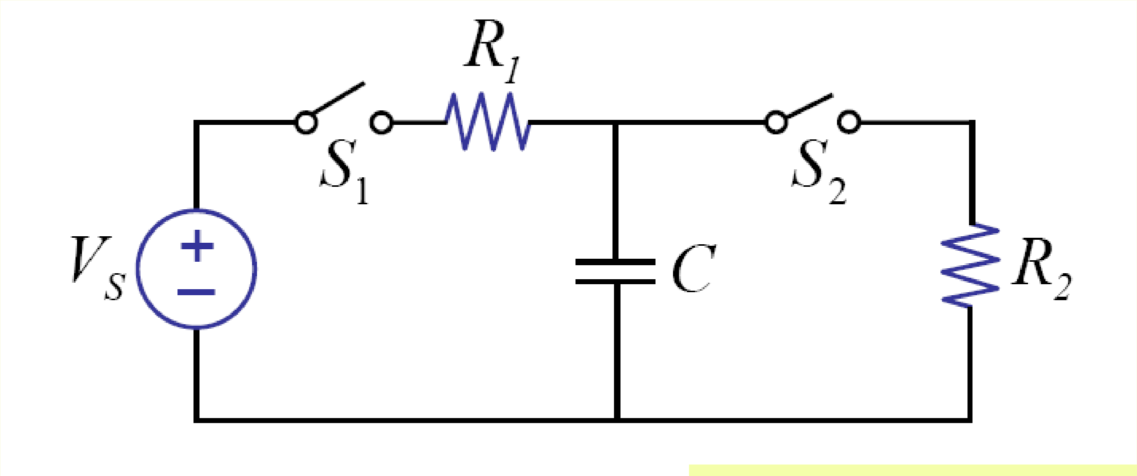
Chap. 11 Energy and Power in Digital Circuits

Average Power in an RC Circuit

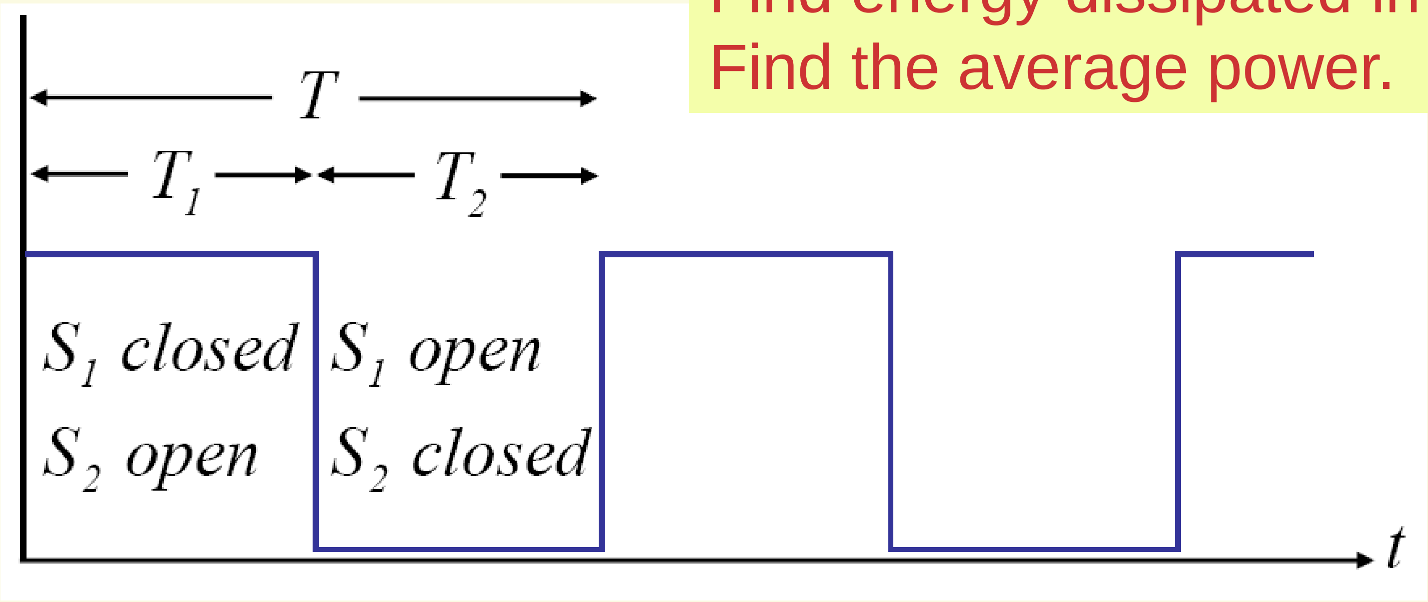
Power Dissipation in Logic Gates

CMOS Logic

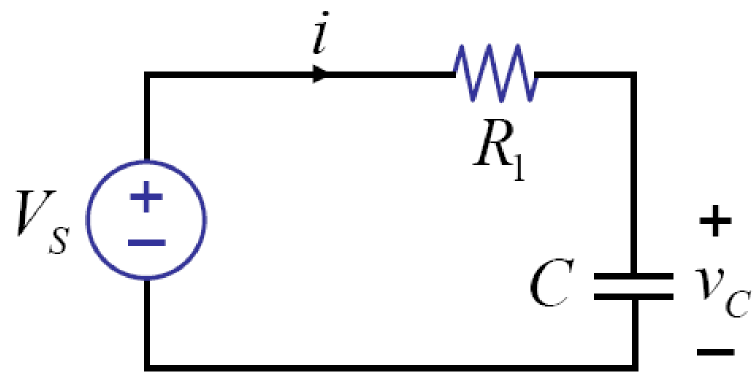
RC Circuit with a Switch



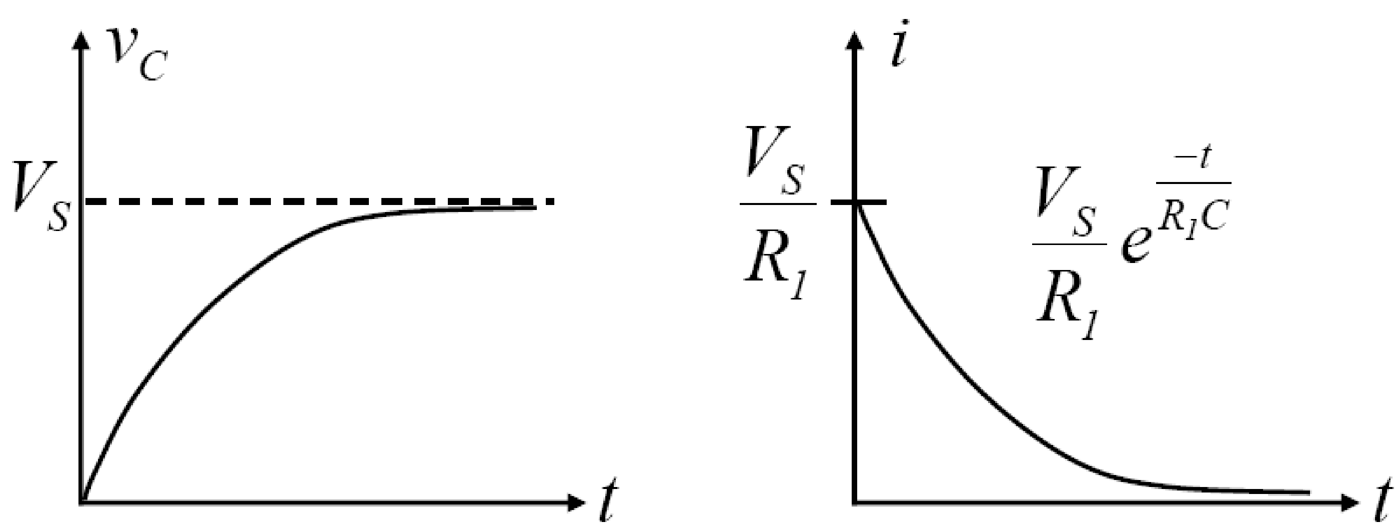
Find energy dissipated in each cycle
Find the average power.



During T1



assume
$v_C = 0$ at $t = 0$



Energy Computation

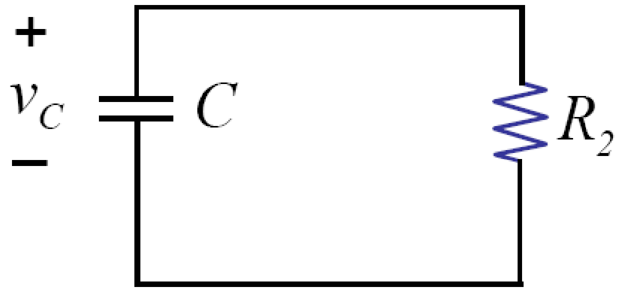
- Total energy provided by the source during T1

$$E = \int_0^{T_1} V_S i \, dt = \int_0^{T_1} V_S \frac{V_S}{R_1} e^{-\frac{t}{R_1 C}} dt = C V_S^{\,2} (1 - e^{-\frac{T_1}{R_1 C}})$$

$$\approx C V_S^2 \quad if \quad T_1 >> R_1 C$$

- Energy stored in the capacitor: $E_C = \frac{1}{2} C V_S^2$
- Energy dissipated in the resistor: $E_1 = \frac{1}{2} C V_S^2$
  - Independent of R

During T2



Initially, $v_C = V_S$ (recall $T_1 >> R_1C$)

If we assume $T_2 >> R_2C$

Capacitor energy is fully discharged =
Energy dissipated by the resister: $E_2 = \frac{1}{2}CV_S^2$

- independent of R

Putting Together

- Energy dissipated in each cycle

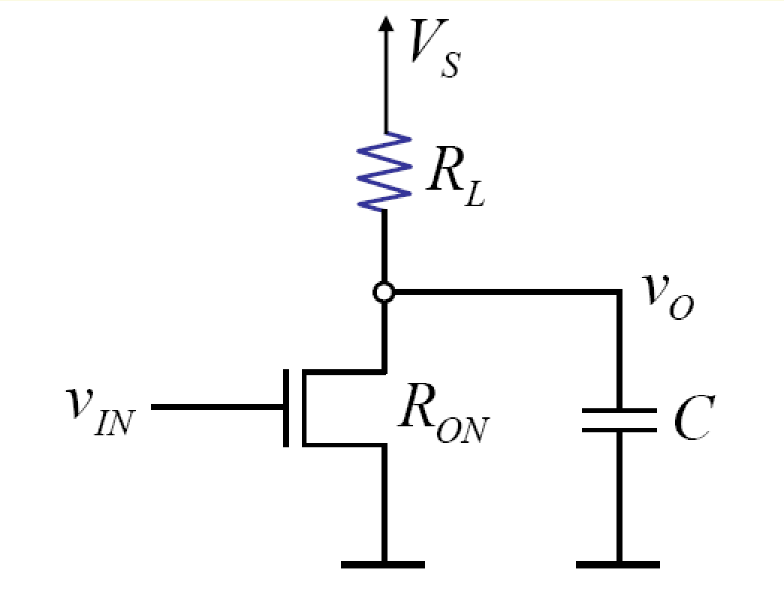
$$E = E_1 + E_2 = CV_s^2$$

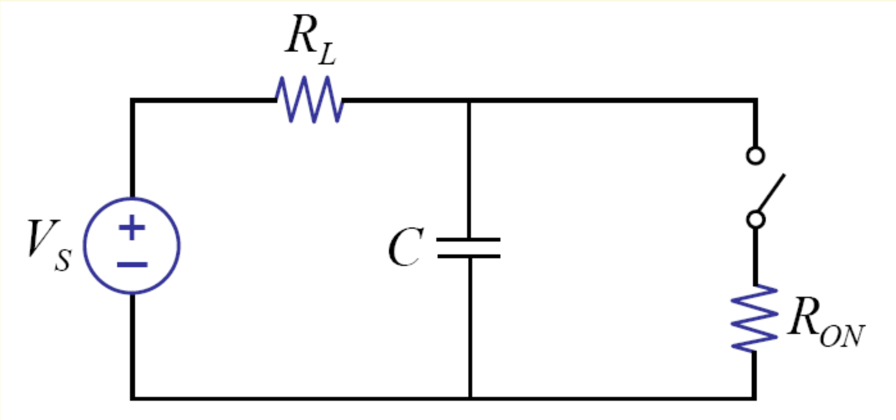
  – assuming that Capacitor is fully charged and discharged during the period.

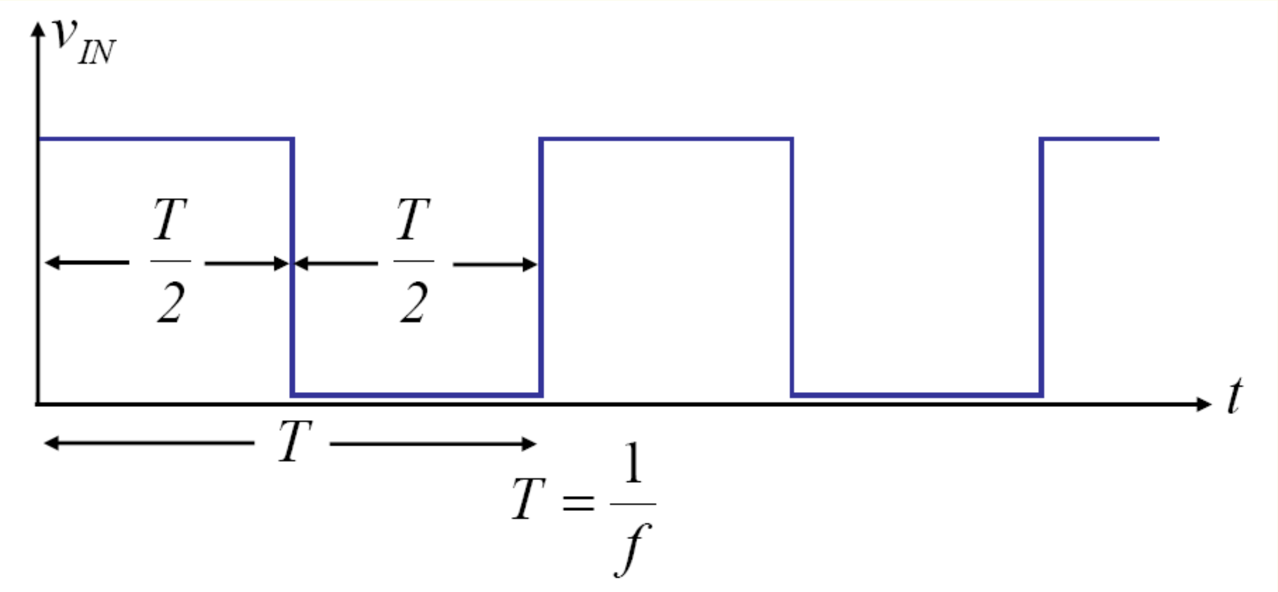
- Average power

$$\overline{P} = \frac{E}{T} = CV_S^2 f$$

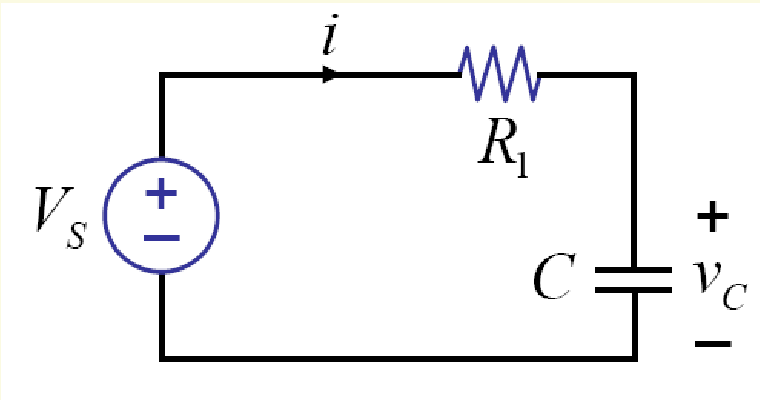
Power Dissipation of Inverter



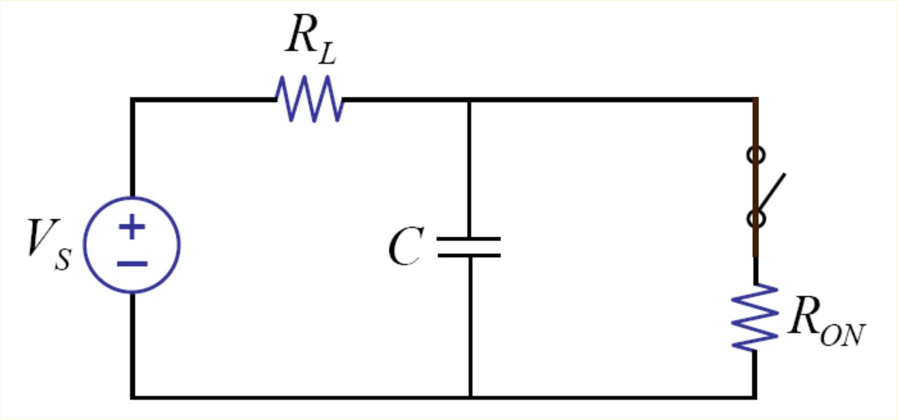




During input is low



During input is high



Easy way to solve

- Average power = static power + dynamic power
  - Static power: independent of the frequency
  - Dynamic power: charging-and-discharging of the capacitor

