Chap. 2 Fundamentals of Electric Circuits

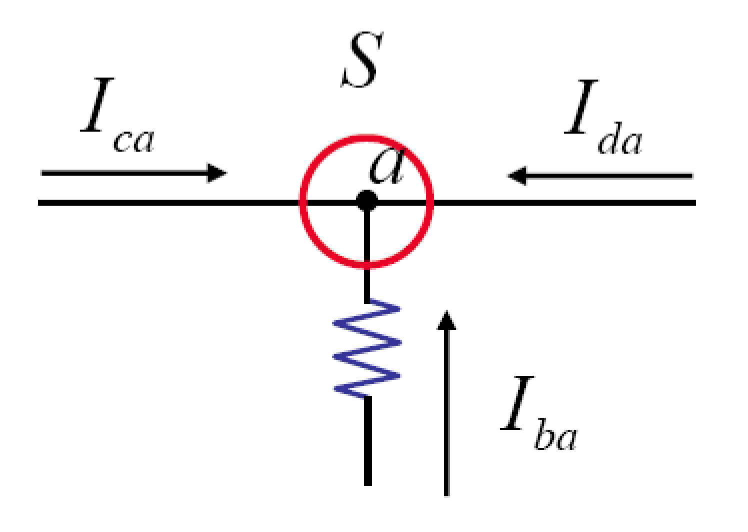
Kirchhoff's laws

Circuit Analysis: Basic Methods

Dependent Sources

KCL (Kirchhoff's Current Law)

- The sum of all branch currents flowing into a node is zero – charge conservation.



$$I_{ca} + I_{ba} + I_{da} = 0$$

Illustration of Kirchhoff's current law

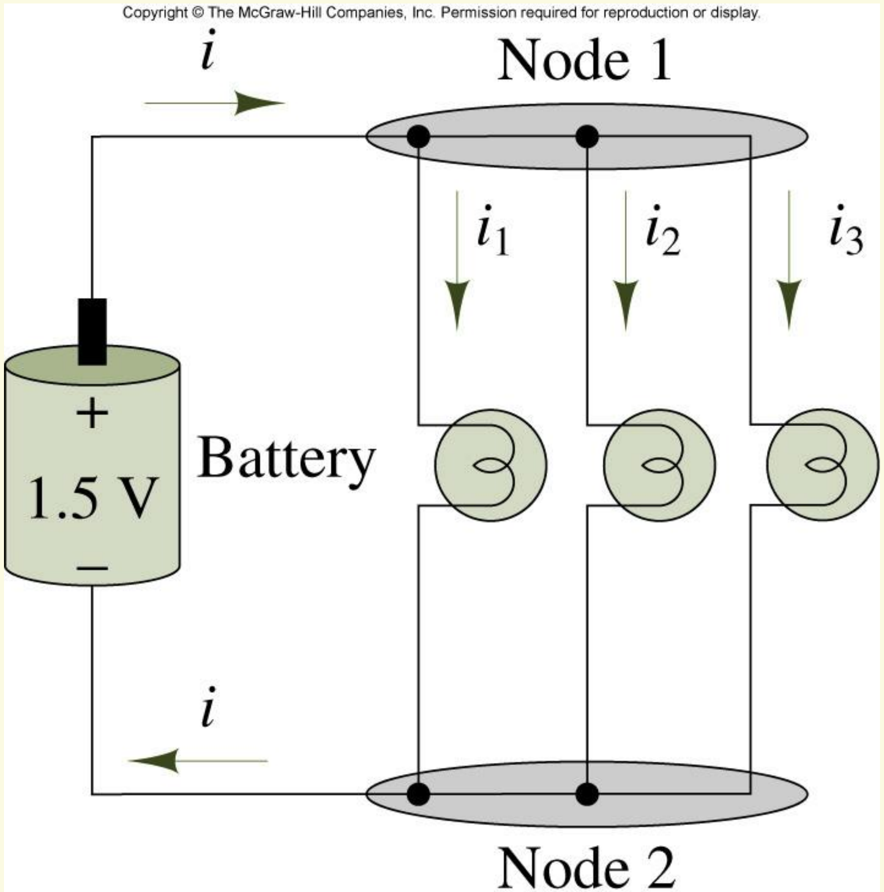
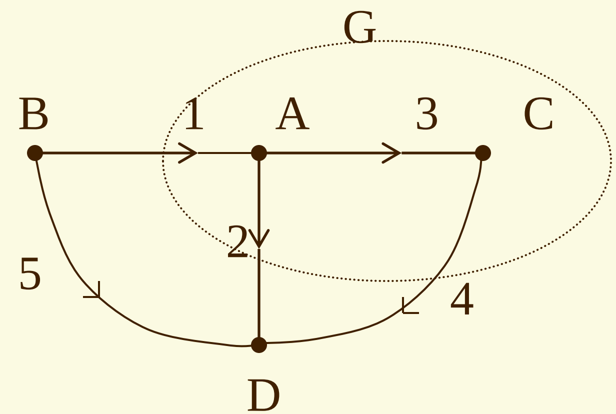


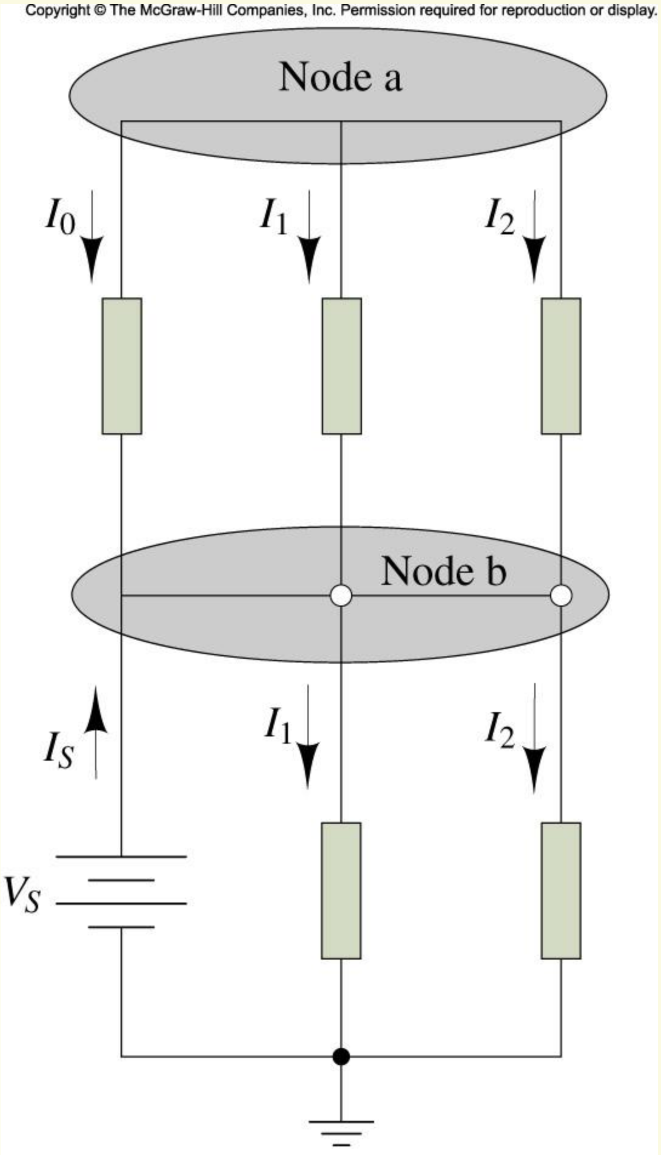
Illustration of KCL at node 1: $-i + i_1 + i_2 + i_3 = 0$



Node: $i_1 - i_2 - i_3 = 0$

G: $i_1 - i_2 - i_4 = 0$

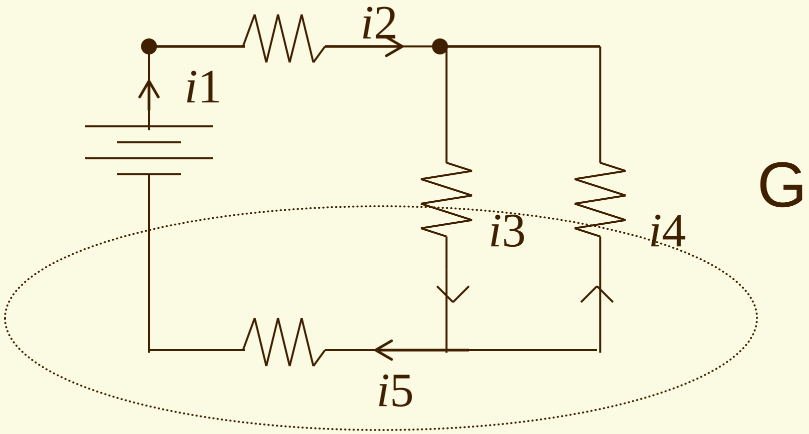
Demonstration of KCL



Is = 5A, I1 = 2A, I2 = -3A, I3 = 1.5 A

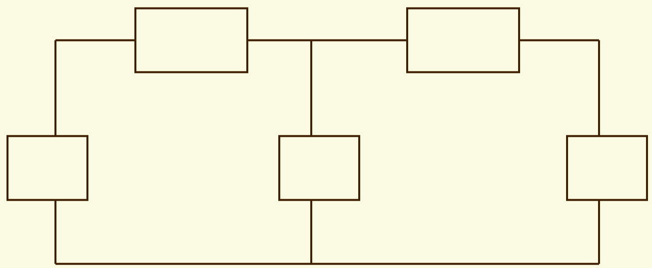
Find I0 and I4

Example

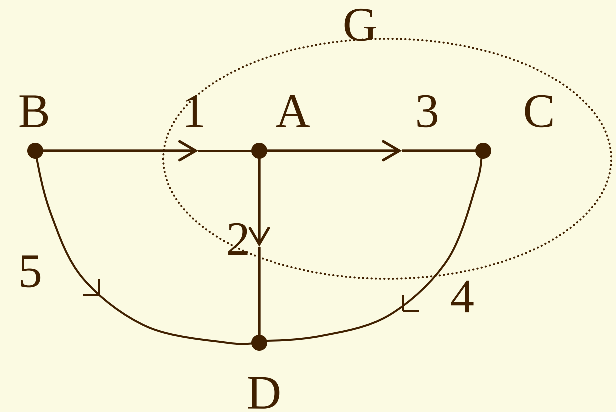


KVL (Kirchhoff's Voltage Law)

- The algebraic sum of the branch voltages around any closed path in a network must be zero



(a)



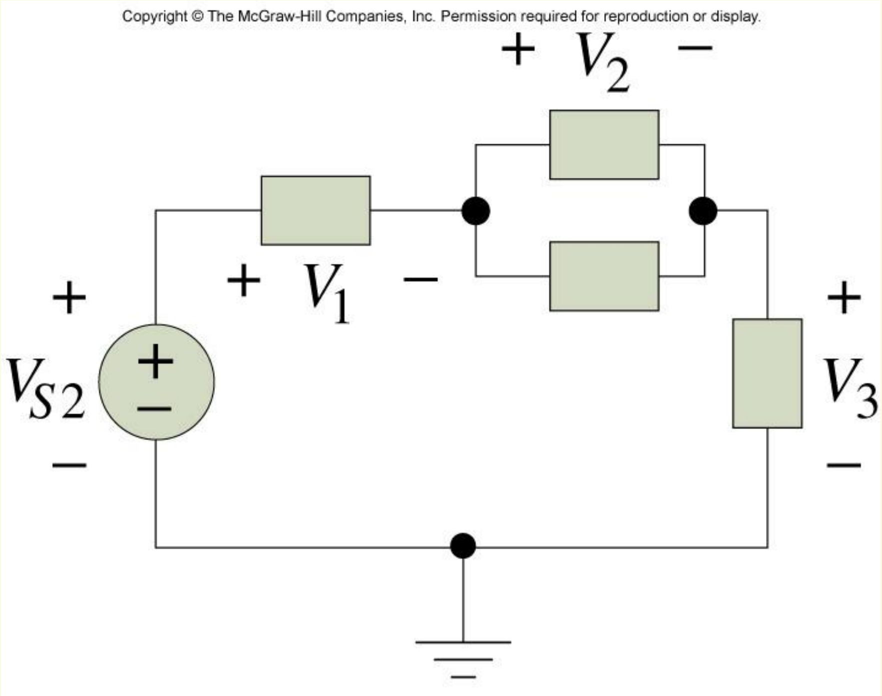
(b)

Node sequence B,A,D,B: $v_{BA} + v_{AD} + v_{DB} = 0$

Using node voltages: $v_{BA} = e_B - e_A, \quad e_D = 0$

(reference node = D, reference voltage = 0)

Example

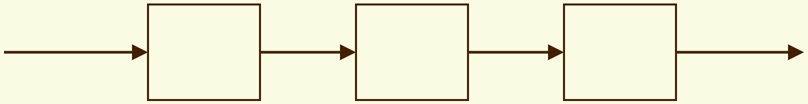


Vs2 = 12 V, V1 = 6V, V3 = 1V

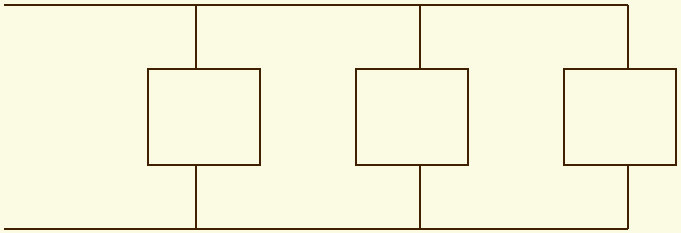
Find V2.

Some Useful Facts

- The branch currents passing through series-connected elements must be the same.

