Fusion Reactor Technology I
(459.760, 3 Credits)

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Contents

Week 1. Magnetic Confinement

Week 2. Fusion Reactor Energetics (Harms 2, 7.1-7.5)

Week 3. How to Build a Tokamak (Dendy 17 by T. N. Todd)

Week 4. Tokamak Operation (I): Startup

Week 5. Tokamak Operation (II):

Basic Tokamak Plasma Parameters (Wood 1.2, 1.3)

Week 7-8. Tokamak Operation (III): Tokamak Operation Mode

Week 9-10. Tokamak Operation Limits (I):

Plasma Instabilities (Kadomtsev 6, 7, Wood 6)

Week 11-12. Tokamak Operation Limits (II):

Plasma Transport (Kadomtsev 8, 9, Wood 3, 4)

Week 13. Heating and Current Drive (Kadomtsev 10)

Week 14. Divertor and Plasma-Wall Interaction

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Plasma as a Complex System

- **High-temperature plasma, confined by a magnetic field, is an exceptionally unusual physical object**
  → complex physical system

- Presence of macroscopic instabilities
- Local entropy production due to local plasma transport
- Rare Coulomb collisions (anomalous transport)
- Non-linear phenomena (noise source) in the edge plasma propagating inside the plasma core leading to transport enhancement
- Heating resulting in additional noise generation

cf) OH heating: drift current velocity of electrons ~ 10^5 m/s

≪ sound velocity (j ~ 1 MA/m^2, n_e ~ 10^{20} m^{-3})

$$V_{Alphen} = \sqrt{\frac{B_0^2}{\mu_0 \rho_0}}, \quad V_{adiabatics\,ound} = \sqrt{\frac{\gamma p_0}{\rho_0}}$$

Plasma as a Complex System

- A rational approach to study complex systems consists of a large number of experiments aimed at understanding empirical laws supported by development of a theoretical description and computer models.
- All this is actively used in modern tokamak studies.
- As experience with other complex systems shows, the general method of scaling and dimensional approach represents a powerful tool for their description.

Dimensional Analysis of Tokamaks

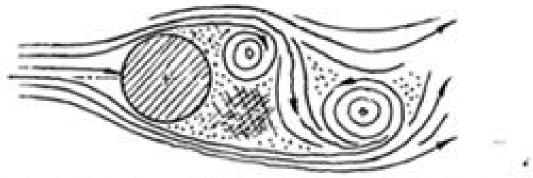
- **Dimensional approach**

- All the laws of physics are based on mechanics.
- Mechanics uses conventionally chosen units for mass, length, and time.
- The objective laws of nature cannot depend on those units. These laws are invariant with respect to variations of measurement units chosen by man.
- This invariance is seen more precisely when non-dimensional combinations of dimensional values are used.
- The non-dimensional parameters define the internal physics of a complex system: indicators of the fundamental state of the system
- Dimensional parameters look like some projection of a given system on the external world.

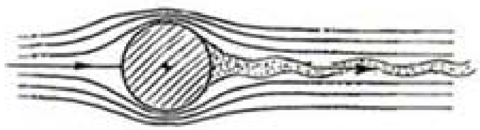
Dimensional Analysis of Tokamaks

- **Dimensional approach - Example**
  - Reynolds number

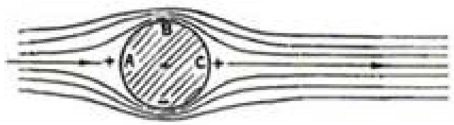
$$\mathrm{Re} = \frac{\rho v_s^2 / L}{\mu v_s / L^2} = \frac{\rho v_s L}{\mu} = \frac{v_s L}{\nu} = \frac{\text{Inertial forces}}{\text{Viscous forces}}$$



(c) CYLINDER BETWEEN $R_d = 10^4$ and 10^5; VORTEX STREET WITH $C_{D} = 1.2$.



(B) CYLINDER ABOVE CRITICAL REYNOLDS NUMBER WITH $C_{D} = 0.3$.



(A) FLOW PATTERN OF CIRCULAR CYLINDER IN NON-VISCOUS FLOW; NO DRAG.

