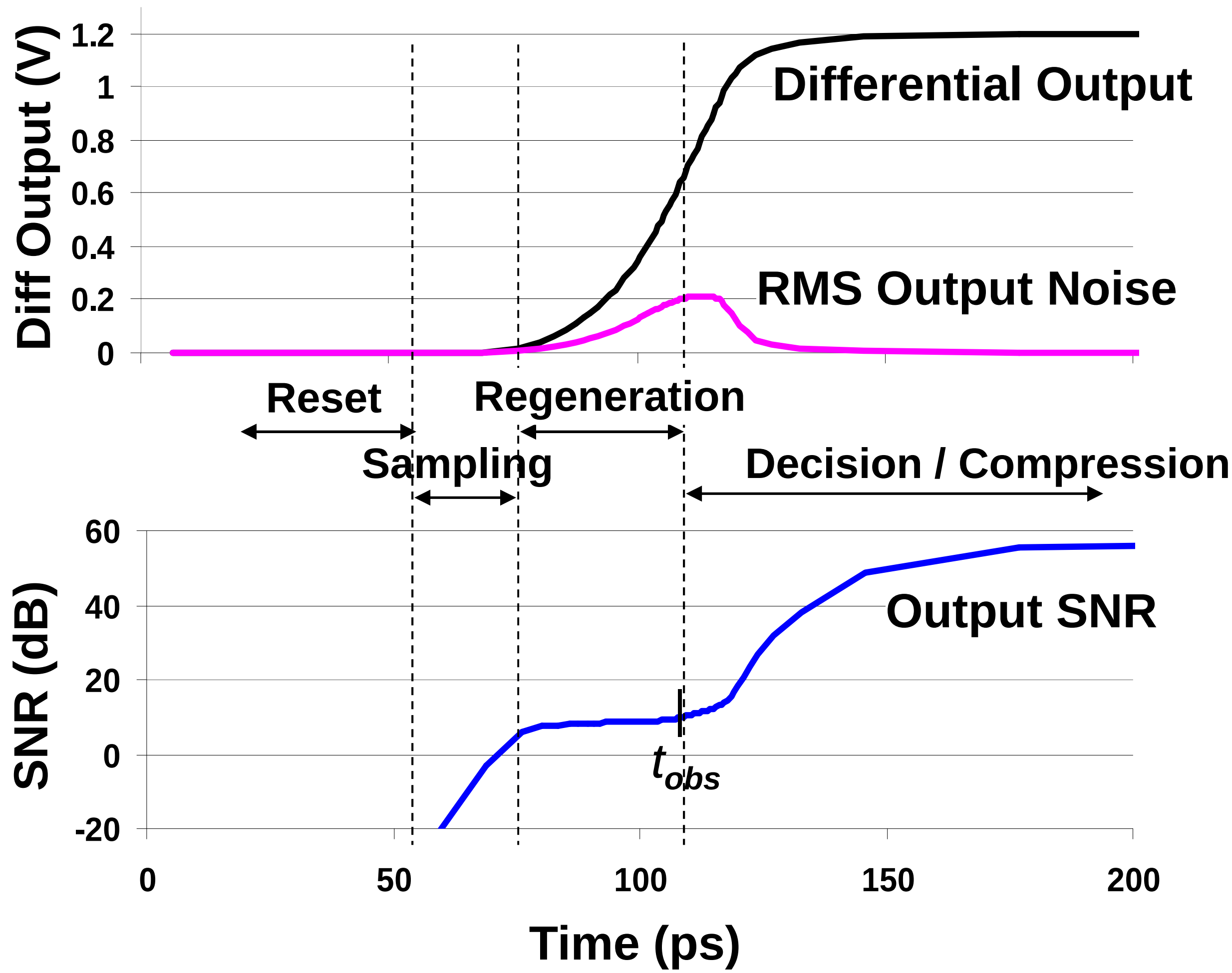
$$\Gamma(\tau) = h\left(t_{obs}, \tau\right) \approx \frac{\partial V_o(t_{obs})}{\partial V_i(\tau)}$$

Comparator Noise (PNOISE)



- Magenta line plots the rms output noise $\sigma(t)$ vs. time, obtained by integrating the noise PSD at each time point
- This is *not* "transient noise analysis"– it's a time sample of cyclo-stationary noise (much more efficient)