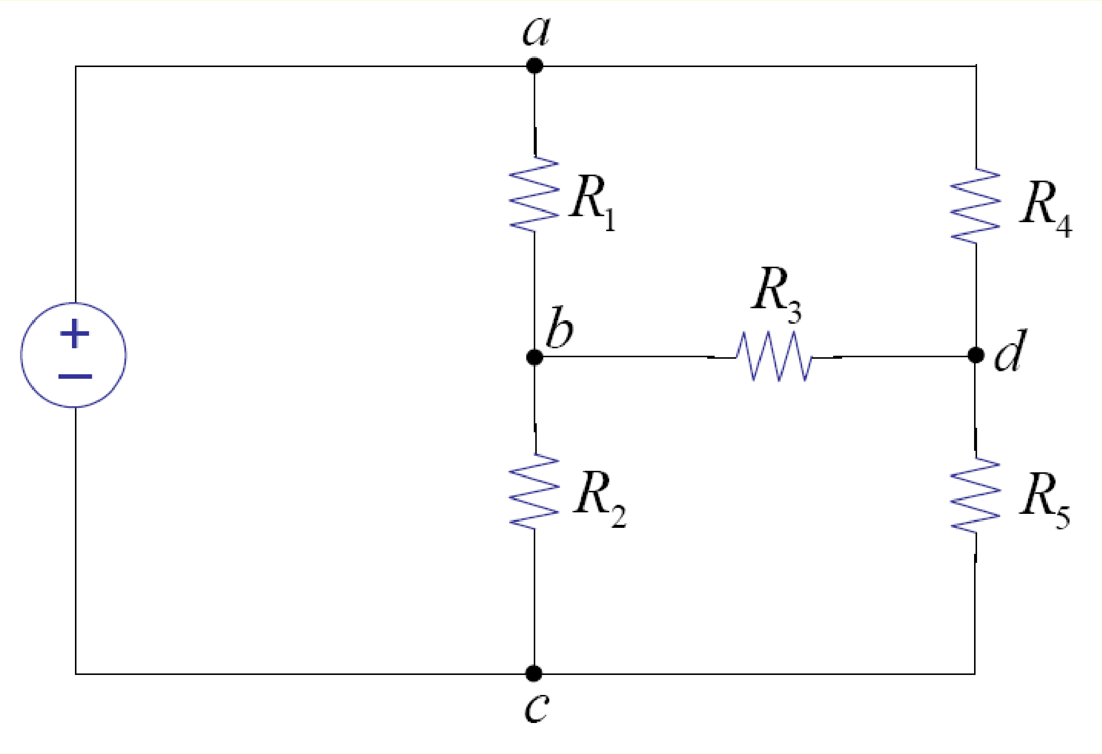


- The voltages across parallel-connected elements must be the same.



Independent Equations

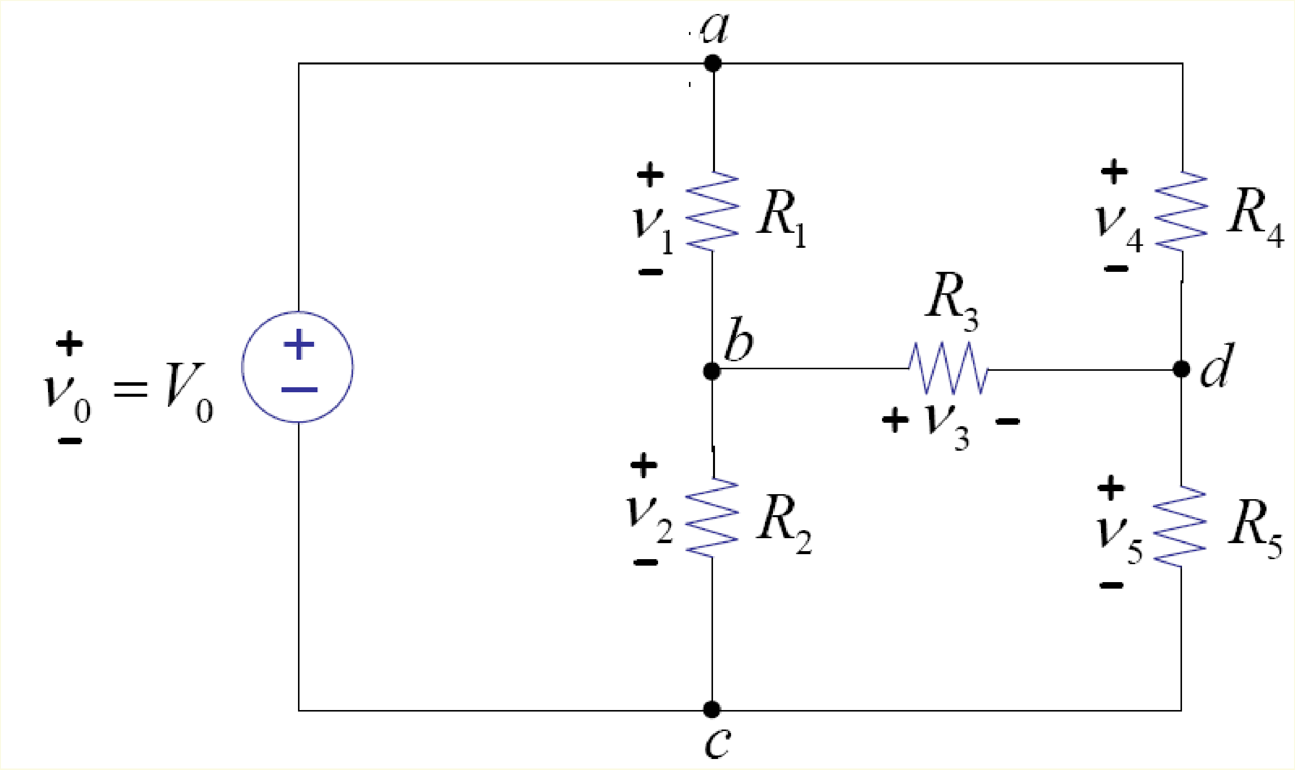
- How many independent KCL equations can be made?
- How many independent KVL equations can be made?



Circuit Analysis: Basic Method

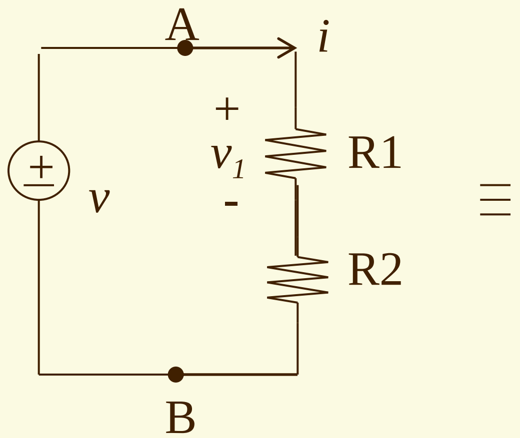
- Goal: find all V and I s for all elements
  - Write element's V-I relationships
  - Write KCL for all nodes
  - Write KVL for all loops

Example

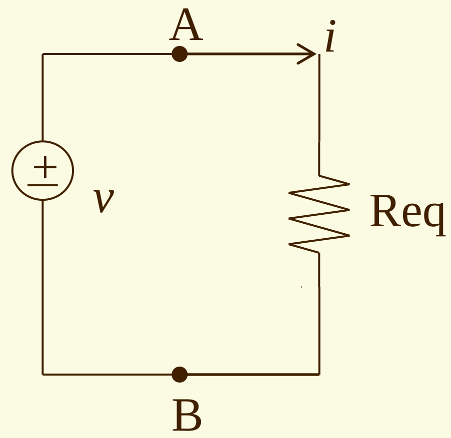


- Element's equations
- KCL
- KVL

Voltage Divider and Series Resistors

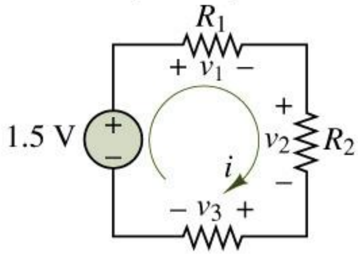


(a) Voltage divider



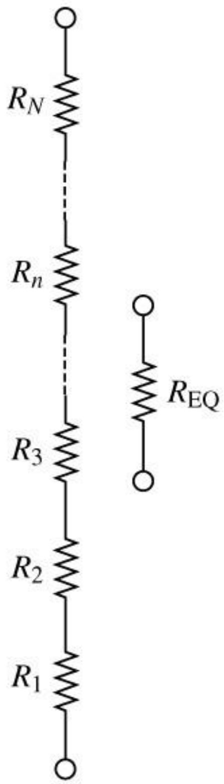
(b) Equivalent resistance

$$v_1 = \frac{R_1}{R_1 + R_2} v$$



The current i flows through each of the four series elements. Thus, by KVL,

$$1.5 = v_1 + v_2 + v_3$$

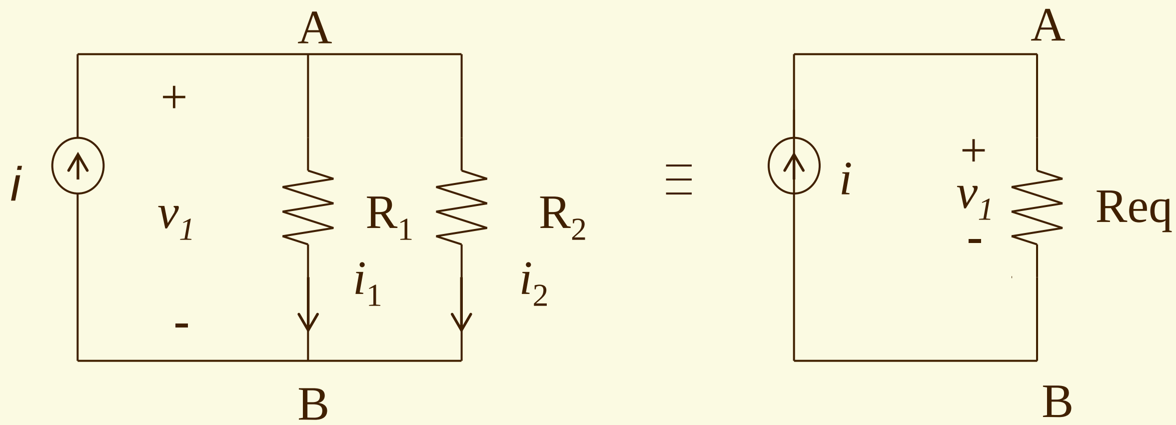


N series resistors are equivalent to a single resistor equal to the sum of the individual resistances.

R1 = 10 kOhm, R2 = 6 kOhm, R3 = 8 kOhm

Find V3.

Current Divider and Parallel Resistors

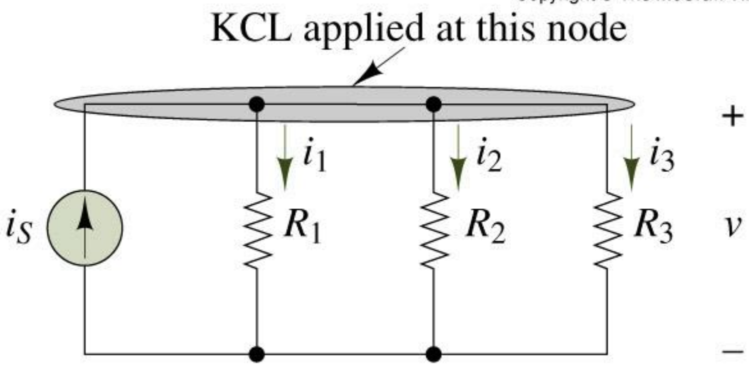


(a) Current divider

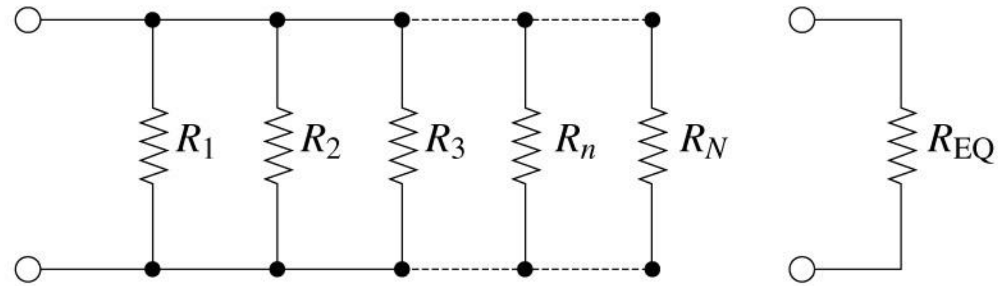
(b) Equivalent resistance

$$i_1 = \frac{G_1}{G_1 + G_2} i = \frac{R_2}{R_1 + R_2} i$$

Parallel circuits

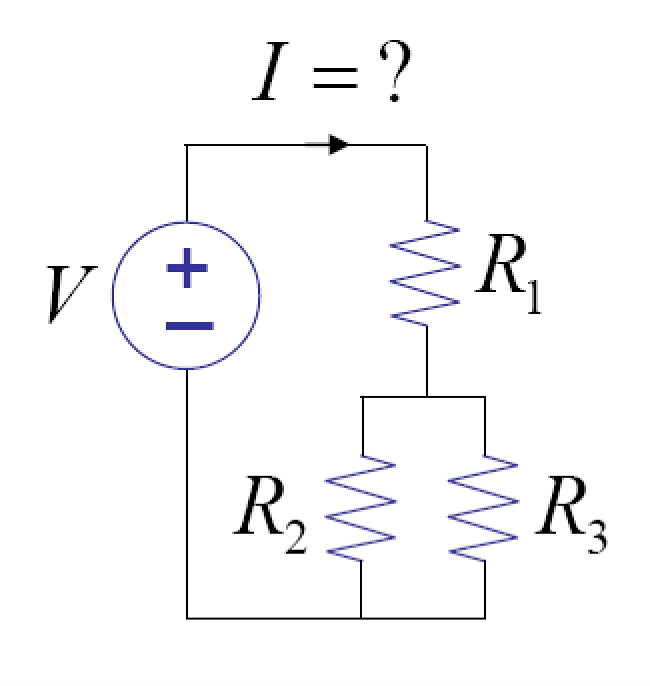


The voltage v appears across each parallel element; by KCL, $i_S = i_1 + i_2 + i_3$



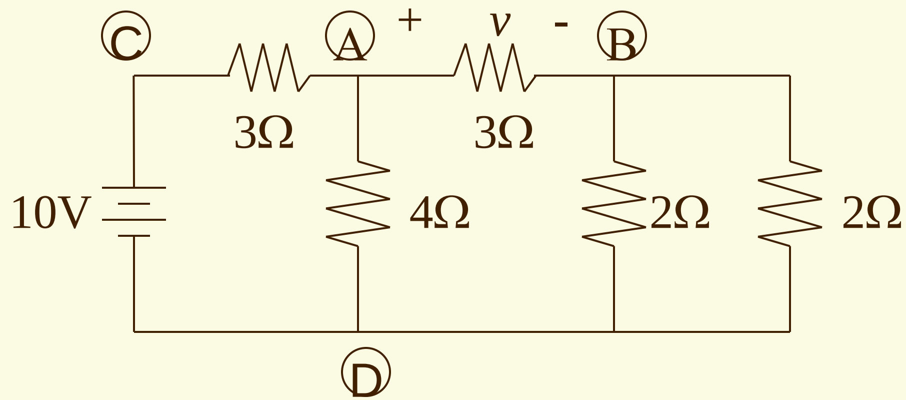
N resistors in parallel are equivalent to a single equivalent resistor with resistance equal to the inverse of the sum of the inverse resistances.

