Dispersion

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Dispersion in SMF

❑ Types of Dispersion

- Chromatic dispersion
  - Material dispersion
  - Waveguide dispersion: usually *smaller* than material dispersion
    - Short wavelength: The effective index is close to n_{core}.
    - Long wavelength: The effective index is close to $n_{cladding}$.

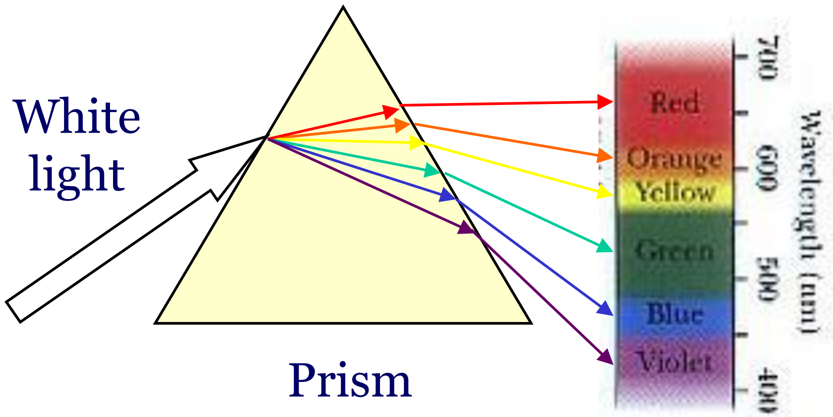
- Modal dispersion
  - Pulse spreading in a multimode fiber

Dispersion is a problem in fiber communications: It limits the *bandwidth* of the fiber.

Material dispersion

White light that is a mixture of colors can be separated into different wavelengths.

Refractive index n is inherently a function of wavelength.

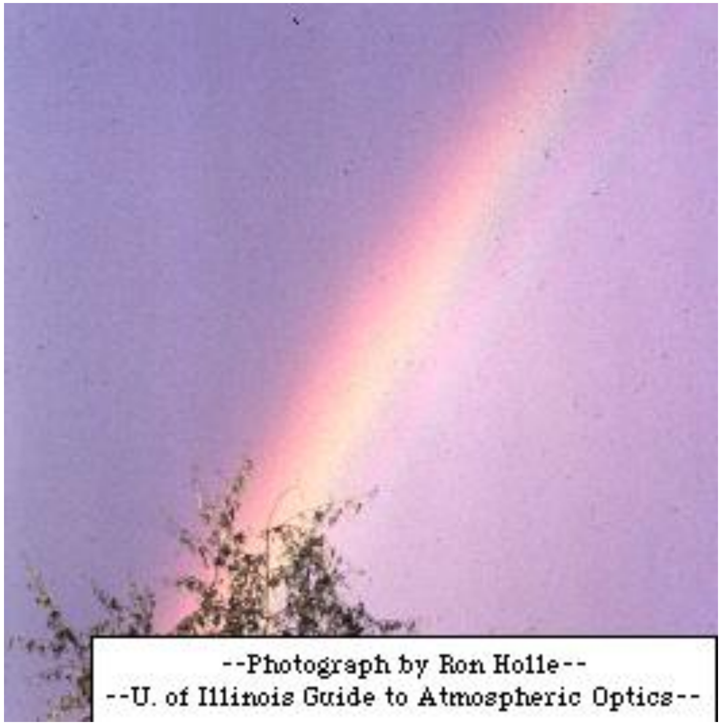


Recall Snell's law!

$$n_i \sin\theta_i = n_t \sin\theta_t$$

■ **Natural Dispersion**

RAINBOW



Material dispersion

e.g. Sellmeier equation:

$$n^2(\omega) = 1 + \sum_{j=1}^{m} \frac{B_j \omega_j^2}{\omega_j^2 - \omega^2}$$

The origin of chromatic dispersion is related to the characteristic resonance frequencies at which the medium absorbs the electromagnetic radiation through oscillations of bound electrons.

$\rightarrow$ Kramers-Kronig relations:

$$\varepsilon(\omega) = \varepsilon_r(\omega) + i\varepsilon_i(\omega)$$

The real and imaginary parts are related to each other.