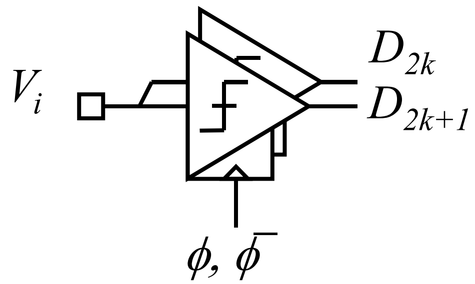
Comparator Output SNR



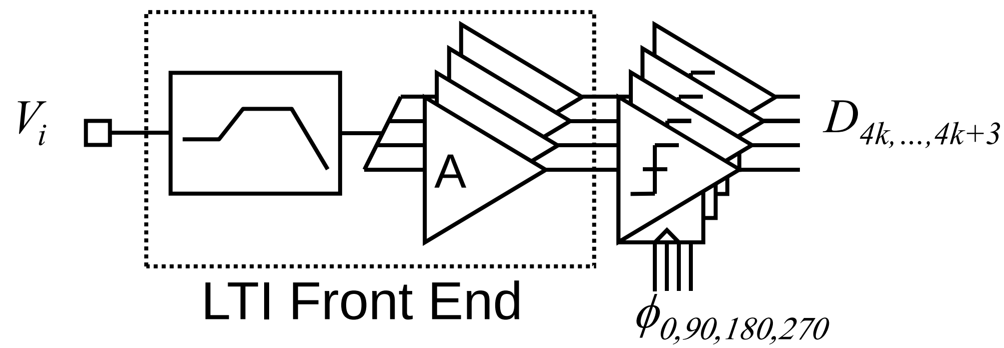
Deciding on t_{obs}

- How to choose t_{obs} that marks the end of regeneration
- Most of the noise is contributed during the sampling phase
  - Noise that enters during the sampling phase sees the full gain
  - Noise that enters later during the regeneration phase sees an exponentially decreasing gain with time
- For the purpose of estimating decision errors, selection of t_{obs} is not critical as long as it's within regeneration phase
  - The SNR and decision error probability stay ~constant
  - I choose t_{obs} when the comparator has the max. small-signal gain (i.e. before the nonlinearity starts suppressing the gain)

Measurement Results



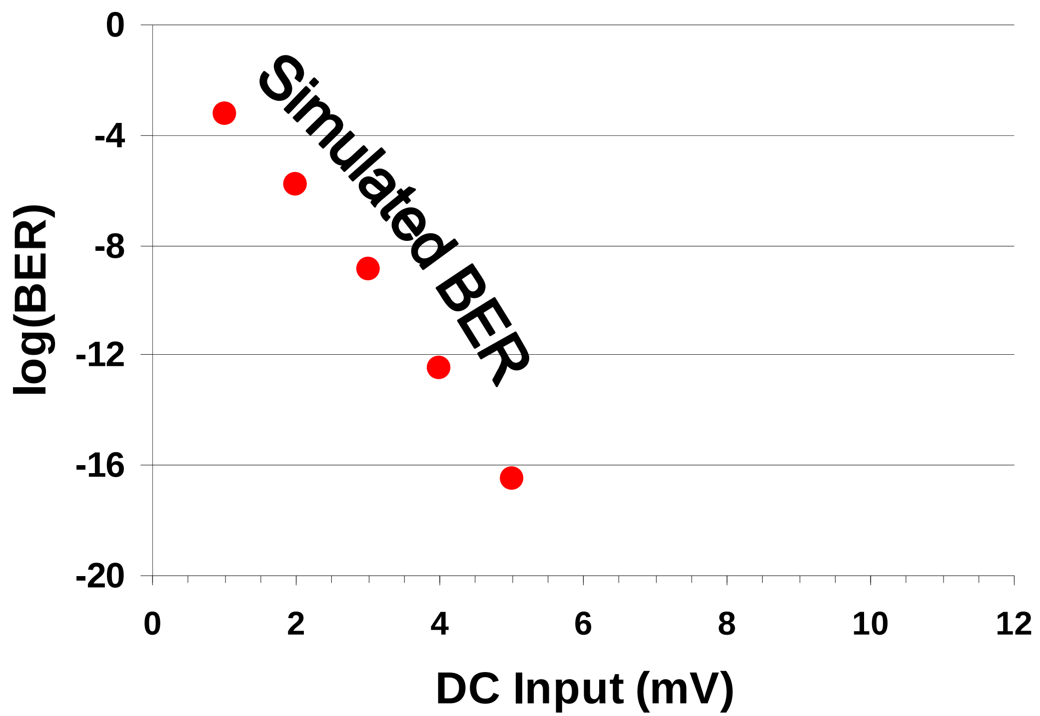
Receiver A (90nm)