Example 1

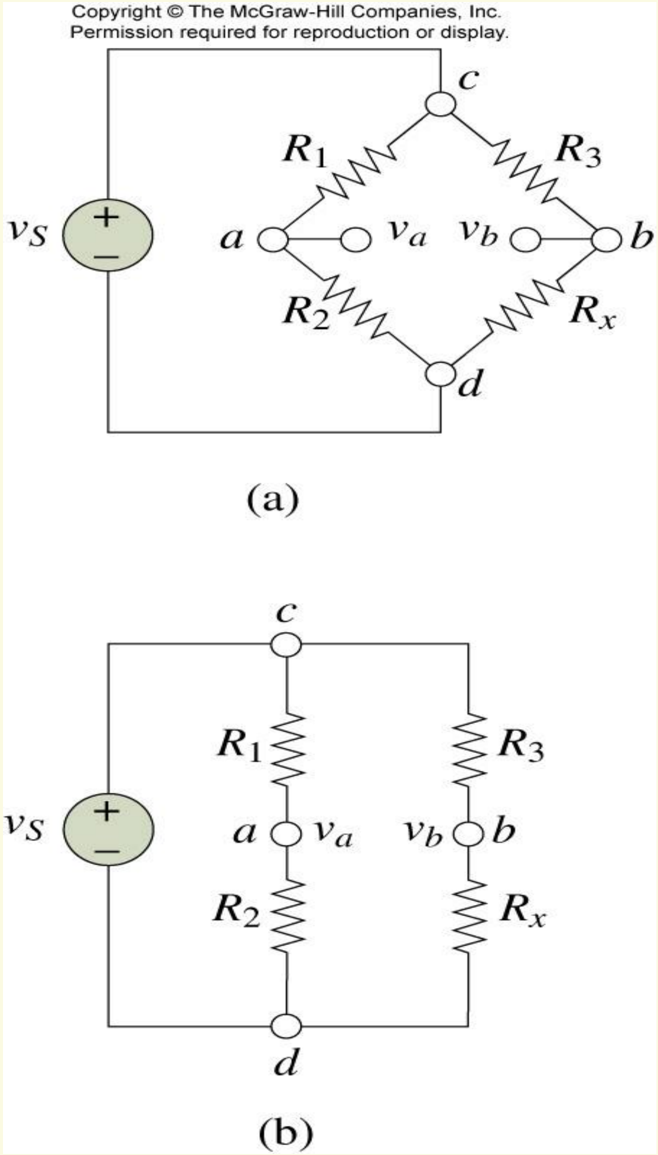


Example 2

Find the voltage (v) between nodes A and B.



Wheatstone bridge circuits

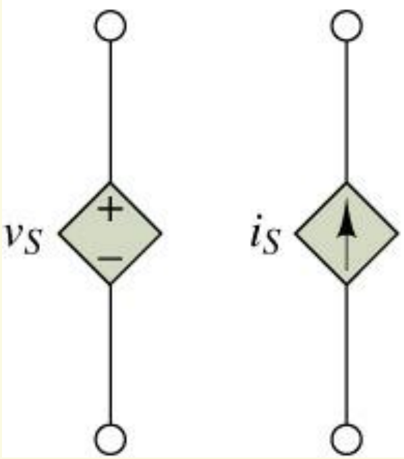


R1 , R2, R3 are known

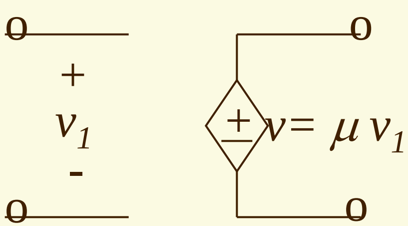
Find Rx by measuring Vab

Symbols for dependent sources

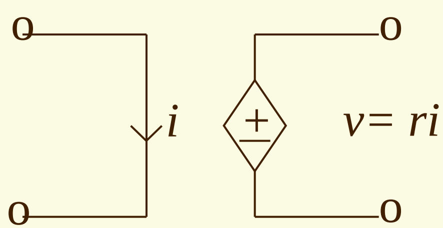
Figure 2.4



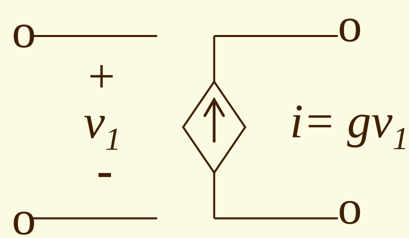
Source type	Relationship
Voltage controlled voltage source (VCVS)	$v_S = \mu v_x$
Current controlled voltage source (CCVS)	$v_S = r i_x$
Voltage controlled current source (VCCS)	$i_S = g v_x$
Current controlled current source (CCCS)	$i_S = \beta i_x$



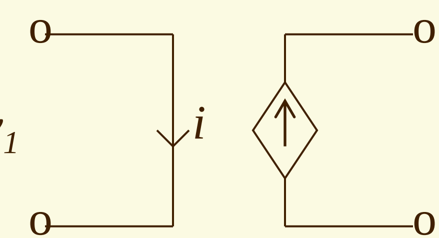
(a)VCVS



(b) CCVS

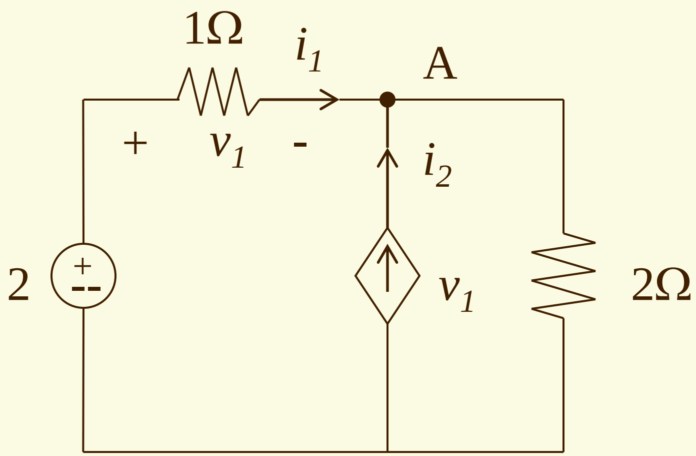


(c) VCCS



(d) CCCS

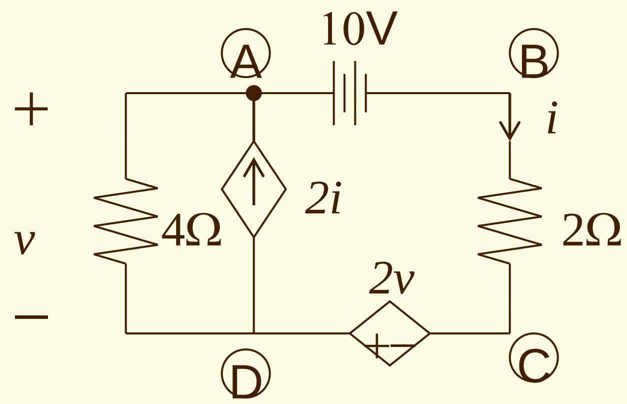
Example 3



What is the current on 2 Ω resistor?

Example 4

Find v. How much power does 4 ohm resistor consume?



Now, the voltage source is changed from 10V to 5V. How much power dies 4 ohm resistor consume?

Conclusion

- Kirchhoff's current and voltage laws
- Basic Circuit Analysis Method
- Series-parallel circuits
- Dependent Sources

Chap. 3 Network Theorems

Node (voltage) analysis

Mesh (current) analysis

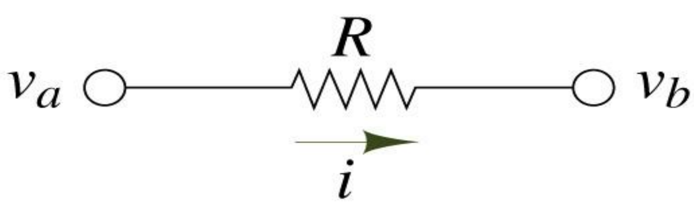
Superposition principle

Thevenin/Norton equivalent circuits

Branch current in Node Analysis

In the node voltage method, we assign the node voltages v_a and v_b; the branch current flowing from a to b is then expressed in terms of these node voltages.

$$i = \frac{v_a - v_b}{R}$$



By KCL: $i_1 - i_2 - i_3 = 0$. In the node voltage method, we express KCL by

$$\frac{v_a - v_b}{R_1} - \frac{v_b - v_c}{R_2} - \frac{v_b - v_d}{R_3} = 0$$

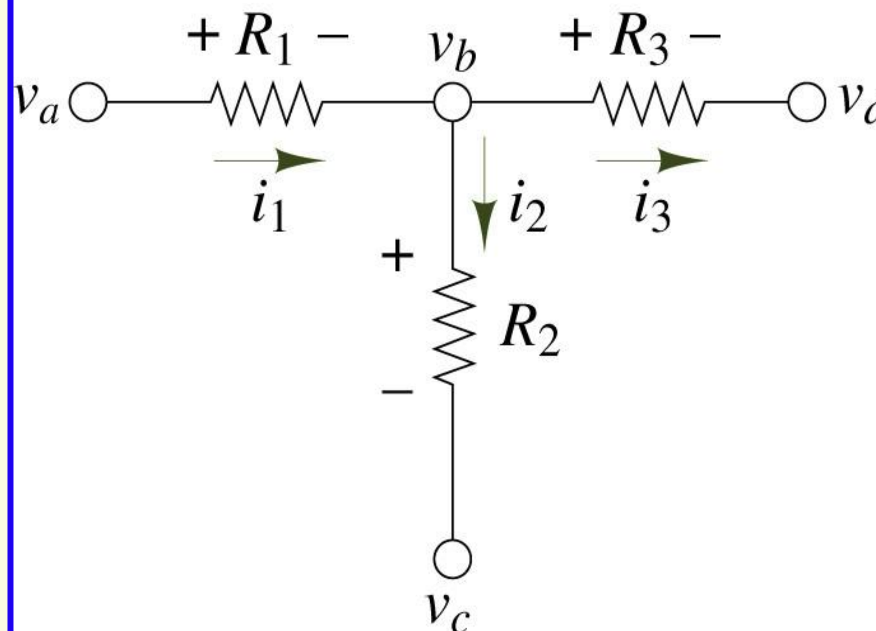
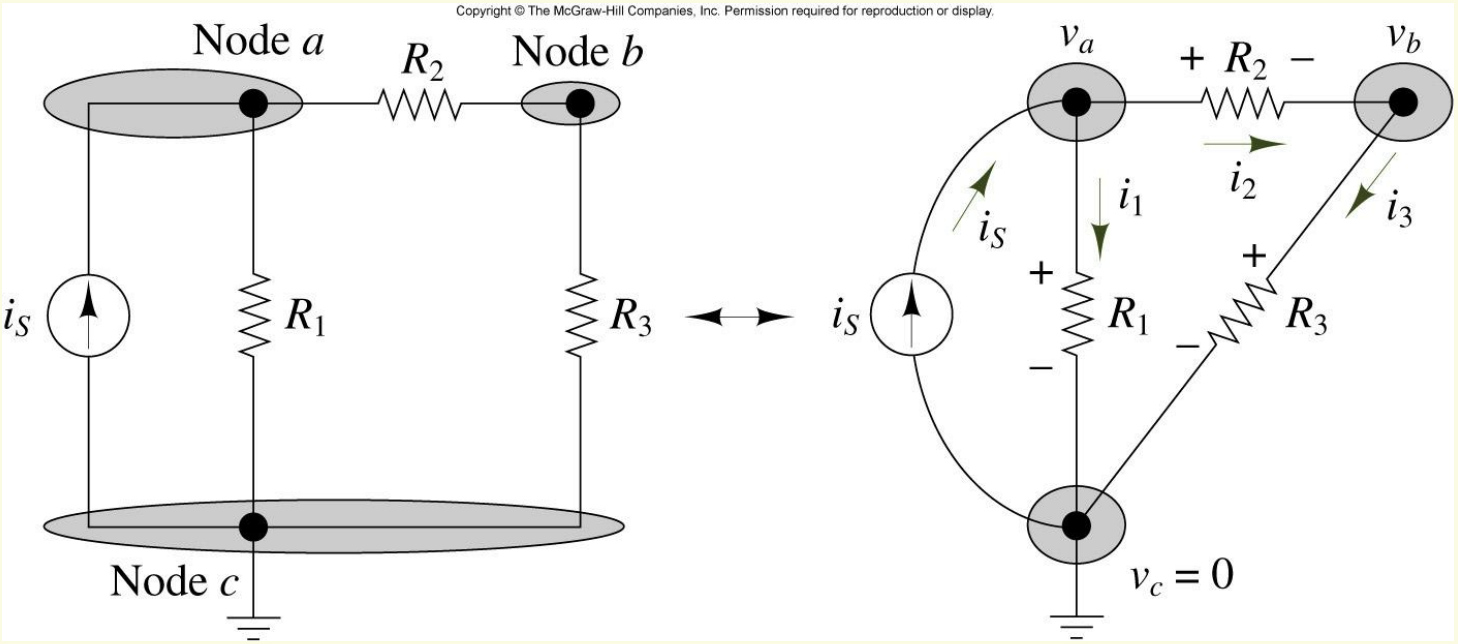


