Chapter I

Shear Strength of Soils

1.1 Background

(1) Principal Stresses and Mohr Circle

- **Principal planes**: Three orthogonal planes on which there are zero shear stresses.

- **Principal stresses:** The normal stresses that act on these three planes

  The largest : major principal stress, σ_1

  The smallest : minor principal stress, σ_3

  The intermediate : intermediate principal stress, σ_2

  At geostatic state in the horizontal ground, the horizontal plane and two vertical planes are principal planes.

  When $K_o(=\sigma'_h/\sigma'_v) < 1$, $\sigma'_v = \sigma'_1$, $\sigma'_h = \sigma'_3$ and $\sigma'_2 = \sigma'_3 = \sigma'_h$

  When $K_o(=\sigma'_h/\sigma'_v) > 1$, $\sigma'_h = \sigma'_1$, $\sigma'_v = \sigma'_3$ and $\sigma'_2 = \sigma'_1 = \sigma'_h$

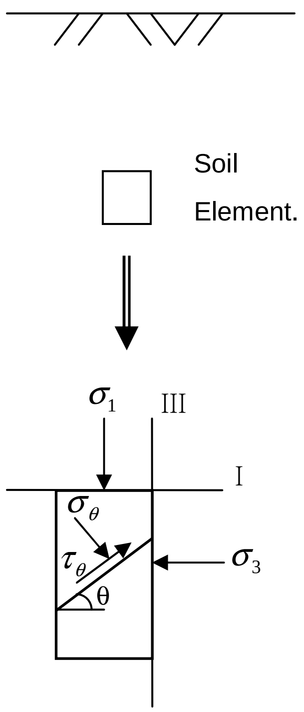
- Some rules on stress description in soil mechanics

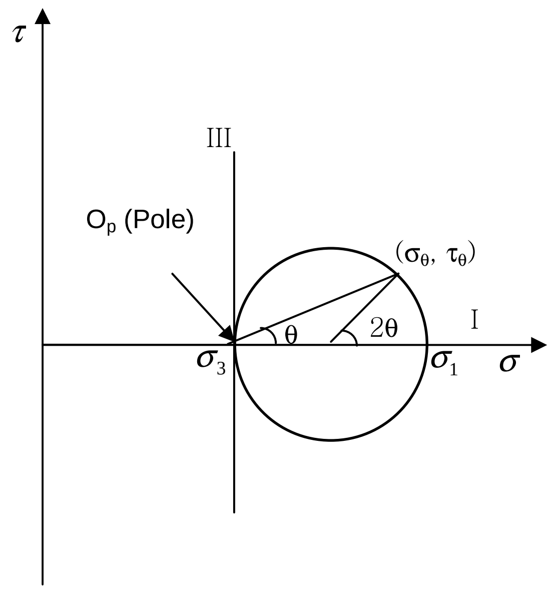
  ① Mostly, $K_o < 1$ and $\sigma_2 = \sigma_3$ or $\sigma'_2 = \sigma'_3$

  (geostatic condition and axisymmetric condition).

  ② Stresses are positive when compressive. Shear stress τ is positive when counterclockwise.

- **Mohr Circle**: The graphical representation of stress state of material element.
  - It can be determined with the normal and shear stresses of two orthogonal planes.
  - Given a Mohr circle, it is possible to find stresses in any direction by graphical construction using the Mohr circle. (Using origin of plane (pole))
  - The origin of plane (or the pole) is a point on the Mohr circle where a line through O_p and any point of the Mohr circle is parallel to the plane on that given point.