ρ: density of the fluid
v_s: mean velocity of the object relative to the fluid
L: travelled length of the fluid
μ: dynamic viscosity
ν: kinetic viscosity

Variation in flow pattern and drag coefficients for cylinders with increase in Reynolds number (Hoerner 1965)

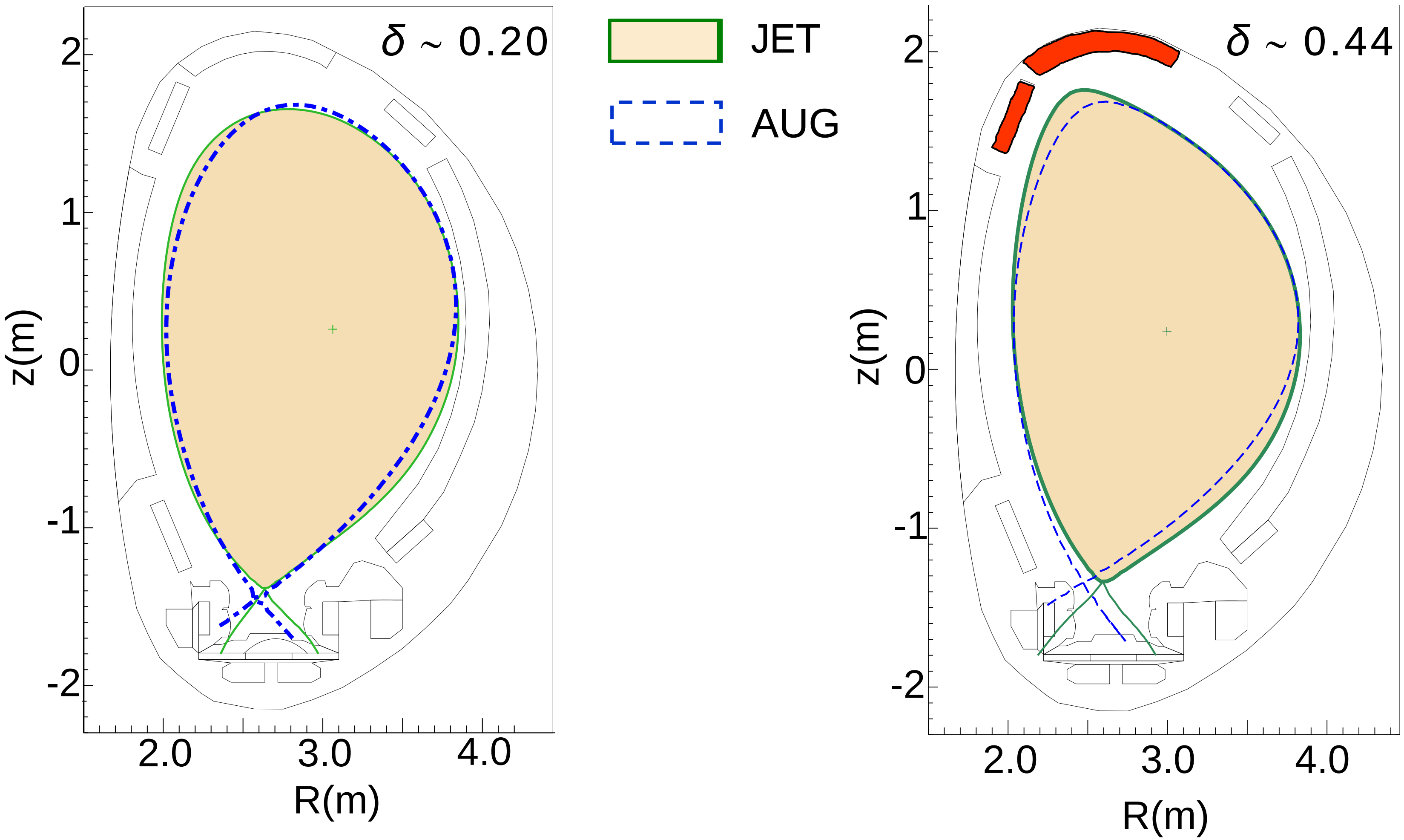
Dimensional Analysis of Tokamaks

- **Dimensional approach**

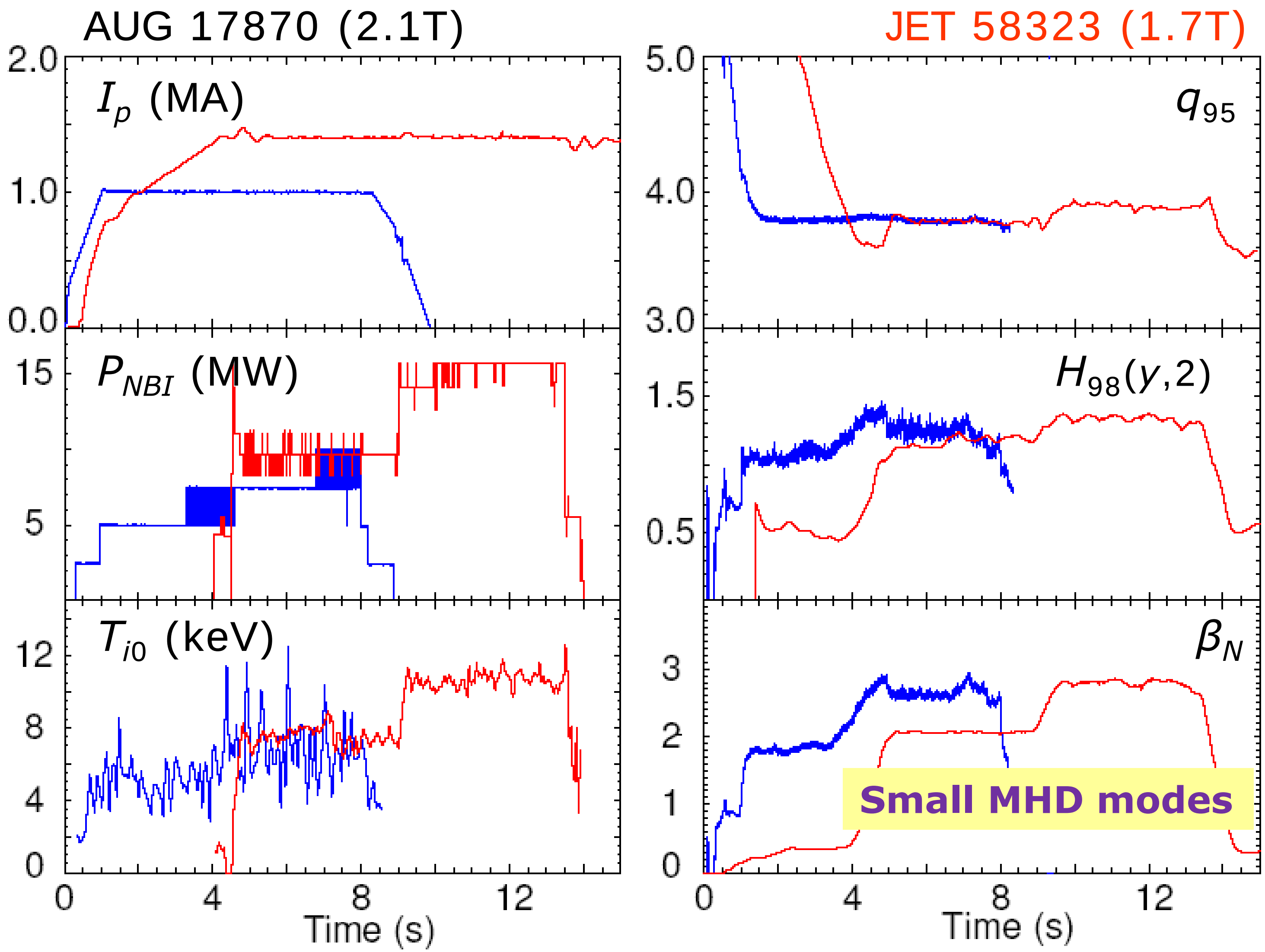
- Being immersed in the external physical world, each complex system can possess a non-unique set of dimensional parameters.
- For a given set of dimensionless parameters the family of systems can exist with different sets of dimensional parameters.
  $\rightarrow$ Self-similarity
- Therefore, all the objective laws of physics may be presented as relations between non-dimensional parameters.
- Dimensional analysis should always be based on reasonable physical parameters which are specific for each particular case. Such an approach can allow us to pick out the most relevant parameters and to drop the unimportant ones.