Average Power

$$\overline{P} = \frac{V_S^{\,2}}{2(R_L + R_{ON})} + CV_S^{\,2} f \frac{R_L^{\,2}}{(R_L + R_{ON})^2} \xrightarrow{R_L \gg R_{ON}} \frac{V_S^{\,2}}{2R_L} + CV_S^{\,2} f$$

Static power + dynamic power

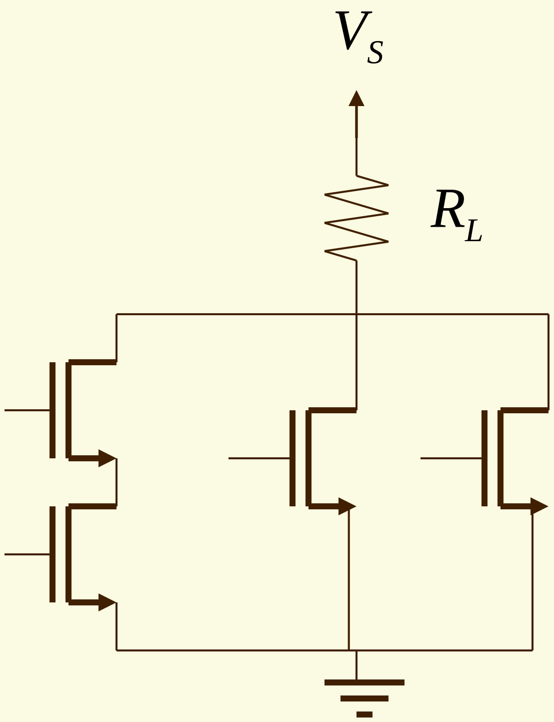
- Static power: Independent of the switching frequency.
- Dynamic power: related to switching capacitor

$$\frac{P_{static}}{P_{dynamic}} = \frac{R_L + R_{ON}}{R_L} \times \frac{T}{2R_L C_L}$$

Example II.1

- Worst static power dissipation

$$R_L = 100k\Omega, \quad R_{ON} = 10k\Omega$$



Some numbers

?

a chip with 10^6 gates clocking at 100 MHZ

$$C = 1\,fF$$

$$R_L = 10\,k\Omega$$

$$f = 100 \times 10^6$$

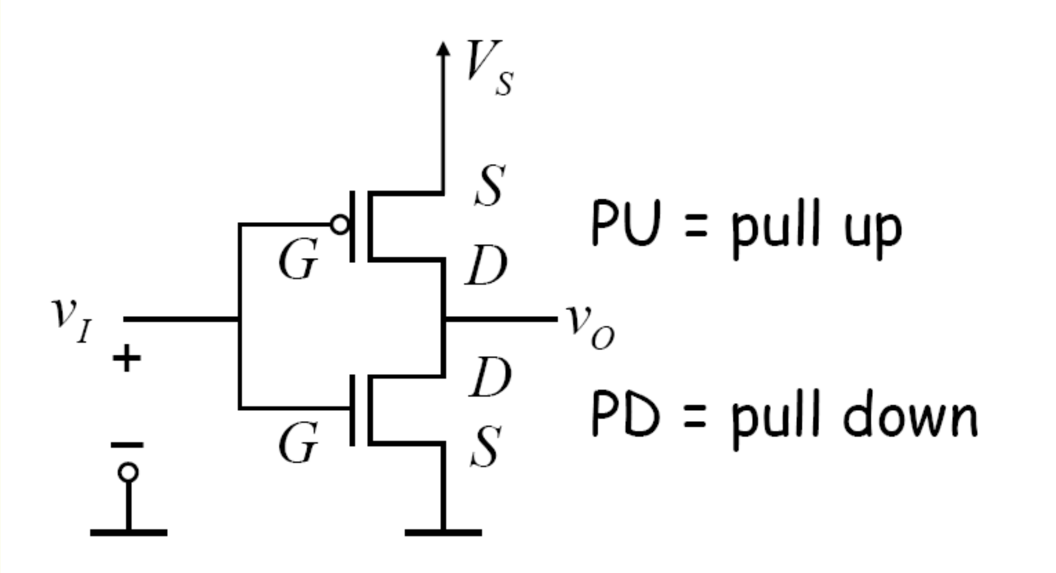
$$V_S = 5V$$

$$\frac{P_{static}}{P_{dynamic}} = \frac{R_L + R_{ON}}{R_L} \times \frac{T}{2R_L C_L}$$

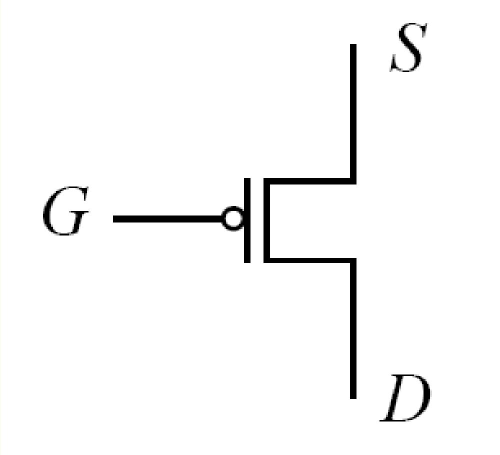
Power Minimization

- Dynamic power
  - Reduce the supply voltage (5V -> 1.5V)
    - Change the voltage on need
  - Reduce the capacitance
  - Reduce the frequency
    - Turn off clock when not in use
- Static power
  - CMOS logic

CMOS Logic

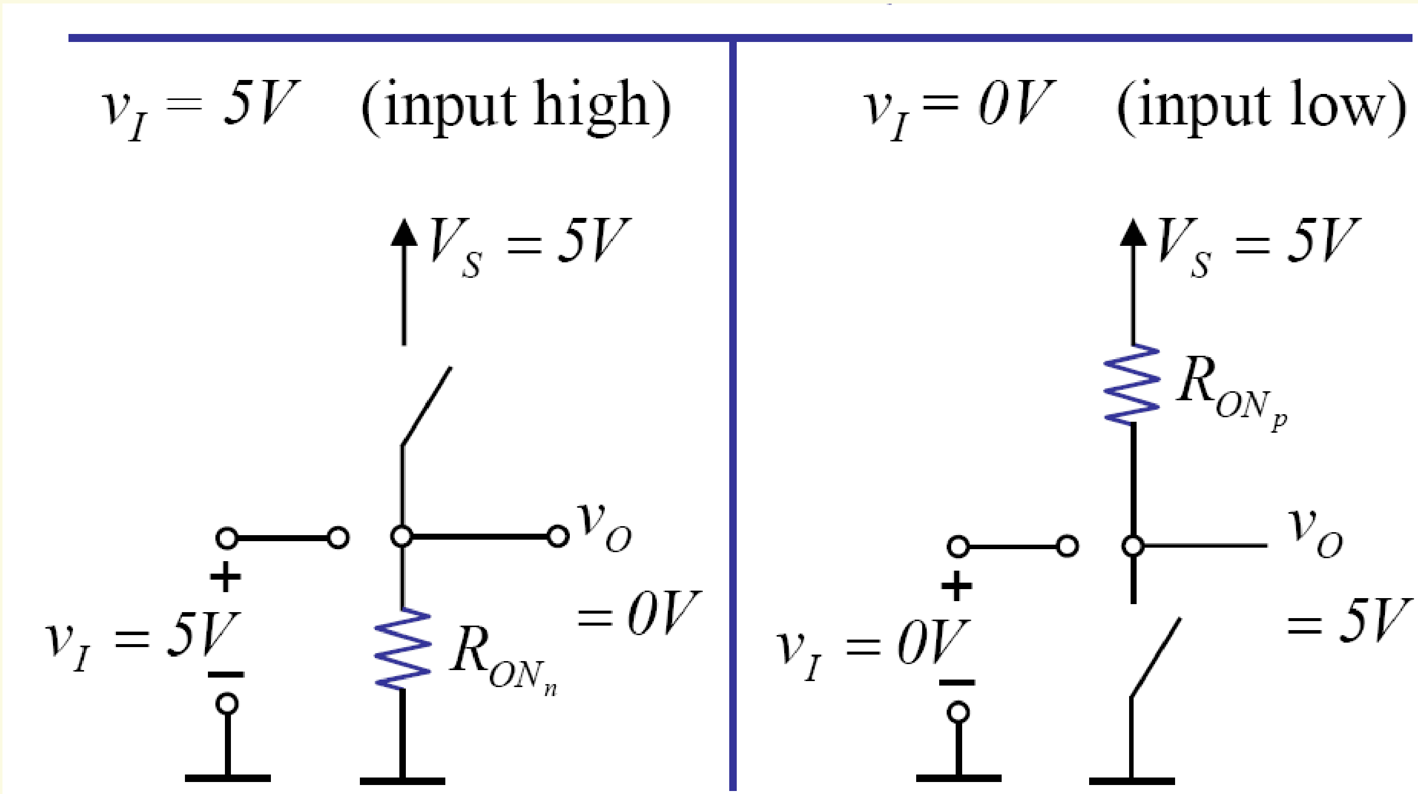


PMOS



on when $v_{GS} \leq V_{TP}$
off when $v_{GS} > V_{TP}$
e.g. $V_{TP} = -1V$

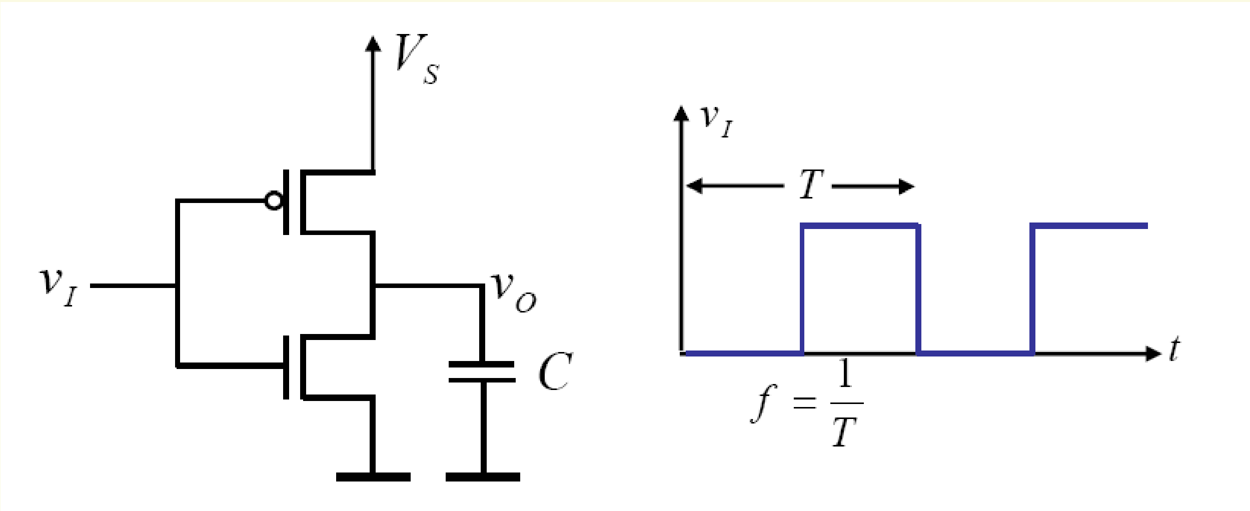
CMOS Inverter

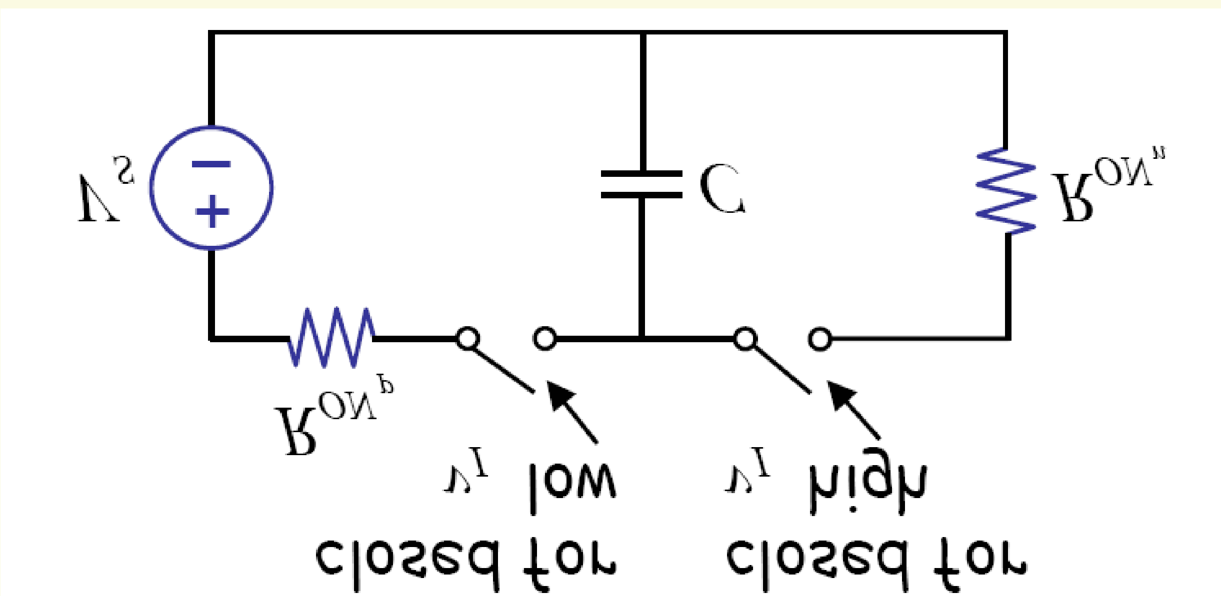


No static power!

