Chap. 9 Energy Storage Elements

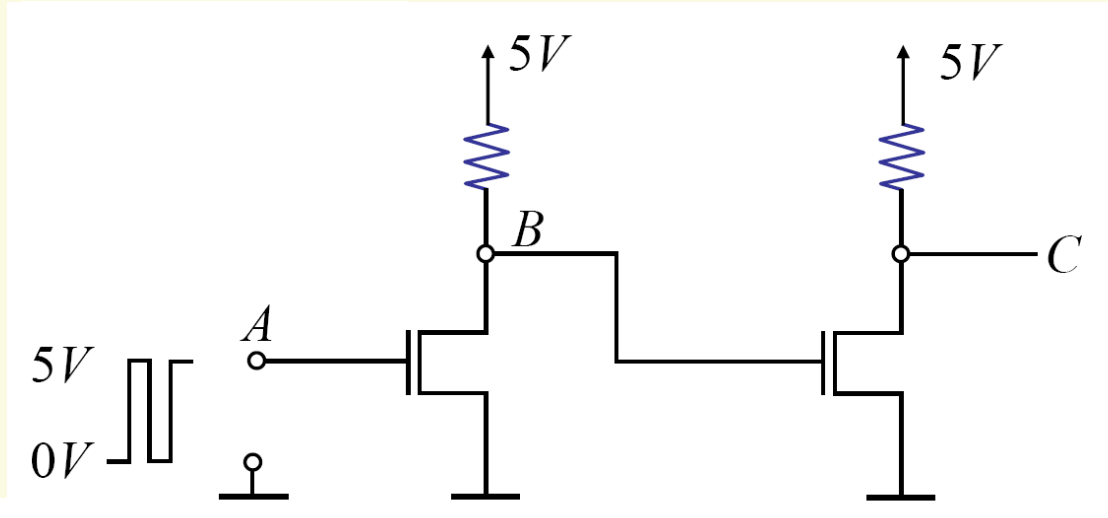
Capacitors and Inductors

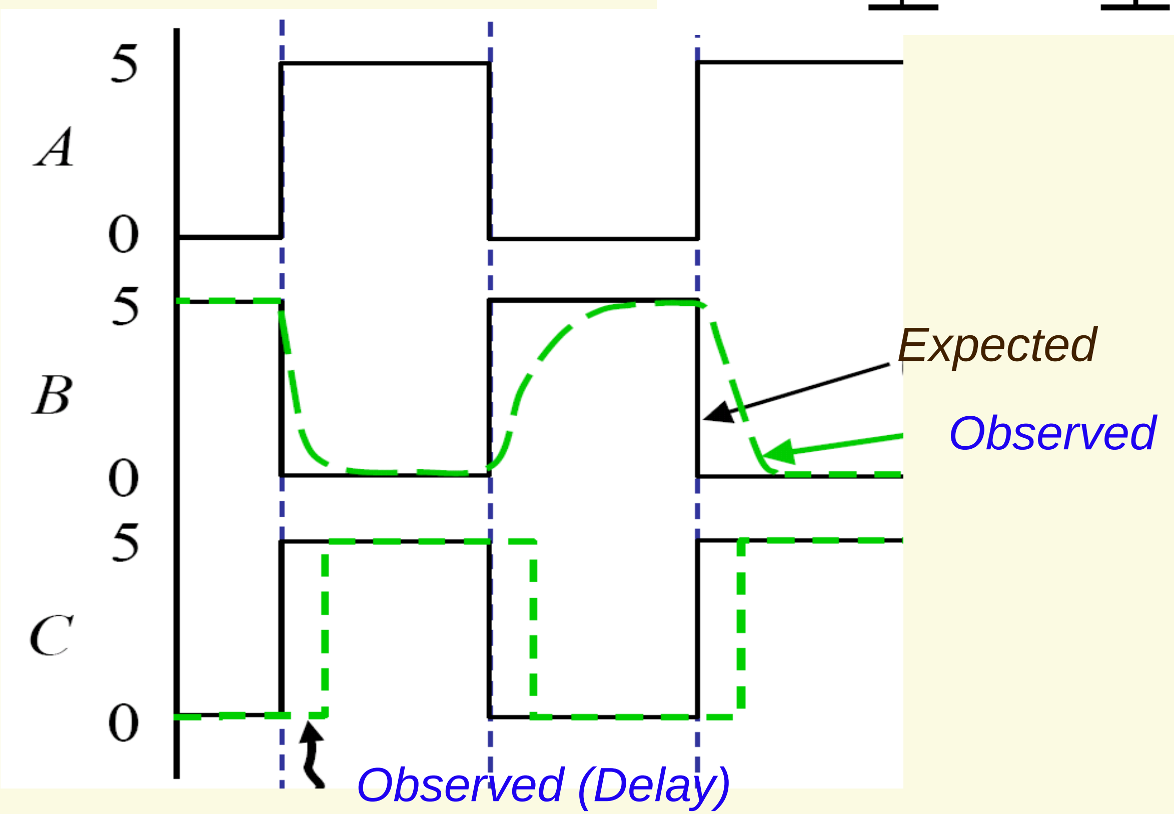
Series and Parallel Connections

Simple Circuit Examples

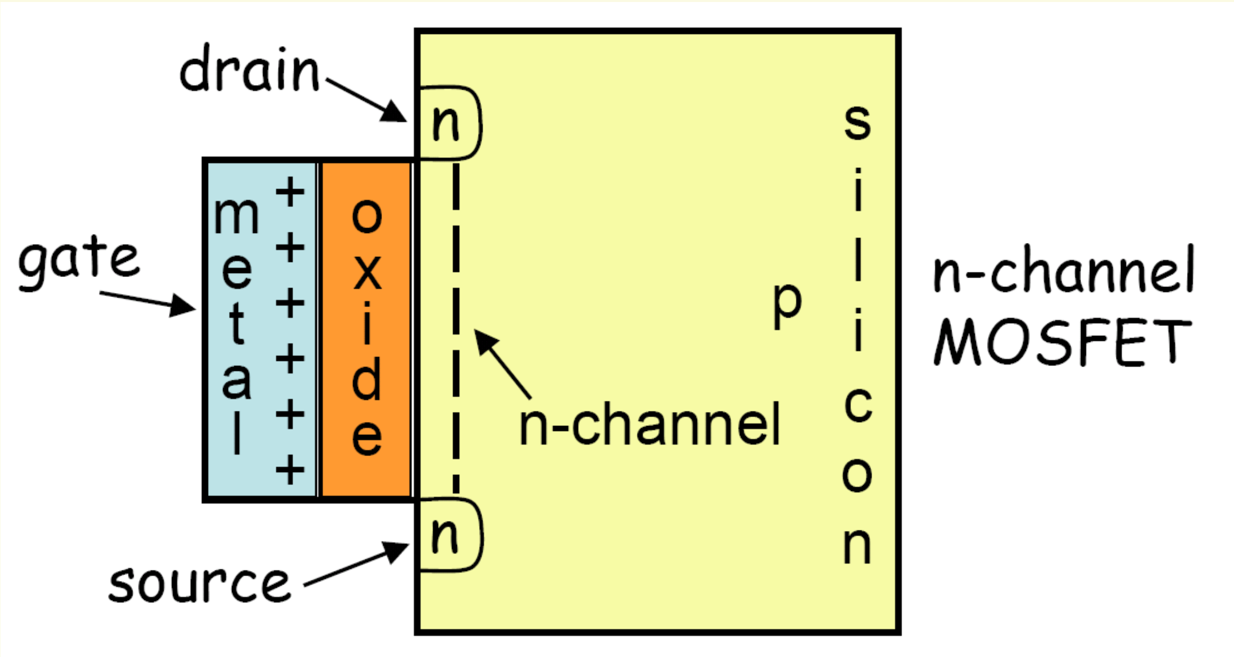
Energy, Charge, and Flux Conservation

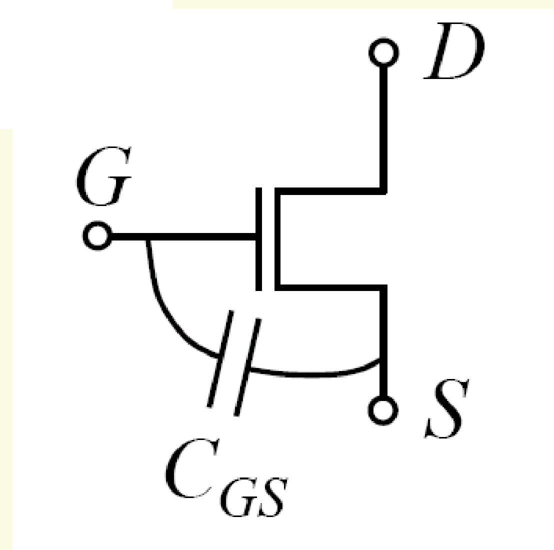
Motivation



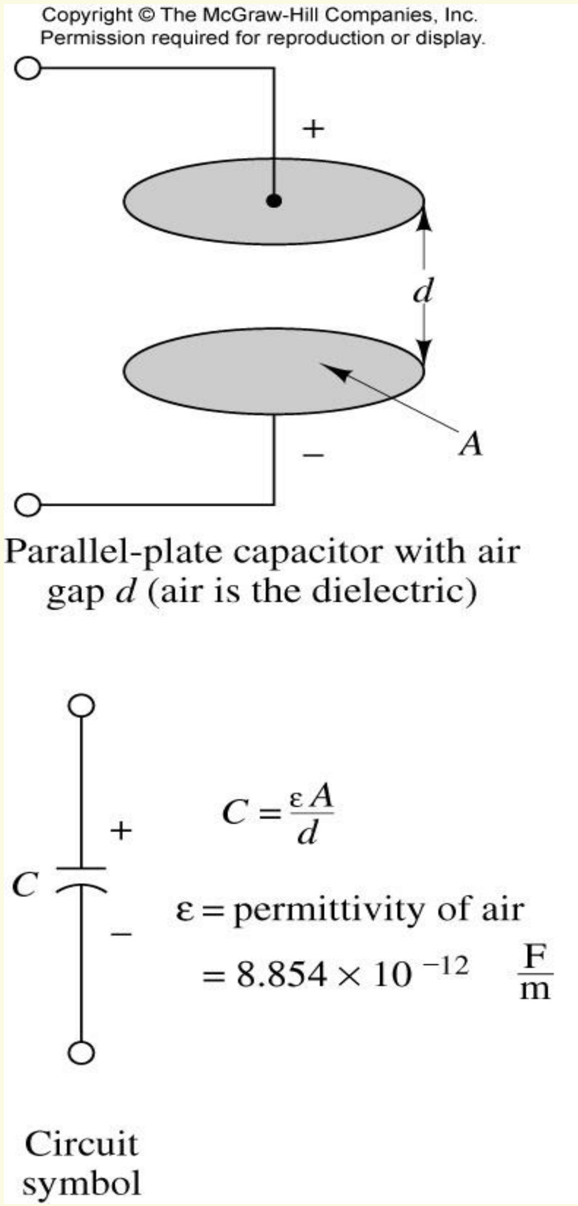


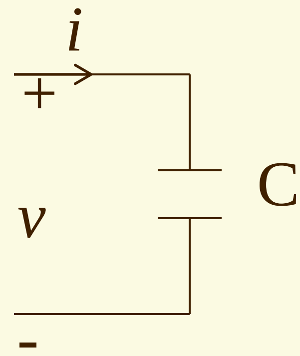
MOSFET Modeling





Ideal Linear Capacitor





$$q = Cv$$

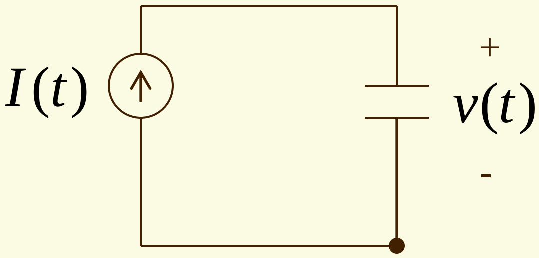
$$\frac{dq}{dt} \equiv i = C\frac{dv}{dt}$$