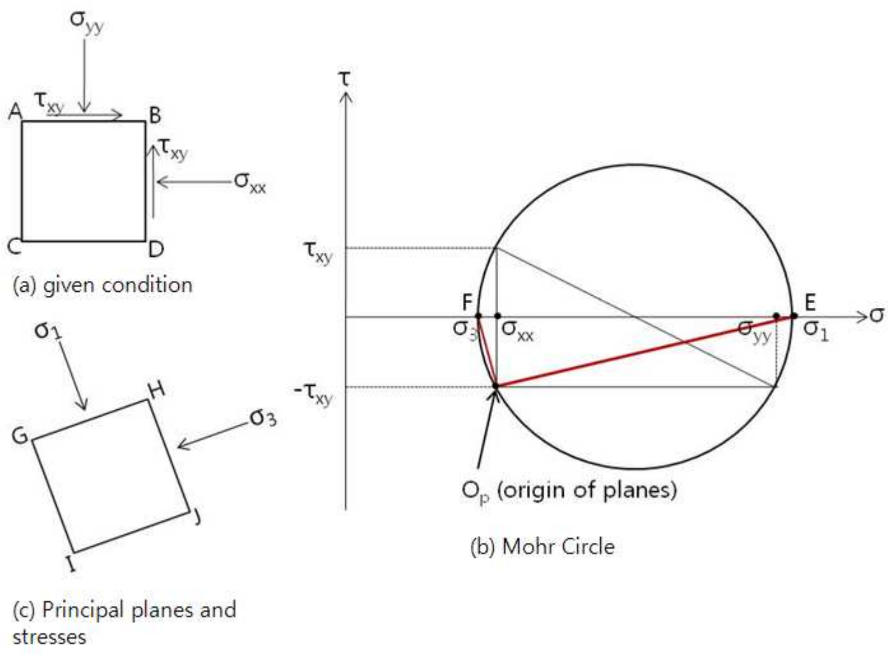


$$\sigma_\theta = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta$$

$$\tau_\theta = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta$$

- The maximum shear stress ($\tau_{max} = (\sigma_1 - \sigma_3)/2$) occurs on planes lying at $\pm 45^o$ to the major principal direction.

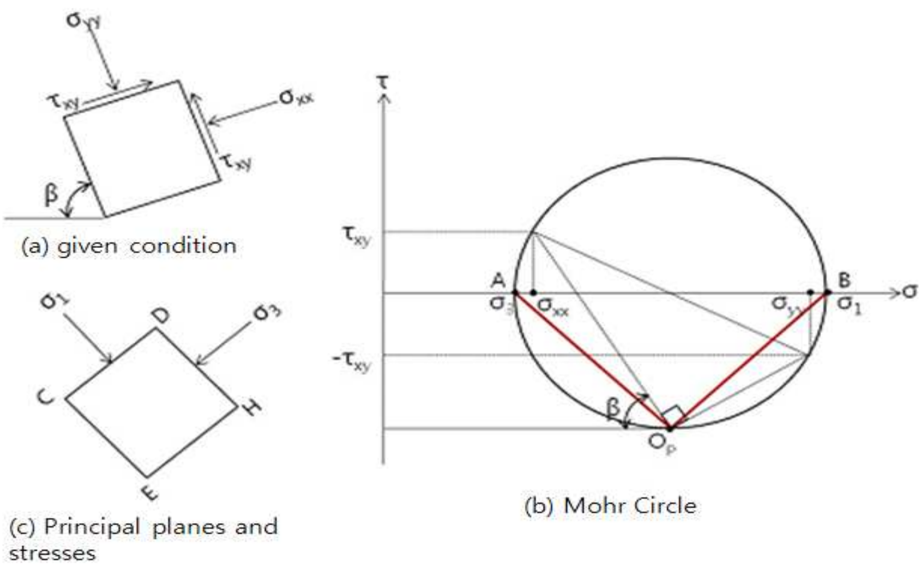
Ex1)



$\overline{GH}$ is parallel to $\overline{O_pE}$ and $\overline{HJ}$ is parallel to $\overline{O_pF}$.

Fig E1. The way to find the plane that σ_1, σ_3 act.

Ex2)



$\overline{CD}$ is parallel to $\overline{O_pB}$ and $\overline{DH}$ is parallel to $\overline{AO_p}$.