(cf. Driven harmonic oscillator)

See also Classical Electrodynamics, J. D. Jackson

Dispersion relation

Mode-propagation constant β in a Taylor series:

$$\beta(\omega) = n(\omega)\frac{\omega}{c} = \beta_0 + \beta_1(\omega-\omega_0) + \frac{1}{2}\beta_2(\omega-\omega_0)^2 + \cdots,$$

$$where \quad \beta_m = \left(\frac{d^m\beta}{d\omega_m}\right)_{\omega=\omega_0} \qquad (m = 1, 2, 3, \ldots)$$

$$\beta_1 = \frac{1}{v_g} = \frac{n_g}{c} = \frac{1}{c}\left(n + \omega\frac{dn}{d\omega}\right), \qquad \rightarrow \text{Group velocity}$$

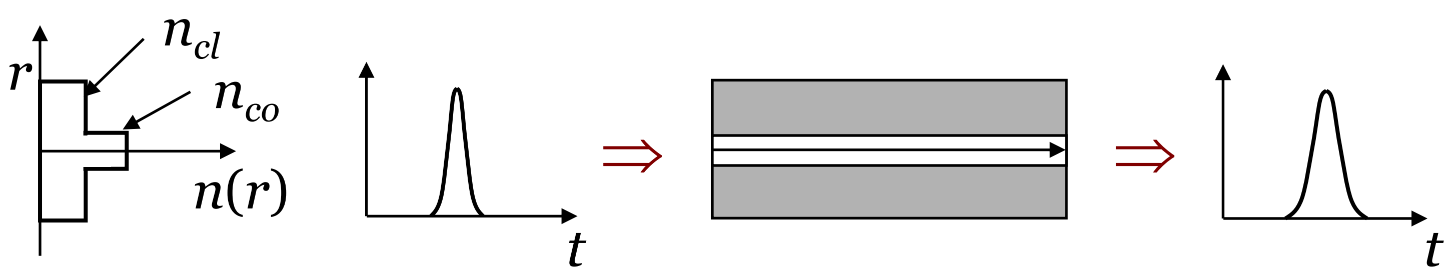
$$\beta_2 = \frac{1}{c}\left(2\frac{dn}{d\omega} + \omega\frac{d^2n}{d\omega^2}\right), \qquad \rightarrow \text{Group velocity dispersion}$$

$$\psi(z,t) = \int_{-\infty}^{\infty} A(\omega) e^{i(\omega t - \beta(\omega)z)} d\omega \qquad \leftarrow \text{Wave packet}$$

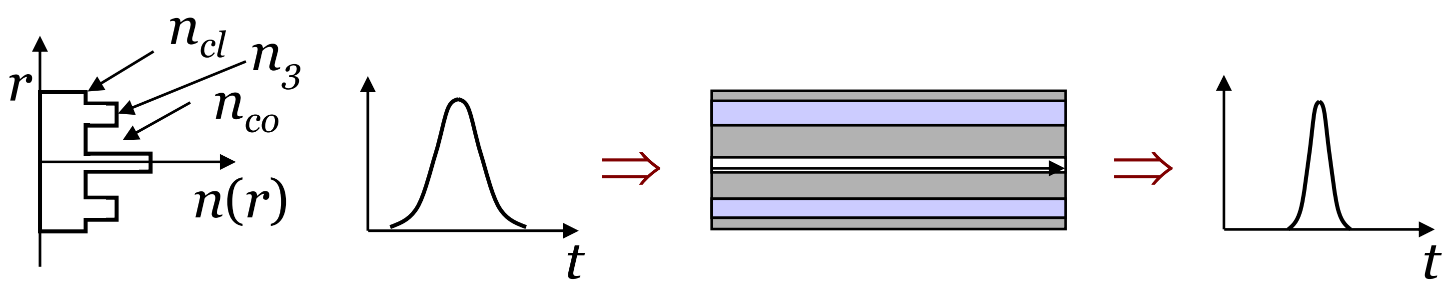
Waveguide dispersion

The effective mode index is slightly lower than the material index $n(\omega)$ of the core, which is also wavelength dependent.