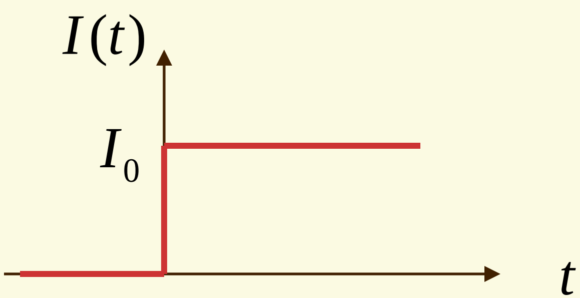
$v(t) =$

Observations from $i = C\, dv/dt$

- For DC voltage: Capacitor is an open circuit
- For i to be finite, v(t) must be continuous
- Typical units: uF ~ pF

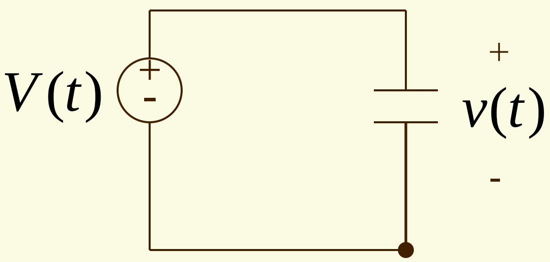
Current Source and Capacitor

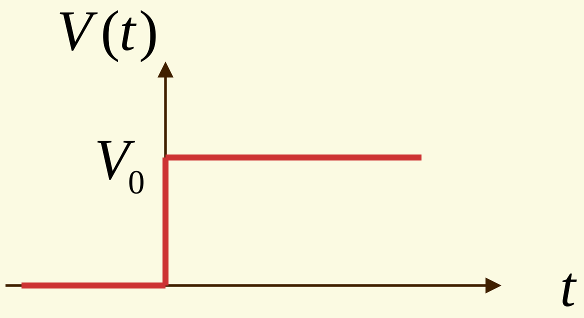




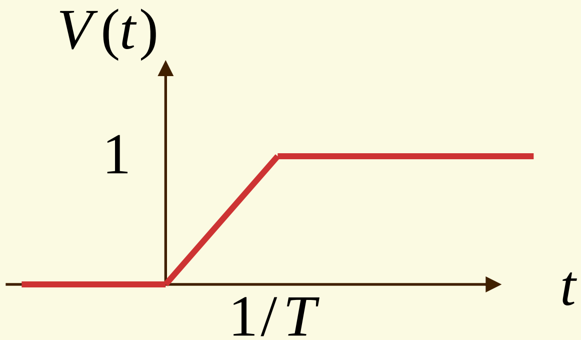
Step input

Voltage Source and Capacitor



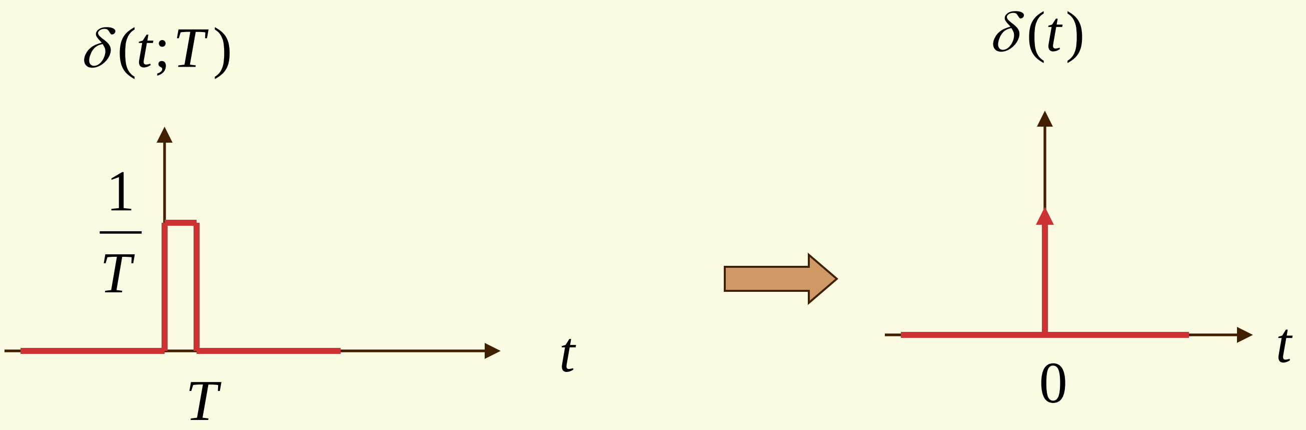


Step input



Ramp input

Unit Impulse Function

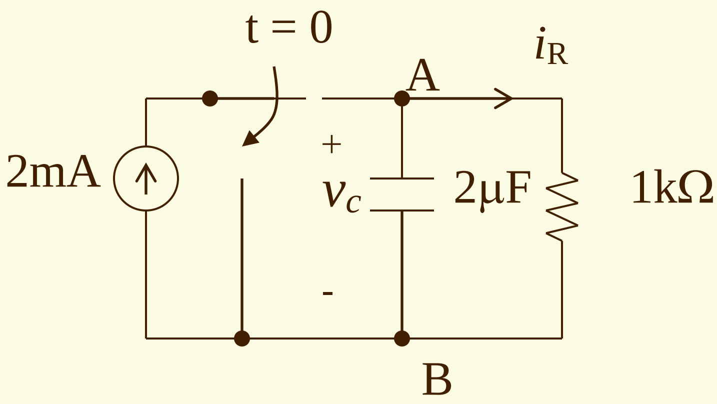


$$\delta(t) = 0 \quad for \quad t \neq 0$$

$$\int_{-\infty}^{t} \delta(t)dt = u(t) \Leftrightarrow \delta(t) = \frac{du(t)}{dt}$$

$$\int_{-\infty}^{+\infty} \delta(t)dt = 1$$

Capacitor Voltage is continuous

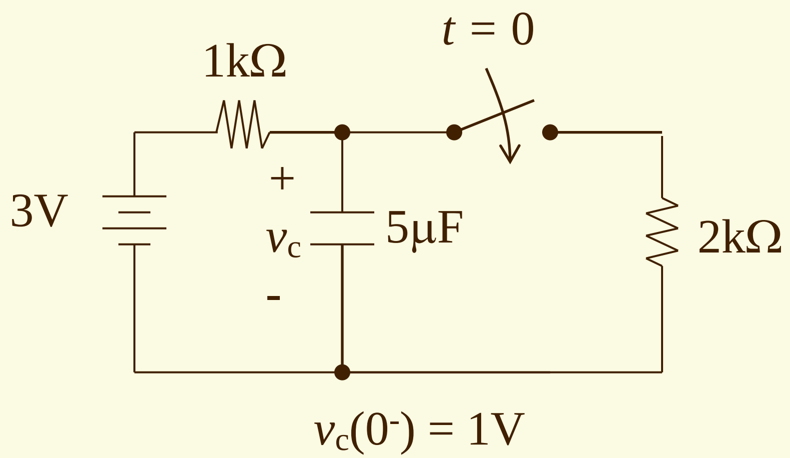


$$i_R(0-) = 1mA$$

$$\frac{dv_C}{dt}(0-) = ? \qquad \frac{dv_C}{dt}(0+) = ?$$

Exercise

Switch is closed at t=0. Find $\dfrac{dv_C}{dt}(0+)$.

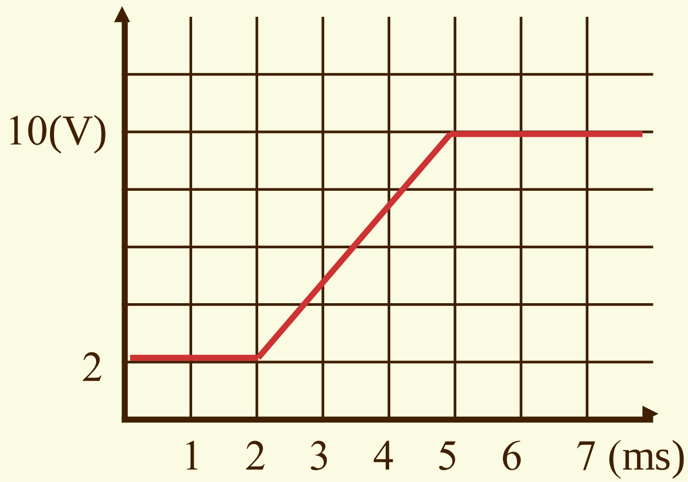


Energy Storage – memory device

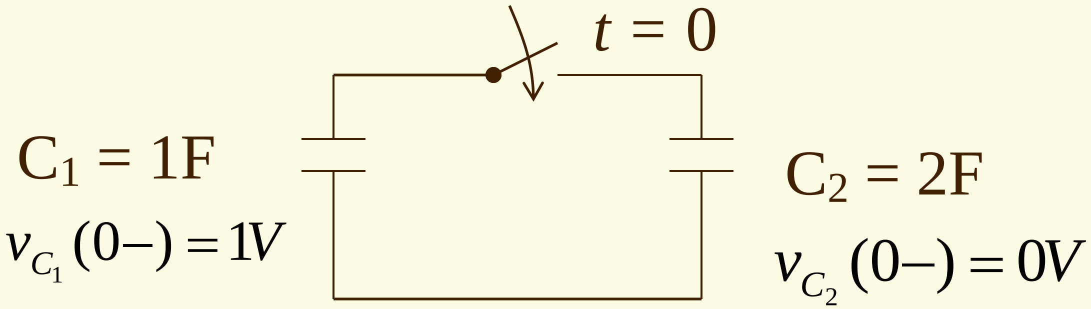
$$\frac{d\omega_E(t)}{dt} = i(t)v(t)$$

$$d\omega_E(t) = v(t)\big(i(t)dt\big) = v(t)dq(t)$$

$$\omega_E = \int_0^q v\,dx = \frac{q^2(t)}{2C} = \frac{Cv(t)^2}{2}$$

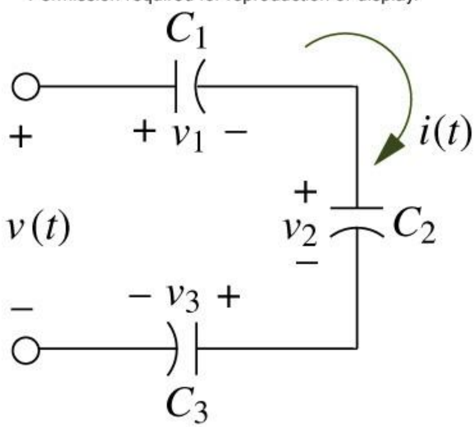


Capacitor Loop: Charge Conservation



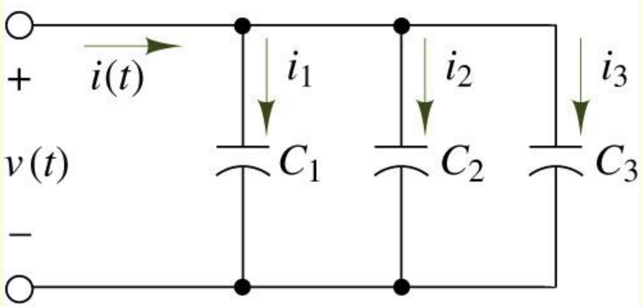
Question: Is energy conserved?