Identity (Similarity) Experiments

- **Plasma shapes used in JET compared to ASDEX Upgrade**



Identity (Similarity) Experiments



Dimensional Analysis of Tokamaks

- **Assumptions**

- Circular CX, steady-state with purely OH heating, $T_e = T_i = T$, no impurities, atomic processes not important (radiation, recycling, etc), fully ionized hot plasma

- Under these conditions, one can expect the existence of self-similar self-organized plasma states, if they have the same macroscopic non-dimensional parameters.

Dimensional Analysis of Tokamaks

- All the dimensional parameters

$$a,\ R,\ B_T,\ B_p,\ m_e,\ m_i,\ e,\ c,\ n,\ T$$

- Non-dimensional parameters for tokamak plasma of circular CX

- independent of each other

$$A = R/a$$

$$q_a = aB_T / RB_p$$

$$m_e / m_i$$

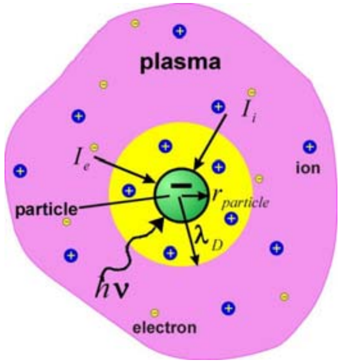
$$\beta = p/2\mu_0 B_T^2 = nT/\mu_0 B_T^2$$