Illustration of Node Analysis

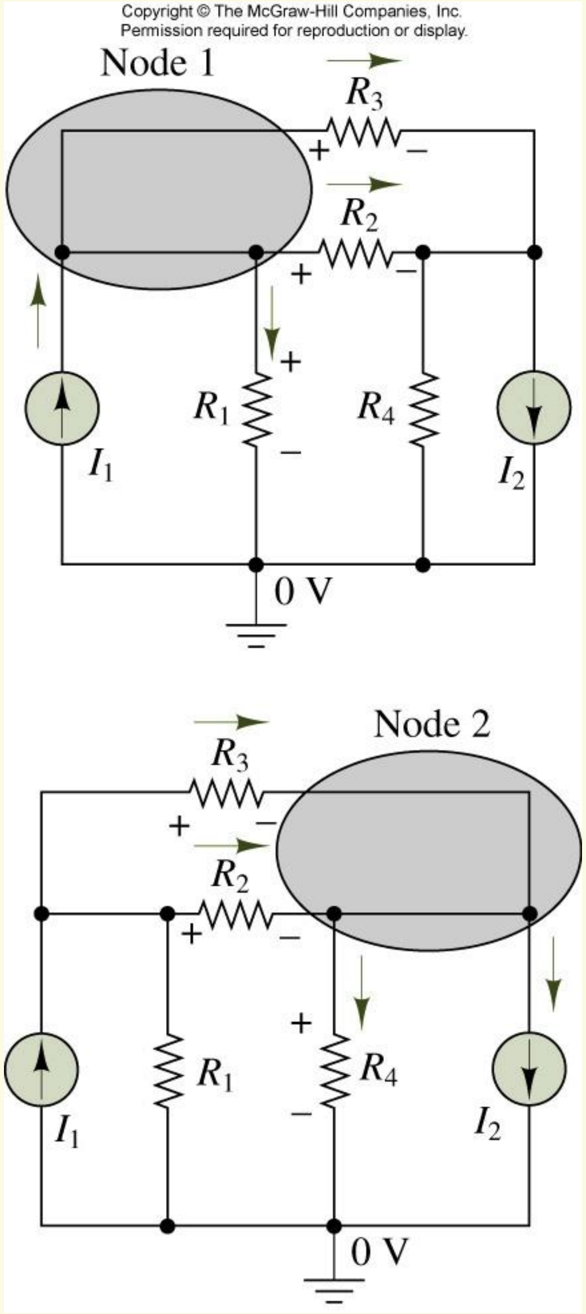


Example

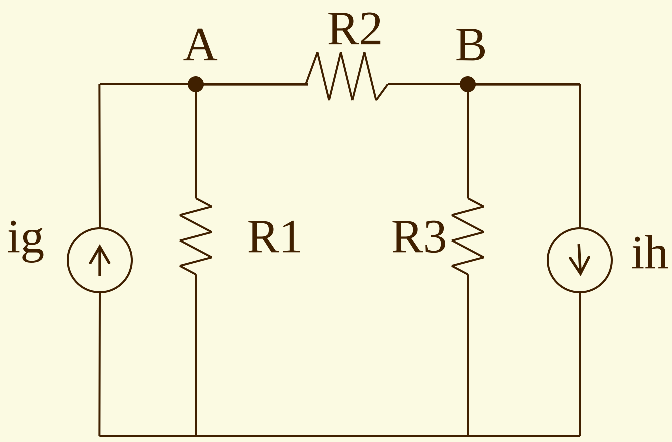
I1=10mA, I2=50mA, R1 = 1k, R2 = 2k

R3 = 10k, R4 = 2k

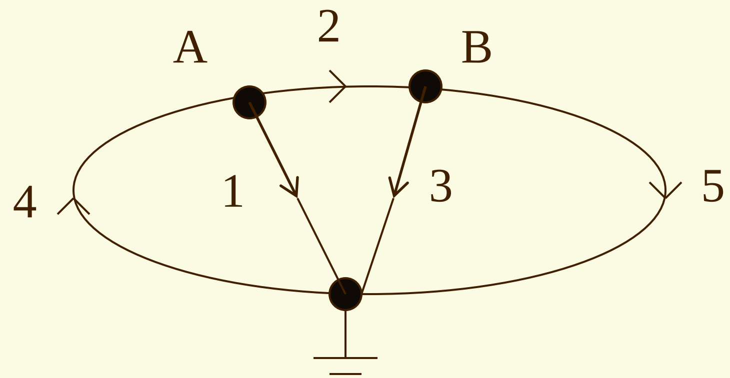
Find all node voltages and branch currents



Matrix Formulation



(a)

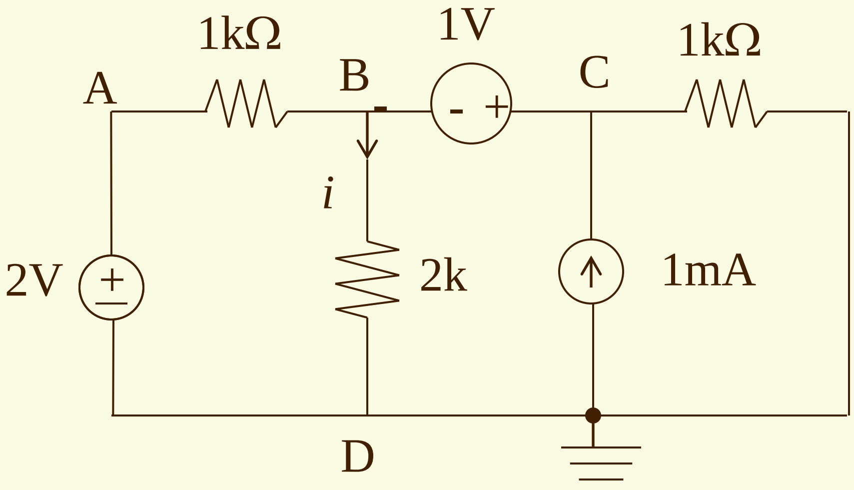


(b)

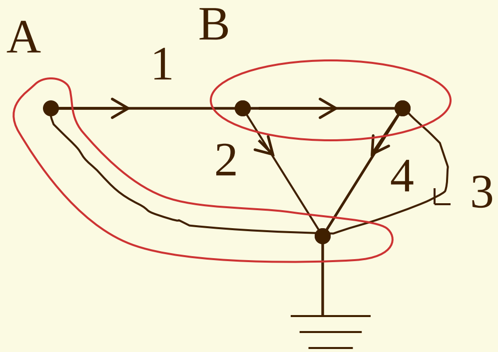
$$G_1 e_A + G_2(e_A - e_B) = i_g$$
$$G_2(e_B - e_A) + G_3 e_B = -i_h$$

$$\Rightarrow \begin{pmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{pmatrix}\begin{pmatrix} e_A \\ e_B \end{pmatrix} = \begin{pmatrix} i_g \\ -i_h \end{pmatrix}$$

Node Analysis with Voltage Source



(a)



(b)

$$\frac{e_B - e_A}{1} + \frac{e_B}{2} + \frac{e_C}{1} = 1 \quad \Rightarrow \quad \frac{e_B - 2}{1} + \frac{e_B}{2} + \frac{e_B + 1}{1} = 1$$

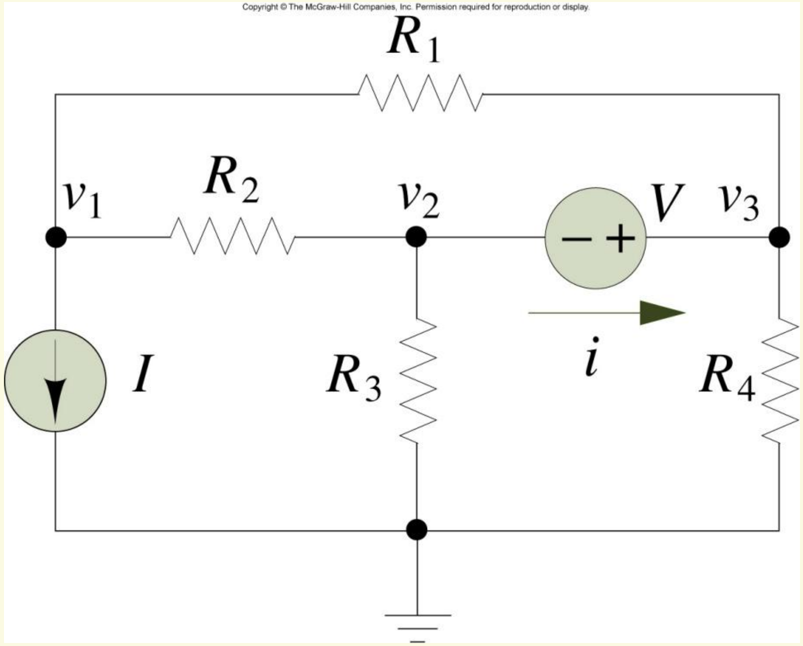
$$e_B = 0.8, \quad i = 0.4(mA)$$

Circuit for Example

I = 1A, V = 3V, R1 = 2, R2 = 2

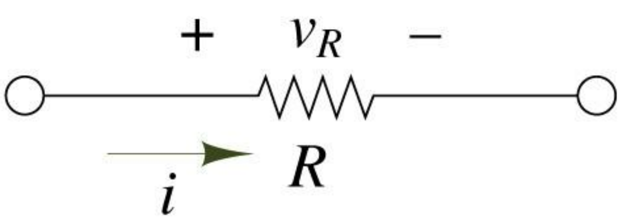
R3 = 4, R4 = 3

Find i

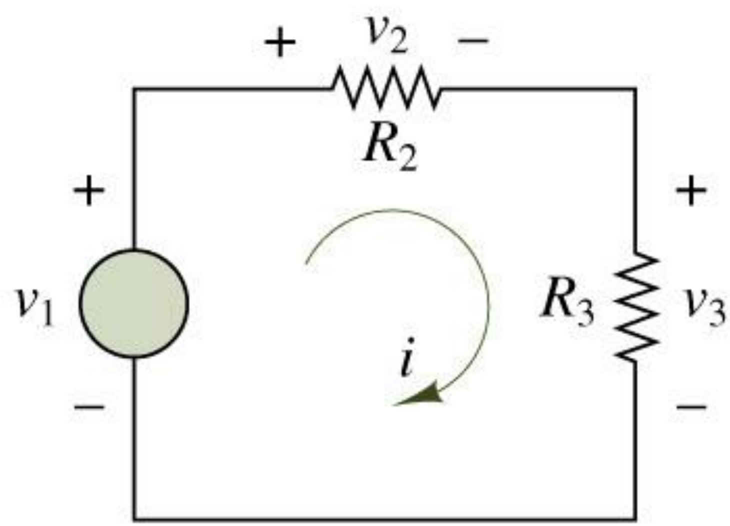


Basic principle of Mesh(Loop) Analysis

The current i, defined as flowing from left to right, establishes the polarity of the voltage across R.



Once the direction of current flow has been selected, KVL requires that $v_1 - v_2 - v_3 = 0$.



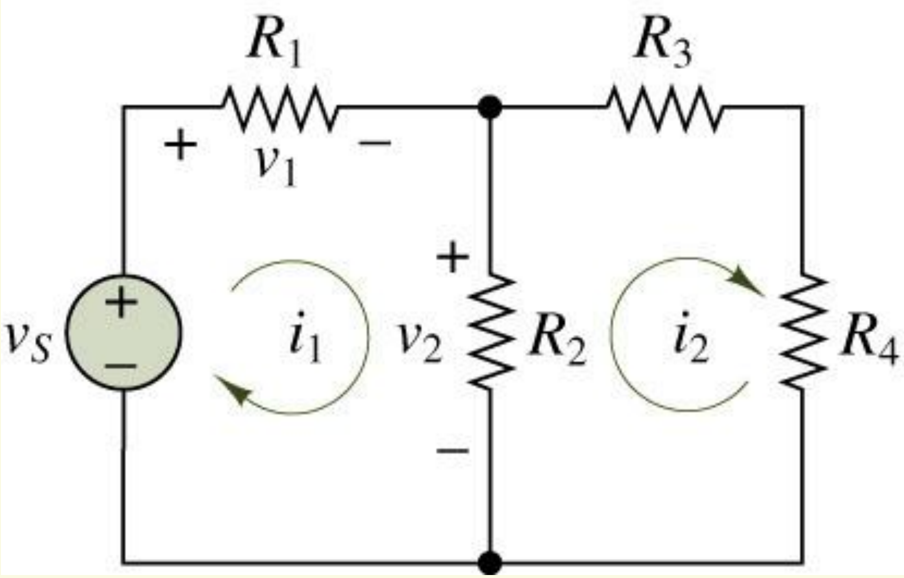
A mesh

A two-mesh circuit

Mesh 1: KVL requires that

$v_S - v_1 - v_2 = 0$, where $v_1 = i_1 R_1$,

$v_2 = (i_1 - i_2) R_1$.