- No direct path from the supply to ground

Dynamic Power of CMOS

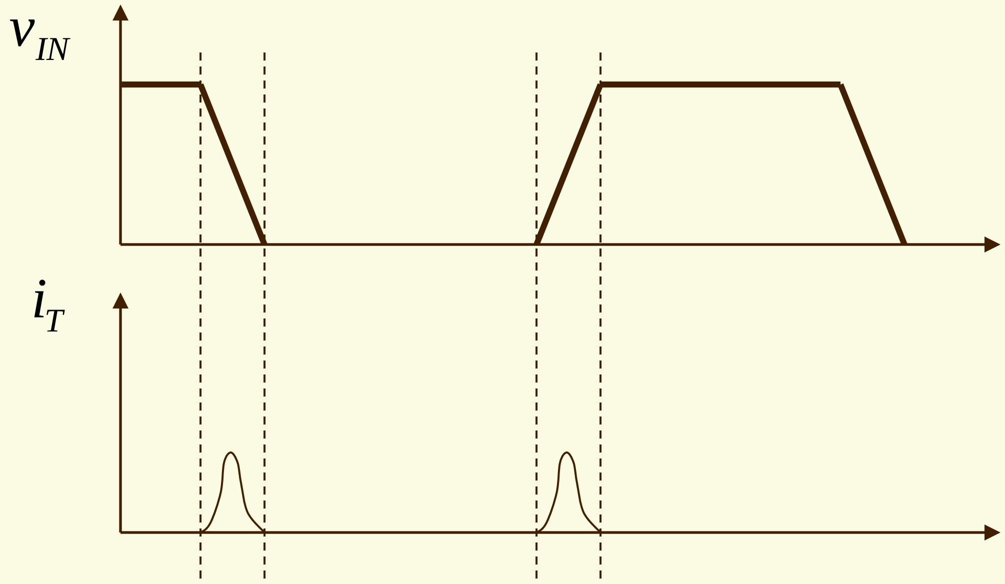




$$\overline{P} = CV_S^2 f$$

Static Power of CMOS

- Leakage power
- Transient current



Some Examples

- Example II.6: Processor SA27E
  - # of gates = 3M, 25% activity

$$C_L = 30\,fF, \quad V_S = 1.5V, \quad f = 425MHz$$

$$P_{dynamic} = (Fraction\ switching) \times (\#\ gates) \times fC_L V_S^2$$

Exercise

- Estimate the power of the following processor (~ P-IV)
  - # of gates = 25M, 25% activity

$$C_L = 1fF, \quad V_S = 1.5V, \quad f = 3GHz$$

Summary

- Total power = static power + dynamic power
- CMOS does not exhibit static loss
- Dynamic power

$$P_{dynamic} = (Fraction\ switching) \times (\#\ gates) \times fC_L V_S^2$$

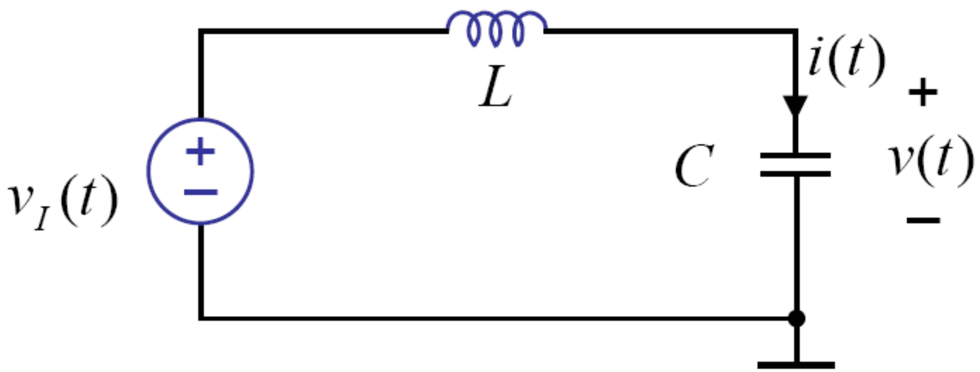
  - Reduce the supply voltage
  - Reduce the switching activities
  - Turn off the logic not in use (clock gating)

Chap. 12 Transients in Second-order Circuits

LC Circuit

RLC Circuits

LC Circuit



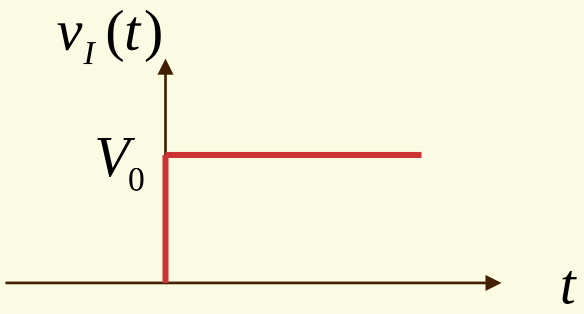
$$\underbrace{LC}_{\text{time}^2}\frac{d^2v}{dt^2}+v=v_I$$

v, i state variables

Solutions

- Find the particular solution
- Find the form of homogeneous solution
- Use initial condition to obtain the total solution

< example >



Zero initial condition:

$$v_C = 0, i_L = 0$$

Homogeneous Solution

- General Form: $v_H(t) = Ae^{st}$

$$LCAs^2e^{st} + Ae^{st} = 0 \quad \Rightarrow \quad s = \pm j\omega_o, \quad \omega_o = \frac{1}{\sqrt{LC}}$$

Total Solution

Total solution

$$v(t) = v_P(t) + v_H(t)$$

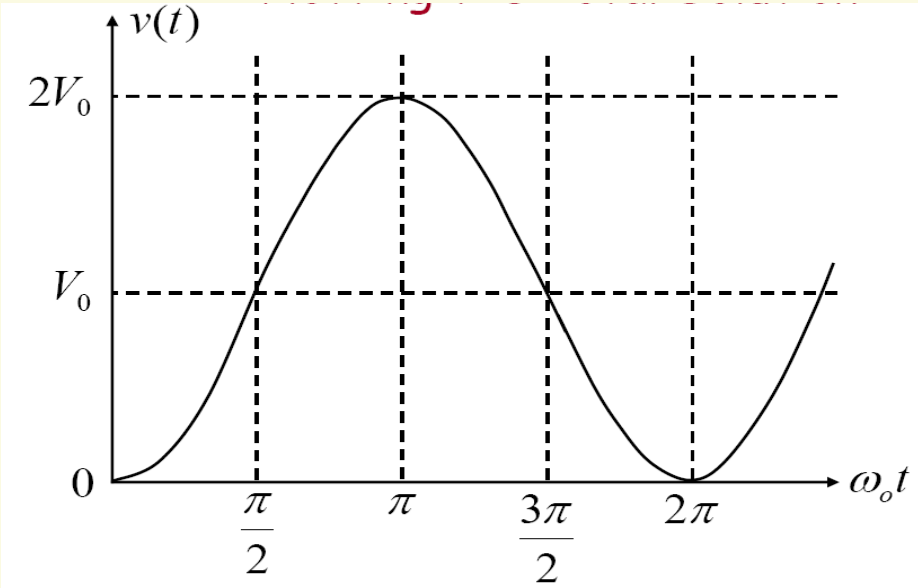
$$v(t) = V_0 + A_1 e^{j\omega_o t} + A_2 e^{-j\omega_o t}$$

$$v(t) = V_0 - \frac{V_0}{2}\left(e^{j\omega_o t} + e^{-j\omega_o t}\right)$$

$$= V_0 - V_0 \cos \omega_o t$$

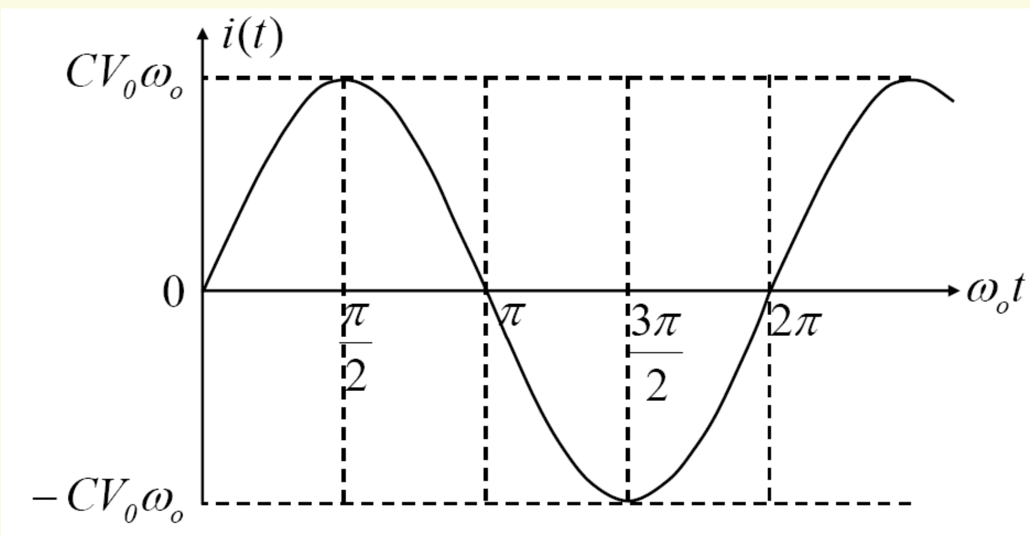
Find A1 and A2 from initial conditions

Total Solution: Plot



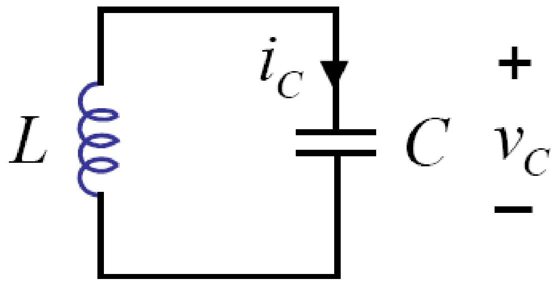
$$v(t) = V_0 - V_0 \cos \omega_o t$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$



$$i(t) = CV_0 \omega_o \sin \omega_o t$$

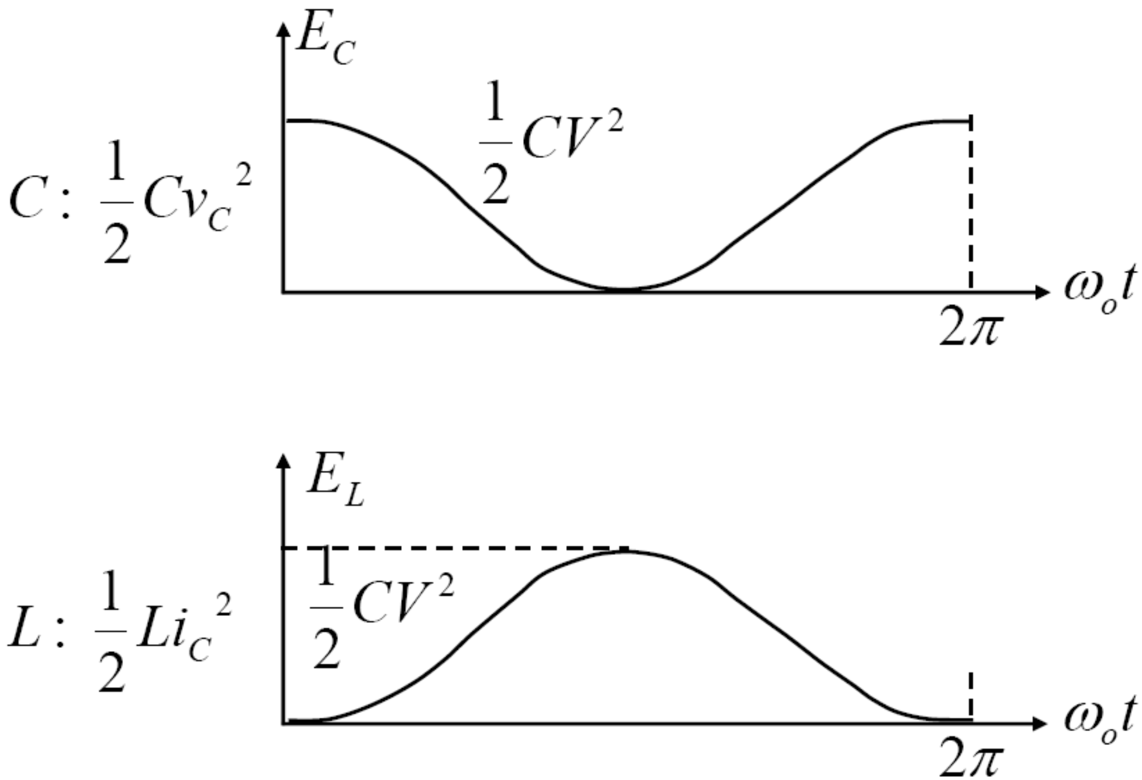
Undriven LC Circuit



$$v_C(0) = V$$
$$i_C(0) = 0$$

$$v_C(t) = A_1 e^{j\omega_o t} + A_2 e^{-j\omega_o t}$$

Energy



Notice $\frac{1}{2}Cv_C{}^2 + \frac{1}{2}Li_C{}^2 = \frac{1}{2}CV^2$

Observations from undriven LC Circuit

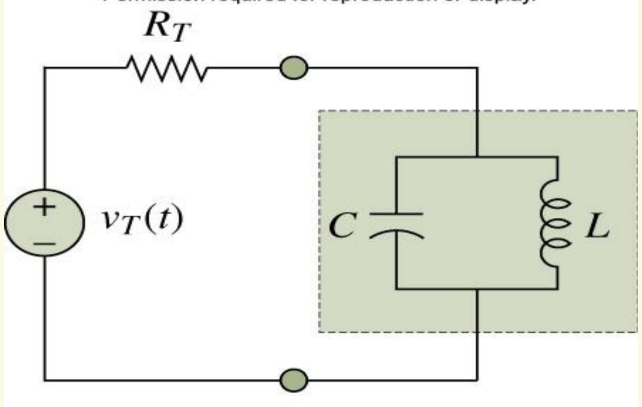
- 1. LC circuits are capable of oscillation
- 2. Important time constant: $\sqrt{LC}$
- 3. Characteristic impedance $\sqrt{\frac{L}{C}}$

$$W_T = \frac{C}{2} v_{C_{peak}}^2 = \frac{L}{2} i_{L_{peak}}^2 \quad \Rightarrow \quad \frac{v_{C_{peak}}}{i_{C_{peak}}} = \sqrt{\frac{L}{C}}$$