Combining capacitors in a circuit



$$C_{EQ} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Capacitances in series combine like resistors in parallel



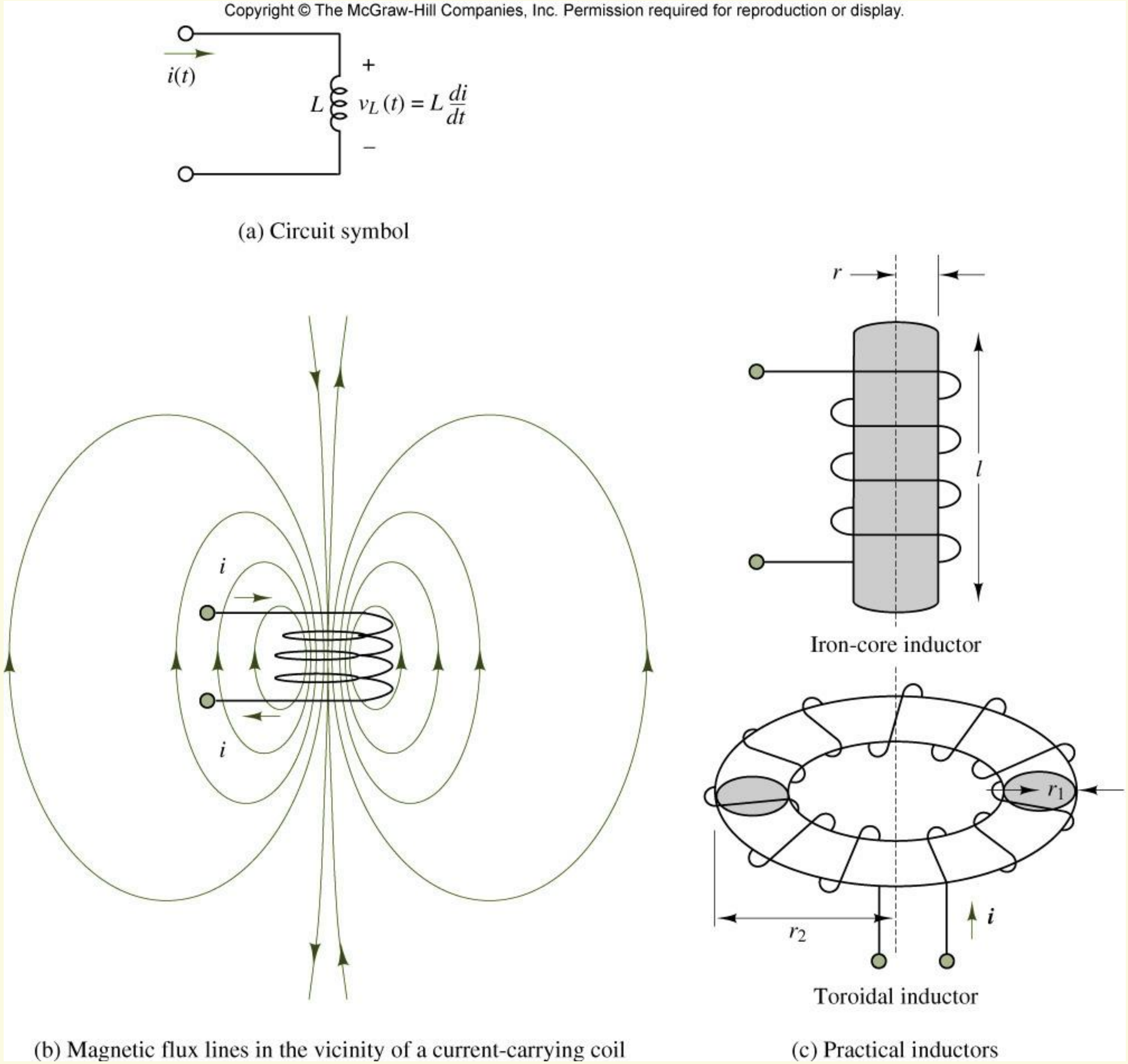
$$C_{EQ} = C_1 + C_2 + C_3$$

Capacitances in parallel add

Series connection:

Parallel connection:

Inductance and practical inductors



V = L di/dt

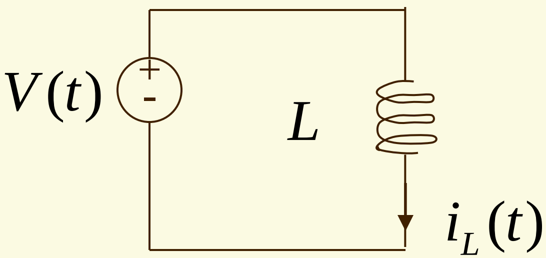
i(t) =

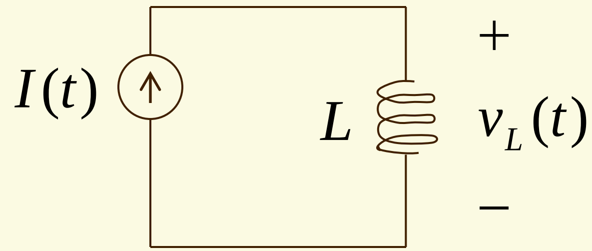
Observations from $v = L\, di/dt$

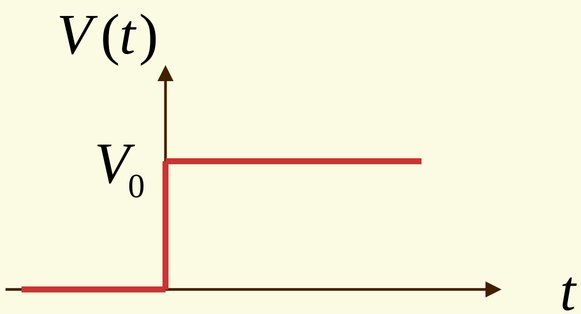
- For DC current: inductor is a short circuit
- For v to be finite, i(t) must be continuous
- Typical units: mH ~ H

