

## Chapter 12 Mixtures

Air consisting of  $N_2, O_2$ , etc  
Though we've assumed it a non-mixture.

Mass fraction  $f_i = \frac{m_i}{m}$   

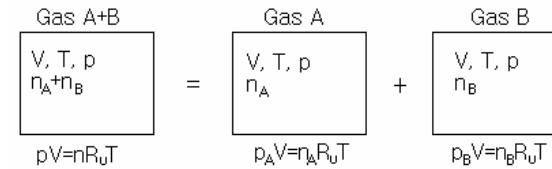
$$\sum_k f_i = 1$$

Number of moles,  $n_1, n_2, \dots, n_i, \dots, n_k$

Mole fraction  $x_i = \frac{n_i}{n}$   

$$\sum_k x_i = 1$$

### Dalton Model (V, T same)

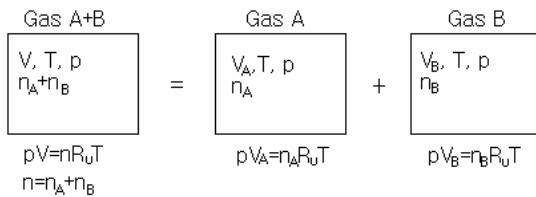


$$\frac{p_A}{p} = \frac{n_A}{n} = x_A \quad ; p_A = \text{partial pressure of component A}$$

$$\frac{p_B}{p} = \frac{n_B}{n} = x_B \quad ; p_B = \text{partial pressure of component B}$$

$$p_A + p_B = p \quad = \text{total pressure}$$

### Amagat Model (p, T same)



$$\frac{V_A}{V} = \frac{n_A}{n} = x_A \quad ; V_A = \text{partial volume of component A}$$

$$\frac{V_B}{V} = \frac{n_B}{n} = x_B \quad ; V_B = \text{partial volume of component B}$$

$$V_A + V_B = V \quad = \text{total volume}$$

$$\therefore \frac{V_A}{V} + \frac{V_B}{V} = 1$$

In general,

$$\frac{p_i}{p} = \frac{V_i}{V} = \frac{n_i}{n} = x_i$$

and

$$\sum_k p_i = p, \quad \sum_k V_i = V$$

- 혼합기체  $n$  몰의 질량,  $m$
- 성분  $i$  의 질량,  $m_i$
- 분자량 (molecular weight),  $M_i$
- 상당 분자량  $M$

$$n_i = \frac{m_i}{M_i} \quad \frac{\text{kg}}{\text{kg}/\text{kmol}} = \text{kmol}$$

$$n = \frac{m}{M}$$

$$\therefore M = \frac{m}{n} = \frac{\sum_k n_i M_i}{n} = \sum_k x_i M_i \quad \frac{\text{kg}}{\text{kmol}}$$

$$n = \sum_k n_i$$

$$\frac{m}{M} = \sum_k \frac{m_i}{M_i}$$

or

$$M = \left( \sum_k \frac{f_i}{M_i} \right)^{-1}$$

**Molecular weight for the mixture**  
(혼합기체에 대하여 상당분자량을 정의하여 순수기체와 같이 취급할 수 있다)

순수공기

		Mole Fraction	Molec. Weight	Mass per kmol of mixture	Mass Fraction
	% Volume	$x_i$	$M_i$	$x_i M_i$	$f_i = \frac{x_i M_i}{M}$
$N_2$	78.03	0.7803	28.016	21.861	0.7547
$O_2$	20.99	0.2099	32.00	6.717	0.2319
$Ar$	0.94	0.0094	39.944	0.376	0.0130
$CO_2$	0.03	0.0003	44.003	0.013	0.0004
$H_2$	0.01	0.0001	2.016	-	-
Sum	100	1		$M = 28.967$ $= \sum x_i M_i$	1

For the mixture

$$R = \frac{R_u}{M} = \frac{8.314 \text{ kJ/kmol.K}}{28.967 \text{ kg/kmol}} = 0.287 \text{ kJ/kg.K}$$

Molec. Wt for mixture

이상기체 혼합물의 엔트로피

$$\left. \begin{array}{l} \text{Gases A, } n_A \text{ 몰, } p, T \\ \text{Gases B, } n_B \text{ 몰, } p, T \end{array} \right\} \text{ Mix} \rightarrow p, T$$

$$\left[ \begin{array}{c} p, T \\ n_A \\ S_A(p, T) \end{array} \right] + \left[ \begin{array}{c} p, T \\ n_B \\ S_B(p, T) \end{array} \right] \Rightarrow \left[ \begin{array}{c} T, p_A \\ n_A \\ S_A(p_A, T) \end{array} \right] + \left[ \begin{array}{c} T, p_B \\ n_B \\ S_B(p_B, T) \end{array} \right]$$