Receiver B (65nm)

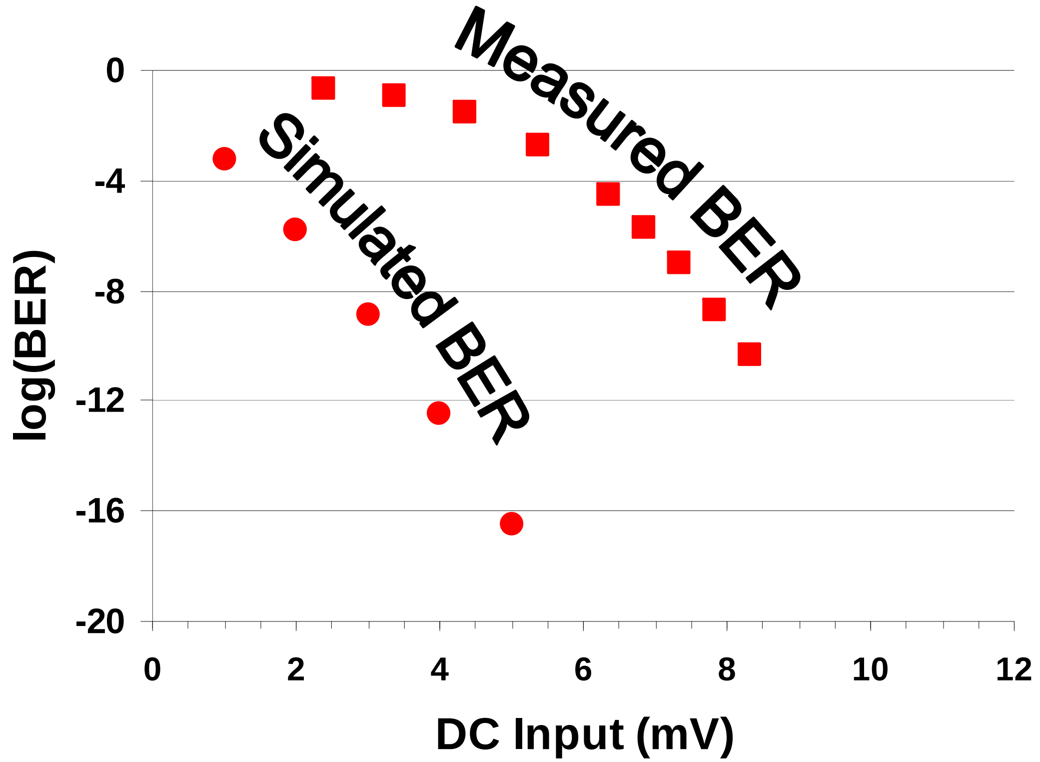
- Both receivers are based on StrongARM comparators
- Differential C_{in} ~ 2pF ⇨ thermal noise from the input termination resistors < 100μVrms
- Excess noise factor γ not spec'd by foundries ⇨ simulated at multiple values