$$N_D = n\left(\varepsilon_0 kT/ne^2\right)^{3/2} \to \infty$$

$$\Pi = a^2/\Lambda^2 = na^2 r_0$$

$$\bar{\nu}^* = avg\left[\left(\frac{R}{r}\right)^{3/2} q(r)R/\lambda(r)\right]$$

$$K = a^2/\rho_i^2 = a^2 e^2 B_T^2 / 2Tm_i >> 1$$



http://www.mpe.mpg.de/pke/PKE/Paper_THOMAS-2000/index.html

$$\Lambda = c/\omega_{pe} = \sqrt{c^2 m_e \varepsilon_0 / ne^2}$$ internal characteristic length

$$r_0 = e^2/c^2 m_e$$ classical electron radius

Dimensional Analysis of Tokamaks

- If only B_T is free

$$\beta \propto nT / B_T^2 \qquad a \propto B_T^{-4/5}$$

$$\Pi \propto na^2 \qquad \Pi \propto B_T^{8/5}$$

$$\overline{\nu}^* \propto an / T^2 \qquad \overline{\nu}^* \propto B_T^{2/5}$$

Dimensional Analysis of Tokamaks

- **Murakami and Hugill Numbers**

- consider atomic processes

$$a,\ R,\ B_T,\ B_p,\ m_e,\ m_i,\ e,\ c,\ n,\ T,\ \hbar$$

- To evaluate the role of atomic processes appropriate power losses with Joule heating power should be compared.

$$P_{OH} = \eta j^2 = \frac{m_e \nu_{ei}}{ne^2}\left(\frac{B_T}{\mu_0 q_a R}\right)^2 = \frac{L\hbar^3}{m_e e^4} F(T_*) \frac{B_T^2}{\mu_0^2 q_a^2 R^2}$$

$$T_* = T/\varepsilon_a$$

$L \sim 12.3$ Coulomb logarithm

$$r_a = \frac{\hbar^2}{m_e e^2},\quad v_a = \frac{e^2}{\hbar},\quad \varepsilon_a = \frac{m_e e^4}{\hbar^2}$$

Atomic units for length, velocity, energy

$$P_R = n^2 r_a^2 v_a \varepsilon_a G(T_*) = n^2 \frac{e^2 \hbar}{m_e} G(T_*)$$

Losses due to radiation and ionization

Dimensional Analysis of Tokamaks

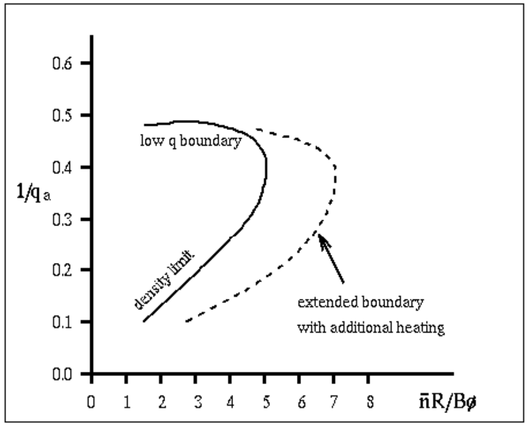
- If P_R becomes comparable to P_{OH}, atomic processes start to play a significant role.

$$P_R / P_{OH} = H^2 G(T_*) / F(T_*) \qquad H = \frac{e\gamma n q_a R}{B_T \sqrt{L}}, \quad \gamma = \frac{e^2}{\hbar c}$$

- The role of atomic processes is defined by a non-dimensional parameter H (Hugill number): as increasing H, the role of atomic processes increases compared with OH heating

$$H = \frac{n q_a R}{B_T}$$

$$M = \frac{nR}{B_T}$$ Murakami number



Operation Limits

- **Density ranges in a tokamak discharge**

- There exist a lower and an upper density limit at a given I_p.

- **Low densities**

- e-i collision frequency not sufficient to prevent
  the generation of run-away or accelerated electrons
- run-away electrons produced by the inductive E field
- run-away electrons spoil the discharge characteristics
  and may be dangerous for the vacuum chamber