Fig E2. The way to find the plane that σ_1 and σ_3 act.

- ***p-q* diagrams** :
  - The stress state is plotted with stress point whose coordinates are

$$p = \frac{\sigma_1 + \sigma_3}{2}$$

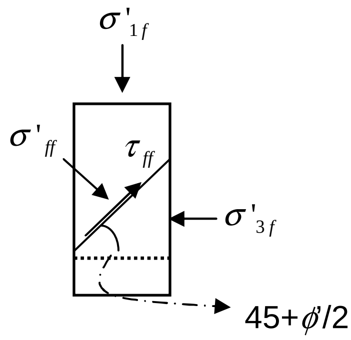
$$q = \pm \frac{\sigma_1 - \sigma_3}{2} \quad \begin{cases} + \text{ if } \sigma_1 \text{ is inclined equal to or} \\ \text{less than } \pm 45^\circ \text{ to the vertical} \\ \\ - \text{ if } \sigma_1 \text{ is inclined less than} \\ \pm 45^\circ \text{ to the horizontal} \end{cases}$$

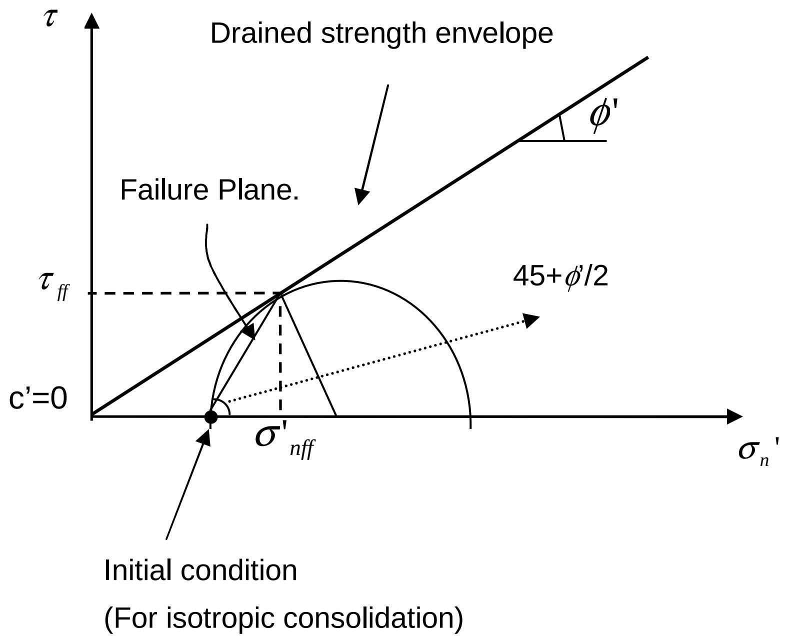
  - Effective way to represent, on a single diagram, many states for a given specimen of soil.

- Representing the change of stress state during loading.
  ① Vector curve
  ② Stress path

