1.2 Shear Strength of Granular Soils

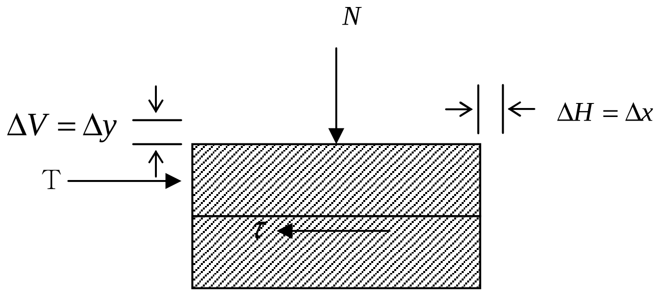
(1) Drained shear strength (ϕ',c'(=0))
(u_e=0)

- **Parameters affecting drained strength**
  1) density
  2) mineral composition
  3) grain size distribution
     ϕ'(well graded) > ϕ'(poorly graded)
  4) grain shape
     ϕ' (angular particle) > ϕ'(rounded particle)

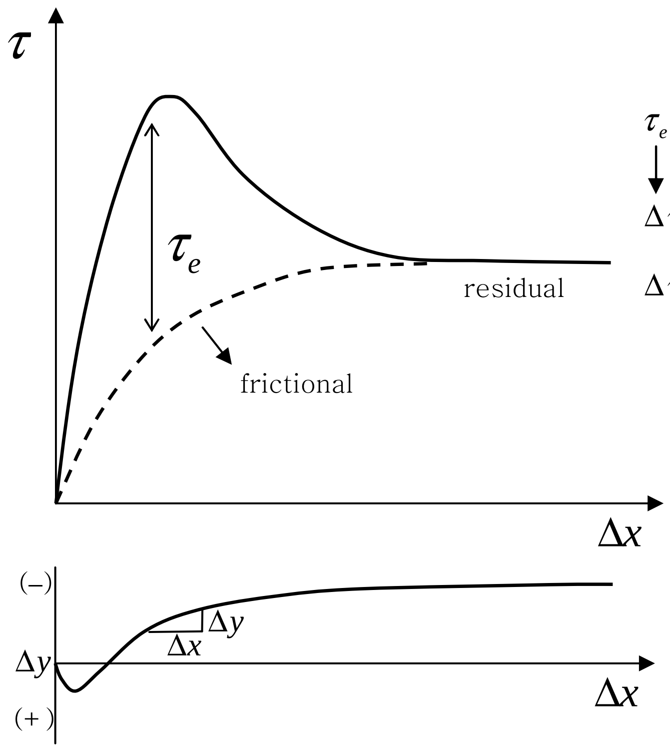
- **Components of drained strength**
  - Energy is expended in 2 ways.
  1) overcome frictional resistance of soil
  2) expand soil against confining pressure.

  (+ particle breakage, particle rearrangement)

- **Taylor and Bishop (Direct shear)**



- For dense sand,



τ_e = shear stress component

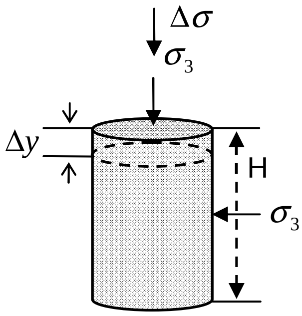
↓ due to volumetric expansion

$$\Delta \tau_e \cdot \Delta x = \Delta \sigma_n \cdot \Delta y$$

$$\Delta \tau_e = \Delta \sigma_n \cdot \frac{\Delta y}{\Delta x}$$

- Similar development can be made for TXC (based on drained test.)

$$(\sigma_c = \sigma_c' = \sigma_3 = \sigma_3')$$



$$\varepsilon_1 = \varepsilon_a = \frac{\Delta y}{H}$$

DS → τ σ_n

TXC → $\sigma_1 (= \sigma_{1e} + \sigma_{1R})$ σ_3

$$\sigma_{1e} d\varepsilon_1 = \sigma_3 \left(\frac{dV}{V_0}\right) \qquad \text{------- Eq. (1)}$$

σ_{1e}: component of σ_1 due to vol. expansion

$\frac{dV}{V_0}$: ε_{vol}

$$\sigma_{1e} = \sigma_3 \frac{dV}{d\varepsilon_1 V_0}$$

⇨ $$\sigma_{1R} = \sigma_1 - \sigma_{1e} = \sigma_1 - \sigma_3 \frac{dV}{d\varepsilon_1 V_0} \qquad \text{------ Eq. (2)}$$

σ_{1R}: (component of σ_1 due to mineral friction and particle rearrangement) → ϕ' at constant volume