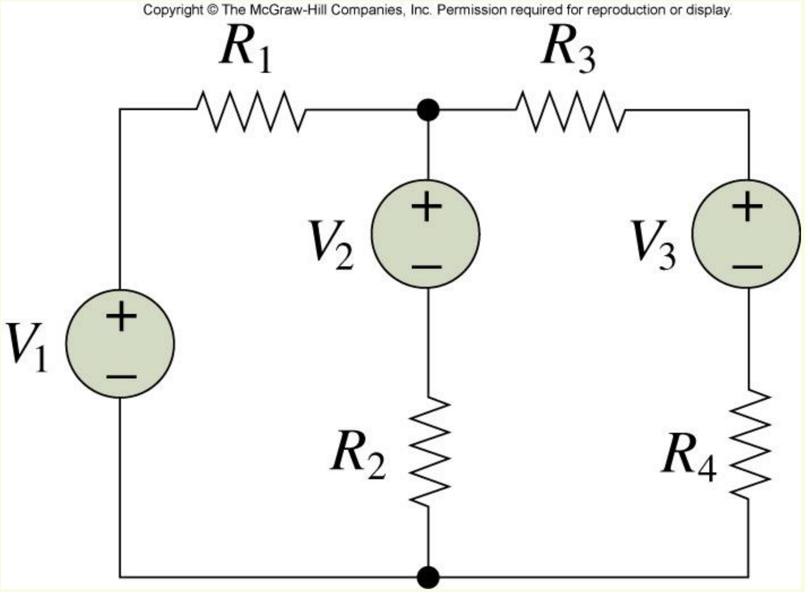


Example

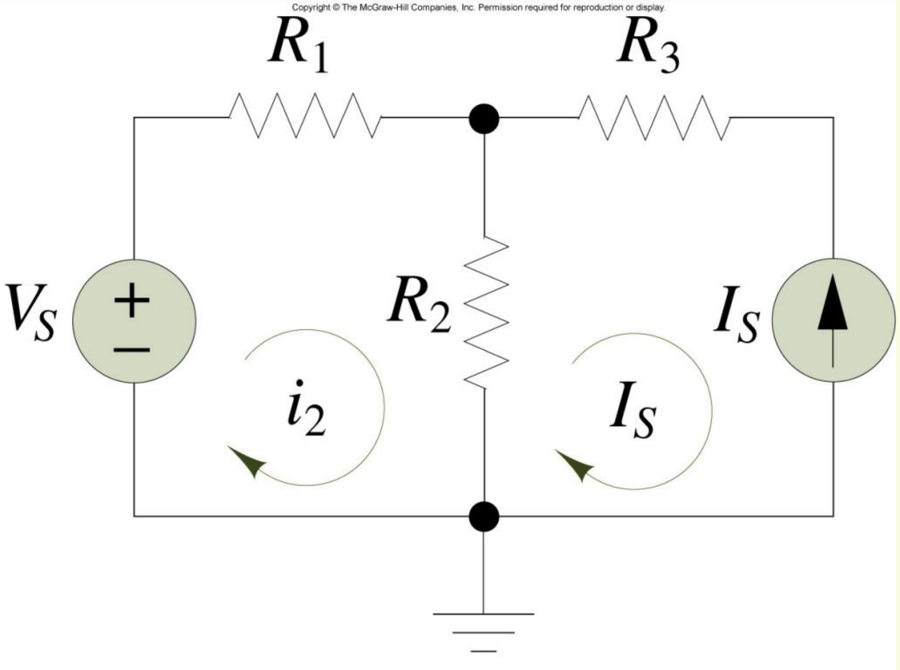
V1 = 10V, V2 = 9V, V3 = 1V,

R1 = 5k, R2 = 10k, R3 = 5k, R4 = 5k

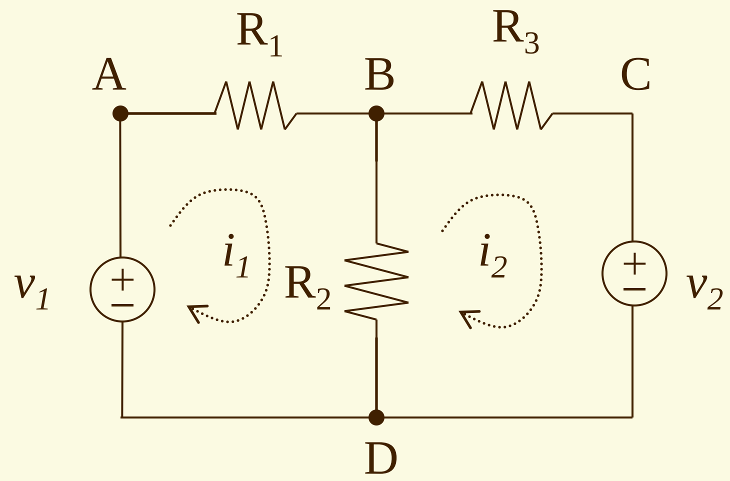
Find i



Mesh Analysis with Current Sources



Matrix Formulation

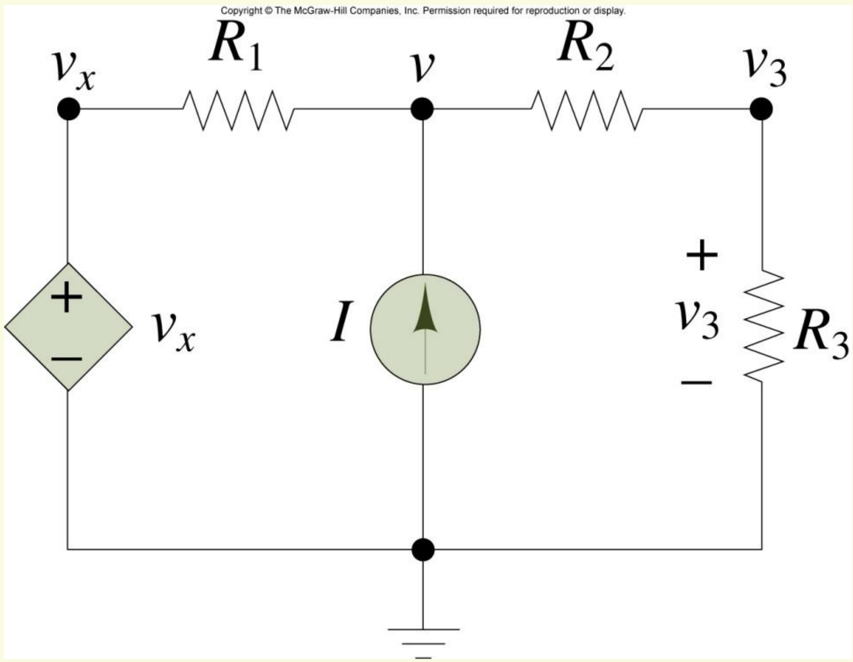


$$R_1 i_1 + R_2(i_1 - i_2) = v_1$$
$$R_2(i_2 - i_1) + R_3 i_2 = -v_2$$

$\Rightarrow$

$$\begin{pmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ -v_2 \end{pmatrix}$$

Node Analysis with Controlled Sources



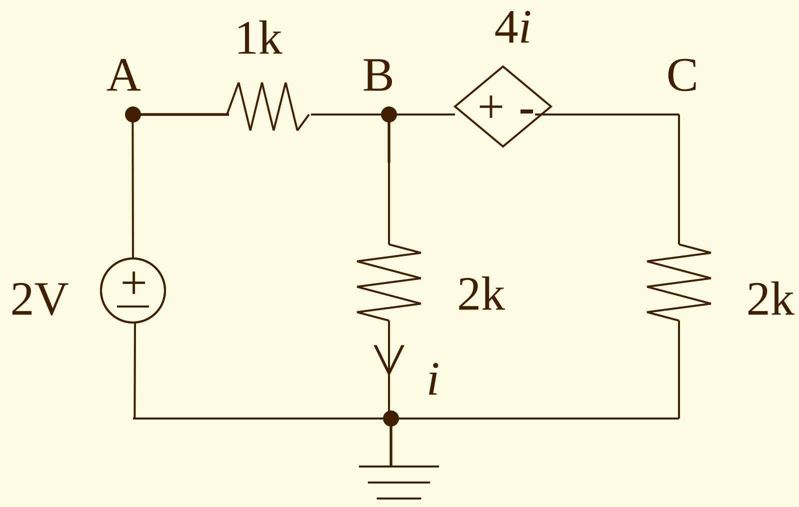
I = 0.5mA, Vx = 2 x v3

R1 = 5k, R2 = 2k, R3 = 4k

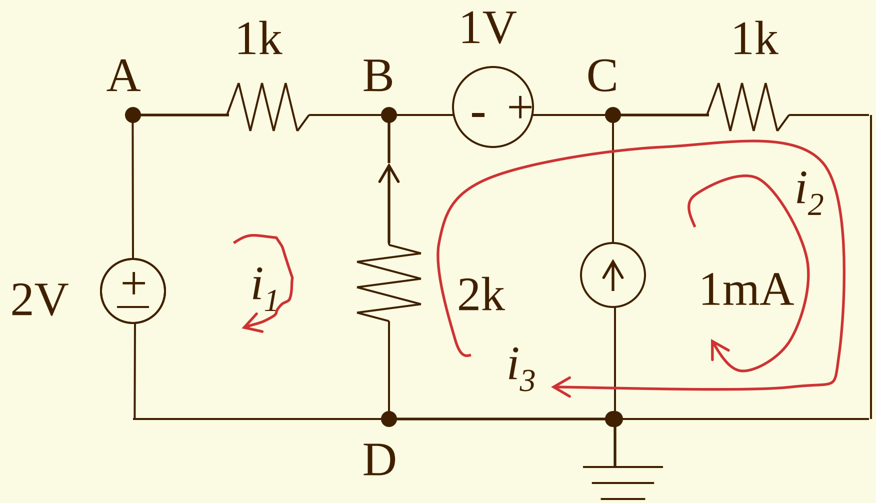
Find   v

Exercise

Find *i* using the node analysis technique



Loop Analysis

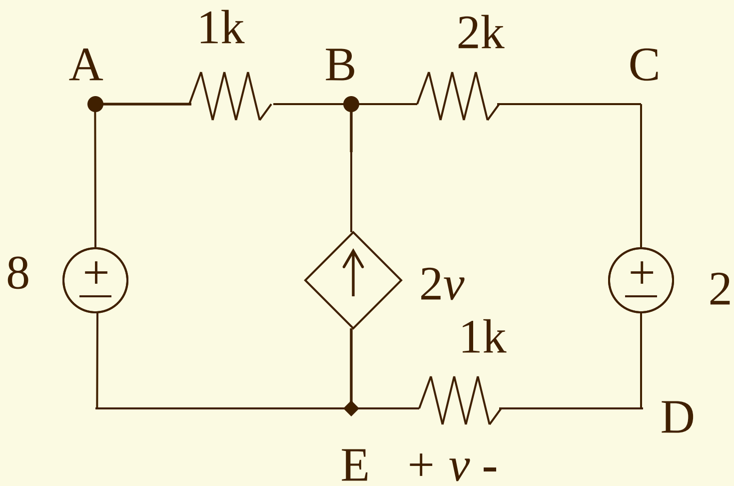


$$\{A, B, D, A\} \quad i_1 + 2(i_1 - i_3) = 2$$

$$\{B, C, D, B\} \quad (i_3 + i_2) + 2(i_3 - i_1) = 1, \quad i_2 = 1$$

Exercise

Find *v* using the loop analysis technique.