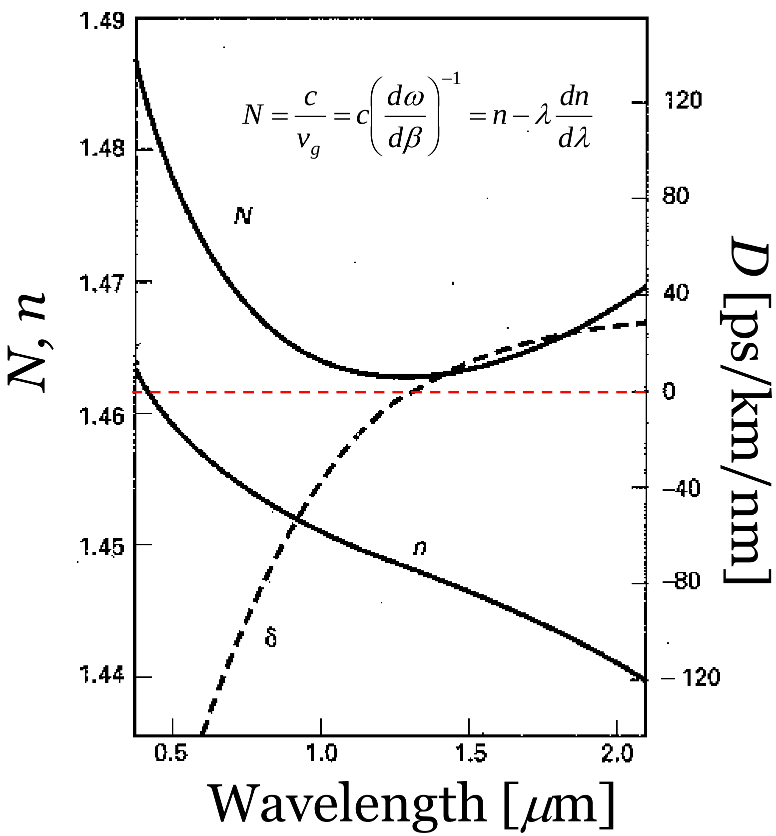
Single-mode fiber (SMF)



Dispersion-compensation Fiber (DCF)



Dispersion in SMF



Source: Nonlinear Fiber Optics, G. P. Agrawal

Dispersion parameter:

$$D = \frac{d\beta_1}{d\lambda} = -\frac{2\pi c}{\lambda^2}\beta_2 \approx \frac{\lambda}{c}\frac{d^2 n}{d\lambda^2}$$

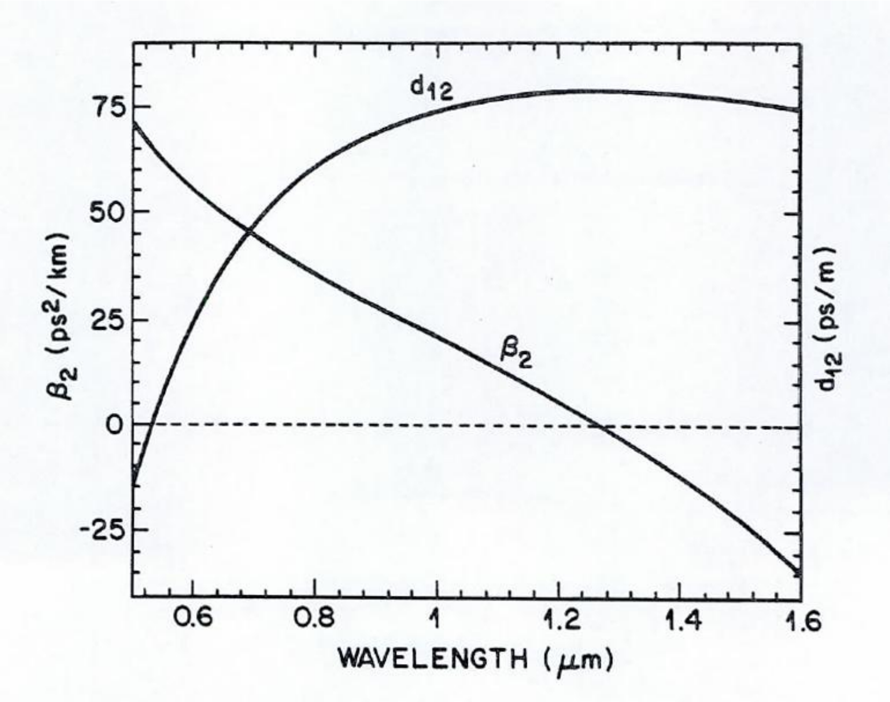
Normal dispersion: $\beta_2 > 0 \; or \; D < 0$