- **High densities**

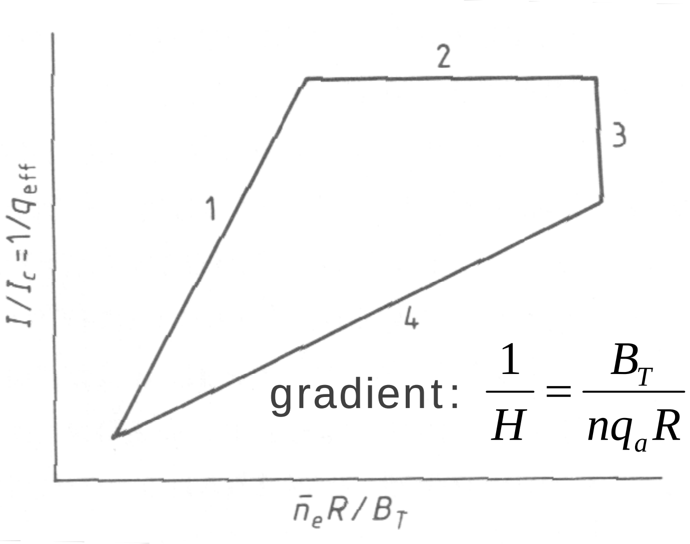
- atomic processes (radiation, CX, neutral atom ionization)
  at the plasma edge become rather important
- atomic processes can lead to contraction of the plasma column
  (decrease of the effective plasma radius)
  → danger of kink instability becomes real

Operation Limits

- **Hugill plot**

- limited operational region on the current-density plane
- non-dimensional current .VS. non-dimensional density

non-dimensional current



gradient: $\frac{1}{H} = \frac{B_T}{n q_a R}$

non-dimensional density

$$q_a = \frac{aB_T}{RB_p} = \frac{aB_T}{R\mu_0 I / 2\pi a} = \frac{2\pi a^2 B_T}{\mu_0 I R}$$

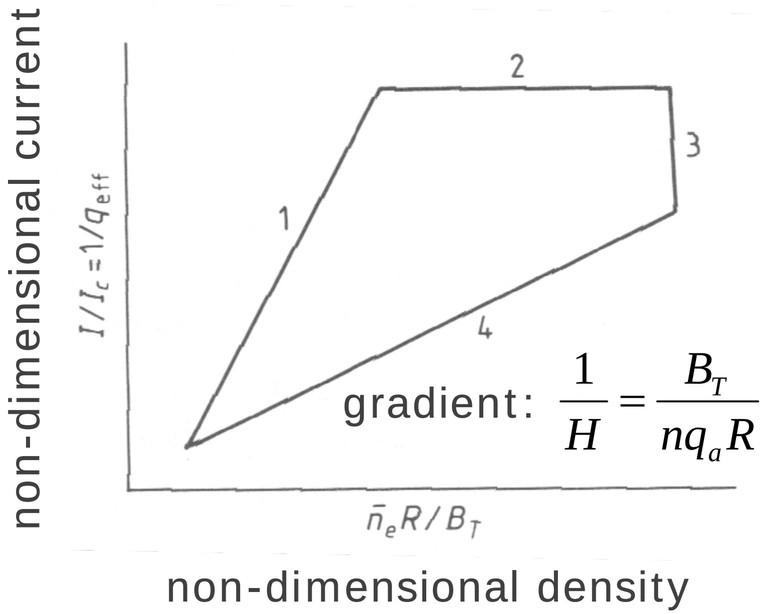
$$\frac{1}{q_a} = \frac{I}{2\pi a^2 B_T / \mu_0 R} = \frac{I}{I_c}$$

$$M = \frac{\bar{n} R}{B_T}$$ Murakami number

Operation Limits

- **Hugill plot**

- Tokamak operational domain on the current-density plane is restricted by four limits.



1: limit of run-away electrons at low density $j/en = v_e^{thermal} = \sqrt{2T/m_e}$

2: current limit due to the MHD-instability

3: Murakami limit at high density (at the maximal permissible plasma current): radiative power balance

