Four-wave mixing I

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Tensor notation for the third-order susceptibility

Constitutive relations for E-field:

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_o\mathbf{E} + \mathbf{P},$$
$$\mathbf{P} = \varepsilon_o\chi\mathbf{E} = \varepsilon_o\left(\chi^{(1)}\mathbf{E} + \chi^{(2)}\mathbf{EE} + \underline{\chi^{(3)}}\mathbf{EEE} + \cdots\right)$$

Tensor notation for $\chi^{(3)}$:

$$\chi^{(3)} \rightarrow \chi^{(3)}_{ijkl}(\omega_4 = \omega_1 + \omega_2 + \omega_3)$$

$$P_{i,NL} \leftarrow \sum_{j,k,l} \chi^{(3)}_{ijkl} E_j E_k E_l$$

Note that we normally omit summation notation.

Tensor nature of the third-order susceptibility

How many elements for $\chi^{(3)}_{ijkl}$? *81 elements*

For isotropic media:

$$\chi^{(3)}_{1111} = \chi^{(3)}_{2222} = \chi^{(3)}_{3333},$$
$$\chi^{(3)}_{1122} = \chi^{(3)}_{1133} = \chi^{(3)}_{2211} = \chi^{(3)}_{2233} = \chi^{(3)}_{3311} = \chi^{(3)}_{3322},$$
$$\chi^{(3)}_{1212} = \chi^{(3)}_{1313} = \chi^{(3)}_{2323} = \chi^{(3)}_{2121} = \chi^{(3)}_{3131} = \chi^{(3)}_{3232},$$
$$\chi^{(3)}_{1221} = \chi^{(3)}_{1331} = \chi^{(3)}_{2112} = \chi^{(3)}_{2332} = \chi^{(3)}_{3113} = \chi^{(3)}_{3223}.$$

$\rightarrow$ *21 nonzero elements*

In addition:

$$\chi^{(3)}_{1111} = \chi^{(3)}_{1122} + \chi^{(3)}_{1212} + \chi^{(3)}_{1221}$$

$\rightarrow$ *Why?*

$\rightarrow$ In the compact form: $\chi^{(3)}_{ijkl} = \chi^{(3)}_{1122}\delta_{ij}\delta_{kl} + \chi^{(3)}_{1212}\delta_{ik}\delta_{jl} + \chi^{(3)}_{1221}\delta_{il}\delta_{jk}$

Cross-phase modulation (XPM)

Suppose that there are two monochromatic waves:

$$\mathbf{E}(r,t) = \frac{1}{2}\hat{x}[E_1 \exp(-i\omega_1 t) + E_2 \exp(-i\omega_2 t)] + c.c.$$

Induced nonlinear polarisation:

$$\begin{aligned}\mathbf{P}_{NL}(r,t) = \varepsilon_0 \chi^{(3)}\mathbf{EEE} = \frac{1}{2}\hat{x}\{&P_{NL}(\omega_1)\exp(-i\omega_1 t) + P_{NL}(\omega_1)\exp(-i\omega_2 t)\\ &+ P_{NL}(2\omega_1 - \omega_2)\exp[-i(2\omega_1 - \omega_2)t]\\ &+ P_{NL}(2\omega_2 - \omega_1)\exp[-i(2\omega_2 - \omega_1)t]\} + c.c\\ &+ \cdots\end{aligned}$$

where

$$P_{NL}(\omega_1) = \chi_{eff}(|E_1|^2 + 2|E_2|^2)E_1,$$

$$P_{NL}(\omega_2) = \chi_{eff}(|E_2|^2 + 2|E_1|^2)E_2,$$

$$\chi_{eff} = \frac{3\varepsilon_0 \chi_{xxxx}^{(3)}}{4}.$$

Nonlinear pulse propagation with XPM

NLSE:

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A = i\gamma |A|^2 A \qquad \textit{for} \ \exp(-i\omega t)$$

Coupled NLSEs:

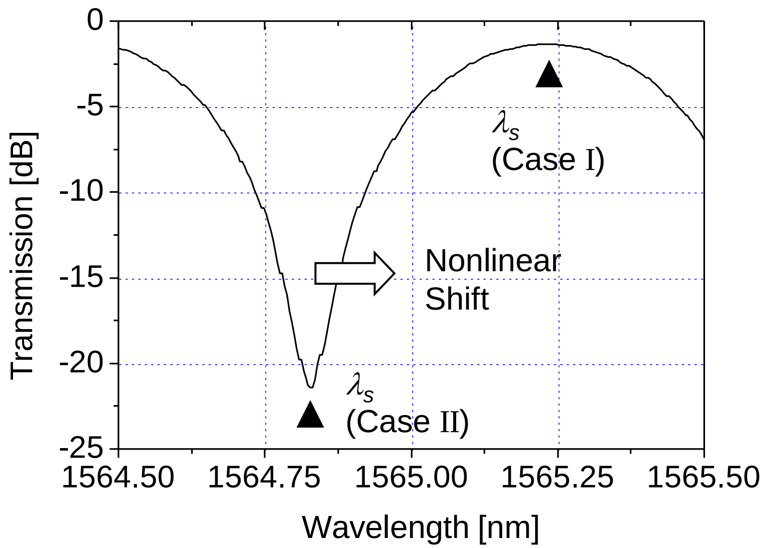
$$\rightarrow \begin{array}{l} \dfrac{\partial A_1}{\partial z} + \beta_{11} \dfrac{\partial A_1}{\partial t} + i\dfrac{\beta_{21}}{2}\dfrac{\partial^2 A_1}{\partial t^2} + \dfrac{\alpha_1}{2} A_1 = i\gamma_1 (|A_1|^2 + 2|A_2|^2) A_1, \\ \dfrac{\partial A_2}{\partial z} + \beta_{12} \dfrac{\partial A_2}{\partial t} + i\dfrac{\beta_{22}}{2}\dfrac{\partial^2 A_2}{\partial t^2} + \dfrac{\alpha_2}{2} A_2 = i\gamma_2 (|A_2|^2 + 2|A_1|^2) A_2. \end{array}$$

Modulation instability?

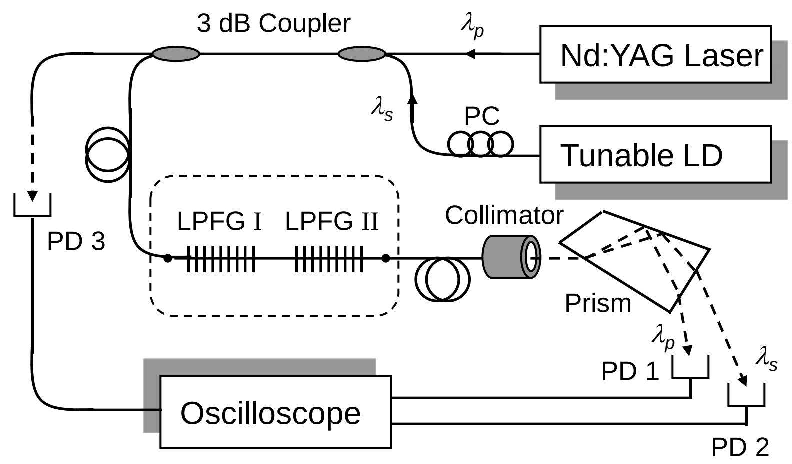
Optical solitons?

All-optical switching via XPM

Transmission spectra



Experimental arrangement



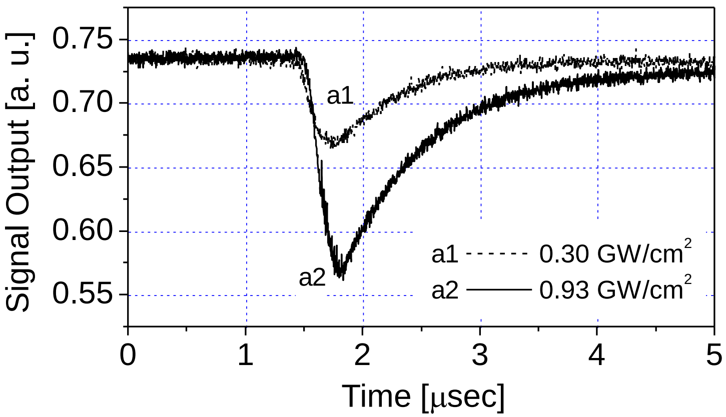
Experiments:

Signal wave: Tunable LD @1565.2 nm (Case I), @1564.8 nm (Case II)

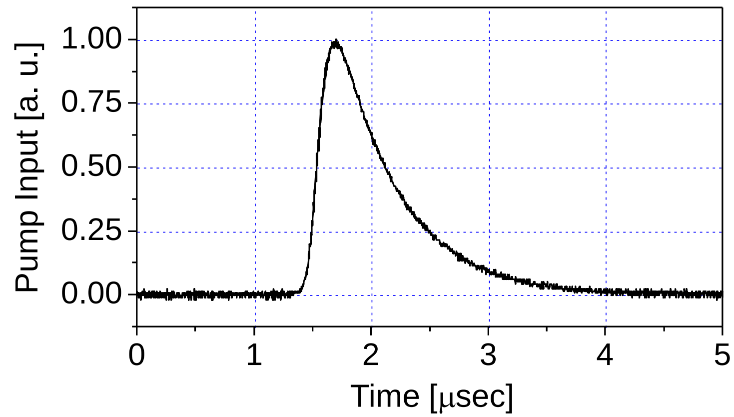
Pump wave: Q-switched Nd:YAG laser (1 kHz)

All-optical switching via XPM

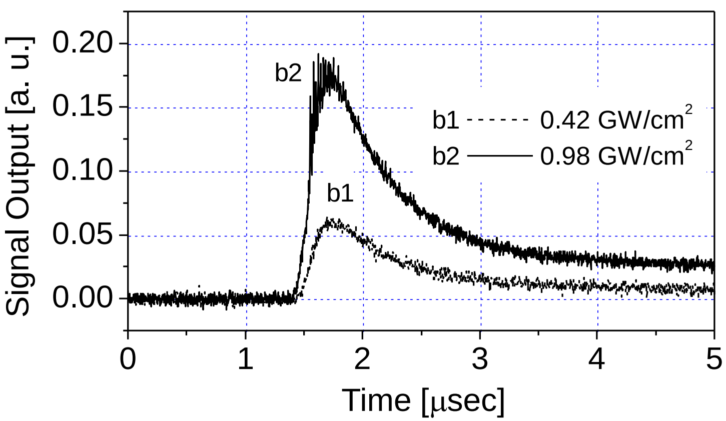
Case I

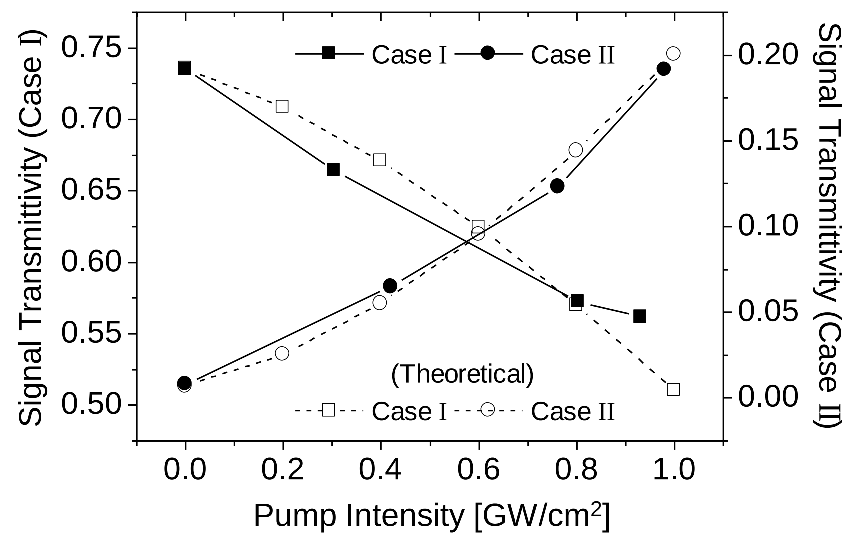


Pump



Case II





$\Delta\lambda_s \sim 0.12$ nm/(GW/cm^2)