Inductor circuit is dual to Capacitor circuit

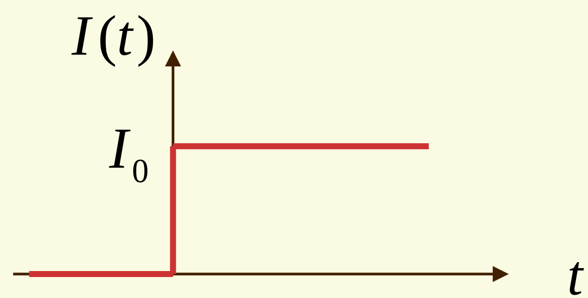
Voltage/Current Source and Inductor







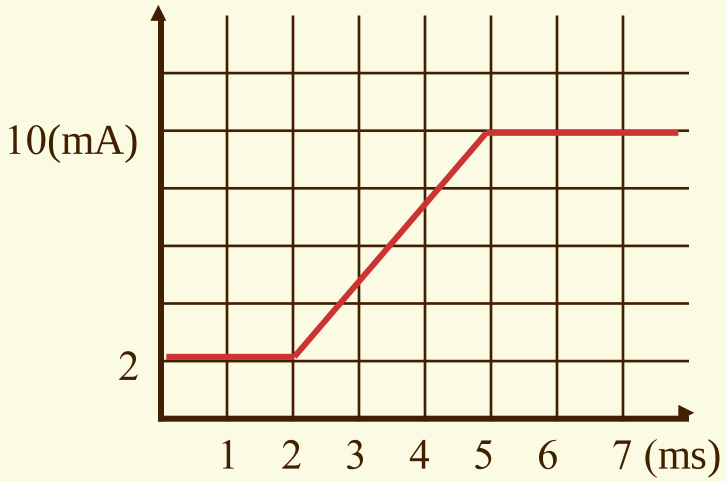
Step input



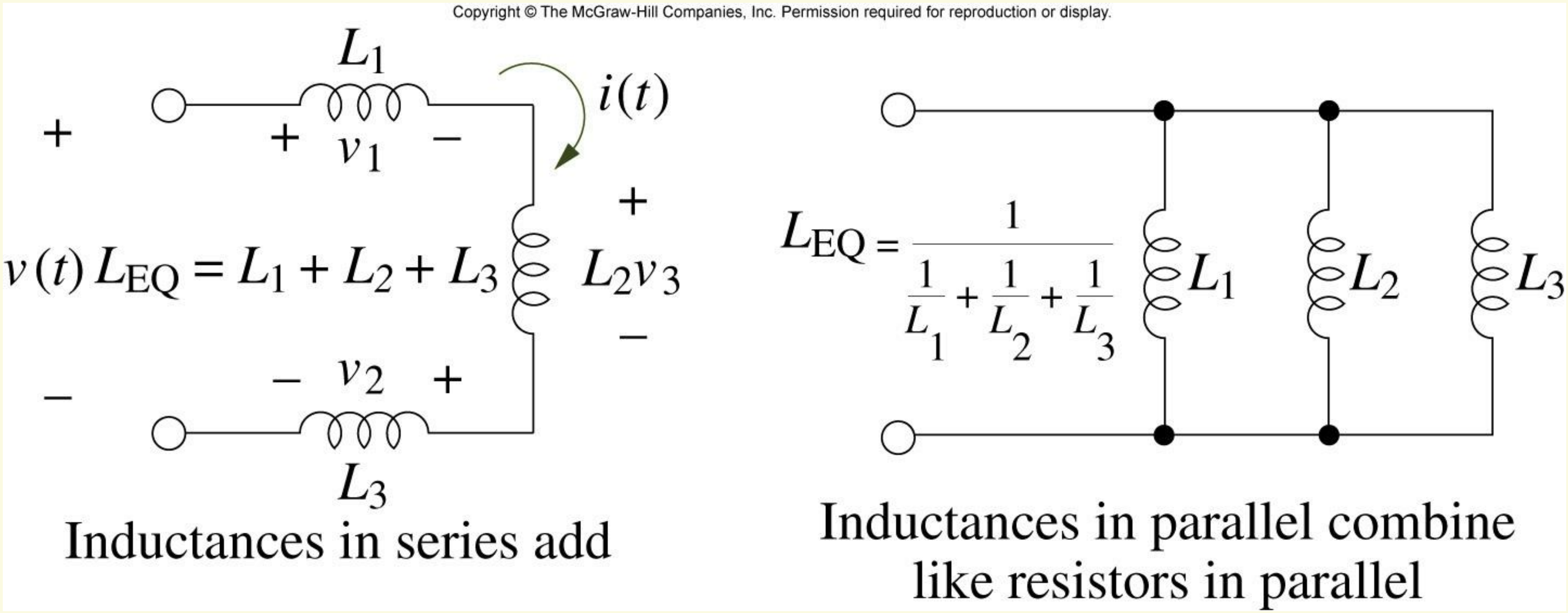
Step input

Energy Storage in an Inductor

- $W_L = \frac{1}{2} L\, i_L^2$



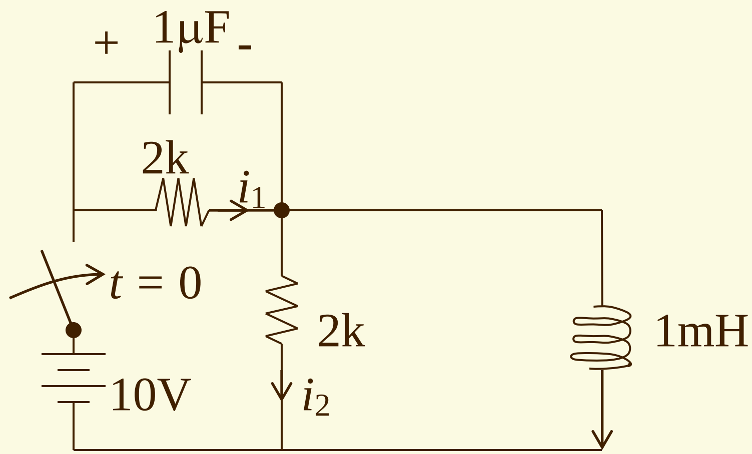
Combining inductors in a circuit



Exercise

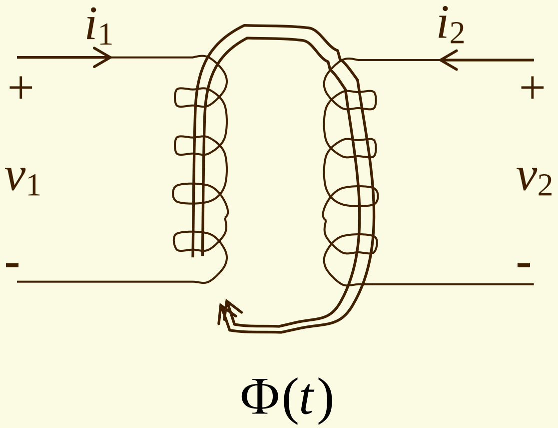
Switch is closed at t = 0. Find the following:

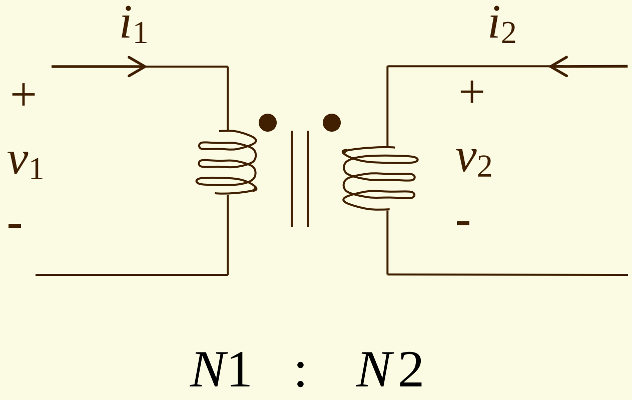
$$i_2(0+),\quad \frac{dv_C}{dt}(0+),\quad \frac{di_2}{dt}(0+)$$



$v_c(0^-) = 5\text{V},\ i_L(0^-) = 5\text{mA}$

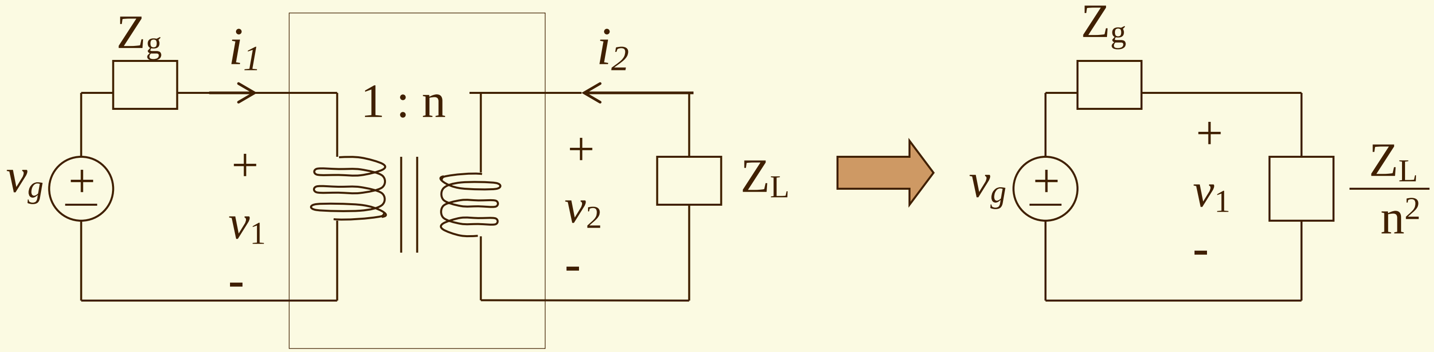
Transformer





$$\frac{v_1(t)}{N_1} = \frac{v_2(t)}{N_2}, \quad N_1 i_1(t) = -N_2 i_2(t)$$

Transformer Circuit



Chap. 10 First-Order Transients in Linear Electrical Networks

Analysis of RC/RL Circuits

Intuitive Analysis

Propagation Delay

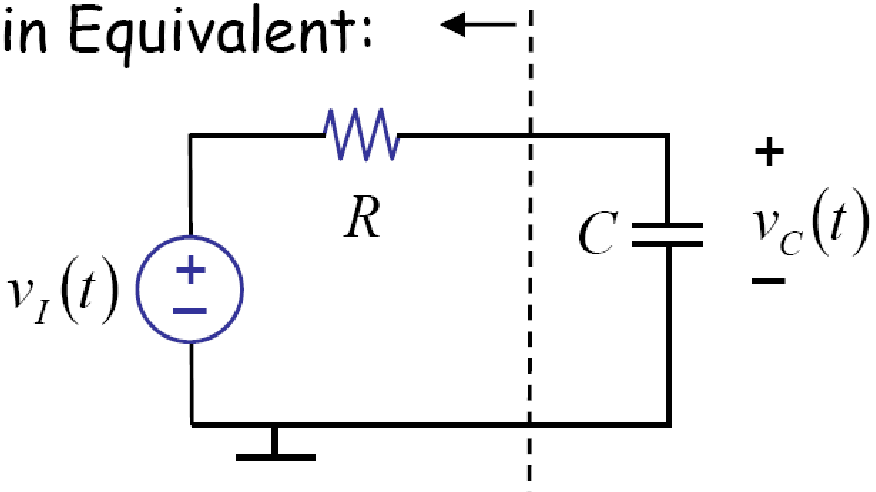
State and State Variables

Additional Examples

Digital Memory

Simple RC Circuit: Series RC Circuit

Thévenin Equivalent: $\leftarrow$



$$\frac{v_C - v_I}{R} + C\frac{dv_C}{dt} = 0$$

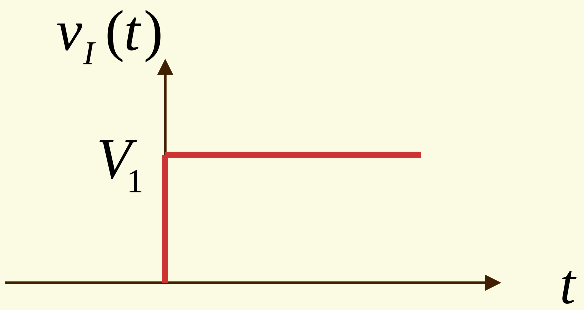
$$RC\frac{dv_C}{dt} + v_C = v_I$$

$v_C(t)$	$=$	$v_{CH}(t)$	$+$	$v_{CP}(t)$
total		homogeneous		particular

Solutions

- Find the particular solution
- Find the form of homogeneous solution
- Use initial condition to obtain the total solution

< example >

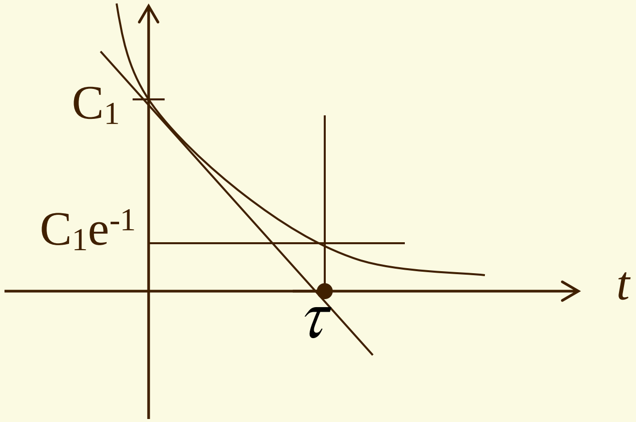


Particular Solution:

Homogeneous Solution

- General Form: $v_{CH}(t) = Ae^{st}$

$$v_{CH}(t) = Ae^{-t/RC} = Ae^{-t/\tau}$$



Time constant: $\tau = RC$

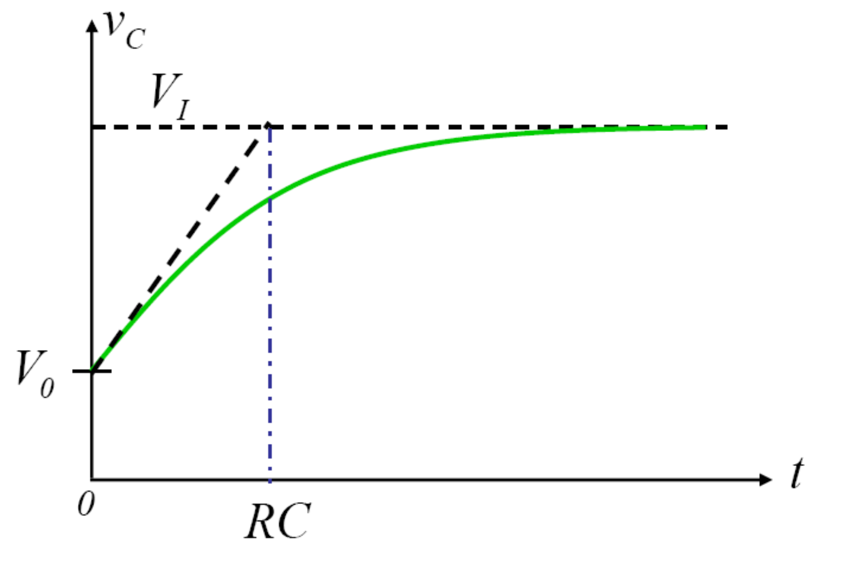
Total Solution

- Assume the following initial condition

$$\text{Given,} \quad v_C = V_0 \quad \text{at} \quad t = 0$$

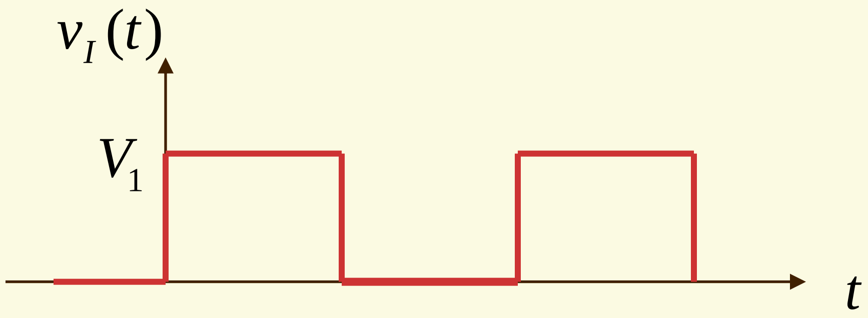
- The total solution becomes

Total Solution (2)



General form: $$x(t) = [x(0+) - x(\infty)]e^{-\frac{t}{\tau}} + x(\infty) \quad t \geq 0$$