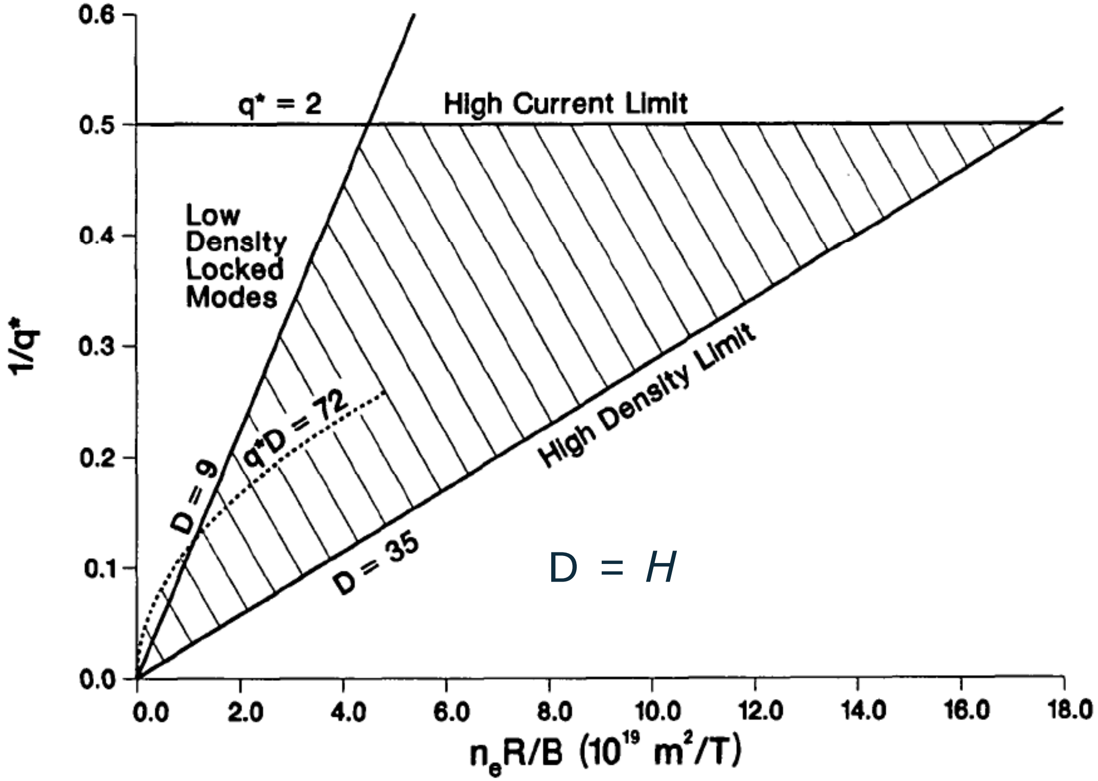
4: Hugill density limit where H ($H = q_{eff}M$) remains constant ($n \sim I$): confinement/disruptive limit

- The limiting density is determined by the power balance on the plasma periphery (by balance of the energy flow from the central region and radiation and ionization losses).
- The density limit usually increases with additional heating as $P^{1/2}$.

Operation Limits

- **Hugill plot**

- limited operational region on the current-density plane
- non-dimensional current .VS. non-dimensional density



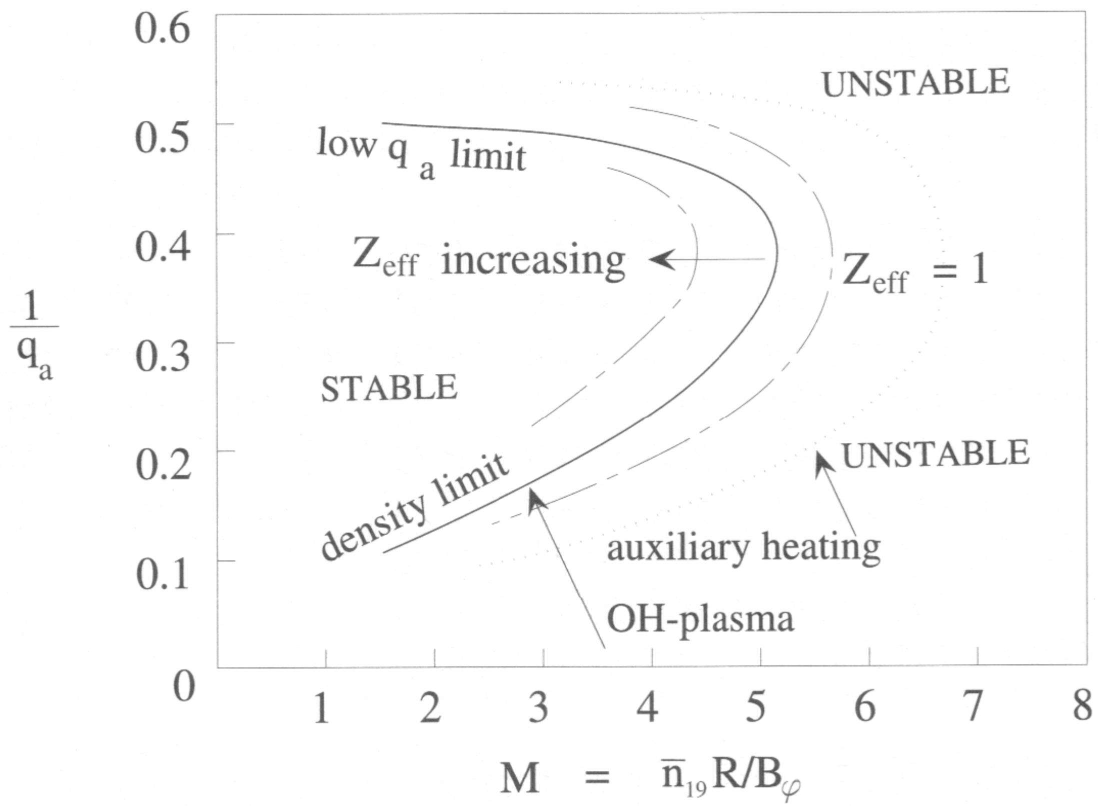
DIII-D NATIONAL FUSION FACILITY SAN DIEGO