Example 12.1

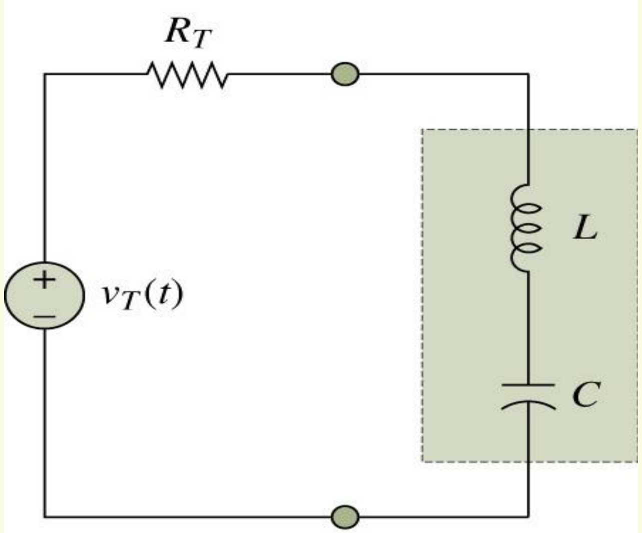
$$C = 1\mu F,\ L = 100\mu H,\ (i_L = 0.5A,\ v_C = 10V\ at\ some\ t)$$

RLC Circuits



Parallel case

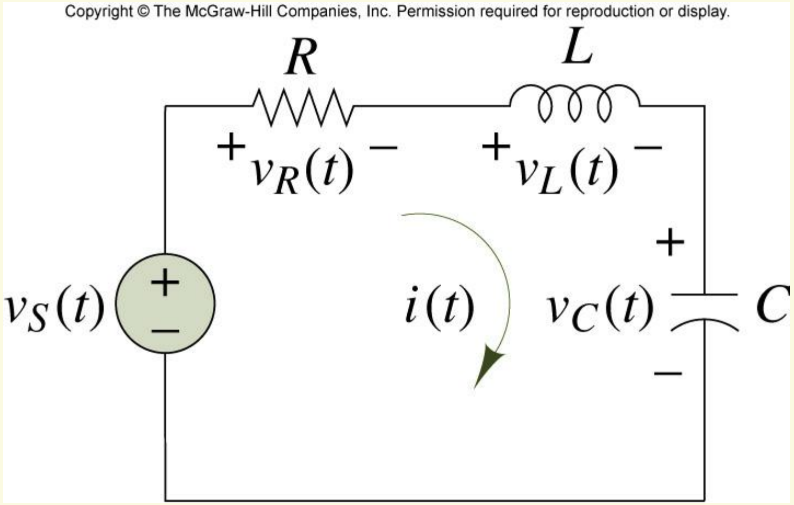
(a)



Series case

(b)

Series RLC Circuit: second order circuit



$$\frac{d^2 v_C(t)}{dt^2} + \frac{R}{L}\frac{dv_C(t)}{dt} + \frac{1}{LC}v_C(t) = \frac{1}{LC}v_S(t)$$

Homogeneous Solution

$$v_C(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} \Rightarrow$$

$$s^2 + 2\alpha s + \omega_o^2 = 0, \quad \alpha \equiv \frac{R}{2L}, \omega_o = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o}$$

- Overdamped case: real and distinct roots

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} \quad (\alpha > w_o)$$

- Critically damped case: real and repeated roots

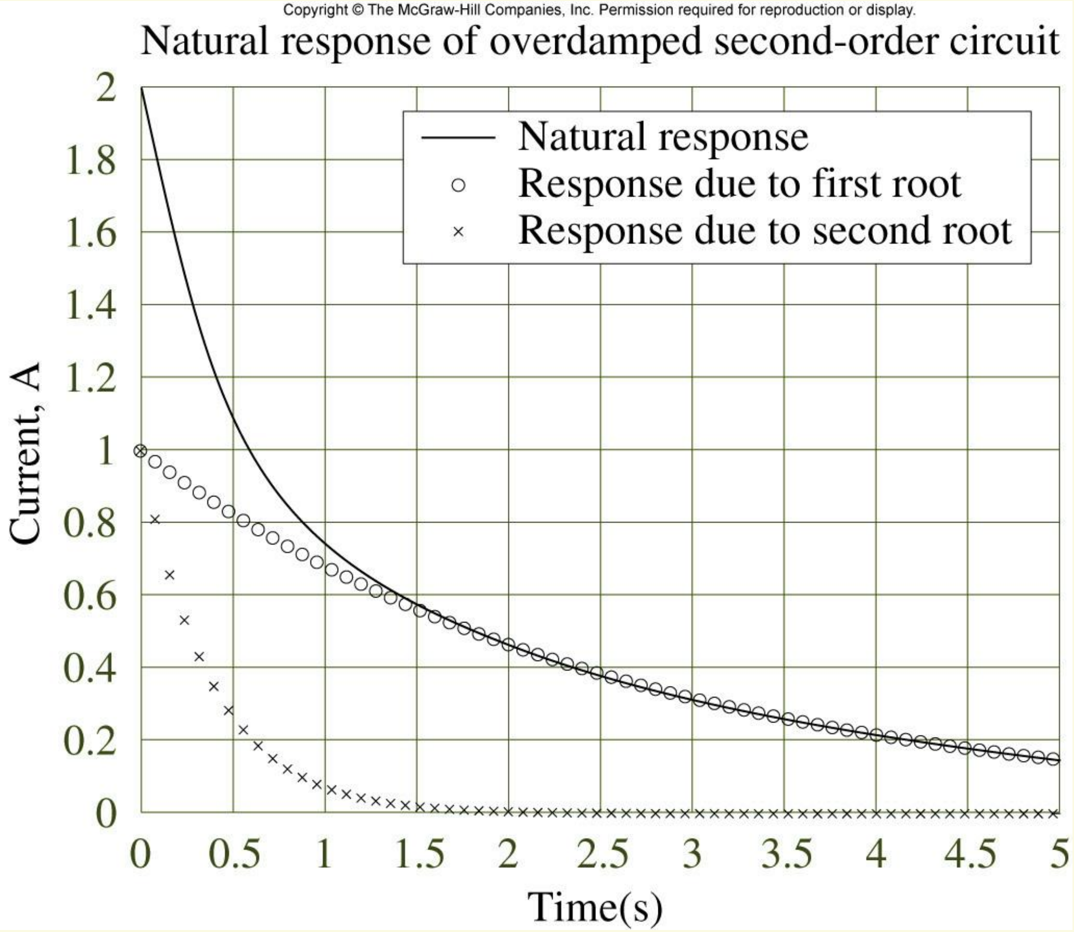
$$s_{1,2} = -\omega_o \quad (\alpha = \omega_o)$$

- Underdamped case: complex conjugate roots

$$s_{1,2} = -\alpha \pm j\omega_{d,} \quad \omega_d = \sqrt{\omega_o^2 - \alpha^2} \quad (\alpha < \omega_o)$$

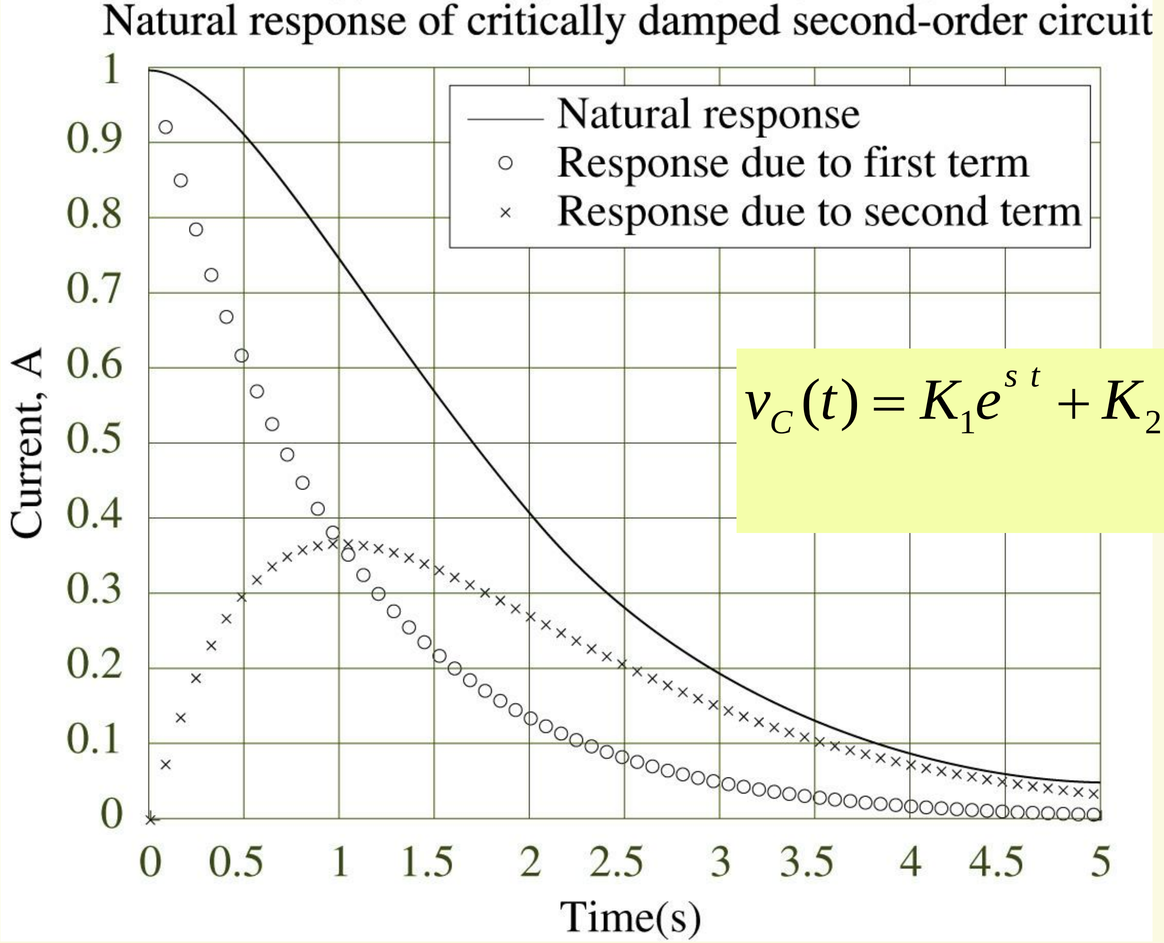
Natural Response: overdamped case

$$K_1 = K_2 = 1, \quad \omega_o = 1, \quad \alpha = 1.5$$



Natural Response: Critically damped case

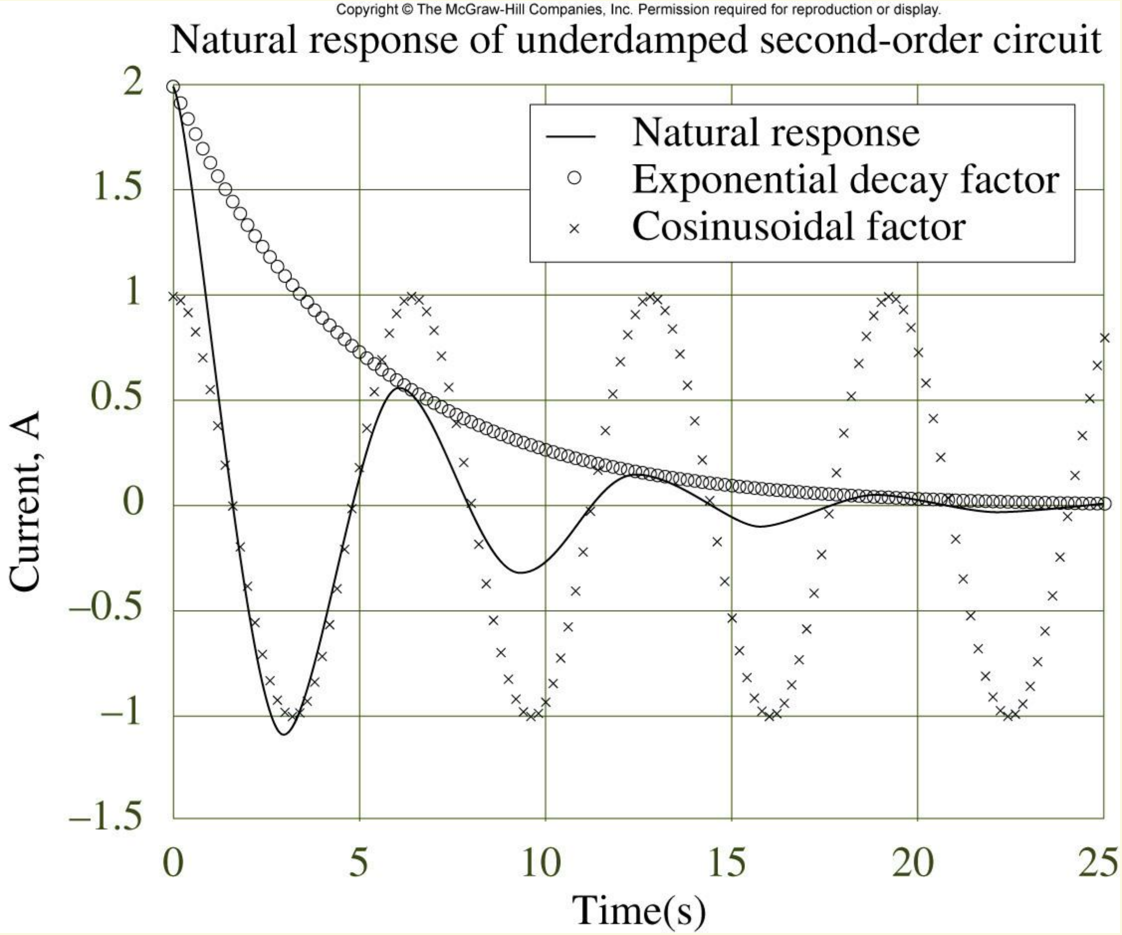
$$K_1 = K_2 = 1, \quad \omega_o = 1, \quad \alpha = 1$$