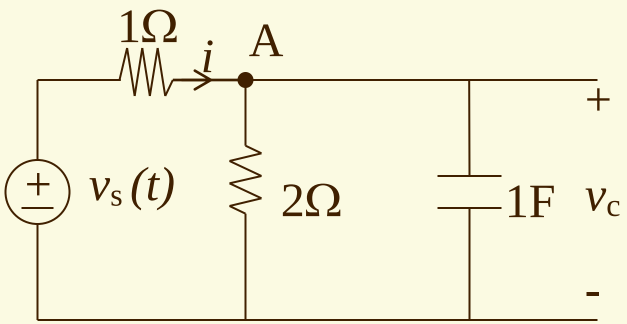
Response to Square Wave



At steady state:

Exercise

Find the differential equation to obtain v_c



DC Steady state

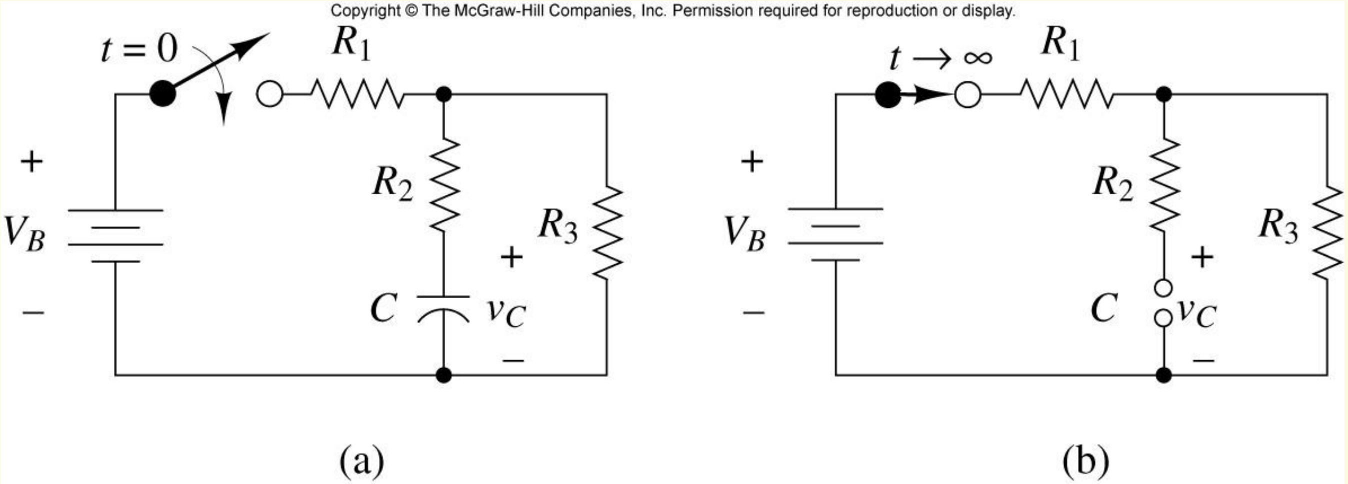
- DC voltage or DC current source
- After a long time:

$$t \rightarrow \infty$$

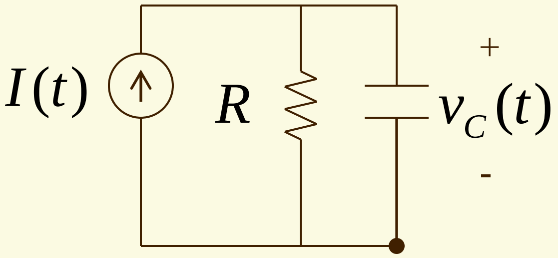
- Capacitor: open circuit
- Inductor: closed circuit

Circuit Example

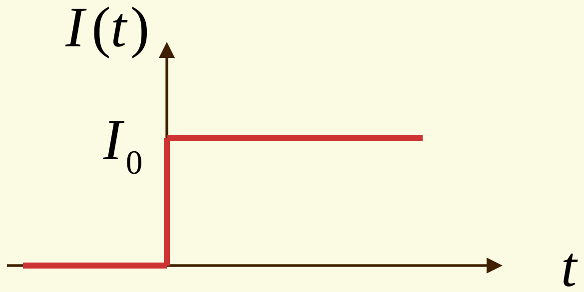
a long time after the switch is closed: final solution



Parallel RC Circuit: Do it yourself !

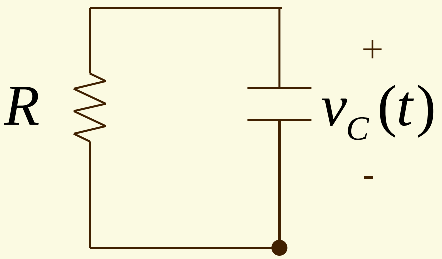


$$v_C(0-) = 0$$



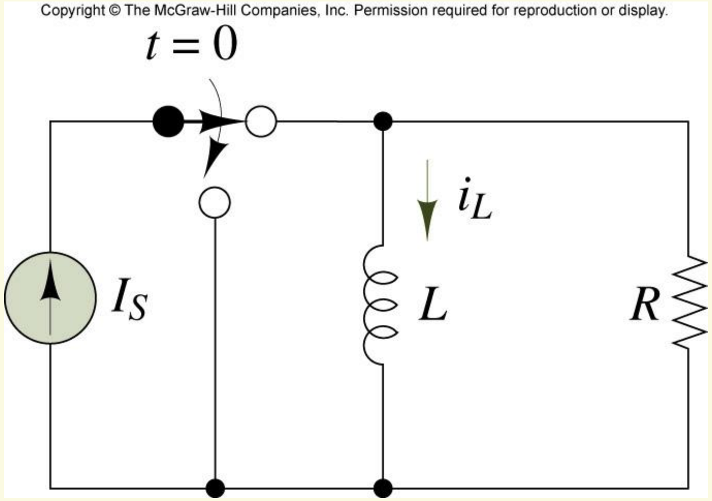
Step input

RC Transient



$$v_C(0-) = V_0$$

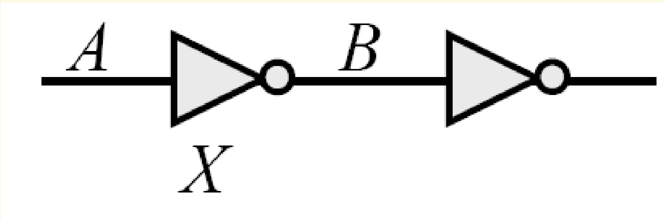
Exercise

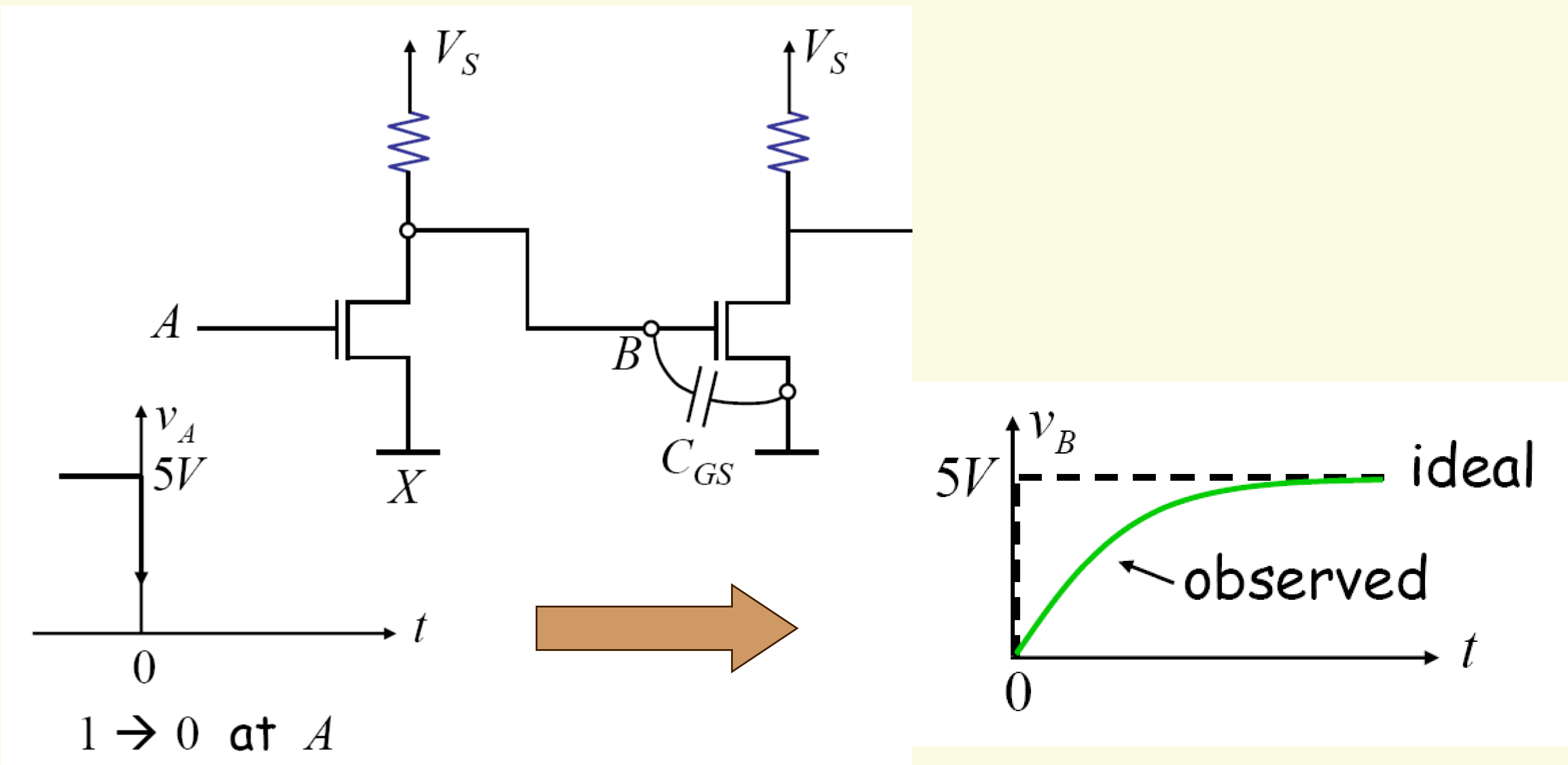


$$i_S = 10mA$$

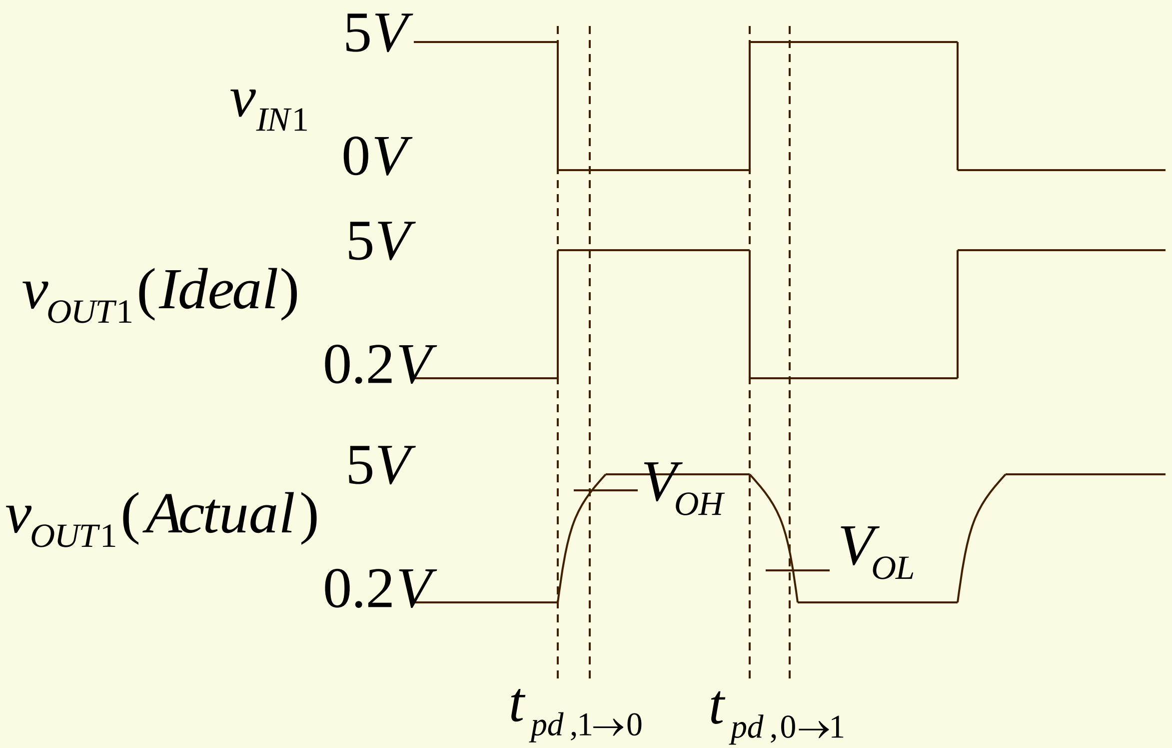
Find i_L $at\ t = 0+\ and\ t \to \infty$