Receiver A – Direct Sampling Front-End



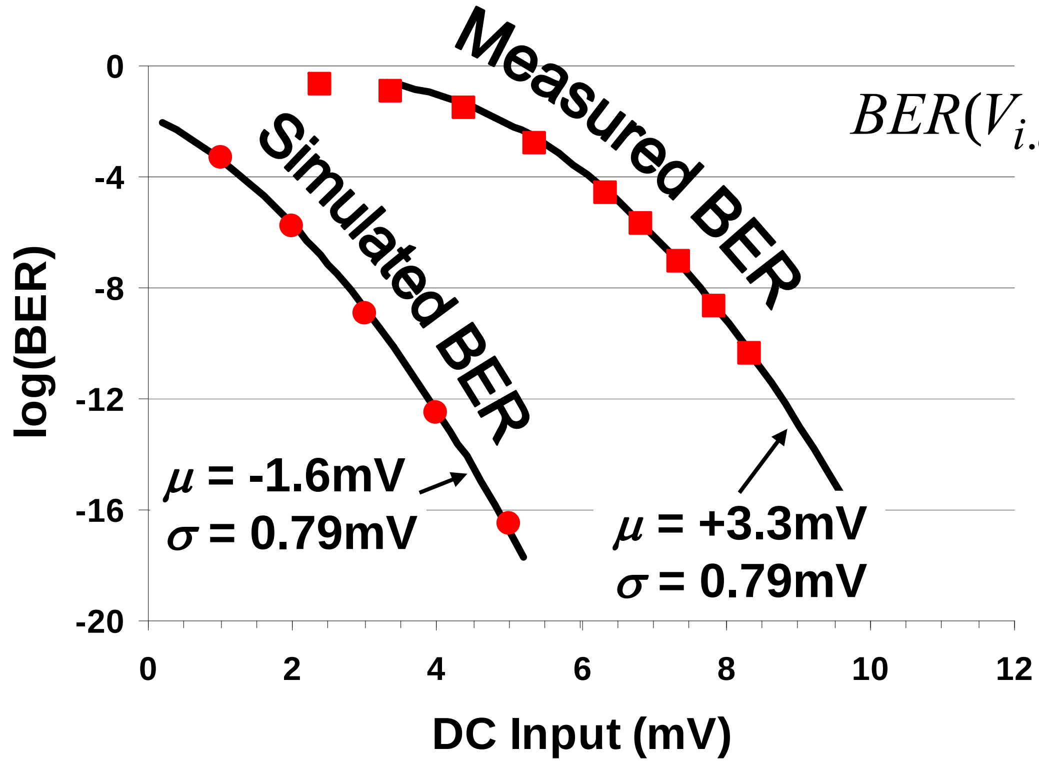
- Simulation of the decision error (BER) = $Q(V_o(t_{obs})/\sigma_o(t_{obs}))$ versus the DC input level (excess noise factor γ=2)

Receiver A – Direct Sampling Front-End



- Measurement of the decision errors (BER) based on the density of the wrong outputs (0's) versus the DC input level