Recall, $(\frac{\sigma_1}{\sigma_3})_f = \tan^2(45^0 + \phi'/2)$

$$\therefore (\frac{\sigma_{1R}}{\sigma_3})_f = \tan^2(45^0 + \phi'_R/2) \qquad \text{------ Eq. (3)}$$

ϕ'_R → ϕ'_{CV} (⇨ ϕ' at constant volume)

And from Eq.(2)

$$(\frac{\sigma_{1R}}{\sigma_3}) = \frac{\sigma_1}{\sigma_3} - \frac{dV}{d\varepsilon_1 V_0},$$

Therefore,

$$\tan^2(45^0 + \phi'_R/2) = \left(\frac{\sigma_1}{\sigma_3}\right)_f - \frac{1}{(d\varepsilon_1)_f}\left(\frac{dV}{V_0}\right)_f$$

- **Component of friction angle**

3 components: ϕ'_μ, ϕ'_{Re}, ϕ'_E — (ϕ'_μ + ϕ'_{Re}) → ϕ'_{CV} =(zero overall volume change)

\+   +

(1 component) ϕ'_B

– ϕ'_μ values (Terzaghi, peck, and Mesri, table 19.1)

Mineral	ϕ'_μ, Range, deg (Typical Value)
Quartz	22–35 (26)
Feldspar	36–38 (37)
Hornblende	31
Calcite	31–34 (33)
Anthracite	31
Chalk	30

- Breakage component (ϕ'_B) of ϕ'

  For quartz sands,

  σ_3 > 500psi, ϕ'_B becomes important.

  For chlorite sands,

  σ_3 > 50psi, ϕ'_B becomes important.

- Accordingly,

  For low σ_3, ϕ'_{CV} and ϕ'_E are important.

  For high σ_3, ϕ'_{CV} and ϕ'_B are important.

- For a given σ_{3c}'

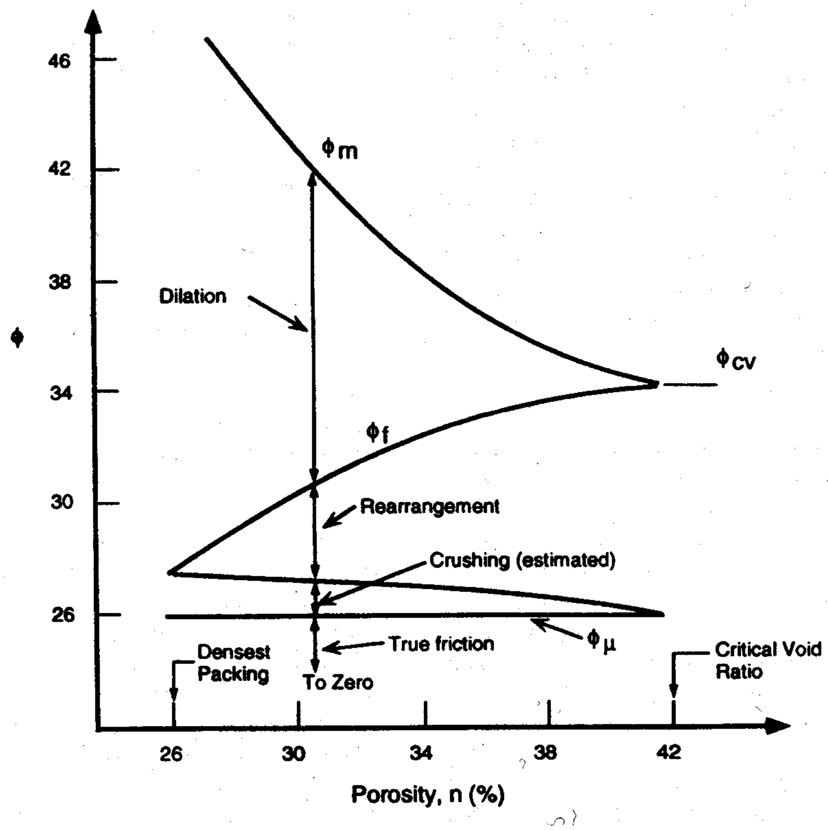
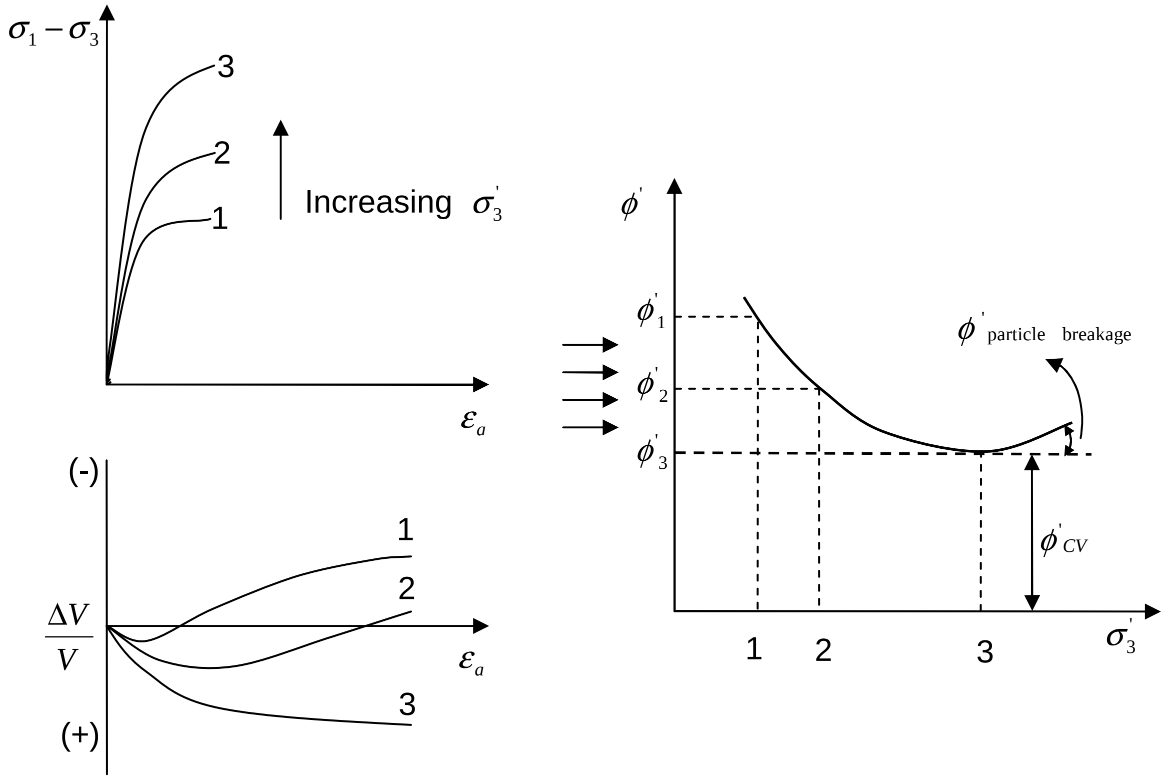