Digital Logic Again



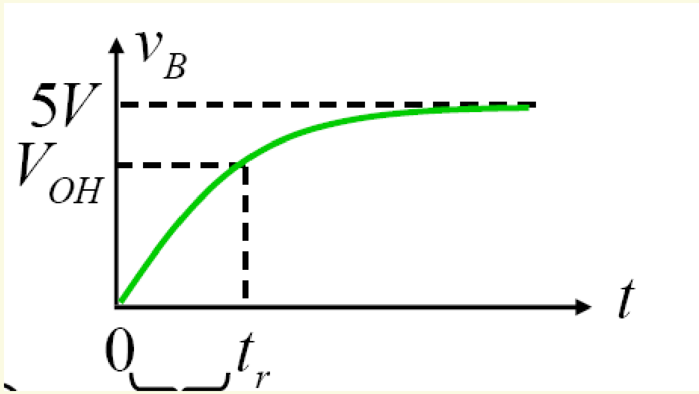


Propagation Delay

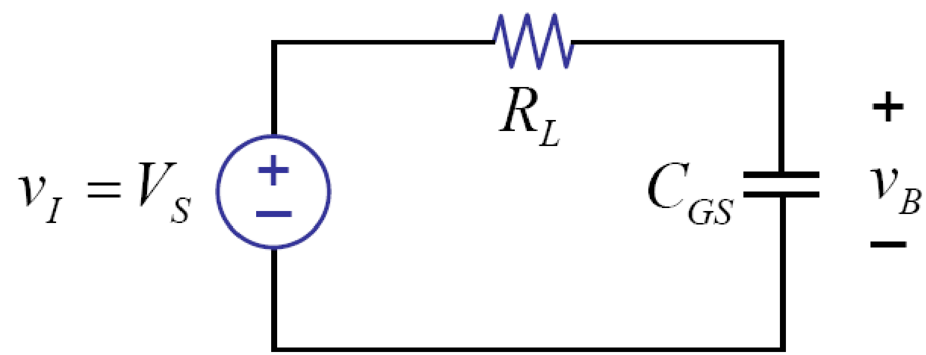


Propagation delay $t_{pd} = \max(t_{pd,1\to 0}, t_{pd,0\to 1})$

Rise time (vs. Fall time)



Equivalent Circuit: $t_{pd,1\to 0}$



$$v_I = V_S \quad \text{for } t \geq 0$$

$$v_B(0) = 0$$

Rise time

$(V_{OL}\ to\ V_{OH})$

Rise time Computation

$$v_B = V_S + \left(0 - V_S\right) e^{\frac{-t}{R_L C_{GS}}}$$

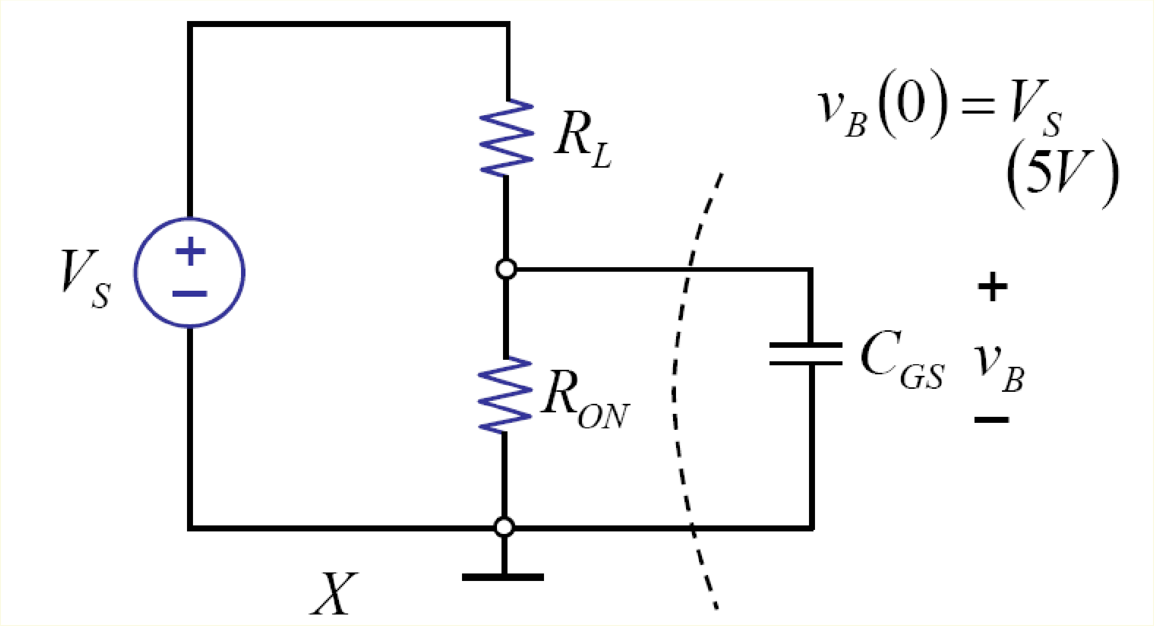
$$v_B = V_{OH} \quad \text{at} \quad t = t_r \; ?$$

example

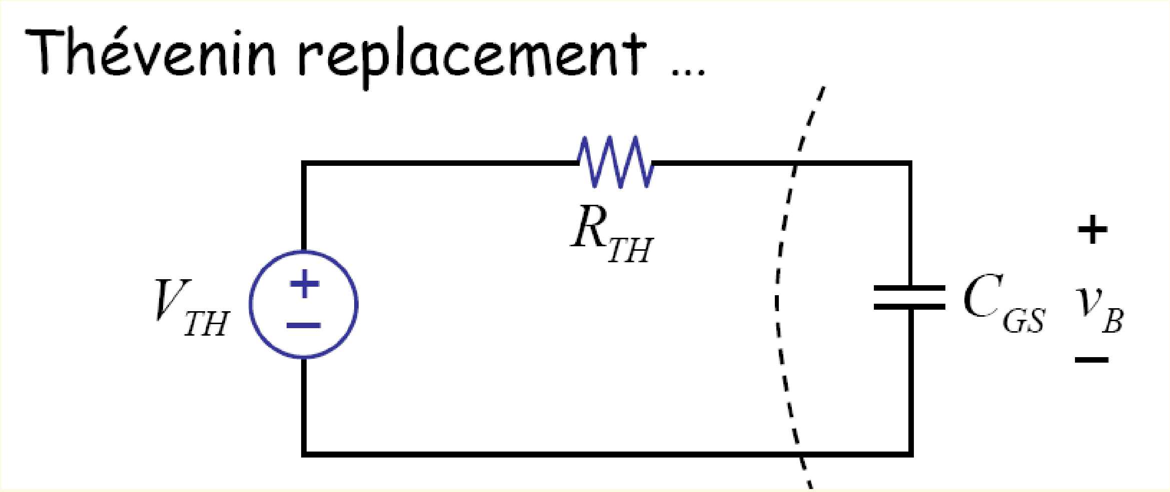
$R_L = 1K$ $\quad V_S = 5V$

$C_{GS} = 0.1\,pF$ $\quad V_{OH} = 4V$

Fall Time: from the SRC Model



$t_{pd,0 \to 1}$



Fall Time Computation

From 5V to V_{OL}

$$V_{OL} = 1V,\ R_L = 10k\Omega,\ R_{ON} = 1k\Omega,\ C_{GS} = 0.1pF$$

Propagation Delay Computation

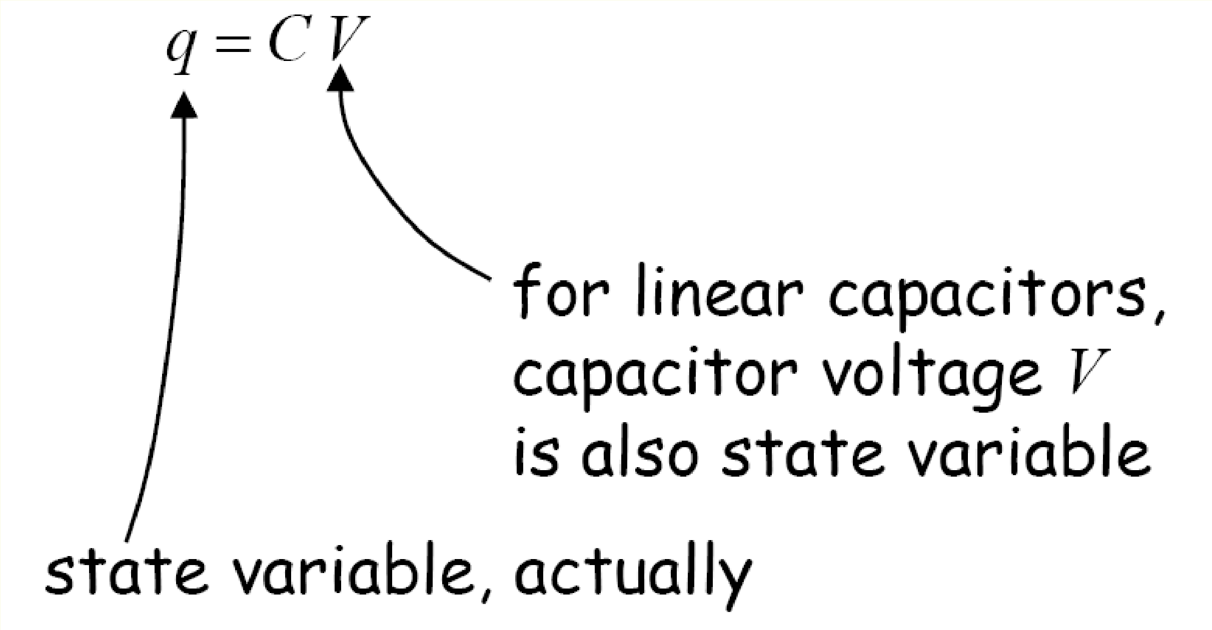
$$t_{pd} = \max(t_{pd,1\to 0}, t_{pd,0\to 1})$$

$$R_L = 1K \qquad V_S = 5V \qquad R_{ON} = 10\Omega$$

$$C_{GS} = 0.1\,pF \qquad V_{OL} = 1V$$

State and State variables

- State
  - Summary of past inputs relevant to predicting the future



Another Interpretation

$$\frac{dv_C}{dt} = -\frac{v_C}{RC} + \frac{v_I(t)}{RC}$$

$\frac{d}{dt}$ (state variable)=K_1(State variable present value) +K_2 (input variable)