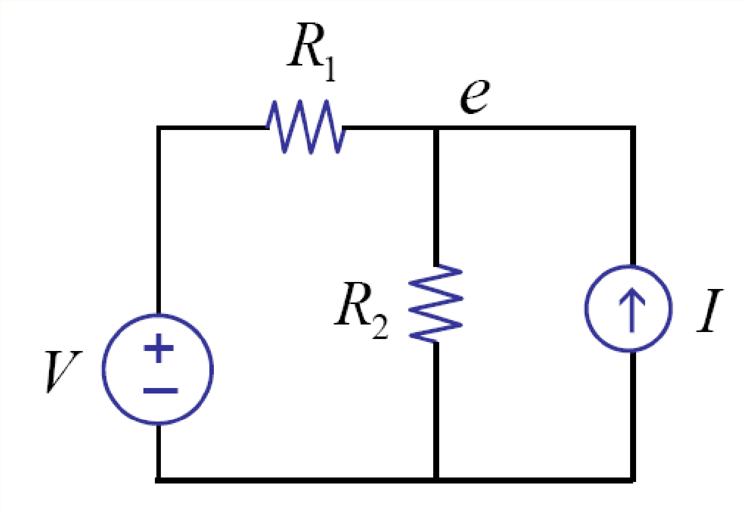


Linearity

- Write node equations

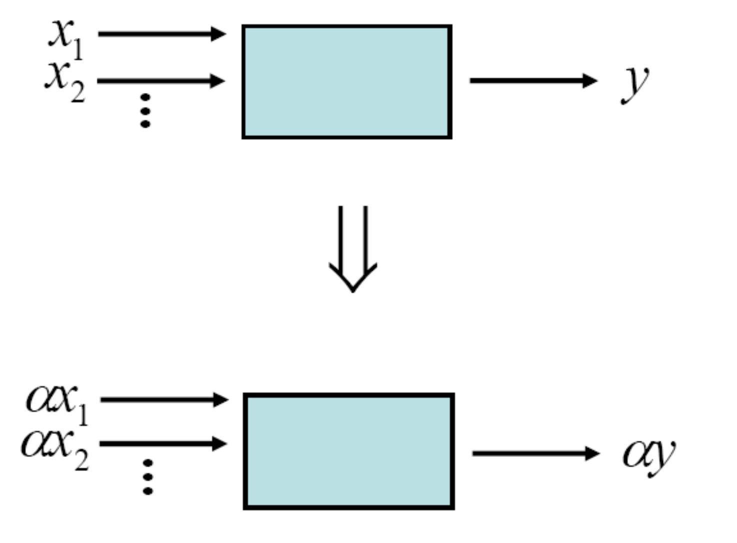
$$e = f_1(R_1, R_2)V + f_2(R_1, R_2)I$$

- Linear in V and I

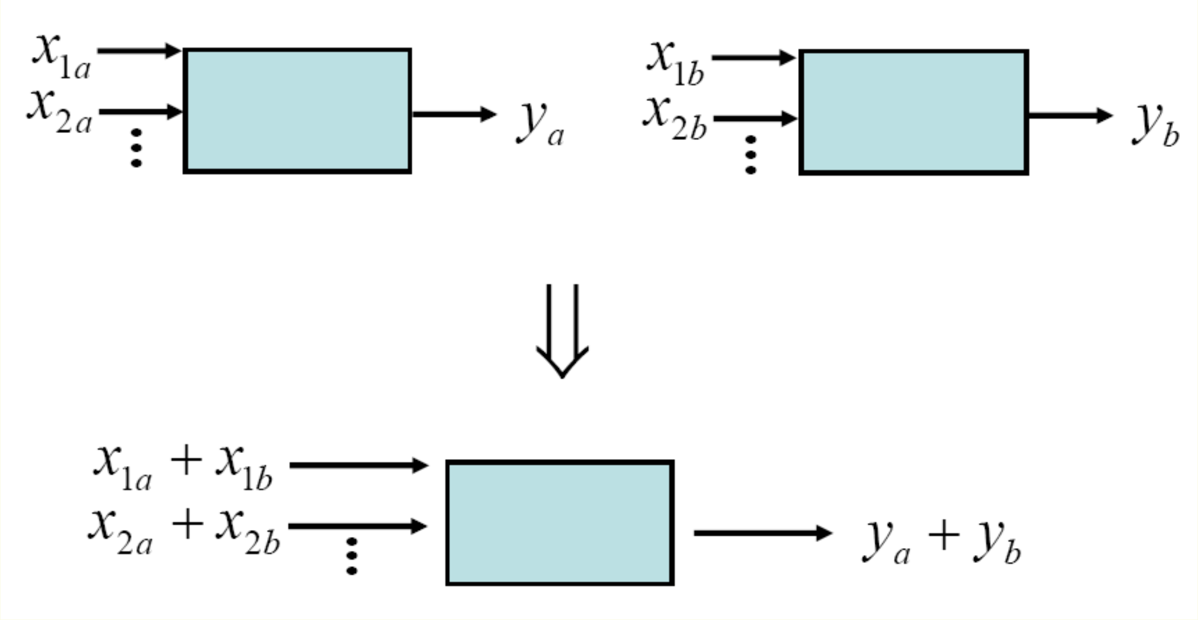


Linearity

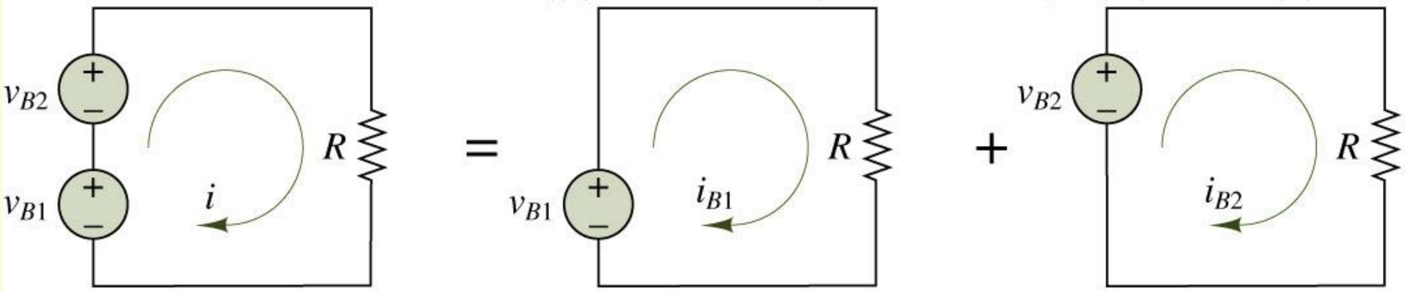
- Homogeneity



- Superposition



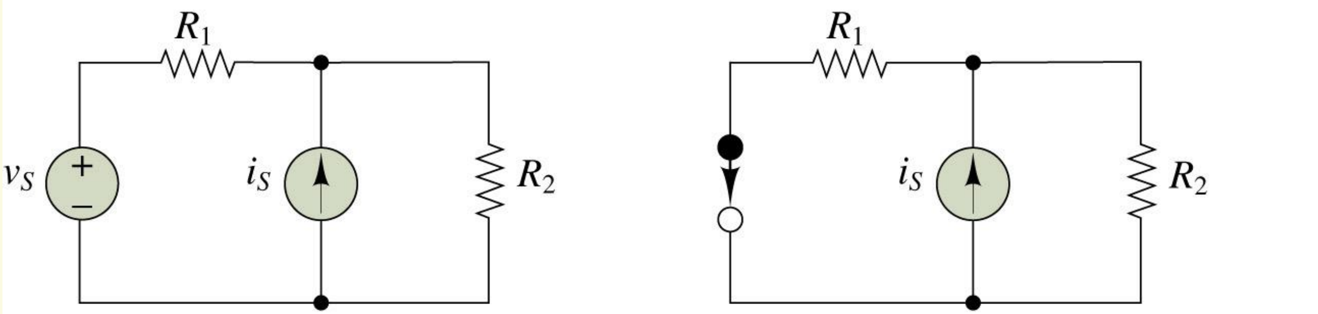
Superposition Principle



The net current through R is the sum of the individual source currents: $i = i_{B1} + i_{B2}$.

Zeroing Voltage and Current Sources

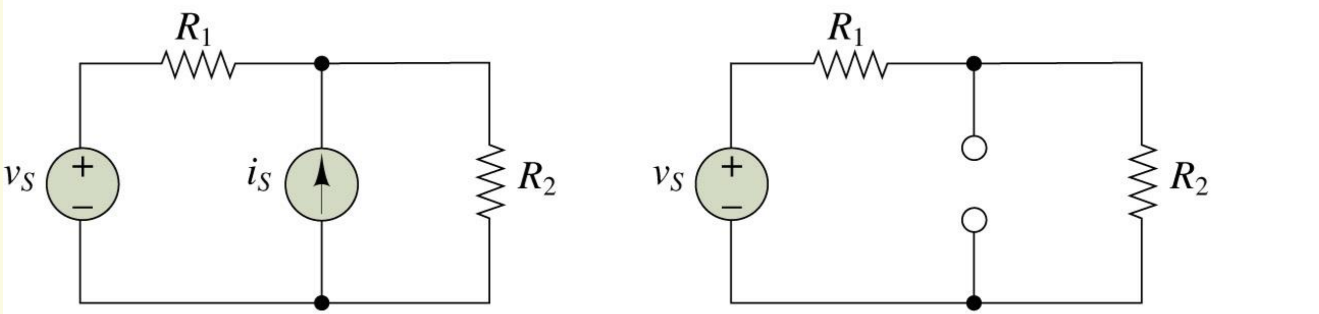
1. In order to set a voltage source equal to zero, we replace it with a short circuit.



A circuit

The same circuit with $v_S = 0$

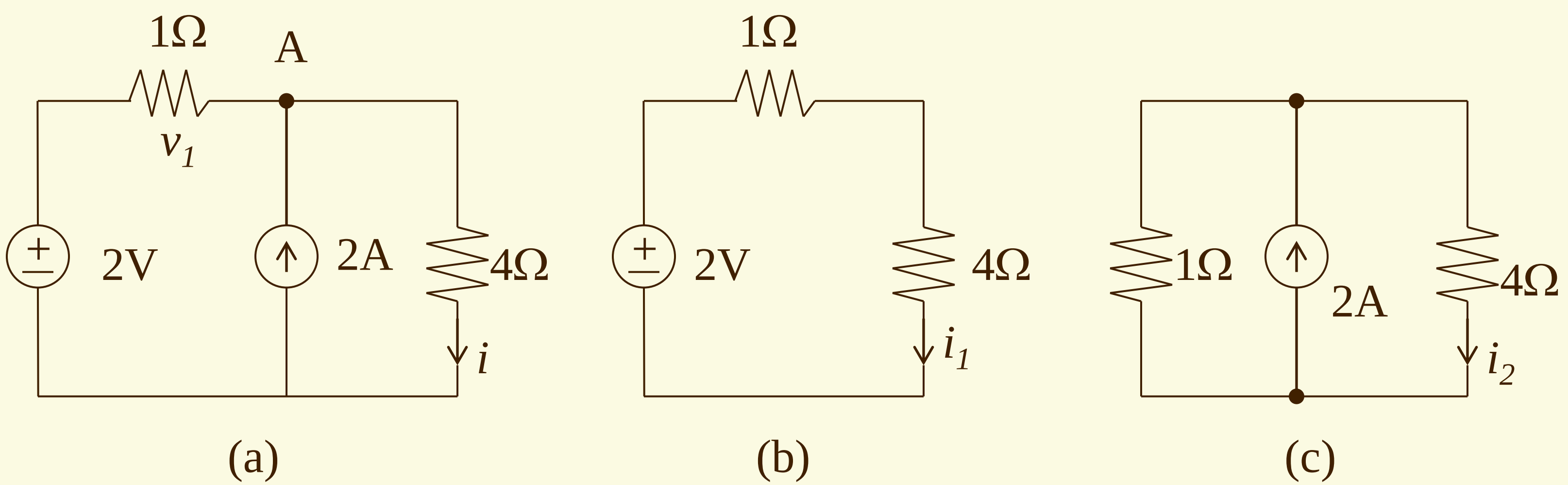
2. In order to set a current source equal to zero, we replace it with an open circuit.



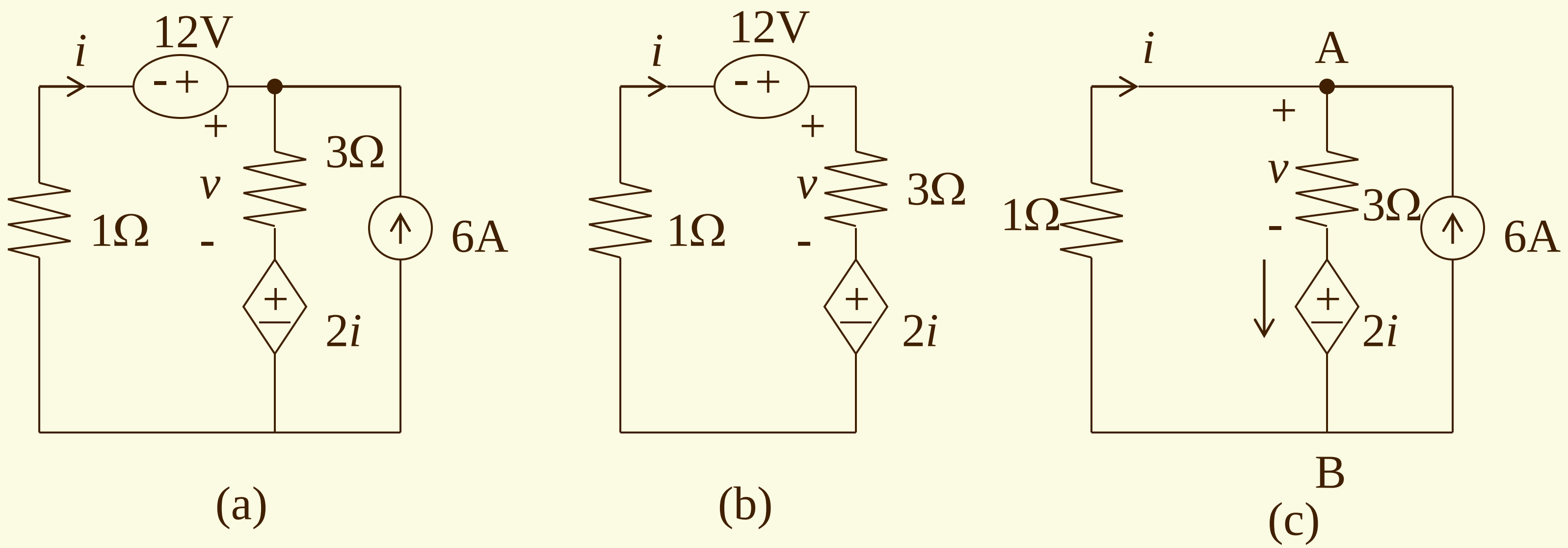
A circuit

The same circuit with $i_S = 0$

Exercise



Superposition with Controlled Sources



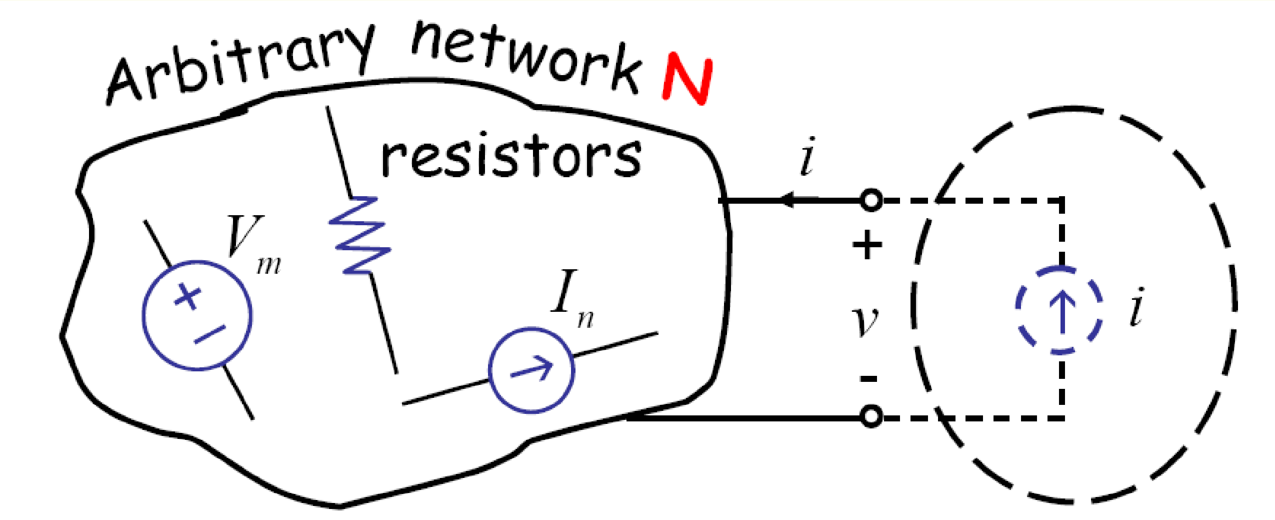
$$v = v_{(b)} + v_{(c)}$$

$$(b): 12 = 3i + 2i + i \quad \rightarrow i = 2, v = 6$$

$$(c): i + 3(i+6) + 2i = 0 \quad \rightarrow i = -3, v = 9$$

Deviation of the Thevenin's Network

- Consider



By superposition

$$v = \underbrace{\sum_m \alpha_m V_m}_{\text{no units}} + \underbrace{\sum_n \beta_n I_n}_{\text{resistance units}} + \underbrace{Ri}_{\text{All}}$$

also independent of external excitement & behaves like a resistor

By setting

$$\forall_n I_n = 0, \quad i = 0 \qquad\qquad \forall_m V_m = 0, \quad i = 0$$

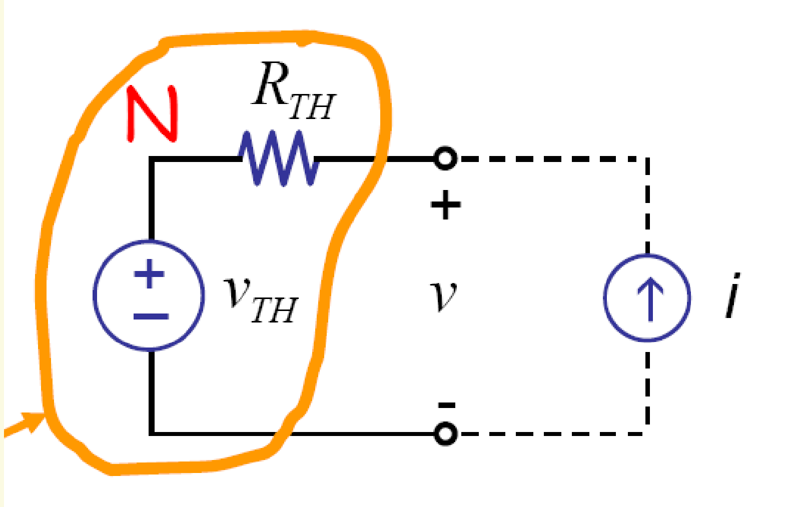
All

$$\forall_n I_n = 0, \quad \forall_m V_m = 0$$

independent of external excitation and behaves like a voltage "v_{TH}"