J. T. Scoville, Nuclear Fusion ***31*** *875 (1991)*

Operation Limits

- **Hugill plot**

- limited operational region on the current-density plane
- non-dimensional current .VS. non-dimensional density



Operation Limits

- **Greenwald density limit**

$$q_a = \frac{aB_T}{RB_p} = \frac{aB_T}{R\mu_0 I / 2\pi a} = \frac{2\pi a^2 B_T}{\mu_0 IR}$$

$$\frac{1}{q_a} = \frac{\mu_0 IR}{2\pi a^2 B_T} \approx \frac{M}{15} = \frac{\bar{n}R}{15B_T}$$

$$\bar{n}_{20} \approx \frac{I}{\pi a^2}$$ Greenwald limit

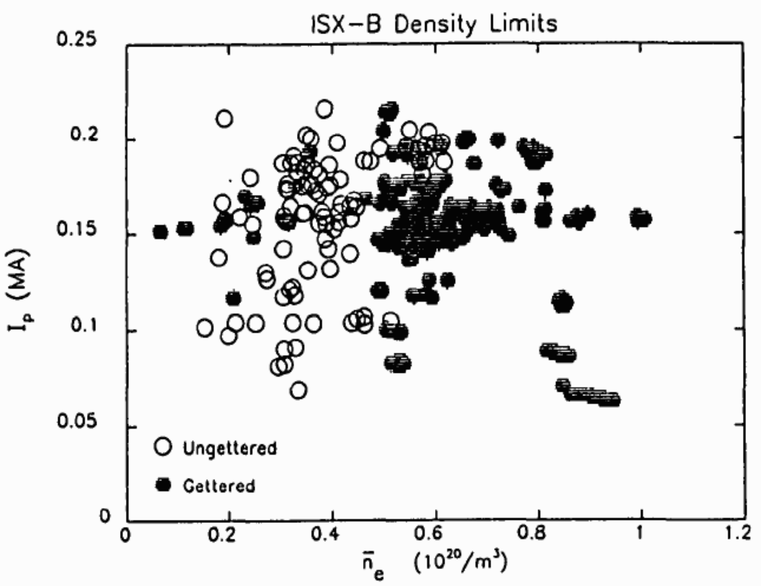


FIG. 10. ISX-B data comparing gettered and non-gettered discharges. While the highest densities are reached in gettered plasmas, the $n \sim I_p$ limit is essentially unaffected.

- For stability observations yield the conditions

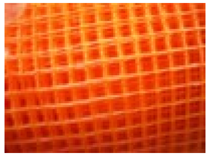
$$\bar{n}_{20} \le \frac{I}{\pi a^2}, \quad q_a > 2$$

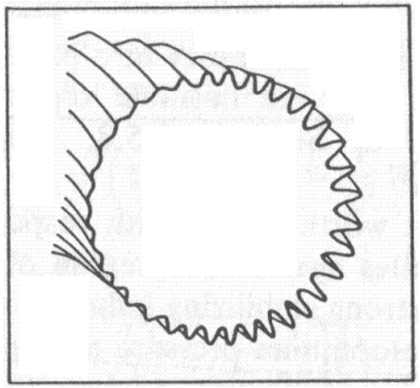
*M. Greenwald, Nuclear Fusion **28** 199 (1988)*

Operation Limits

- **Pressure Limit**

- related to the ballooning instability occurring due to convex magnetic lines of the outer region:
  swelling on magnetic surface at the high pressures
- force balance between the cause for swelling (plasma pressure gradient) and the magnetic tension





$$\frac{p}{a} = \frac{B_T^2}{2\mu_0 qR}$$

$$\beta_c = \frac{p}{B_T^2/2\mu_0} = \frac{a}{qR} = \frac{a}{2\pi a^2 B_T R/\mu_0 IR}$$