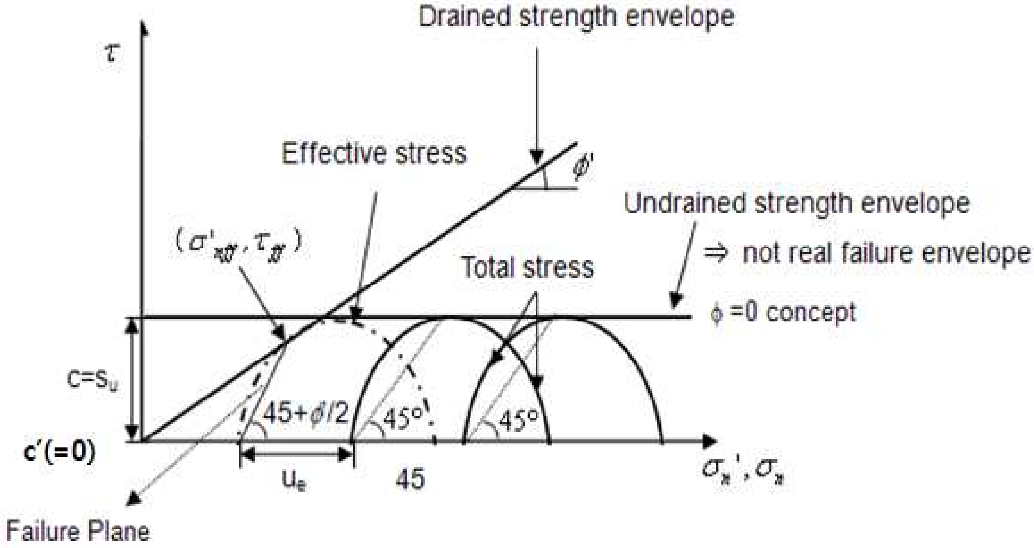
- Locations of failure plane and failure stress conditions are defined in terms of effective stresses. (→ Based on drained strength envelope)

① Drained tests (CD) (σ_1 acts on horizontal plane.)





② UU tests



Notes : $s_u = \dfrac{\sigma_{1f} - \sigma_{3f}}{2}$

τ_{ff} = shear stress at failure at failure plane = $s_u \cos\phi < s_u$

(2) Vector Curves

- **Vector Curves** : " Locus of stress states (shearing stresses and effective normal stresses) on a potential failure plane for loading to failure."

Shearing Phase (**D** : Drained, **U** : Undrained)

Loading Method (**C** : Compression, **E** : Extension)

Ex) **CID TXC**

Testing device (**TX** : Triaxial, **PS** : Plane Strain)

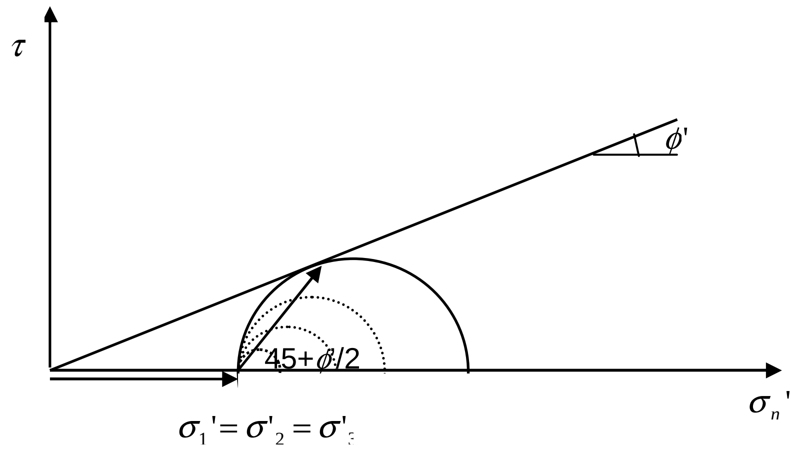
Consolidation phase

CI : Isotropic Consolidation ($\sigma_1=\sigma_2=\sigma_3=\sigma_c$)

CA : Anisotropic Consolidation ($\sigma_1 \neq \sigma_2=\sigma_3=\sigma_c$)