Thevenin equivalent network

$$v = v_{TH} + R_{TH} i$$



Thevenin equivalent network

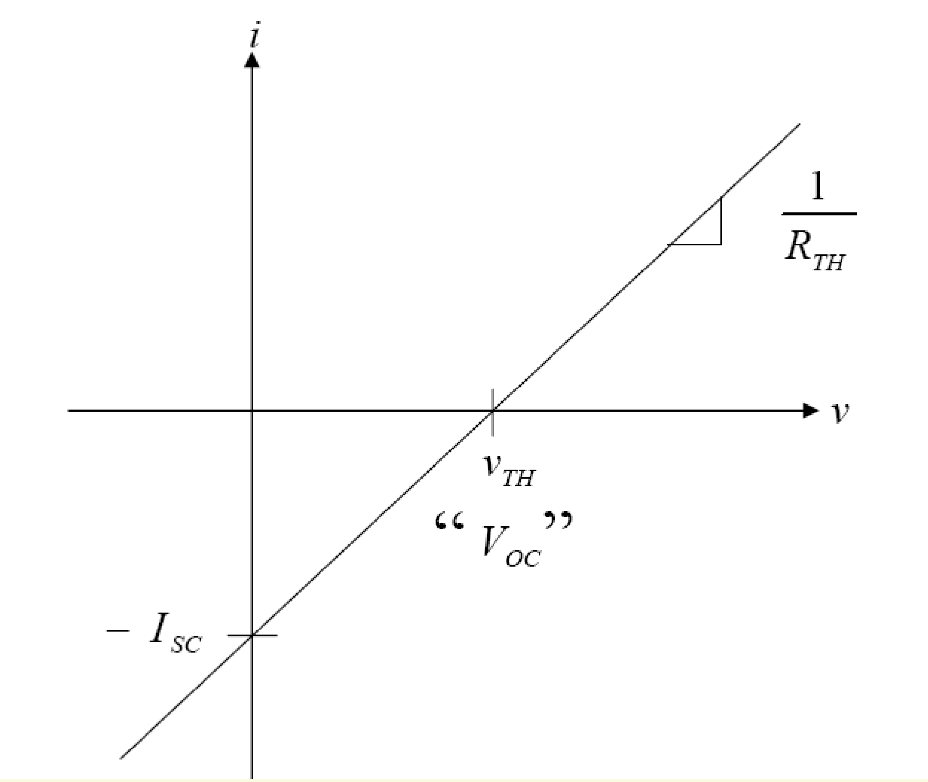
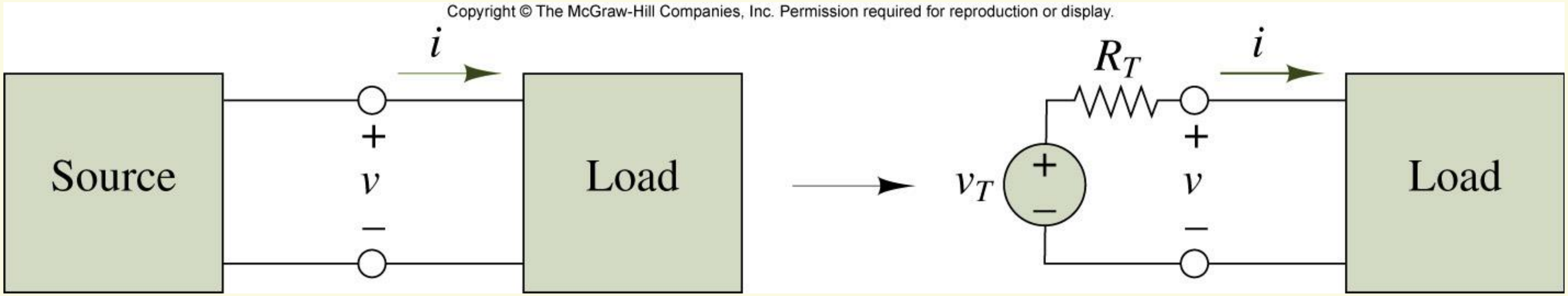
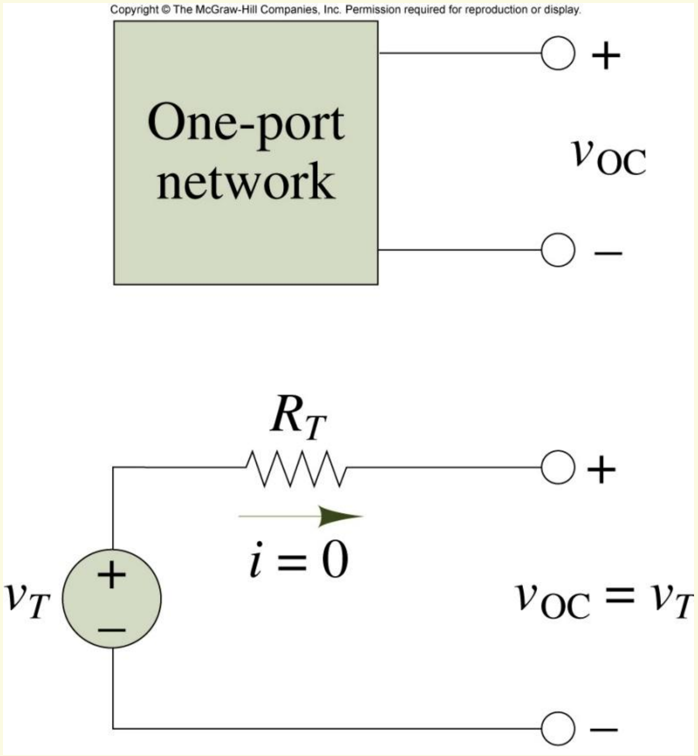


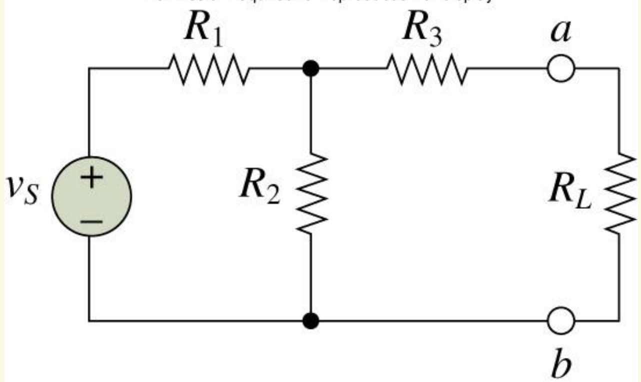
Illustration of Thévenin theorem



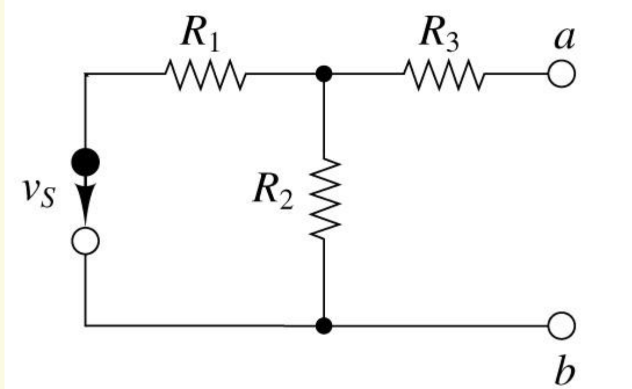
Open-circuit and Thévenin voltage



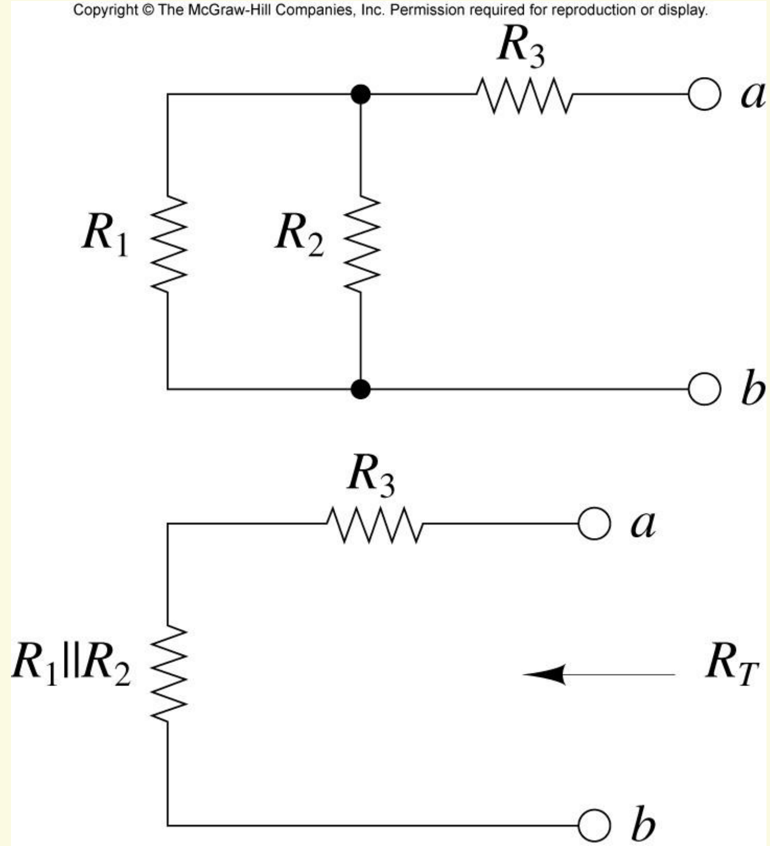
Computation of Thévenin resistance



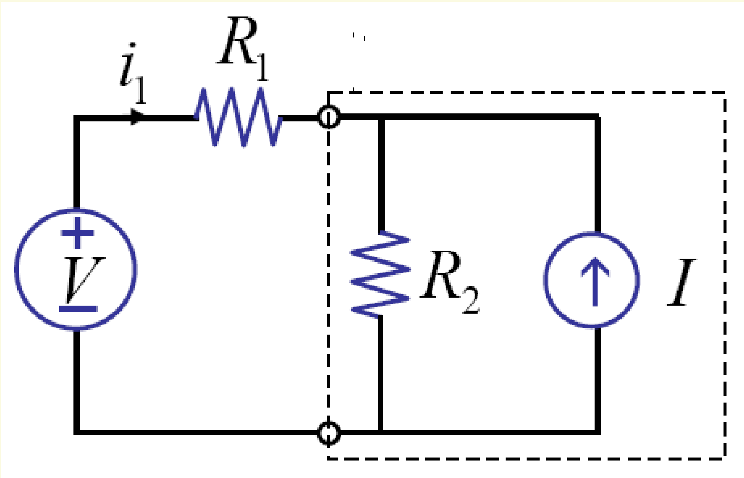
Complete circuit



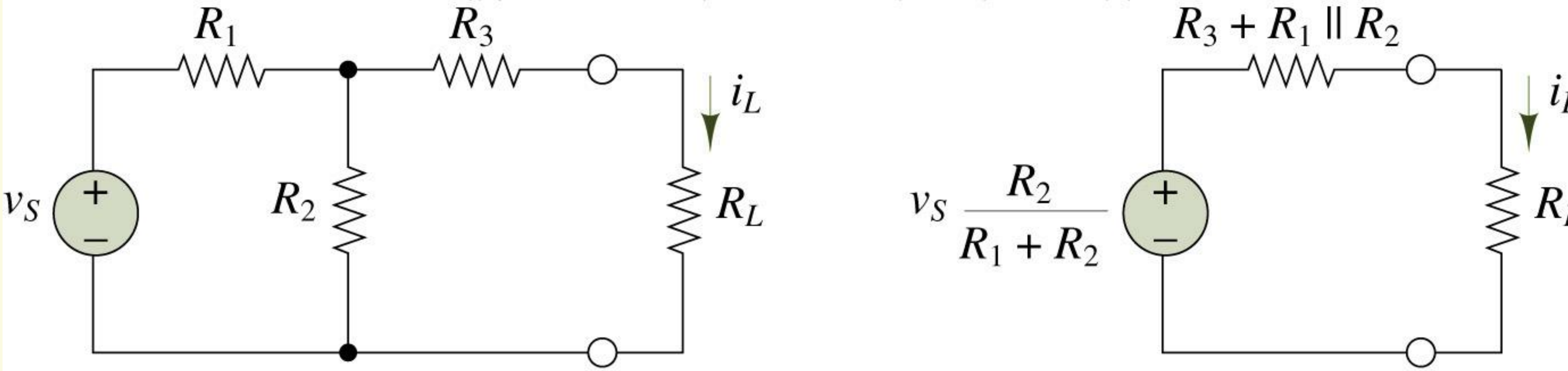
Circuit with load removed for computation of R_T. The voltage source is replaced by a short circuit.