Receiver A – Direct Sampling Front-End



$$BER(V_{i.dc}) = Q\left(\frac{V_{i,dc} - \mu_i}{\sigma_i}\right)$$

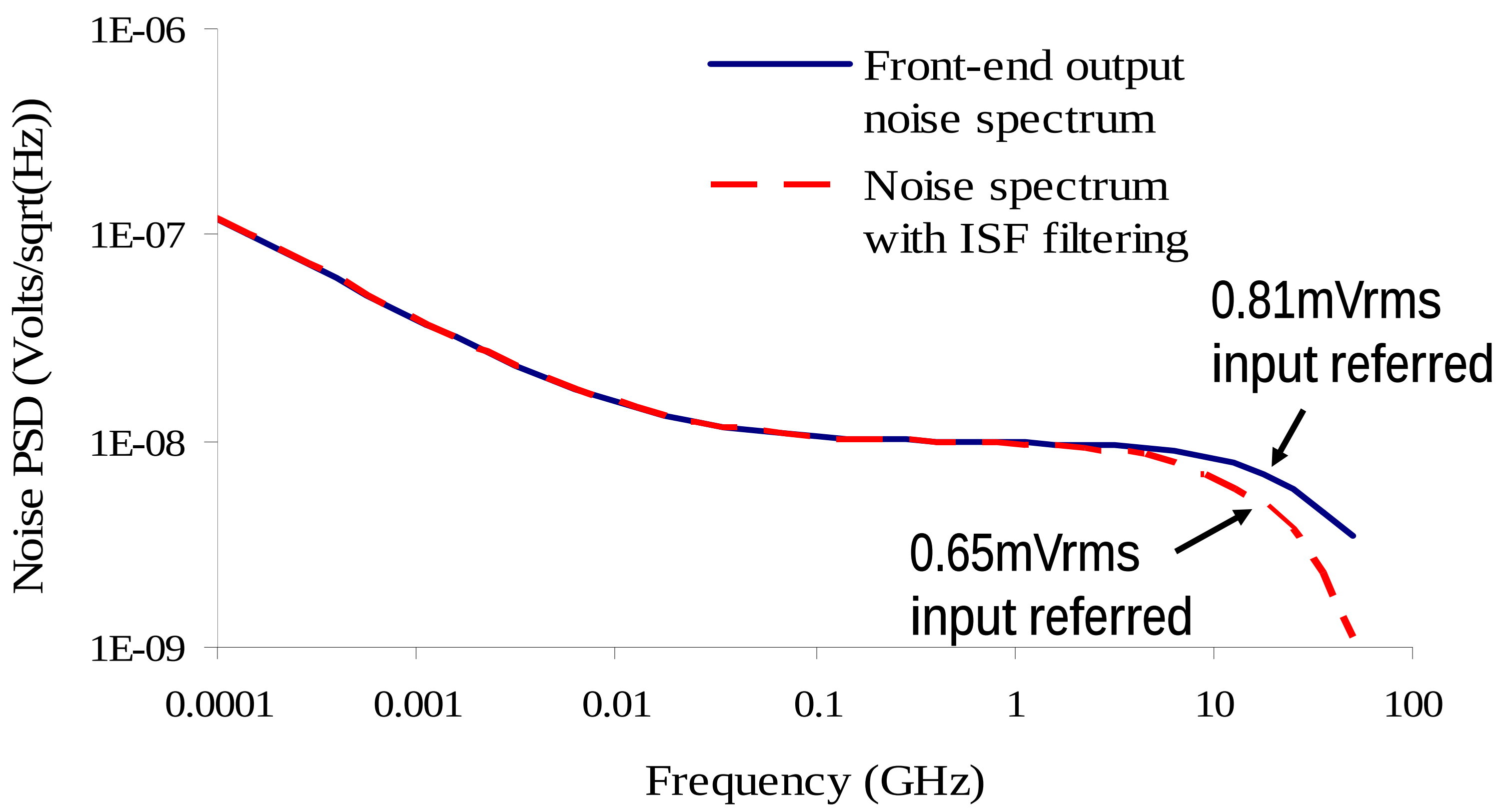
- Fit both sets of points to the Gaussian BER model
- Compare the estimated σ's (input-referred rms noise)

Simulation vs. Measurement

Receiver	Simulated (mV,rms) $\gamma=1$	$\gamma=2$	$\gamma=3$	$\gamma=4$	Measured (mV,rms) (Pos. / Neg. / Avg.)
(A) 90nm Direct Sampling Front-End	0.59	0.79	0.94		0.79 / 0.65 / 0.72
(B) 65nm w/ Linear Front-End		0.62	0.73	0.83	0.87 / 0.83 / 0.85

- (Pos. / Neg. / Avg.) refers to measurement results for positive V_{IN}, negative V_{IN}, and their average

Noise Filtering via Finite Aperture (ISF)



- In receiver B, the noise contributed by the linear front-end is filtered by the finite aperture of the comparator