Fig 1-5. Friction angle vs. Porosity (Redrawn after Rowe(1962))

- **To test our hypothesis**

Run tests on dense samples with varying **σ_3'**.



- **The effect of relative density or void ratio, grain shape, grain size distribution, and particle size on ϕ'**

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Table 1-2. Angle of Internal Friction of Cohesionless Soils

No.	General Description	Grain Shape	D_{10} (mm)	C_u	Loose e	Loose ϕ(deg)	Dense e	Dense ϕ(deg)
1	Ottawa standard sand	Well rounded	0.56	1.2	0.70	28	0.53	35
2	Sand from St. Peter sandstone	Rounded	0.16	1.7	0.69	31	0.47	37†
3	Beach sand from Plymouth, MA	Rounded	0.18	1.5	0.89	29	—	—
4	Silty sand from Franklin Falls Dam site, NH	Subrounded	0.03	2.1	0.85	33	0.65	37
5	Silty sand from vicinity of John Martin Dam, CO	Subangular to subrounded	0.04	4.1	0.65	36	0.45	40
6	Slightly silty sand from the shoulders of Ft. Peck Dam, MT	Subangular to subrounded	0.13	1.8	0.84	34	0.54	42
7	Screened glacial sand, Manchester, NH	Subangular	0.22	1.4	0.85	33	0.60	43
8‡	Sand from beach of hydraulic fill dam, Quabbin Project, MA	Subangular	0.07	2.7	0.81	35	0.54	46
9	Artificial, well-graded mixture of gravel with sands No. 7 and No. 3	Subrounded to subangular	0.16	68	0.41	42	0.12	57
10	Sand for Great Salt Lake fill (dust gritty)	Angular	0.07	4.5	0.82	38	0.53	47
11	Well-graded, compacted crushed rock	Angular	—	—	—	—	0.18	60

Table 1-3. Summary of Factors Affecting ϕ'

Factor	Effect
Void ratio e	$e\uparrow$, $\phi\downarrow$
Angularity A	$A\uparrow$, $\phi\uparrow$
Grain size distribution	$C_u\uparrow$, $\phi\uparrow$
Surface roughness R	$R\uparrow$, $\phi\uparrow$
Water W	$W\uparrow$, $\phi\downarrow$ slightly
Particle size S	No effect (with constant e)
Intermediate principal stress	$\phi_{ps} \geq \phi_{tx}$ (see Eqs. 11-5a, b)
Overconsolidation or prestress	Little effect