Before mixing,

$$\bar{s}_A(p, T) = \int_{T_0}^T \bar{c}_{p_A} \frac{dT}{T} - R_u \ln \frac{p}{p_0} + \bar{s}_{0A}$$

$$\bar{s}_B(p, T) = \int_{T_0}^T \bar{c}_{p_B} \frac{dT}{T} - R_u \ln \frac{p}{p_0} + \bar{s}_{0B}$$

$$\text{Total } S = n_A \bar{s}_A(p, T) + n_B \bar{s}_B(p, T)$$

In other words, for mixing,

$$s_{mix} = n_A \bar{s}_A(p_A, T) + n_B \bar{s}_B(p_B, T)$$

Where

$$\bar{s}_A(p_A, T) = \int_{T_0}^T \bar{c}_{p_A} \frac{dT}{T} - R_u \ln \frac{p_A}{p_0} + \bar{s}_{0A}$$

$$\bar{s}_B(p_B, T) = \int_{T_0}^T \bar{c}_{p_B} \frac{dT}{T} - R_u \ln \frac{p_B}{p_0} + \bar{s}_{0B}$$

Upon substitution and some rearranging,

$$s_{mix} - s = -nR_u (x_A \ln x_A + x_B \ln x_B)$$

Or more generally for  $k$  number of gases,

$$s_{mix} - s = -nR_u \sum_k x_i \ln x_i > 0$$

$\ln x$  그림참조, ( $x < 1$ )

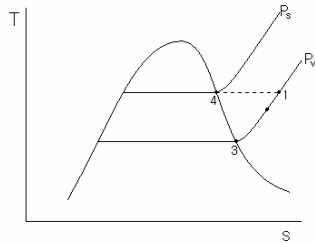
$$\because 0 < x_i < 1, \ln x_i < 0$$

$$\therefore s_{mix} > s$$

혼합과정에서 엔트로피 증가

**기체와 수증기의 혼합물 (Mixture of gases and vapor)**

Atmosphere = dry air + 수증기



State 1: the vapor of the mixture is initially superheated at state 1.

State 3: the mixture cools at constant p, the partial pressure of the vapor remains constant until state 3, condensation begins. = Dew point temperature.

We are familiar with the condensation of water from the atmosphere when it cools on a summer day.

Dew point of a gas-vapor mixture:

T- at which the vapor condenses or solidifies when it cools at constant pressure.

**Relative humidity**

The ratio of the mole fraction of the vapor in the mixture to the mole fraction of vapor in a saturated mixture at the same temperature and total pressure.

Since ideal gas, only one pressure ratio used!

$$\begin{aligned} \phi &= \frac{p_v}{p_{sat}} \quad @ \text{ T constant} \\ &= \frac{p_1}{p_4} \quad (\text{since vapor is ideal}) \\ &= \frac{\rho_v}{\rho_s} = \frac{v_s}{v_v} \end{aligned}$$

**Absolute humidity (~ specific humidity 비습도)**

$$\omega = \frac{m_v}{m_a} = \frac{\text{mass of water vapor}}{\text{mass of dry air}}$$

Since both air and vapor are considered ideal,

$$m_v = \frac{p_v V}{R_v T} = \frac{p_v V M_v}{R T}$$

$$m_a = \frac{p_a V}{R_a T} = \frac{p_a V M_a}{R T}$$

Then

$$\omega = \frac{p_v V / R_v T}{p_a V / R_a T} = \frac{R_a p_v}{R_v p_a} = \frac{M_v p_v}{M_a p_a}$$

For air-vapor mixture, we know

$$\frac{m_v}{m_a} = \omega = 0.622 \frac{p_v}{p_a}$$

We can also derive a relationship between  $\omega$  and  $\phi$

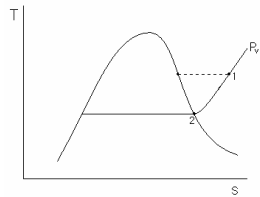
$$\phi = \frac{\omega p_a}{0.622 p_s}$$

or using  $p_a = p - p_v$  and  $\phi = \frac{p_v}{p_s}$

$$\omega = \frac{0.622 \phi}{\left( \frac{p}{p_s} \right) - \phi}$$

• Example

Consider  $100\text{m}^3$  of air–water vapor mixture at 0,1 Mpa,  $35^\circ\text{C}$ , 70% relative humidity. Calculate the specific humidity,  $\omega$ , dew point temperature, mass of air, and mass of vapor.



$$p = p_a + p_v$$

$$\phi = \frac{p_v}{p_s} = 0.70$$

Look up  $p_s$  at  $T_s = 35^\circ\text{C}$

$$p_s = 5.625\text{kPa}$$

$$\rightarrow p_v = (0.7)(5.628) = 3.94\text{kPa}$$

$$p = p_a + p_v$$

The dew point, is found by finding saturated temperature at constant partial pressure of vapor  $p_v$ .

$$T_2 = 28.6^\circ\text{C}$$

Absolute humidity,

$$\begin{aligned} \omega &= 0.622 \frac{p_v}{p_a} \\ &= 0.622 \left( \frac{3.94\text{kPa}}{p - p_v} \right) \\ &= 0.622 \left( \frac{3.94\text{kPa}}{96.06\text{kPa}} \right) = 0.0255 \end{aligned}$$

Mass of air (ideal gas)

$$m_a = \frac{p_a V}{R_a T} = \frac{96.06 \times 100}{0.287 \times 308.2} = 108.6\text{kg}$$

The mass of vapor,

$$\begin{aligned} m_v &= \omega m_a \\ &= 0.0255(108.6\text{kg}) = 2.77\text{kg} \end{aligned}$$