$$v_C(t) = K_1 e^{s\,t} + K_2 t e^{st} \Rightarrow v_C(t) = e^{-t} + t e^{-t}$$

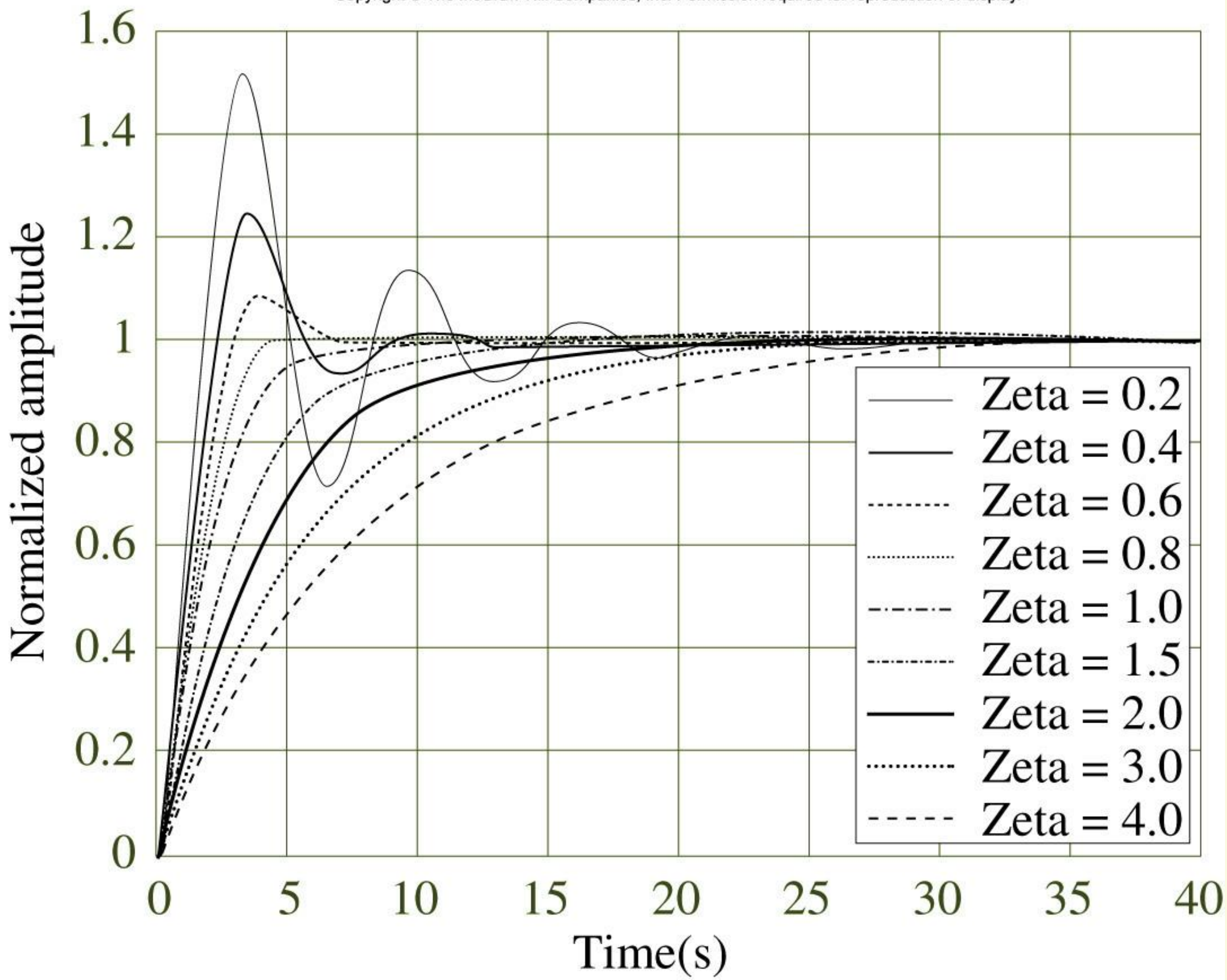
Natural Response: Underdamped case

$$K_1 = K_2 = 1, \quad \omega_o = 1, \quad \alpha = 0.2$$

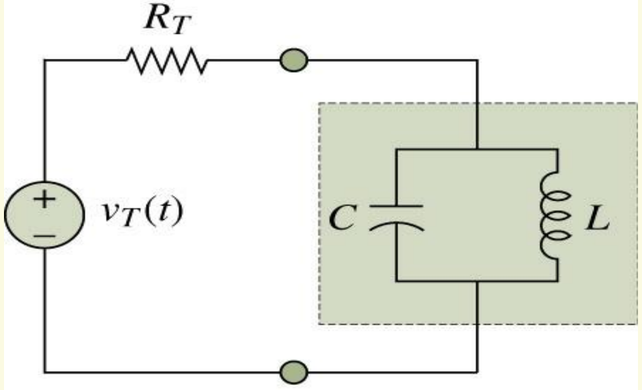


Response to step input voltage

$$\omega_o = 1, \quad \alpha = 0.2 \sim 4 \equiv Zeta$$



Exercise: Parallel RLC Circuit



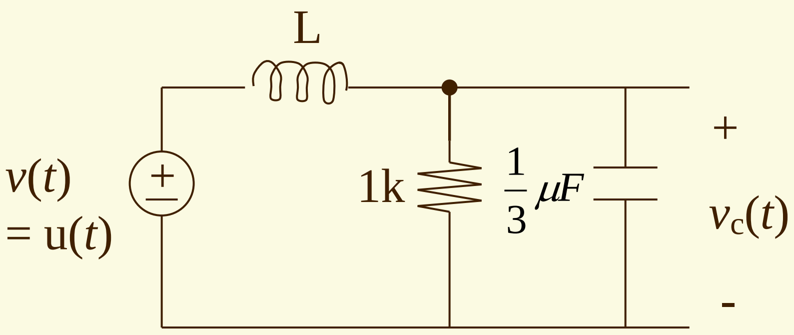
Parallel case

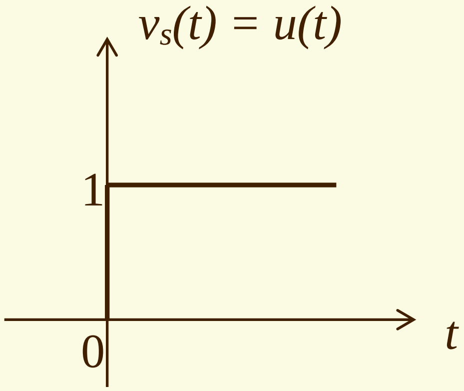
Find $i_L(t)$
assume zero initial state

$$R = 5k\Omega, C = 1\mu F, L = 1H, V_S = 25V$$

Exercise

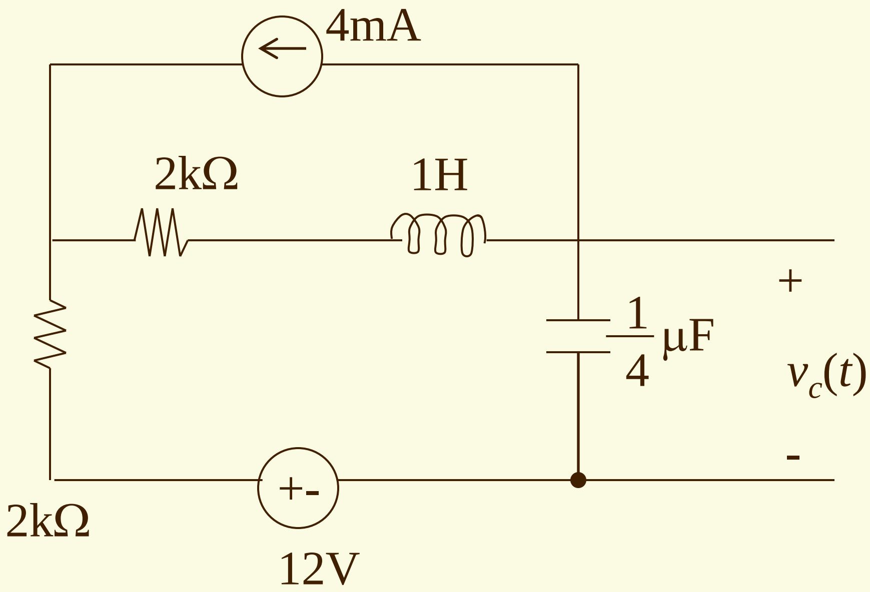
Find the range of L to make $v_C(t)$ to be overdamped



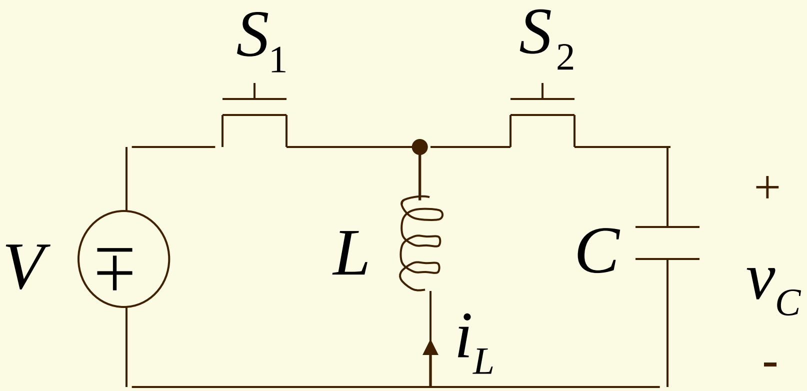


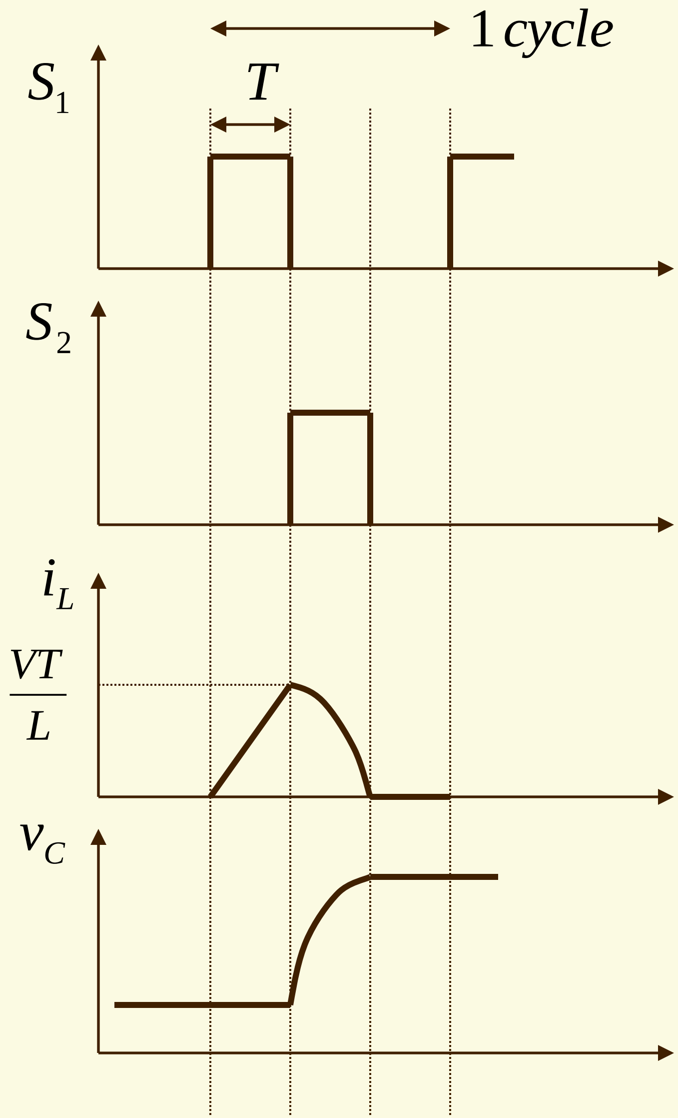
Exercise

Find the differential equation for capacitor voltage. Find the total response with the following initial condition.
$i_L(0) = 4$ mA, $v_C(0) = 0$ (V)



Example 12.4 Ideal Switched Power Supply





(1) S1 on, S2 off

(2) S1 off, S2 on until inductor current becomes zero

(3) S1 off, S2 off

Summary

- LC Circuit
- RLC Circuit
  - Three kinds of transient responses
    - Over-damped
    - Critically damped
    - Under-damped

Chap. 13 Sinusoidal Steady State: Imdepance and Frequency Response

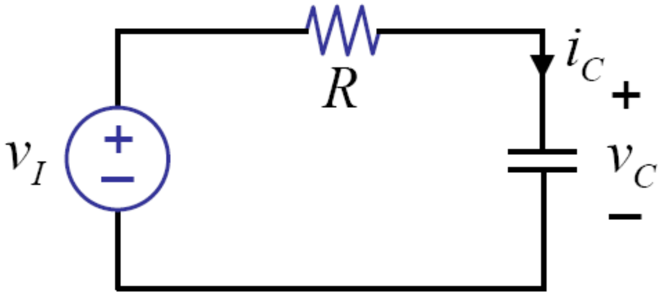
Sinusoidal Steady-state Response

Impedance

Frequency Response

Filter

Sinusoidal Response of RC Circuit



$$v_I(t) = V_i \cos \omega t \quad \text{for} \quad t \geq 0 \quad (V_i \text{ real})$$
$$= 0 \quad \text{for} \quad t < 0$$

$$v_C(0) = 0 \quad \text{for} \quad t = 0$$

$$v_I(t) = V_i \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

Find the particular solution to $V_i e^{j\omega t}$

Total solution

$$v_C = \boxed{v_P} + v_H$$

$$v_C = \frac{V_i}{\sqrt{1+\omega^2 R^2 C^2}} cos(\omega t + \phi) + Ae^{-\frac{t}{RC}}$$

$$where \ \ \phi = tan^{-1}(-\omega RC)$$

Given $v_C(0) = 0$ for $t = 0$

so,

$$A = -\frac{V_i}{\sqrt{1+\omega^2 R^2 C^2}} \cos(\phi)$$

Sinusoidal Steady State (SSS)

- Particular solution for sinusoidal input after transients have died

$$v_C = \frac{V_i}{\sqrt{1+\omega^2 R^2 C^2}} \cos(\omega t + \phi) + A e^{-\frac{t}{RC}}$$