CK$_0$: Consolidation with zero lateral strain

U : Unconsolidation



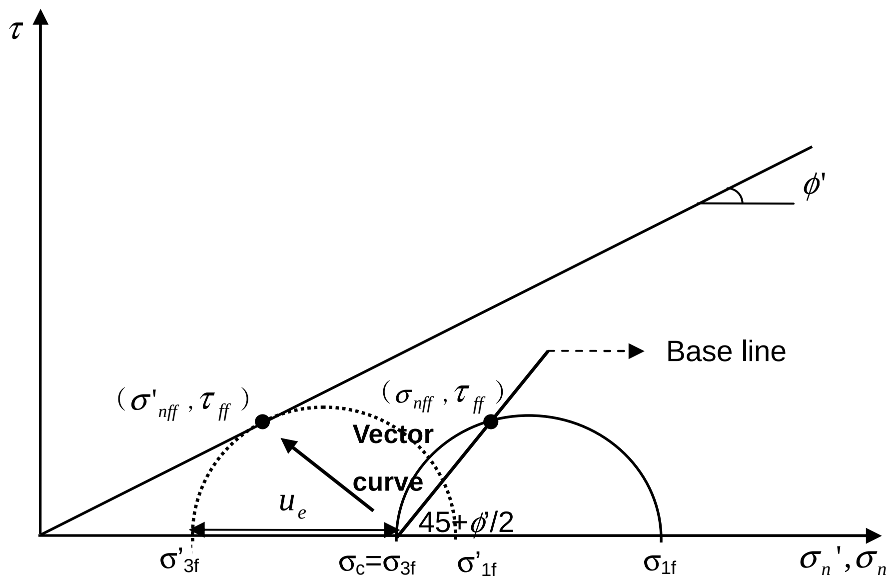
To plot the curves,

1. Assume ϕ'.
2. Plot Mohr circle for each load. (σ_1' and σ_3' are given from tests)
3. Find τ, σ_n' on potential failure plane for each circle.
4. Connect points.
5. Redraw if vector curve at peak or at large strain dose not match with assumed failure envelope.

↗ Pore pressure measurement

Ex) CIU TXC (σ_1' , σ_3' and u_e are given from tests)

Look at 1 Mohr circle (at failure).

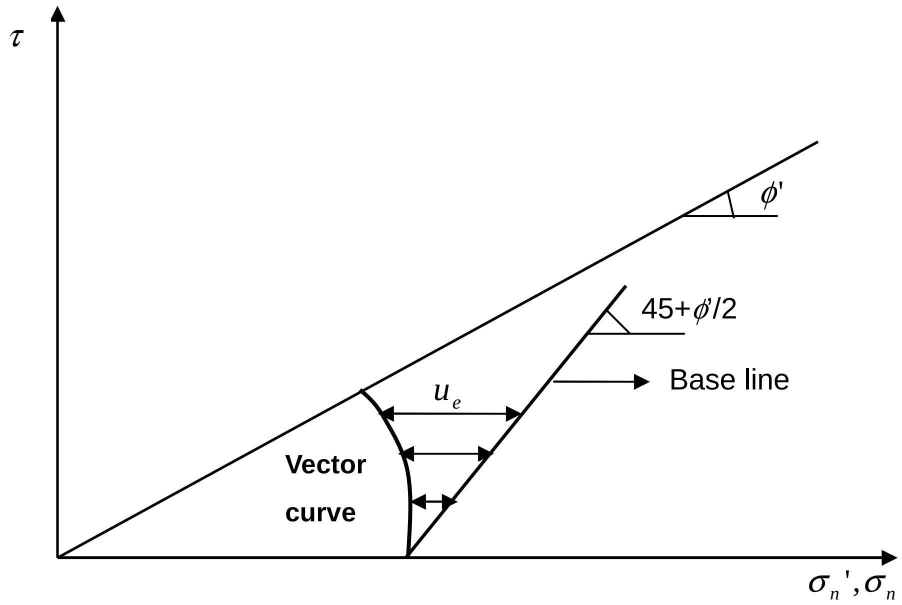


1. Assume ϕ'.
2. Plot total Mohr circle.
3. Find $\tau(=\tau_{ff})$, $\sigma_n(=\sigma_{nff})$. (→which is located at base line)
4. Find $\tau(=\tau_{ff})$, $\sigma'_n(=\sigma'_{nff})$ with measured pore pressure.

$$\sigma'_3(=\sigma'_{3f}) = \sigma_c(=\sigma_{3f}) - u_e \quad \rightarrow \quad \sigma'_n(=\sigma'_{nff}) = \sigma_n(=\sigma_{3nff}) - u_e$$