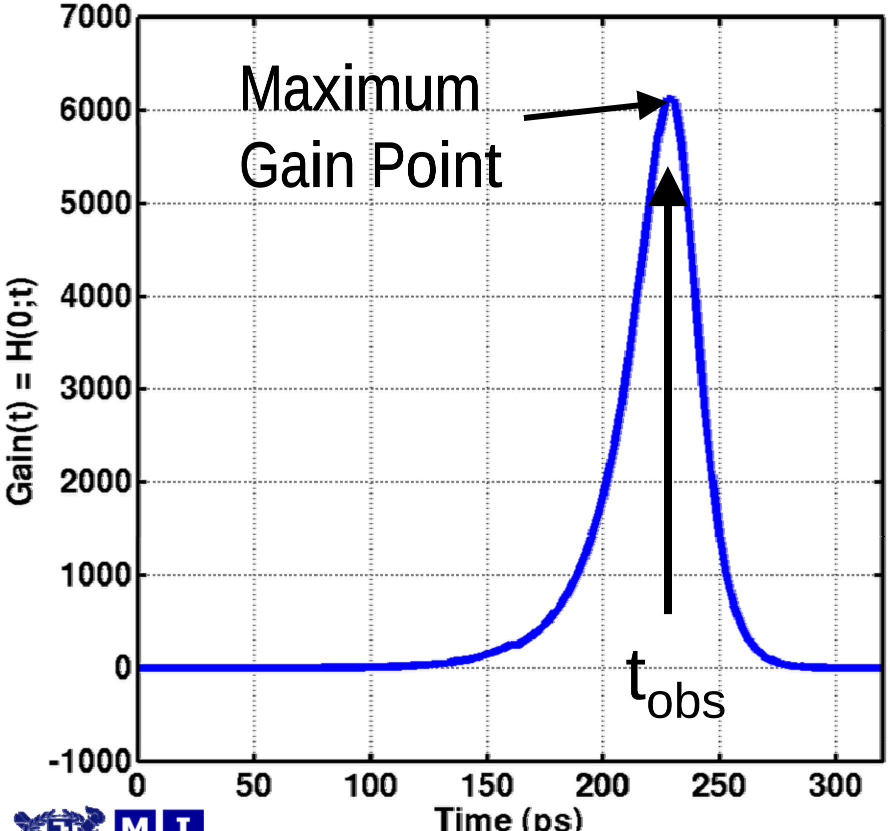
Conclusions

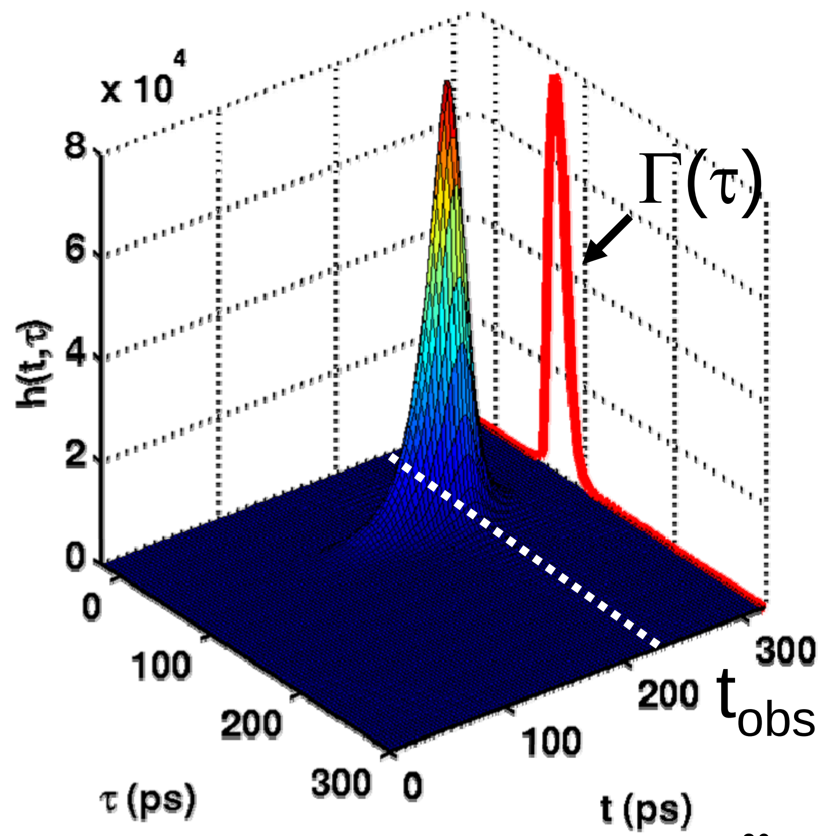
- The linear time-varying (LTV) system model is a good tool for understanding the key characteristics of clocked comparators
  - Sampling aperture and bandwidth
  - Regeneration gain and metastability
  - Random decision errors and input-referred noise
- The impulse sensitivity function (ISF) has a central role in it:
  - As it did for oscillators
  - Guides design trade-offs between noise, bandwidth, gain, etc.
- The LTV framework is demonstrated on the analysis, simulation, and measurement of clocked comparators