$$\text{where } \phi = \tan^{-1}(-\omega RC)$$

$$|V_C| \qquad\qquad \angle V_C$$

- All information of SSS is contained in the complex amplitude

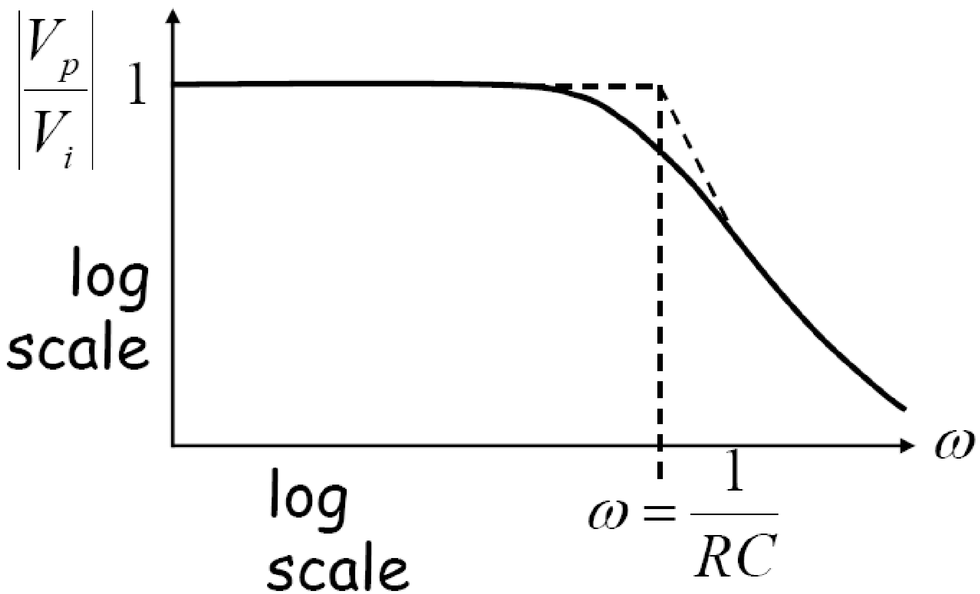
$$\frac{V_i}{1 + j\omega RC}$$

Magnitude Plot

transfer function

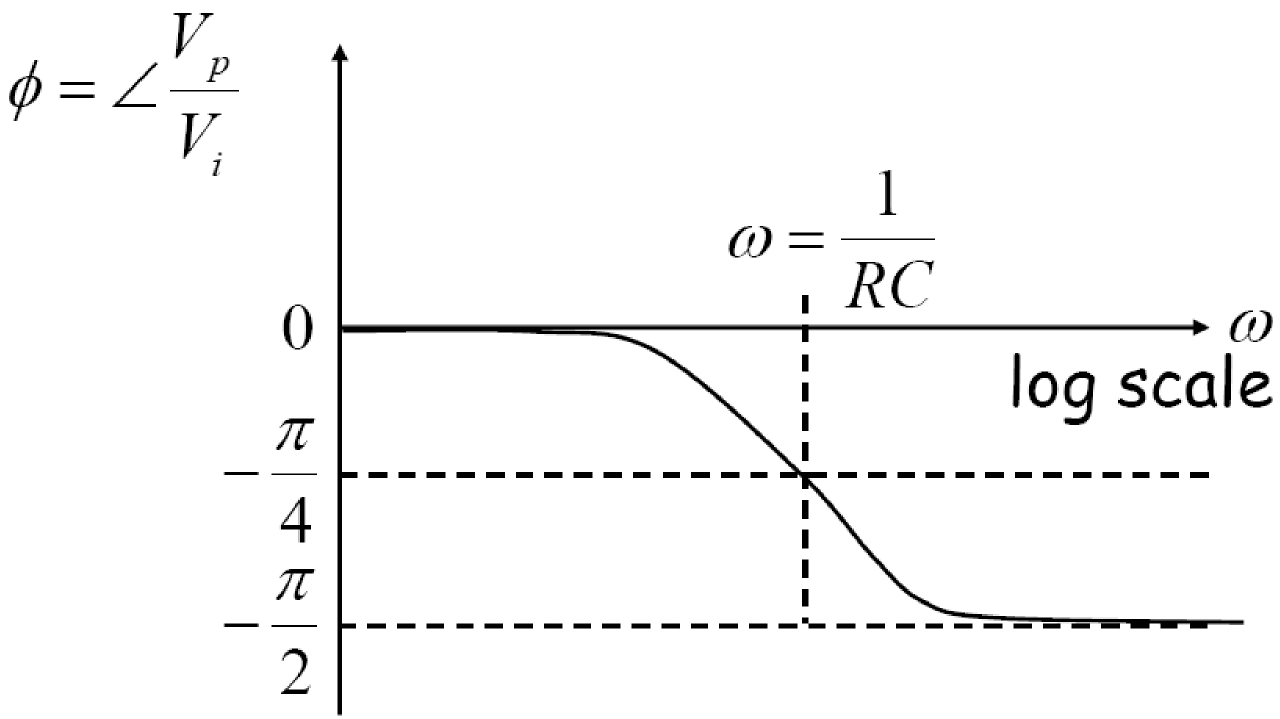
$$H(j\omega) = \frac{V_p}{V_i}$$

$$\left|\frac{V_p}{V_i}\right| = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}}$$

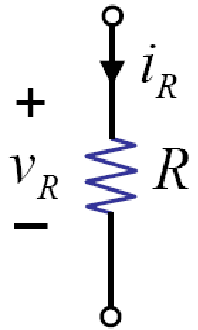


Phase Plot

$$\phi = tan^{-1} - \omega RC$$



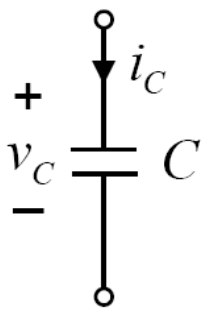
Impedance Model



$$i_R = I_r e^{j\omega t} \qquad v_R = R i_R$$

$$v_R = V_r e^{j\omega t} \qquad V_r \cancel{e^{j\omega t}} = R I_r \cancel{e^{j\omega t}}$$

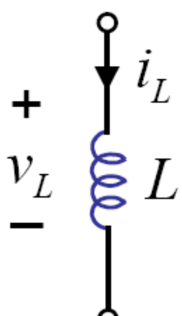
Resistor $\boxed{V_r = R I_r}$



$$i_C = I_C e^{j\omega t} \qquad i_C = C \frac{dv_C}{dt}$$

$$v_C = V_C e^{j\omega t} \qquad I_C \cancel{e^{j\omega t}} = C V_C j\omega \cancel{e^{j\omega t}}$$

Capacitor $\boxed{V_C = \underbrace{\frac{1}{j\omega C}}_{Z_C} I_C}$



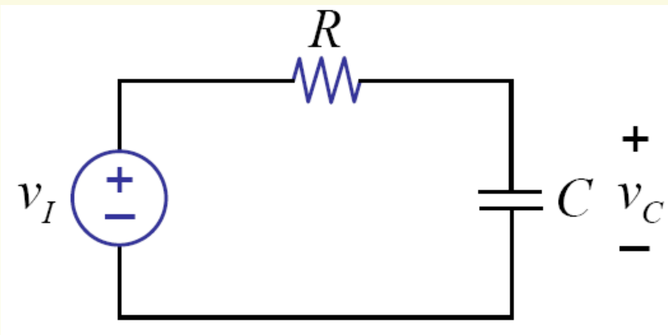
$$i_L = I_l e^{j\omega t} \qquad v_L = L \frac{di_L}{dt}$$

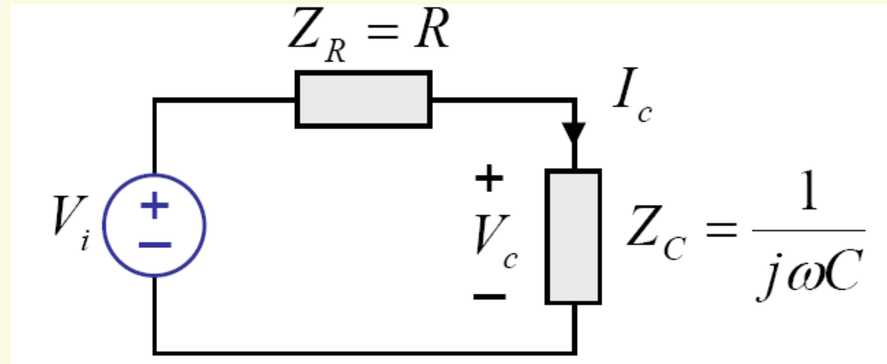
$$v_L = V_l e^{j\omega t} \qquad V_l \cancel{e^{j\omega t}} = L I_l j\omega \cancel{e^{j\omega t}}$$

Inductor $\boxed{V_l = j\omega L \; I_l}$

Impedance Model

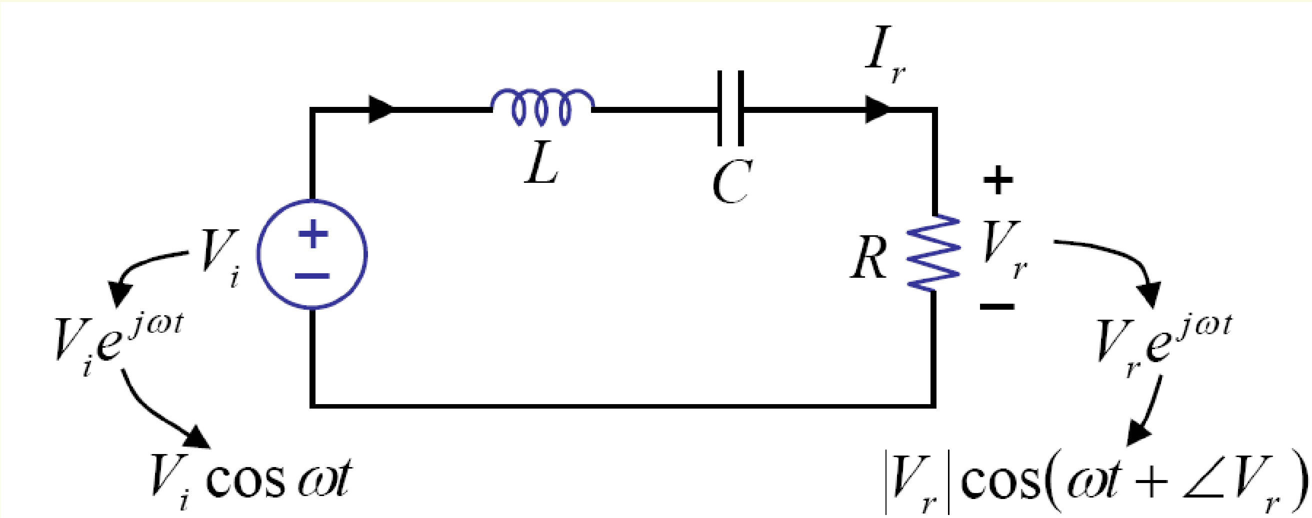
- Circuit corresponding to a complex exponential input



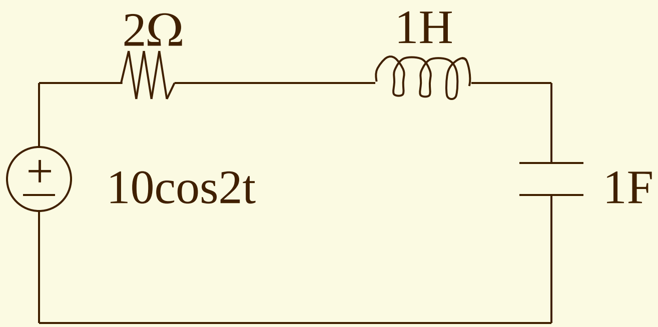


$$\frac{V_C}{V_I}$$

Series RLC Circuit



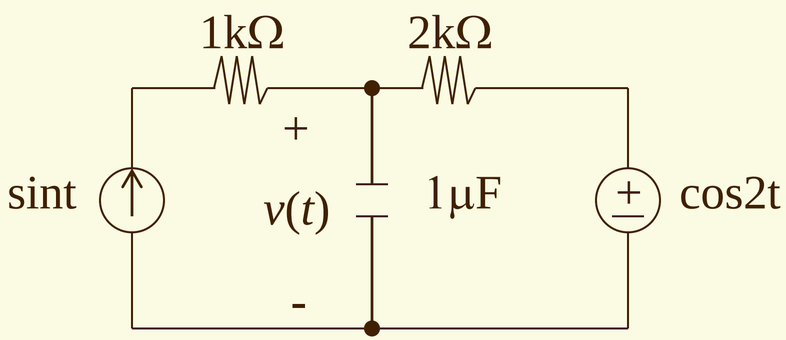
Exercise 1



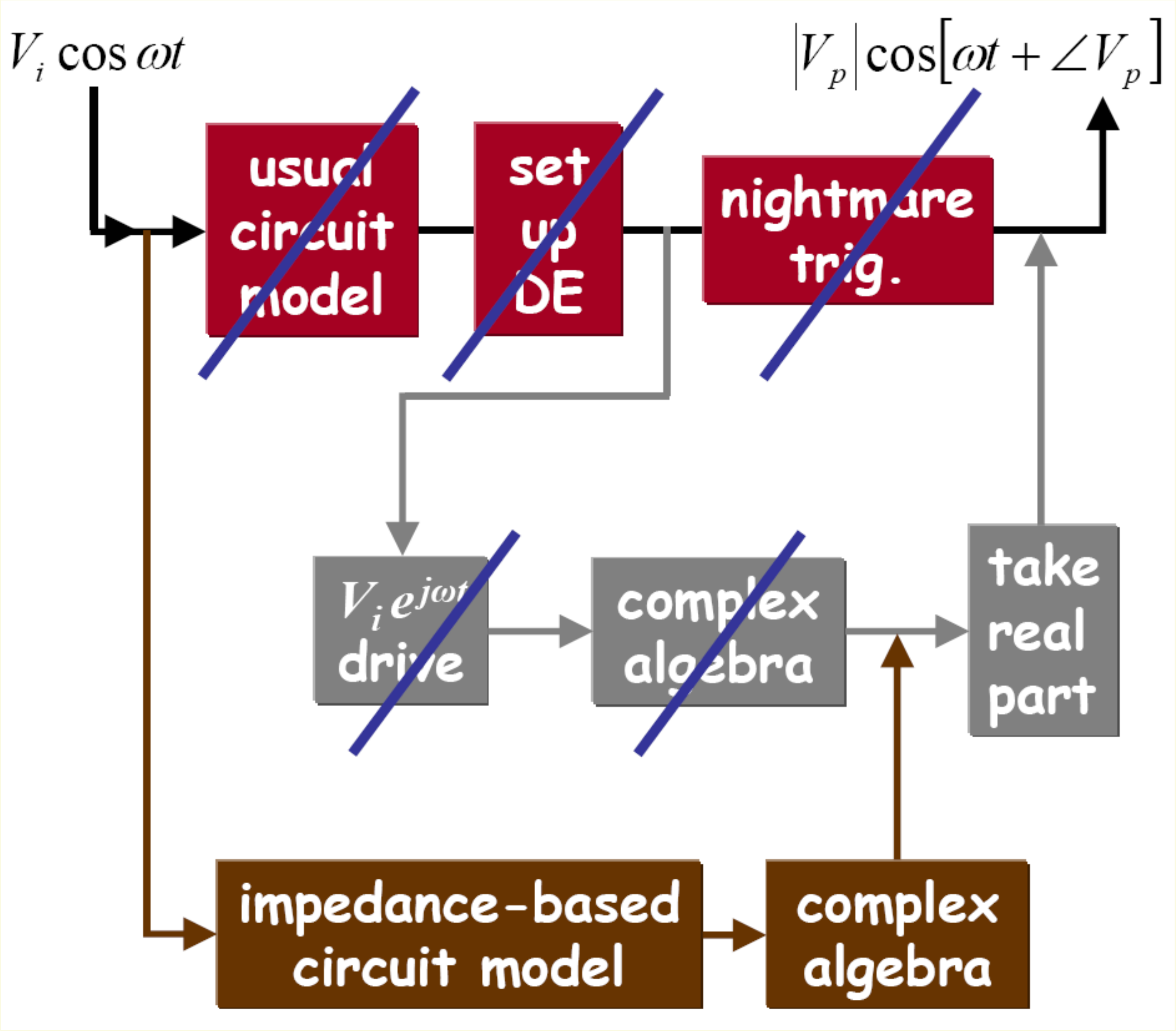
find v_C

Exercise 2

Find *v(t)* at steady state (time unit: *msec*)

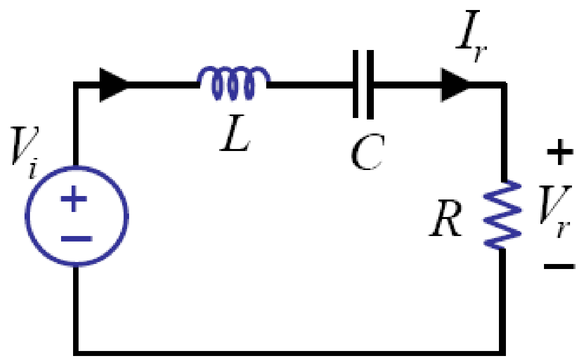


What we are doing? The Big Picture



Back to

$$\frac{V_r}{V_i} = \frac{j\omega RC}{1 + j\omega RC - \omega^2 LC}$$



$$\left|\frac{V_r}{V_i}\right| = \frac{\omega RC}{\sqrt{\left(1 - \omega^2 LC\right)^2 + \left(\omega RC\right)^2}}$$