Example



A circuit and its Thévenin equivalent



A circuit

Its Thévenin equivalent

Norton equivalent circuit

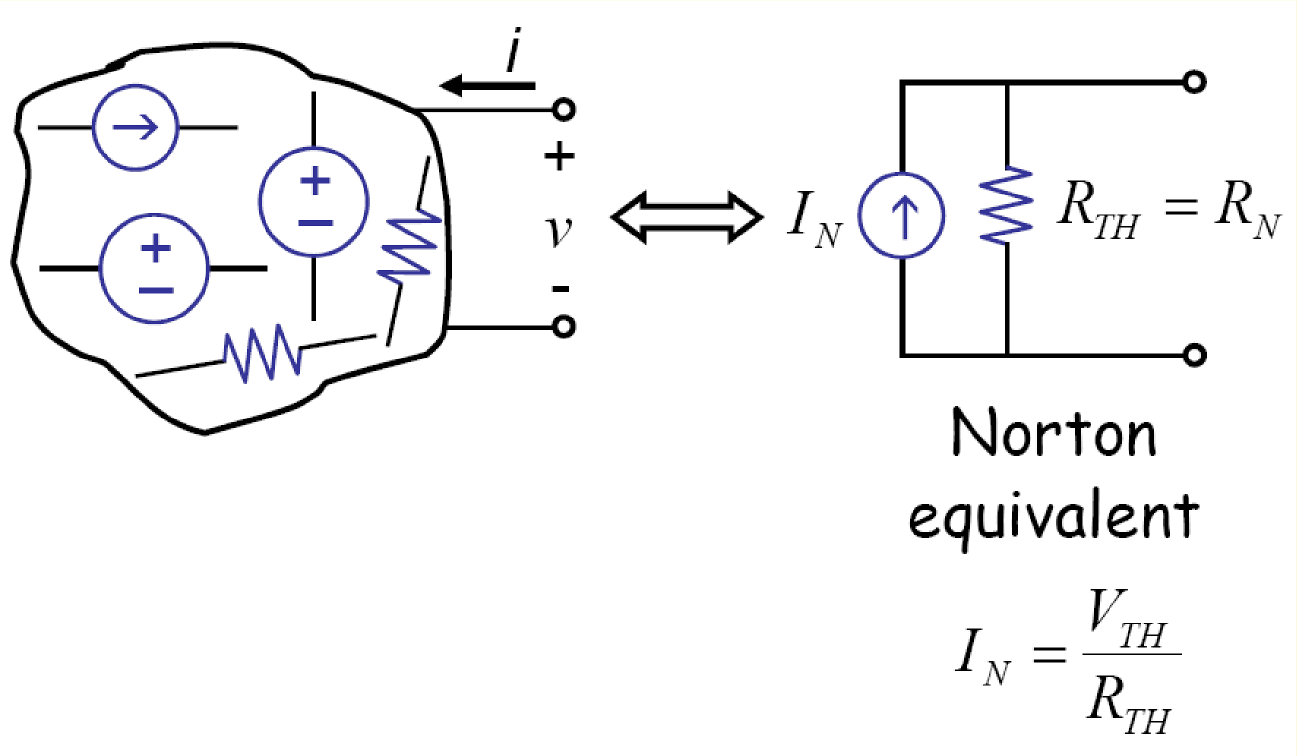
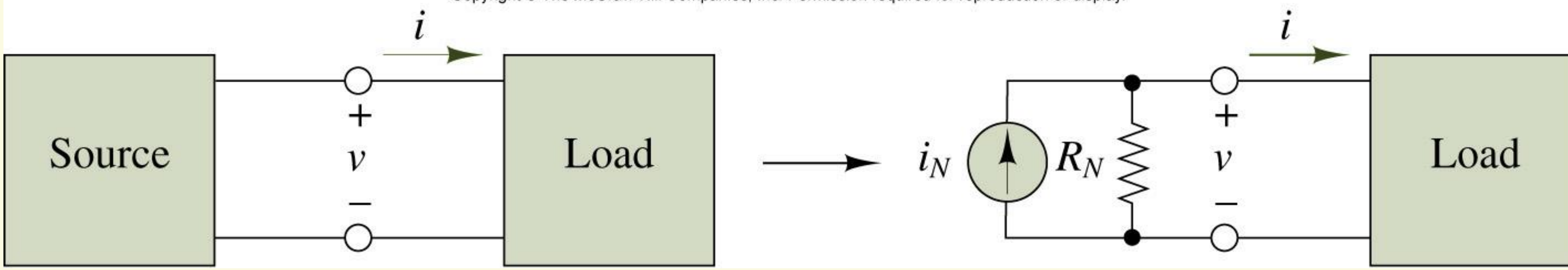
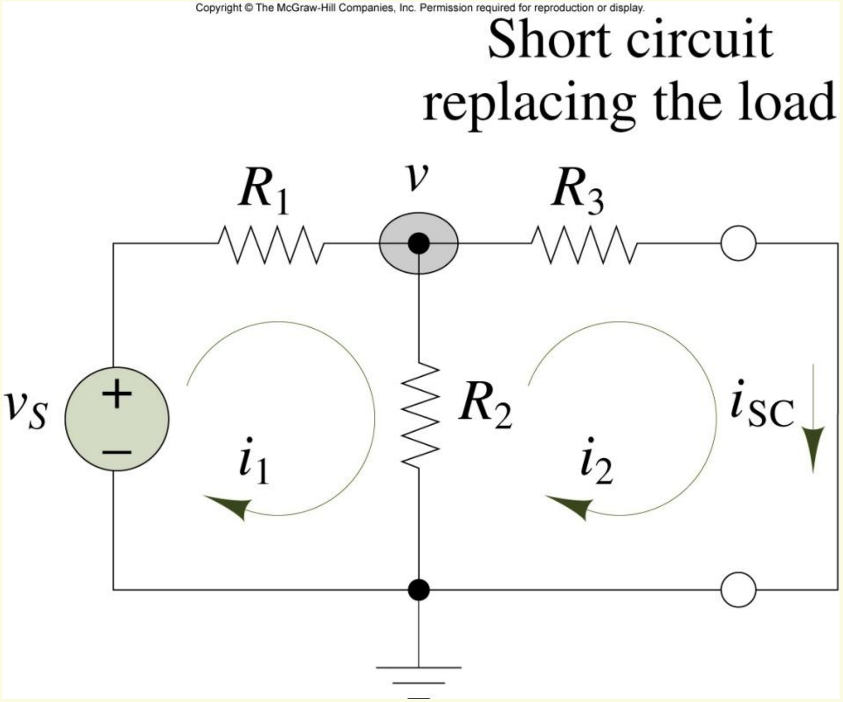


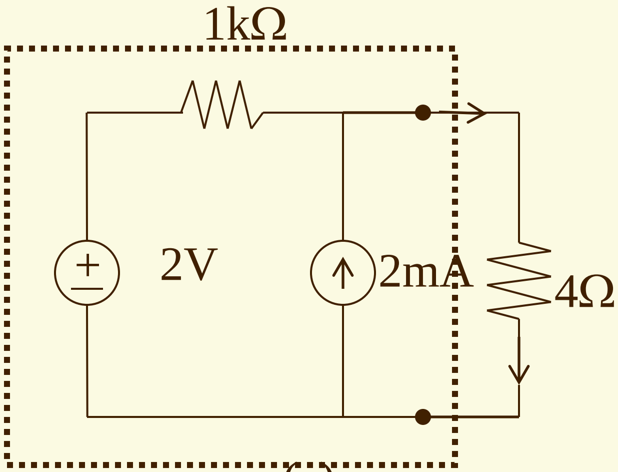
Illustration of Norton theorem



Computation of Norton current



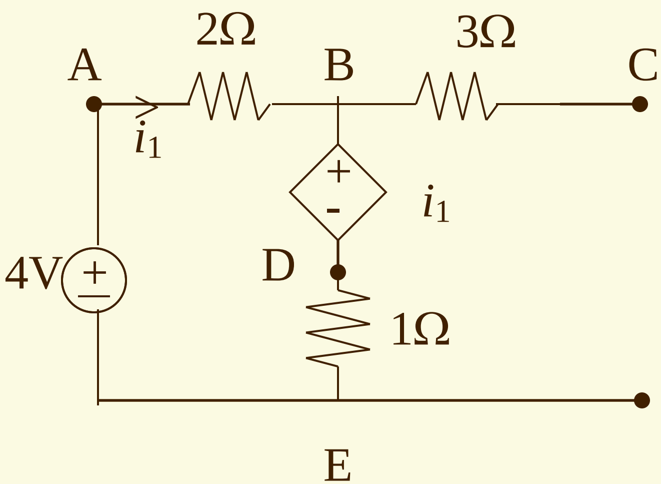
Example

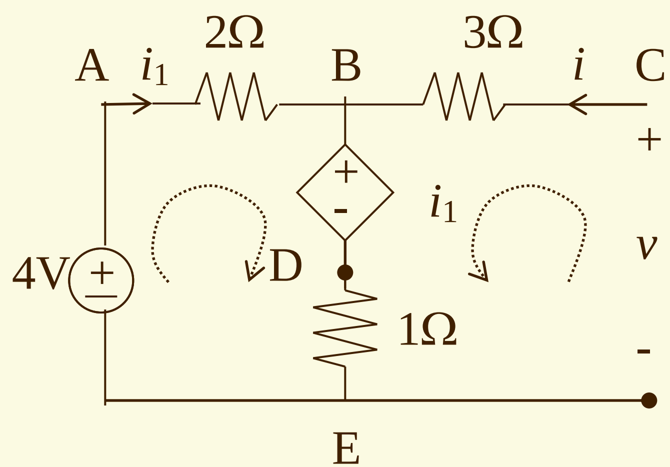


(a)

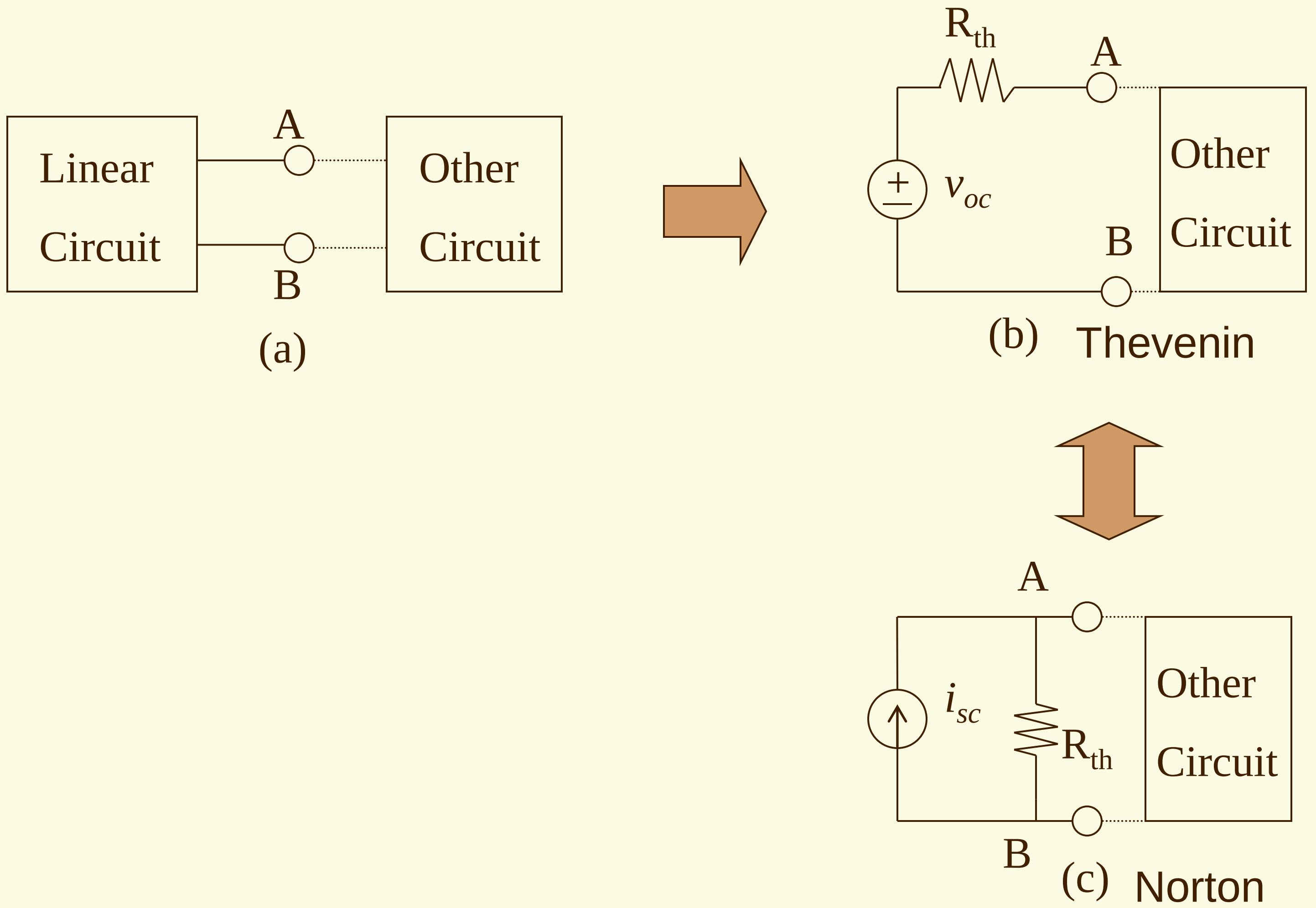
Exercise

Obtain Thevenin and Norton equivalent circuits

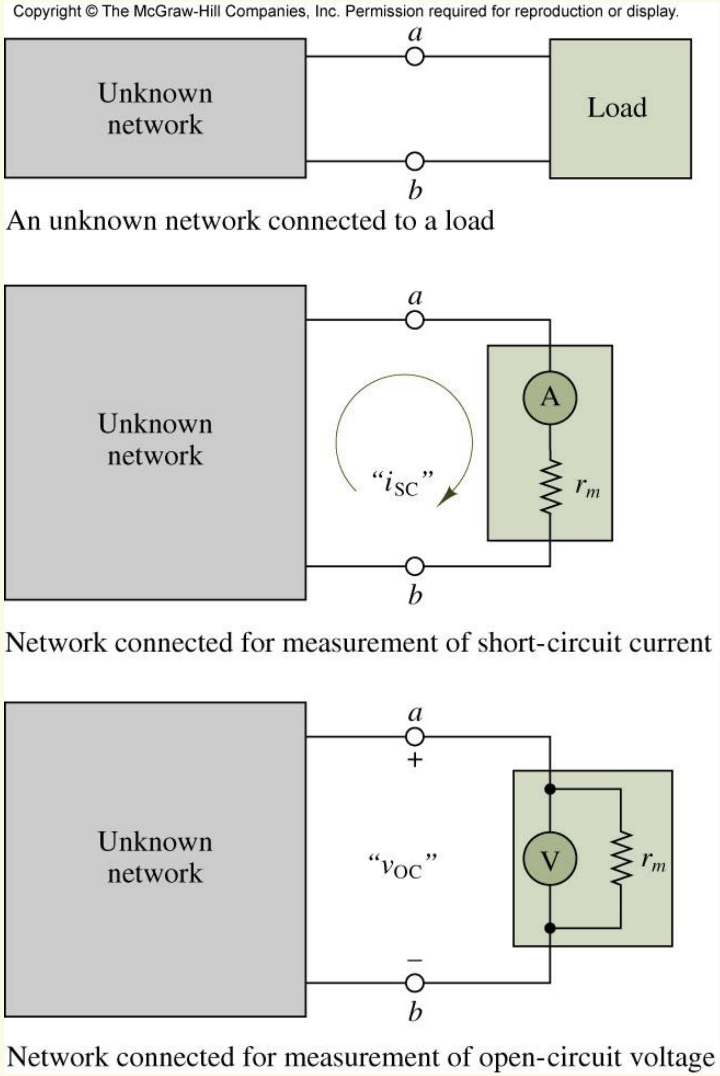




Source Transformation



Measurement of open-circuit voltage and short-circuit current



An unknown network connected to a load

Network connected for measurement of short-circuit current

Network connected for measurement of open-circuit voltage

Conclusion

- Network analysis: node analysis and mesh analysis
- Superposition principle: linear circuit
- Thevenin and Norton equivalent circuits