High-freq. components slower than low-freq. components

Anomalous dispersion: $\beta_2 < 0 \; or \; D > 0$

High-freq. components faster than low-freq. components

Walk-off in SMF



Source: Nonlinear Fiber Optics, G. P. Agrawal

Walk-off parameter:

$$d_{12} = \beta_1(\lambda_1) - \beta_2(\lambda_2) = \frac{1}{v_g(\lambda_1)} - \frac{1}{v_g(\lambda_2)}$$

Walk-off length:

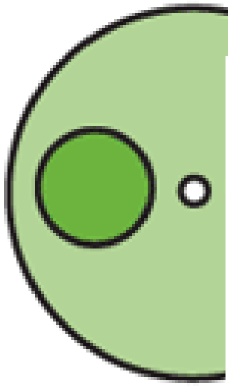
$$L_W = \frac{T_0}{|d_{12}|}$$

e.g. $\lambda_1 = 532\ nm,\ \lambda_2 = 1064\ nm$

$d_{12} \approx 80\ ps/m$

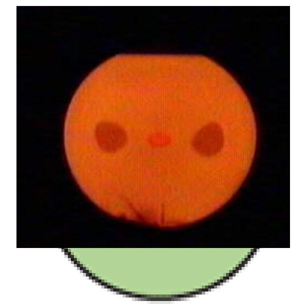
$L_W = 25\, cm\ \ for\ T_0 = 20\ ps$

Polarisation-mode dispersion (PMD)



Fibre details

- **Fibre details: hi-bi Yb-doped fibre with borosilicate stress rods**
- **Core 25 μm, NA <0.06, OD 380 μm**
- **Birefringence ~2×10^{-4}**
- (*cf.* SMF: ~10^{-8})



Source: www.nufern.com

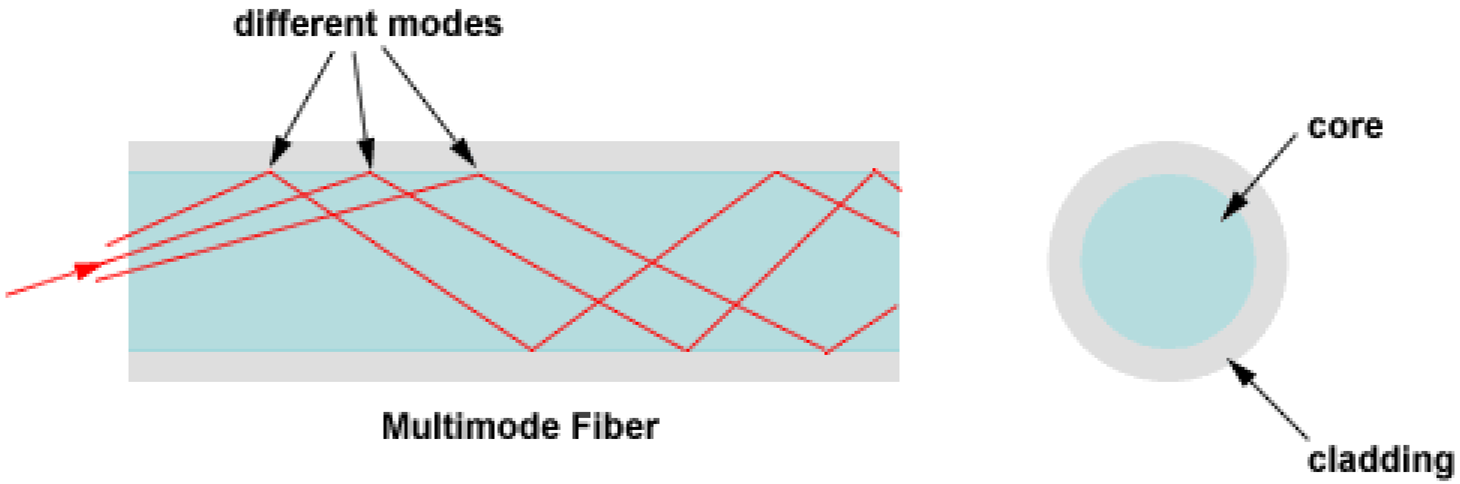
Birefringence:

$$B_m = \frac{|\beta_x - \beta_y|}{k_0} = |n_x - n_y|$$

Beat length:

$$L_B = \frac{2\pi}{|\beta_x - \beta_y|} = \frac{\lambda}{B_m}$$

Modal dispersion



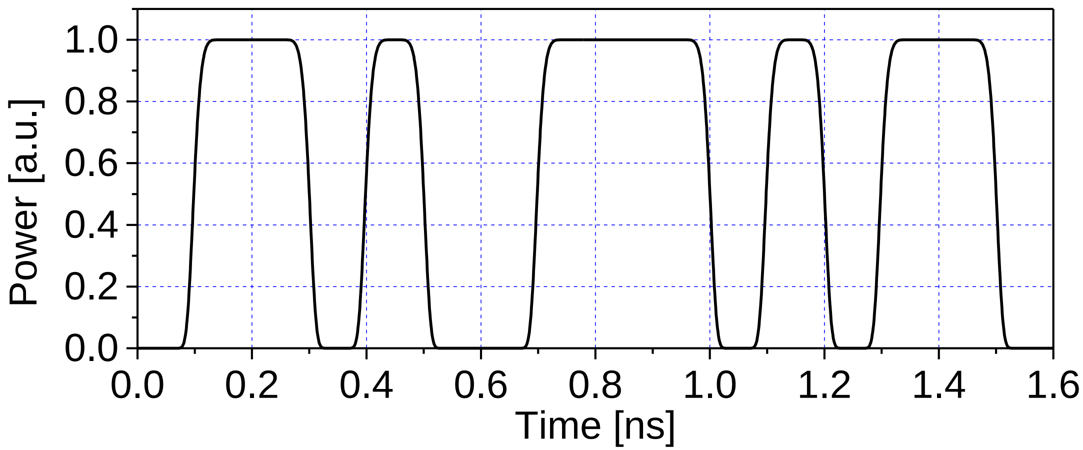
Source: www.fiberoptics4sale.com

Optical path differences among modes → different group velocity.

$$v_{g,m} = \frac{d\omega}{d\beta_m}$$

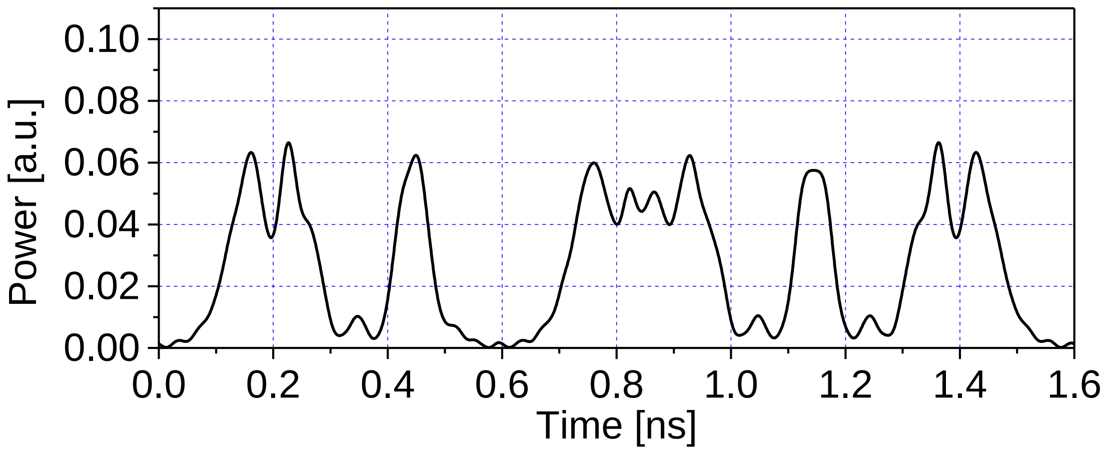
Data Transmission over SMF

- Initial Optical Pulses (10 Gbps, 0 dBm)



- Group velocity dispersion (GVD)
  ⇒ Frequency chirp
- Nonlinear effect
  ⇒ Four-wave mixing (FWM)

- After 50-km Transmission



- Power Spectrum