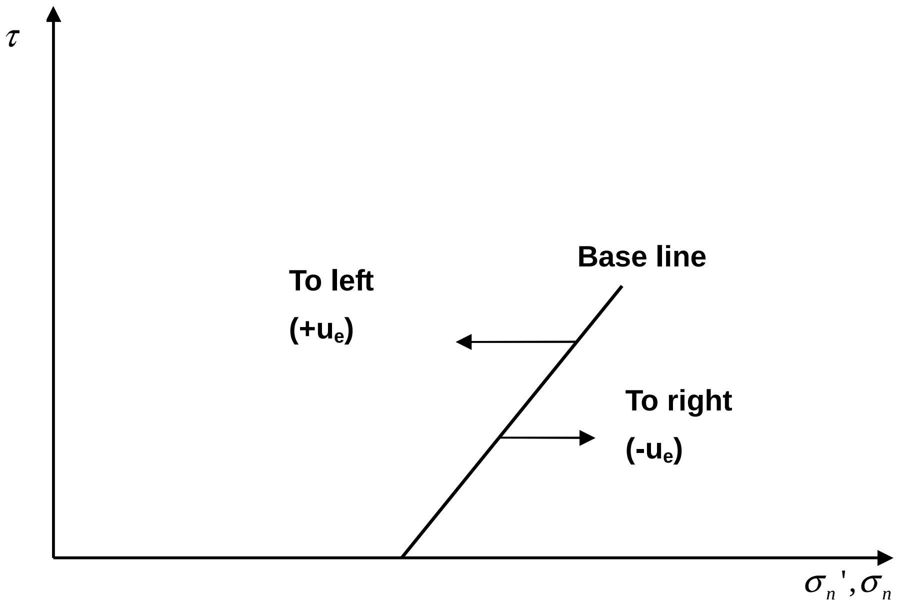
To draw vector curve,

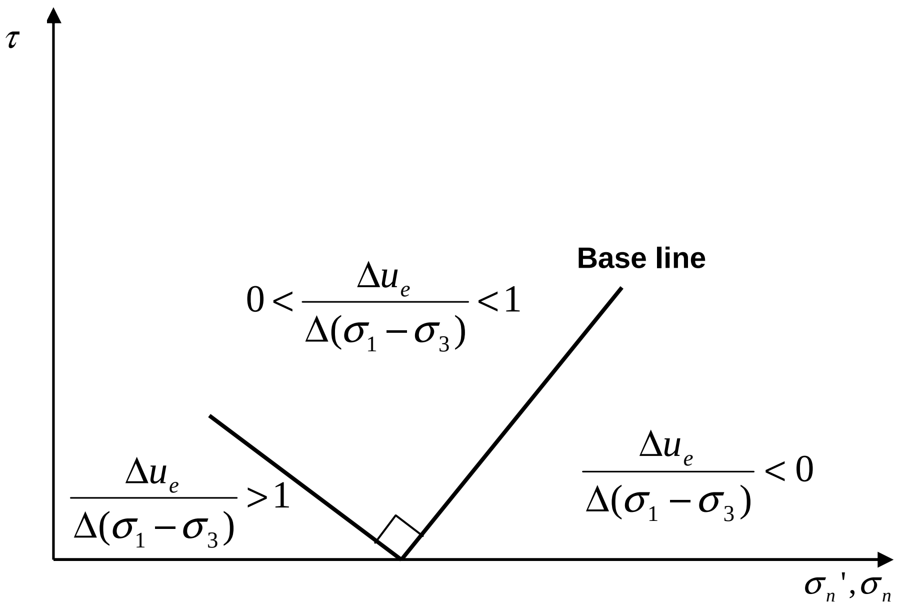
1. Draw base line based on assumed ϕ'.
2. Account for u_e at each stress level. → Vector curve.