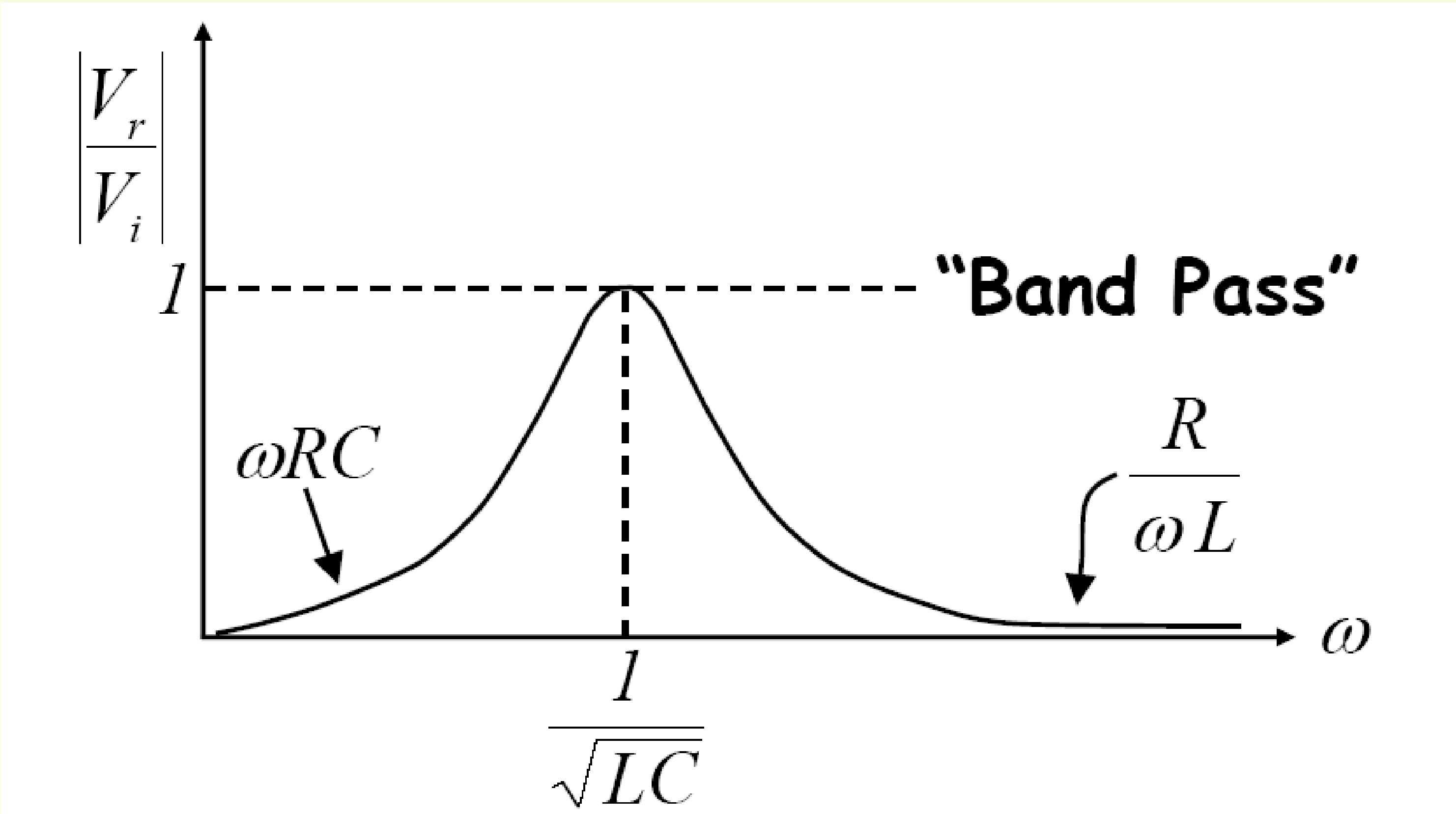
Observe

$$\text{Low } \omega: \quad \approx \omega RC$$

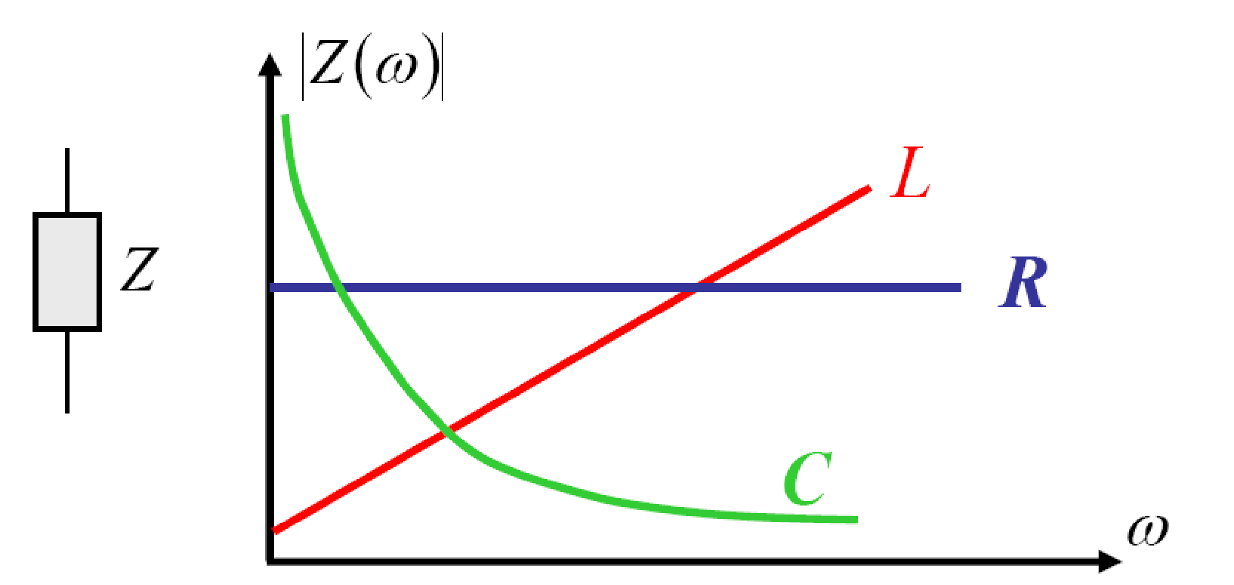
$$\text{High } \omega: \quad \approx \frac{R}{\omega L}$$

$$\omega\sqrt{LC} = 1: \quad \approx 1$$

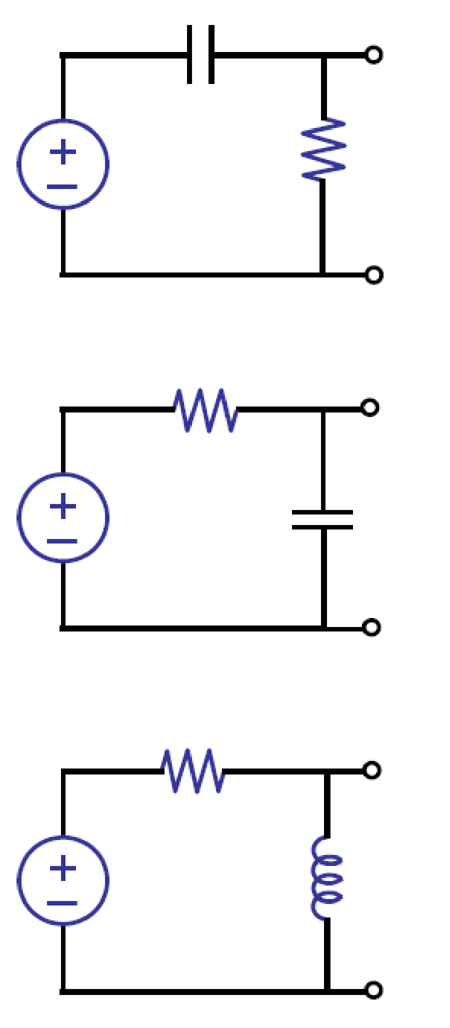
Frequency Response

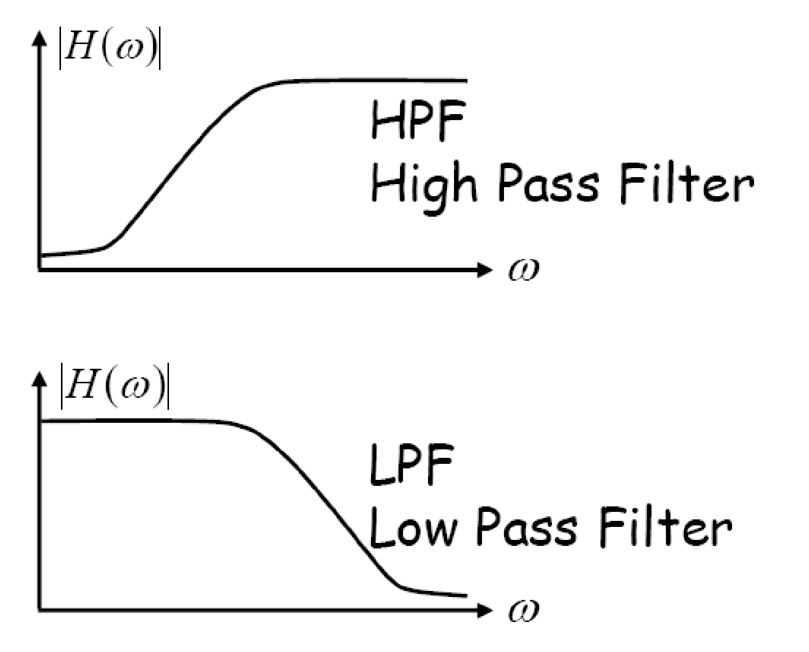


Frequency Response of Basic Elements



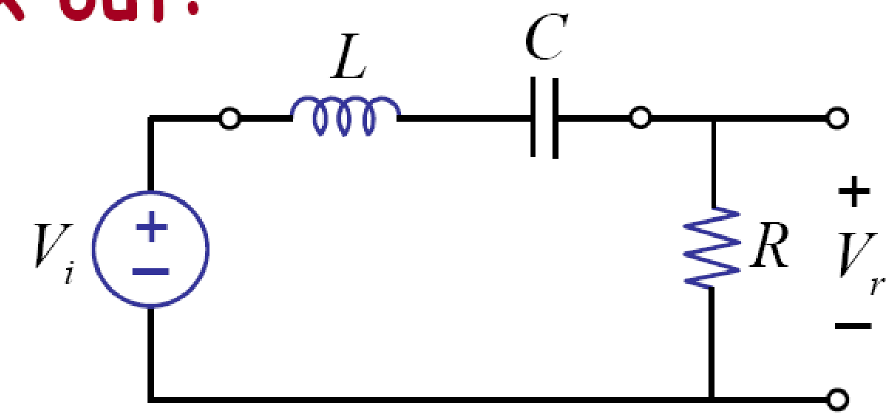
RC and RL Circuits



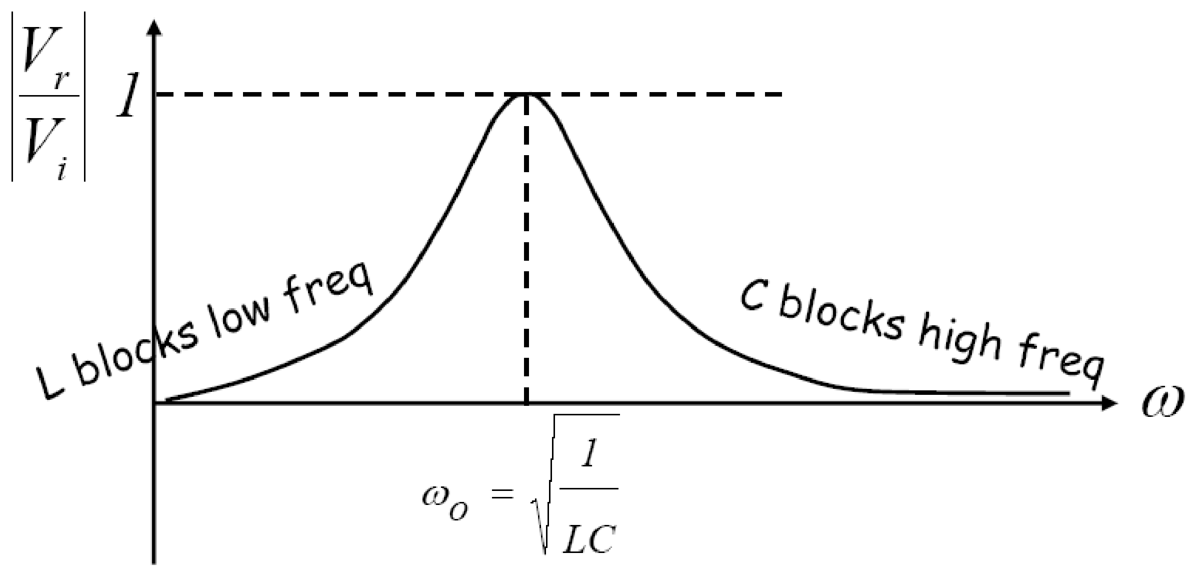


RLC Circuits

Check out:



Intuitively:



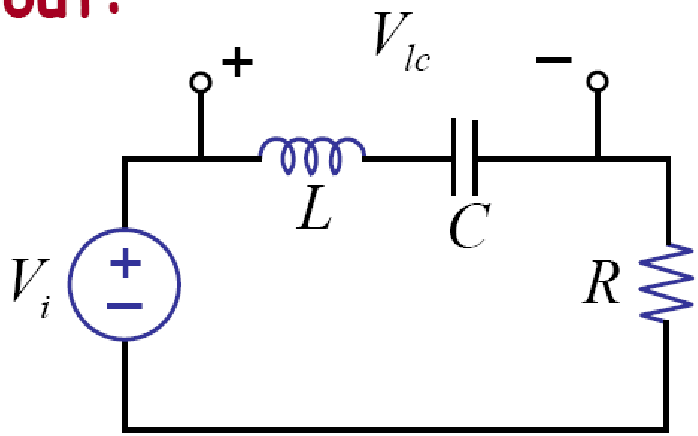
Resonance

- Equivalent impedance = Resistor

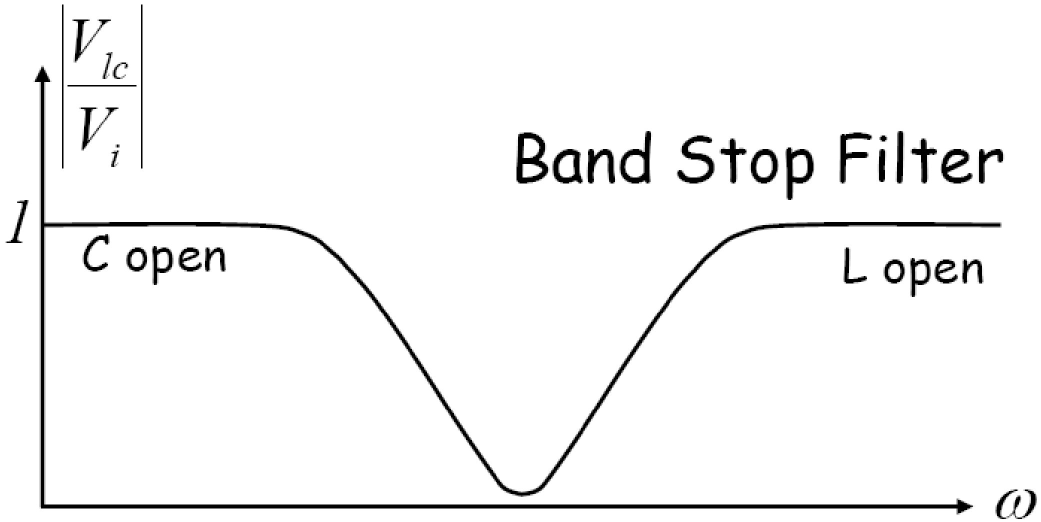
$$Z_L + Z_C = 0$$

- Circuit response is maximum

What about:

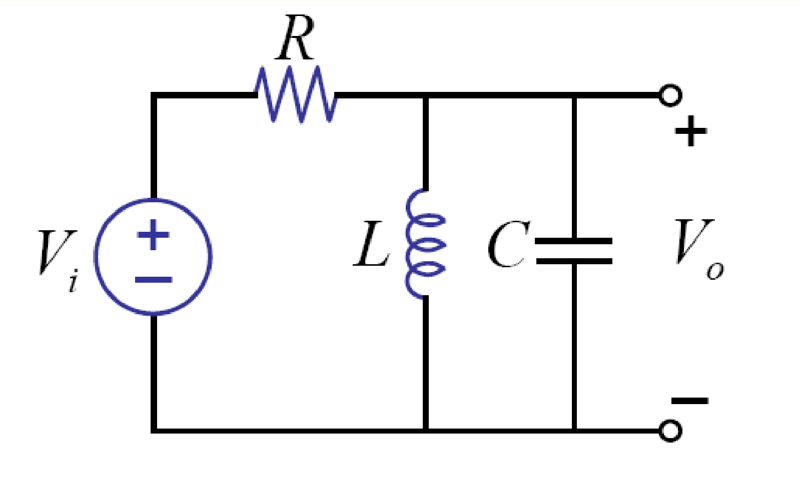


+
V_{lc}
−
L
C
V_i
R

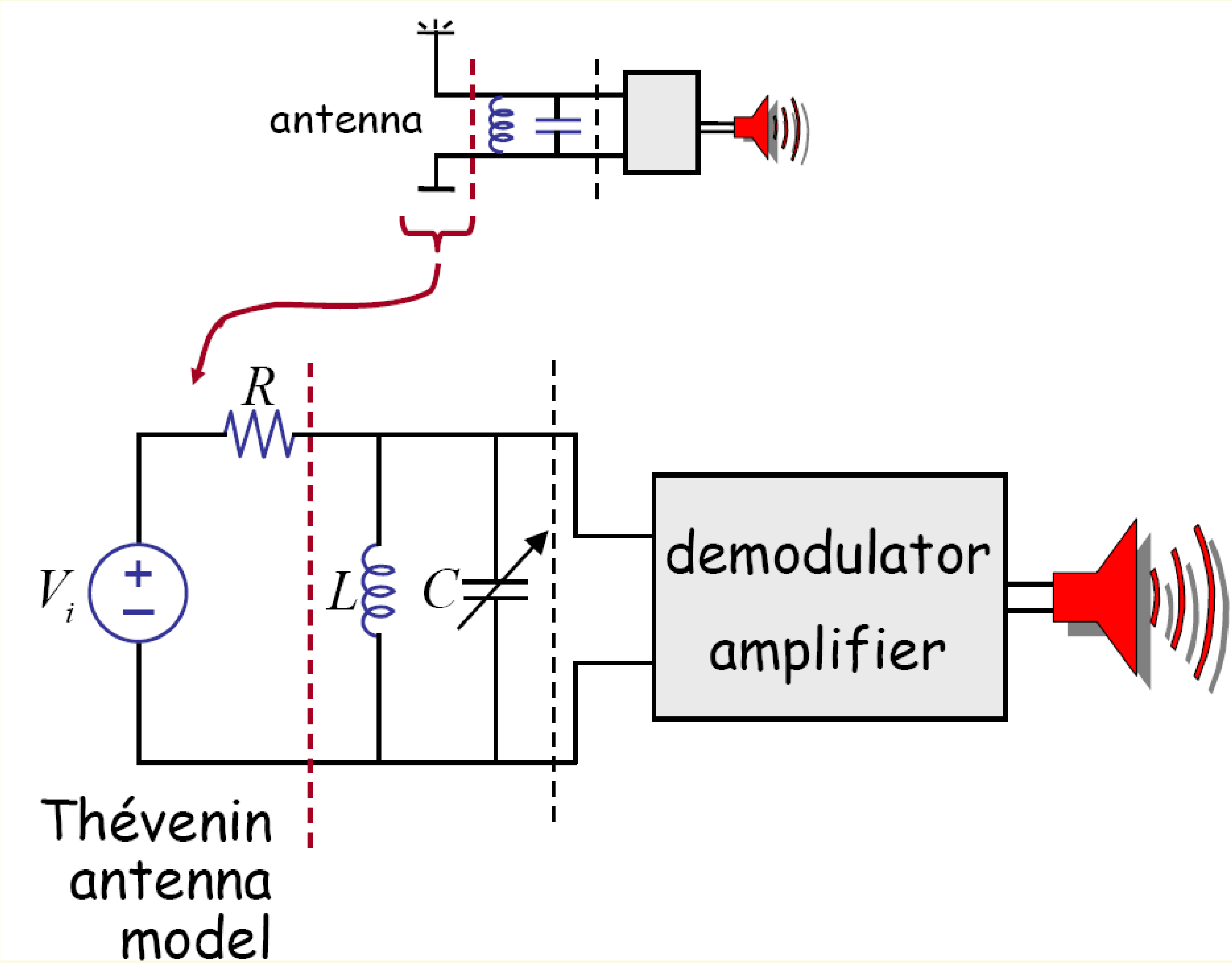


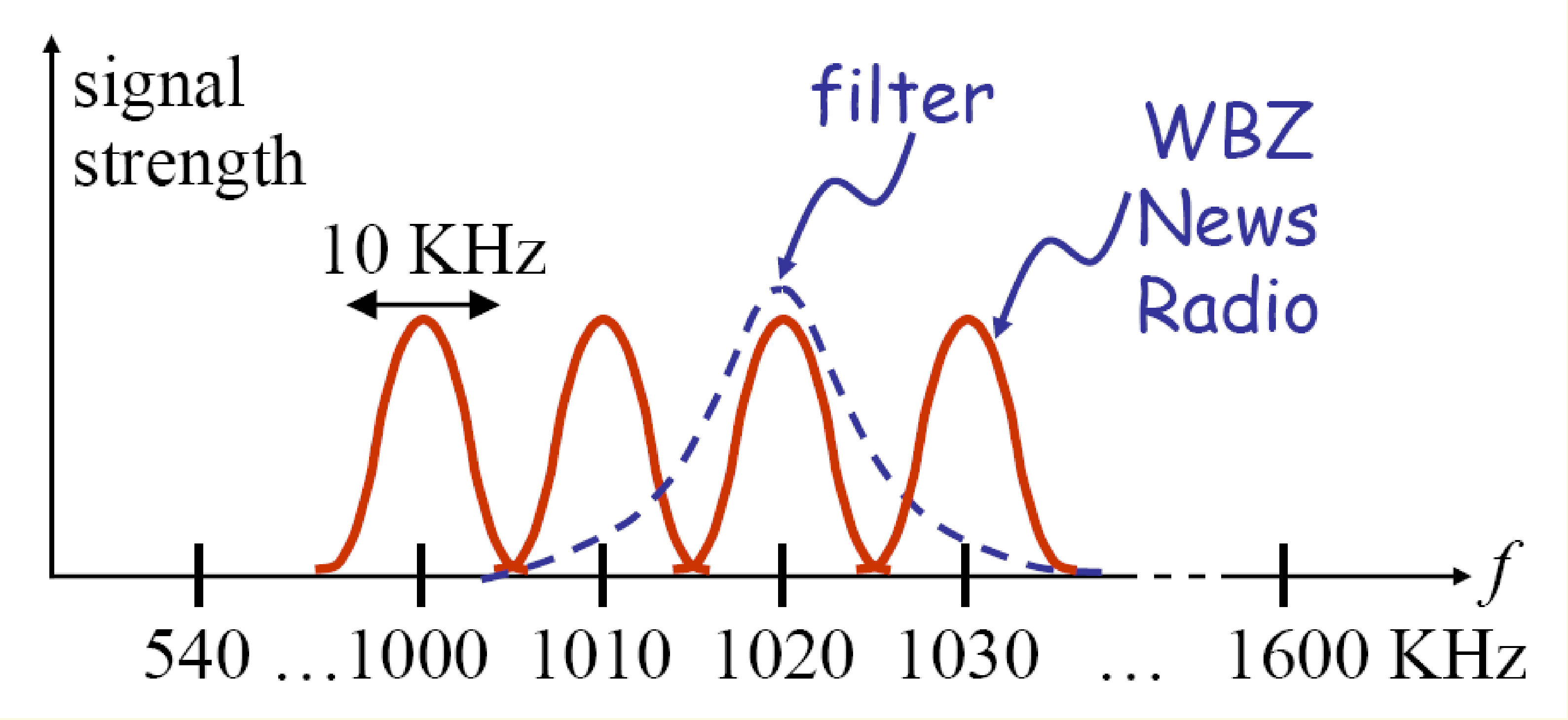
$\left|\frac{V_{lc}}{V_i}\right|$
Band Stop Filter
1
C open
L open
ω

Another Example



AM Radio Receiver

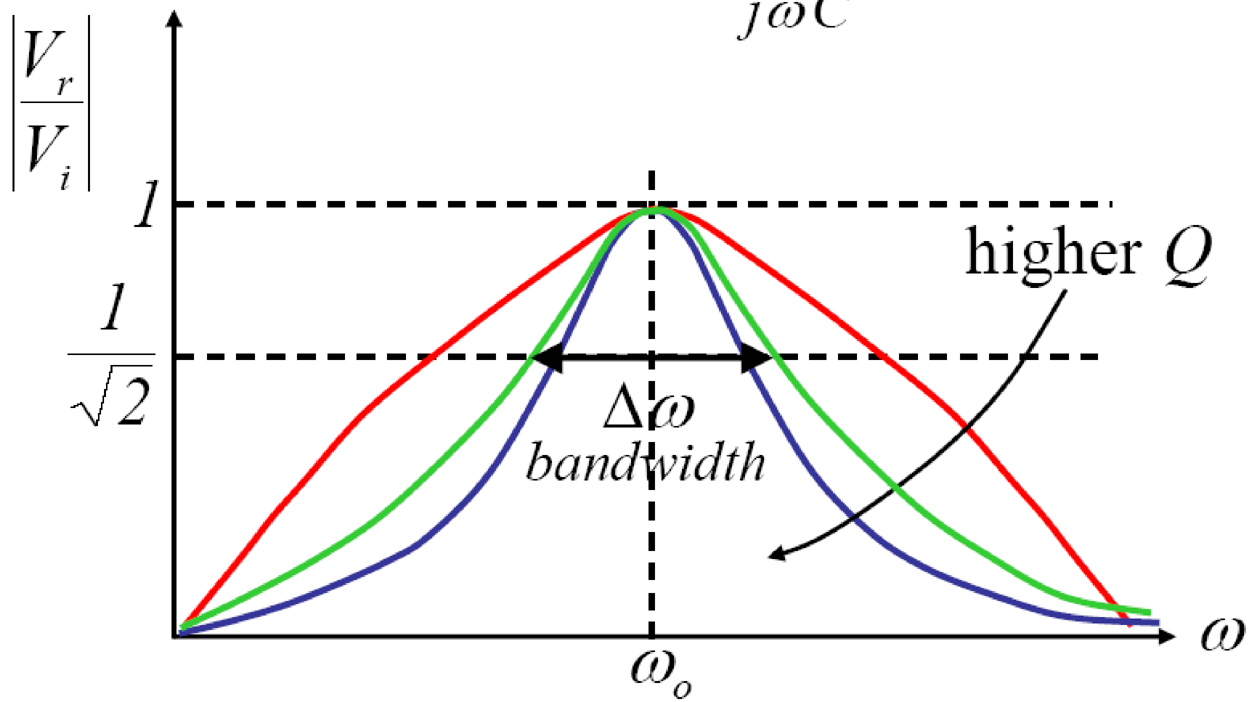




signal
strength
10 KHz
filter
WBZ
News
Radio
f
540
…1000
1010
1020
1030
…
1600 KHz

Selectivity

Recall, $$\frac{V_r}{V_i} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}}$$



Define $Q = \frac{\omega_o}{\Delta\omega}$ *quality factor*

high Q $\Rightarrow$ more selective

Quality Factor: Q

$\Delta\omega$:

Note that abs magnitude is $\frac{1}{\sqrt{2}}$

when $$\frac{V_r}{V_i} = \frac{1}{1 + j\left(\omega\frac{L}{R} - \frac{1}{\omega CR}\right)} = \frac{1}{1 \pm j1}$$

i.e. when $$\frac{\omega L}{R} - \frac{1}{\omega CR} = \pm 1$$

$$\Delta\omega = \omega_1 - \omega_2 = \frac{R}{L}$$

Quality Factor

$$Q = \frac{\omega_o}{\frac{R}{L}} = \frac{\omega_o L}{R} \qquad \omega_o = \frac{1}{\sqrt{LC}}$$

- Lower R, higher Q
- Another interpretation

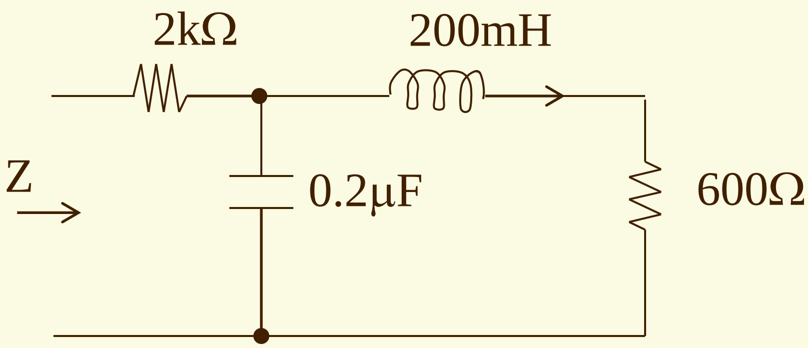
$$Q = 2\pi \frac{\text{energy stored}}{\text{energy lost per cycle}}$$

$$= 2\pi \frac{\frac{1}{2} L \left| I_r \right|^2}{\frac{1}{2} \left| I_r \right|^2 R \frac{2\pi}{\omega_0}}$$

$$Q = \frac{\omega_o L}{R}$$

Exercise

Find the resonance frequency and the equivalent impedance at resonance.



Summary

- Sinusoidal Steady-state Response
- Impedance
- Frequency Response
- Filter