Back-Up Slides

Extracting ISF from $h(t,\tau)$

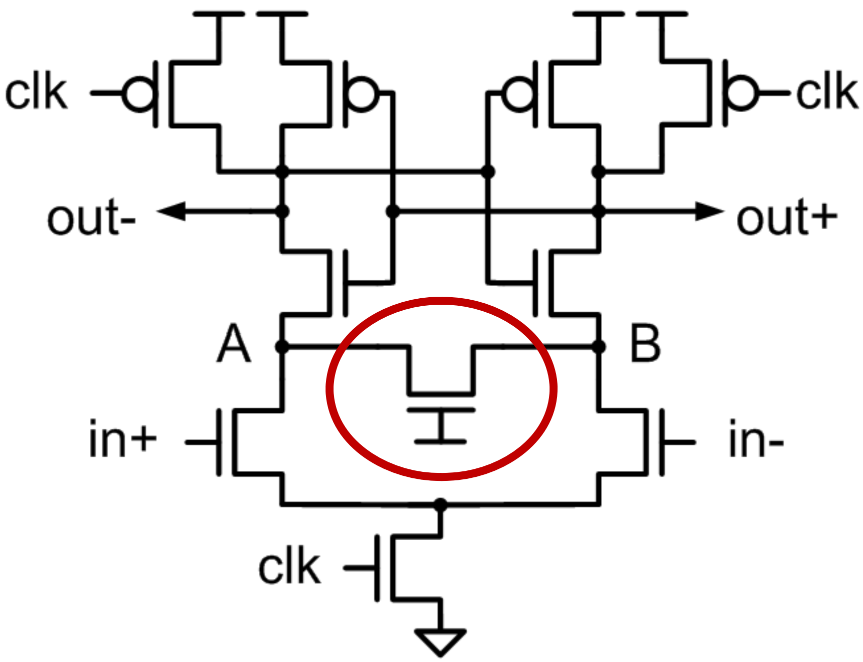
- Choose t_{obs} as the maximum small-signal gain point
- ISF: $\Gamma(\tau) = h(t_{obs},\tau)$

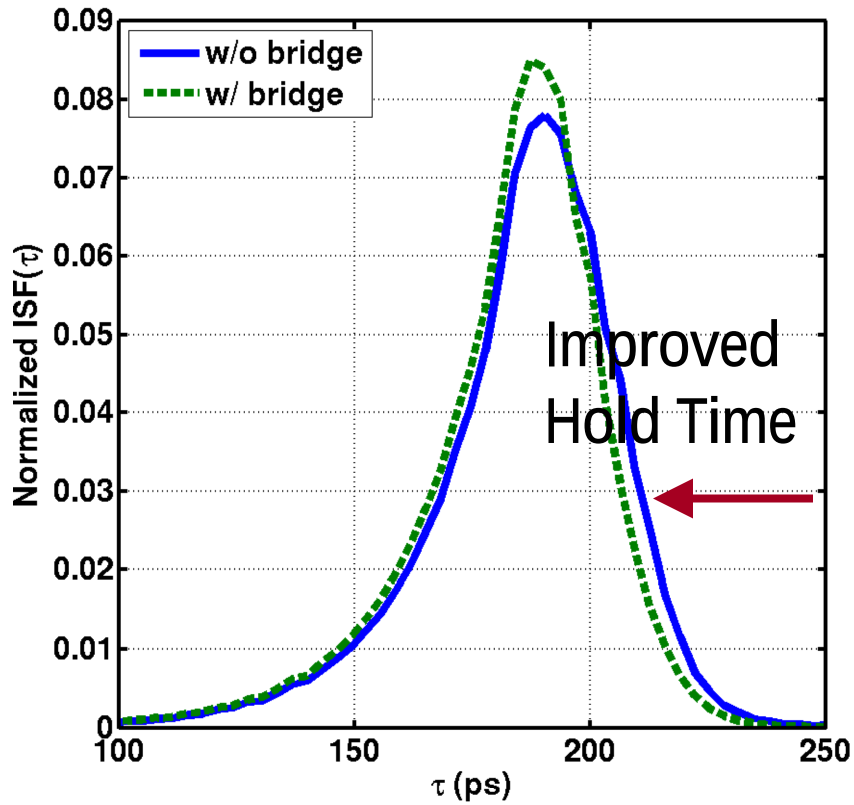




Effects of the Bridging Device

- Improves hold time and metastability





Effects of Input and Output Loading