- **Major Points**

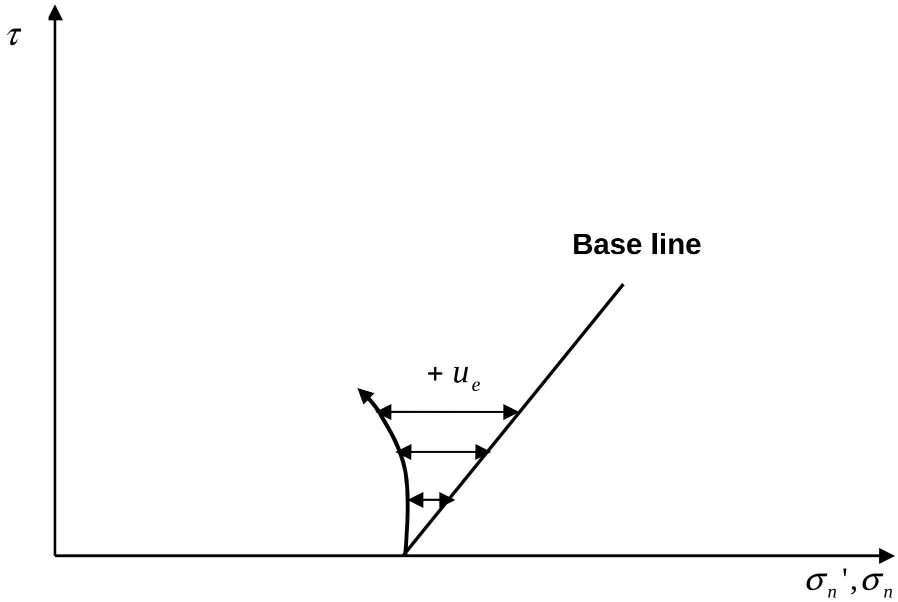
① Pore pressure development



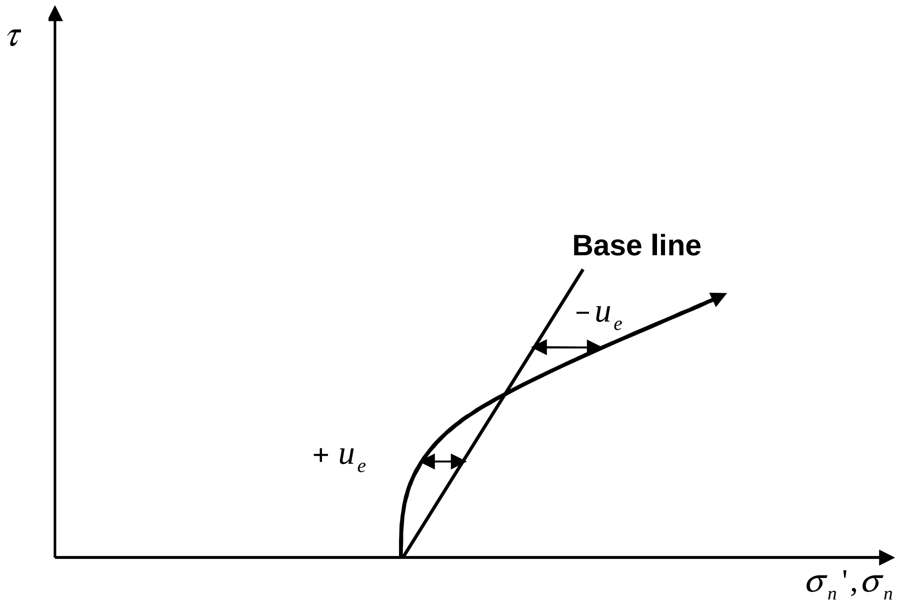


② Typical behavior in CIU TXC tests

- Loose sands,
  Normally Consolidated Clays (NC)



- Dense sands,
  Heavily Overconsolidated Clays



- Typical test (CIU) results for clay samples by varying consolidation `pressure.