Total Solution = zero-input response + zero-state response

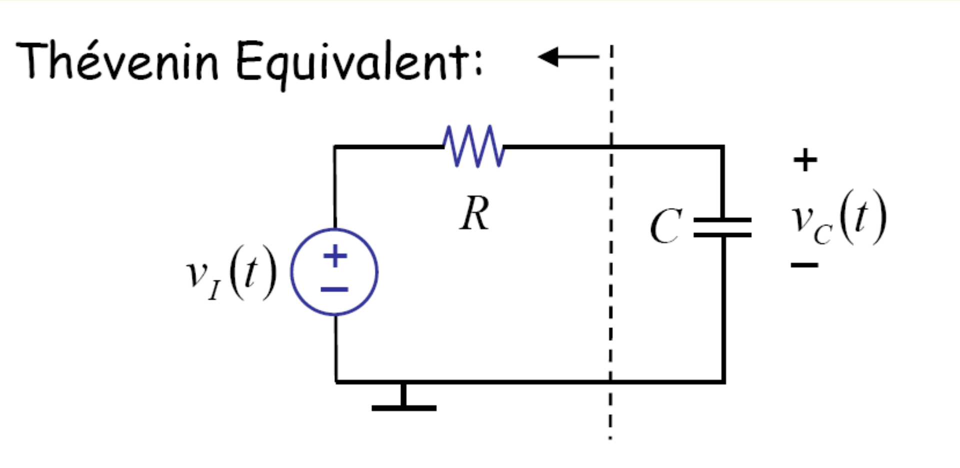
Total Solution

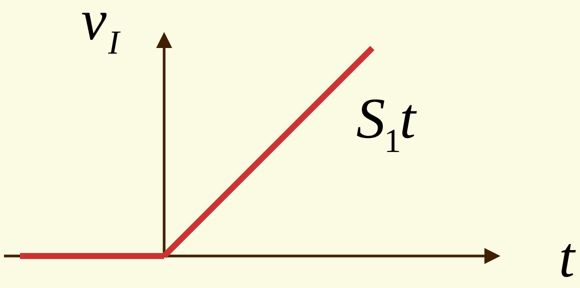
$$\frac{dv_C}{dt} = -\frac{v_C}{RC} + \frac{v_I(t)}{RC}$$

Zero-input response: $$\frac{dv_C}{dt} = -\frac{v_C}{RC}$$

Zero-state response: $$\frac{dv_C}{dt} = -\frac{v_C}{RC} + \frac{v_I(t)}{RC}$$

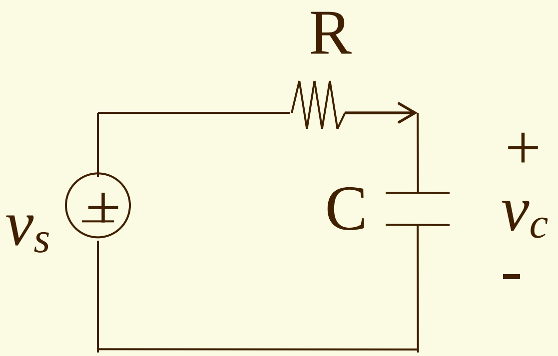
Ramp Input As an Example





Ramp input

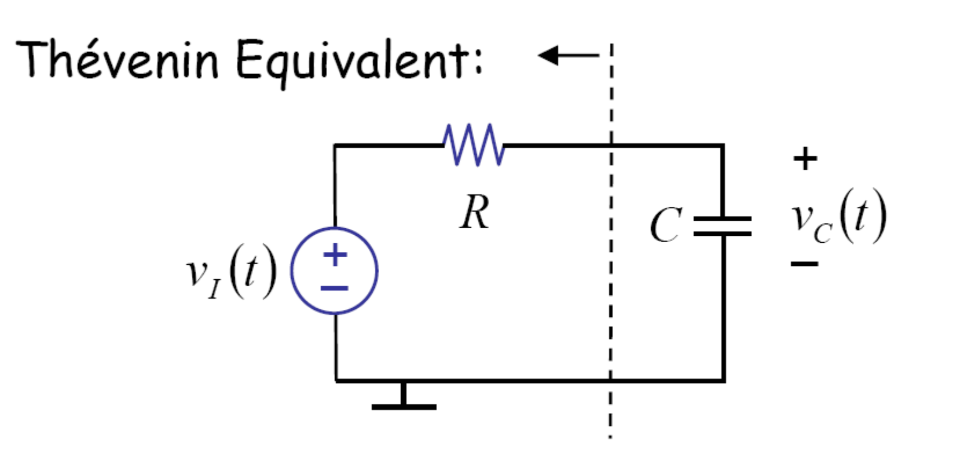
Sine wave input

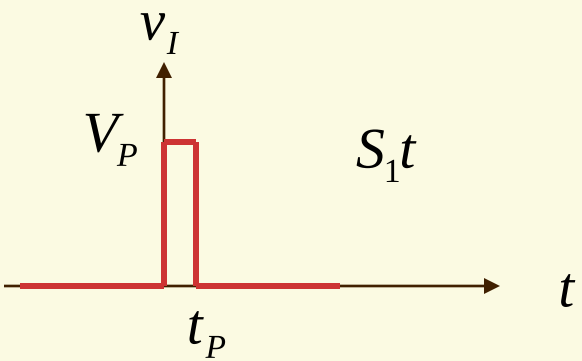


Sinusoical signal waveform:

$v_S(t) = V \cos(\omega\, t)$

Impulse Response



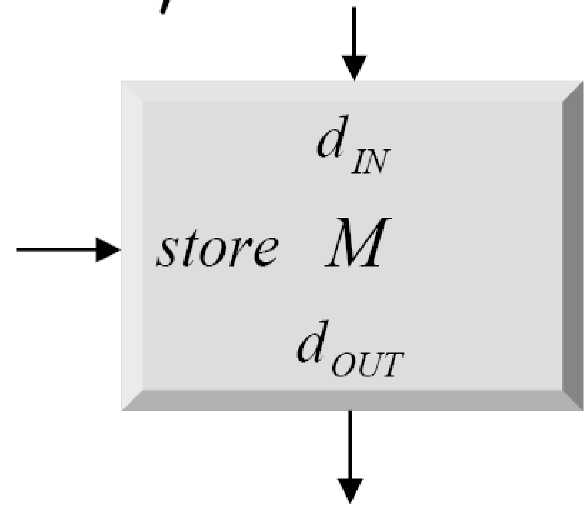


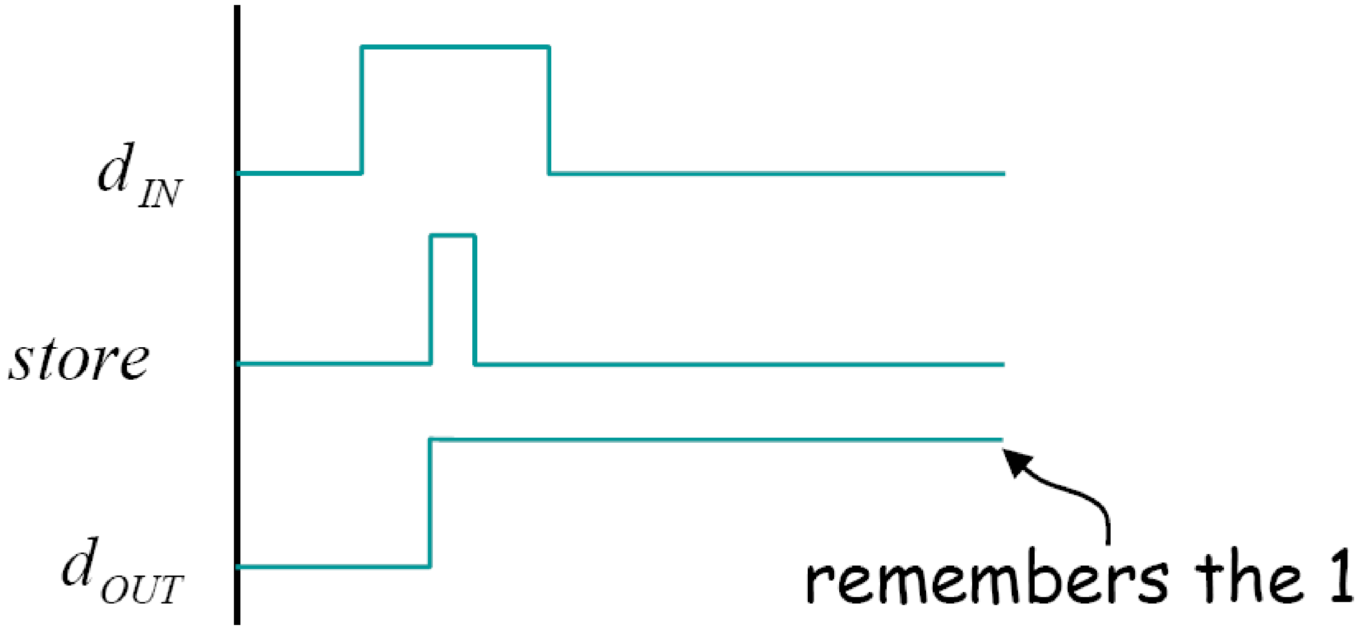
Pulse input -> impulse input

$$t_P \to 0,\ V_P t_p = A \Rightarrow A\delta(t)$$

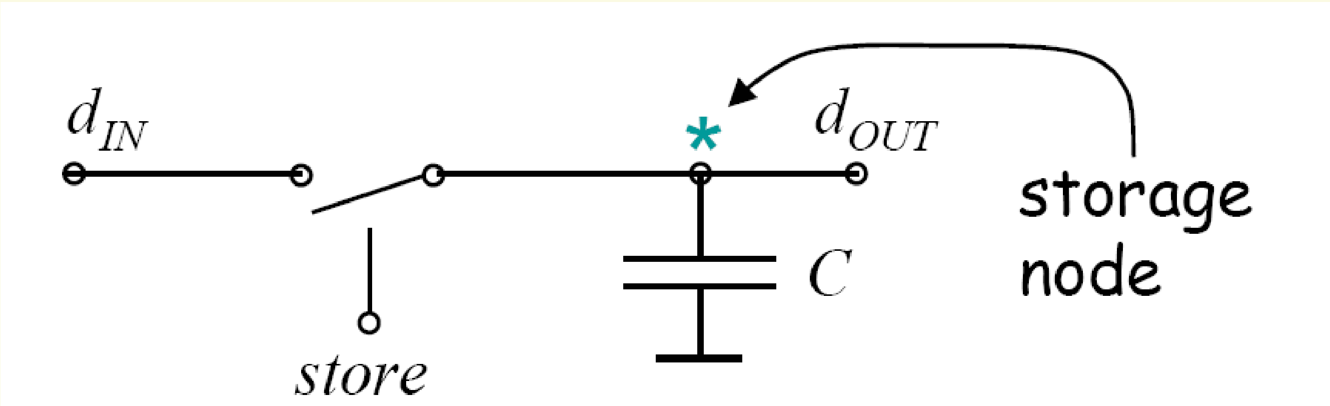
Memory Abstraction

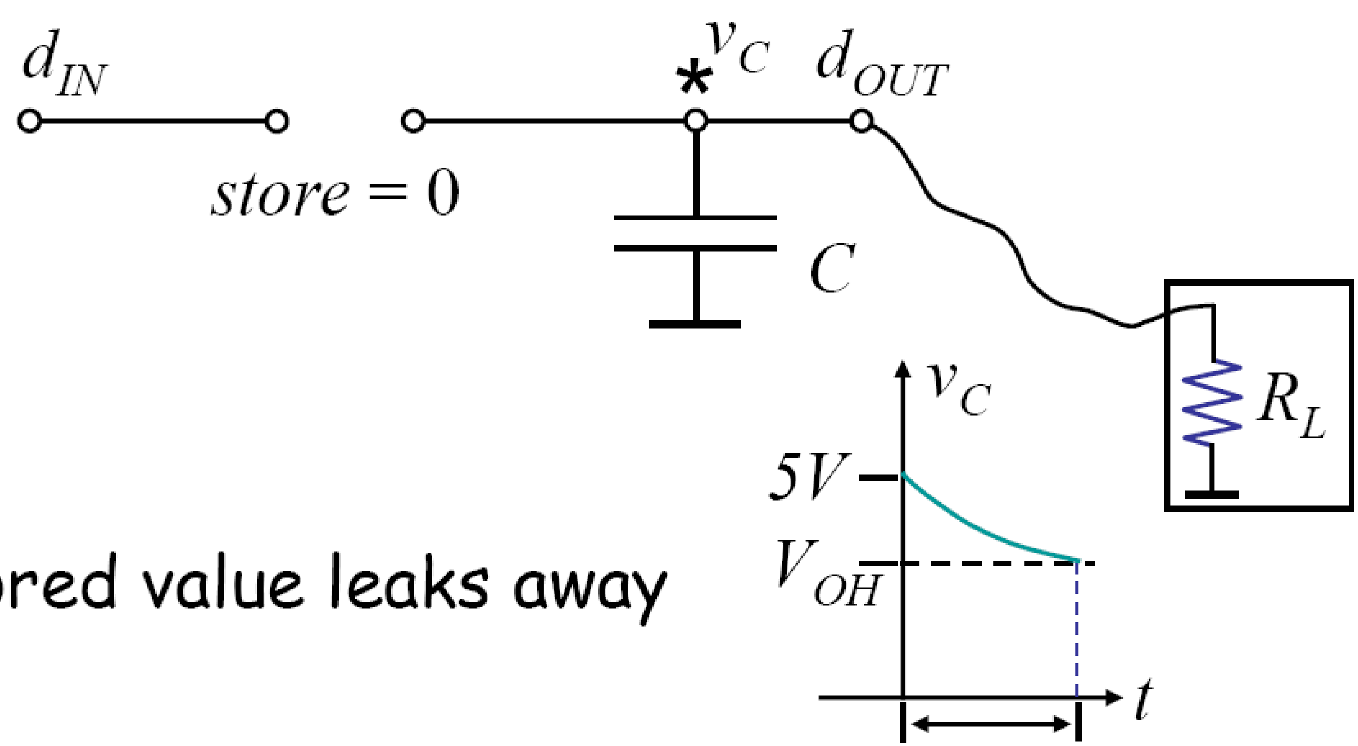
A 1-bit memory element





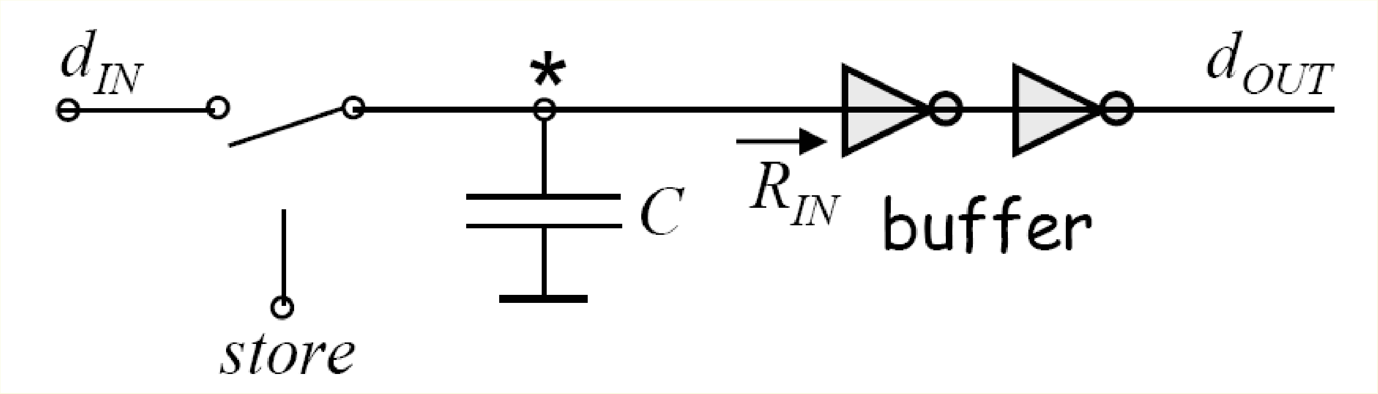
Building a memory element: First Attempt





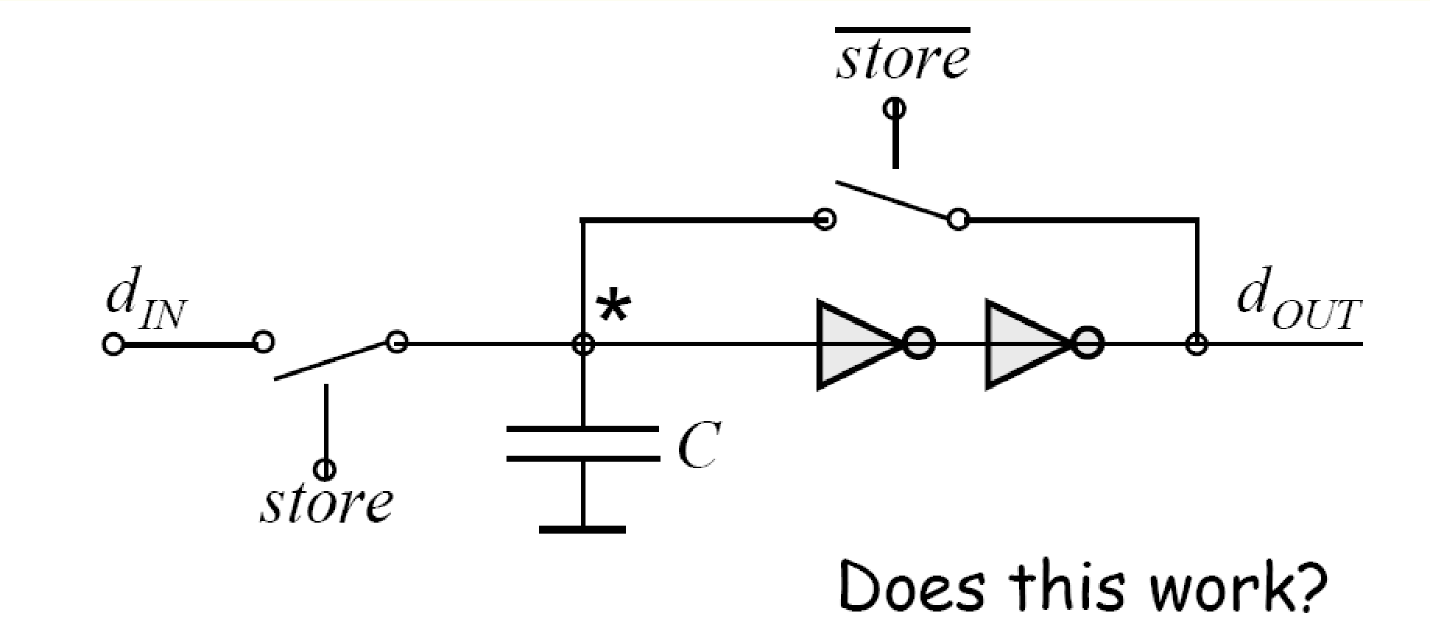
Stored value leaks away

Second Attempt: buffer

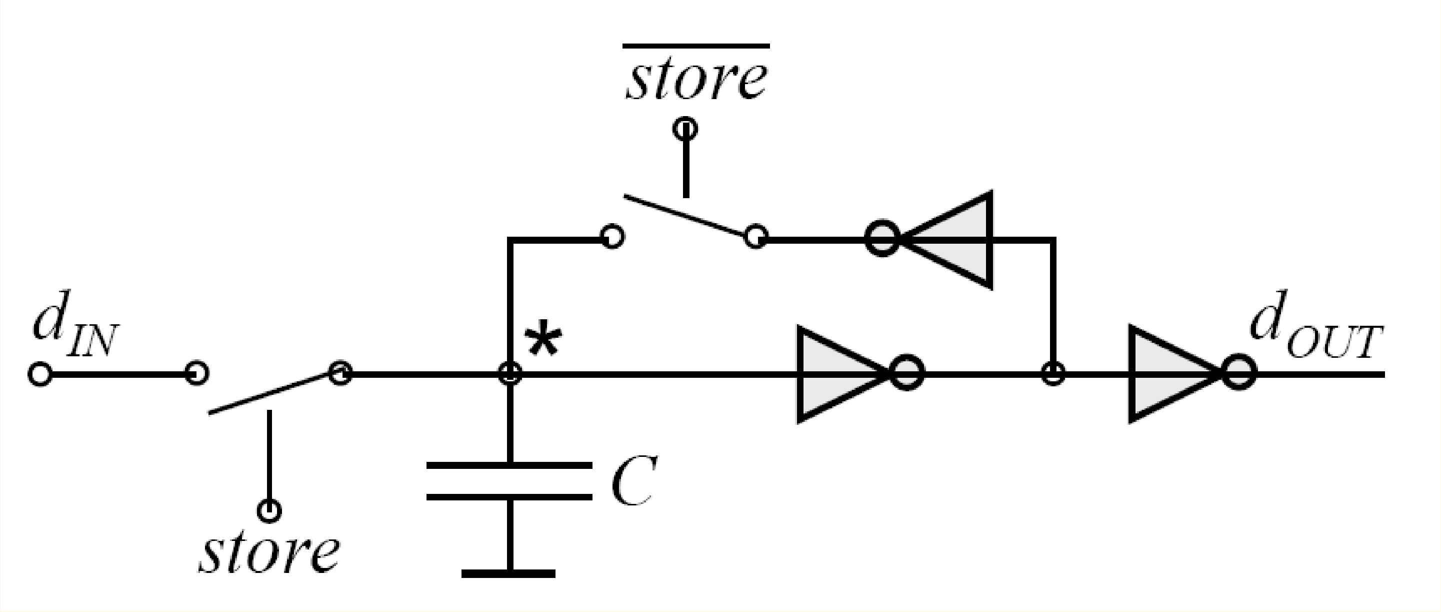


R_{IN} is much larger than R_L

Third attempt: buffer + refresh



Finally: buffer + decoupled refresh



Refresh Period?

Conclusion

- First order circuit
  - Time constant
  - DC steady state
- Propagation delay
  - Rise time and fall time
- State and state variables
- Memoy element