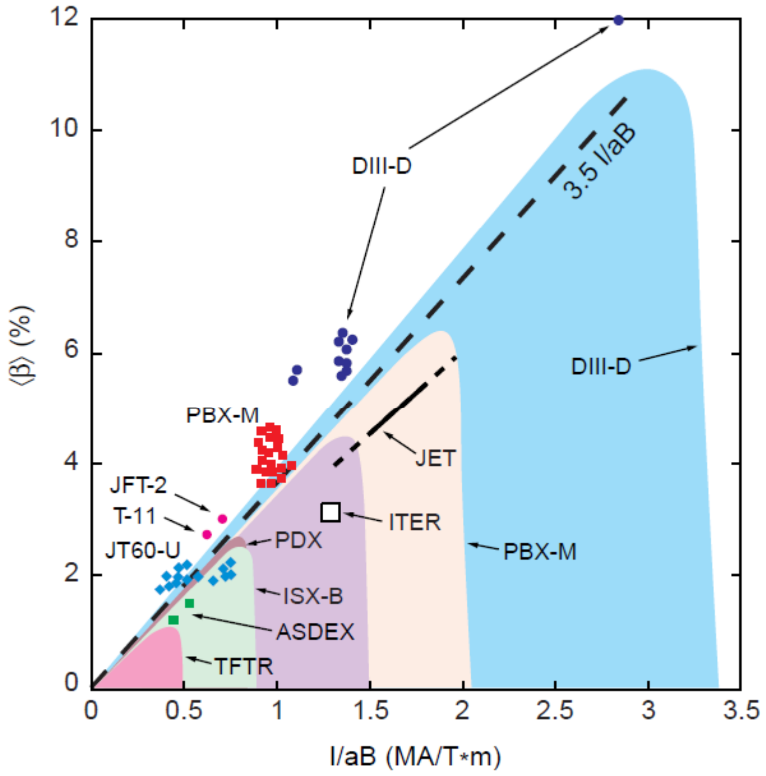
$$= g\frac{I}{aB_T} \equiv gI_N$$

$$I_N = \frac{I}{aB_T} = \frac{I}{I_c}\frac{5b}{R}$$

$g = \beta_N$: Troyon factor

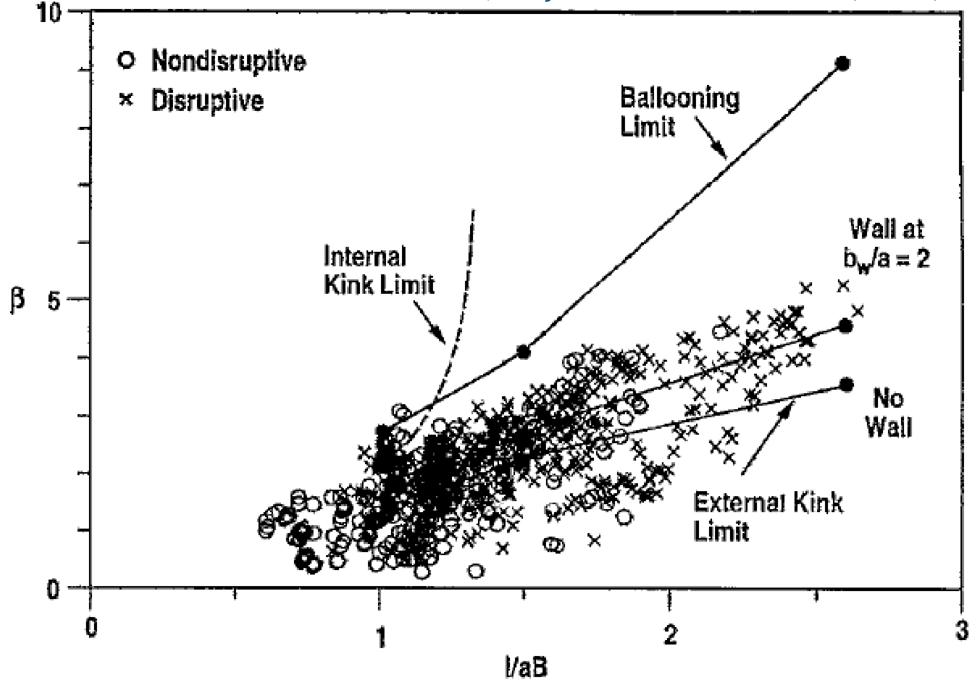
Operation Limits

$$\beta_c = \beta_N \frac{I}{aB_T}$$



ITER Physics Basis, Nuclear Fusion ***39*** *2261 (1999)*

E. J. Strait, Phys. Plasmas ***1*** *1415 (1994)*



Operating range of high beta discharges in PBX compared to calculated stability limits, including the ideal n = 1 external kink limit with a conducting wall at twice the plasma minor radius and without a wall

Operation Limits

- **Pressure Limit**

- If g is constant, for the most effective use of the B_T it is preferable to have the values of β as high as possible.

→ high I_N

$$\beta_c = gI_N = g\frac{I}{I_c}\frac{5b}{R}$$

- Since I/I_c is limited by the upper current limit on the Hugill diagram, b/R should be maximized.

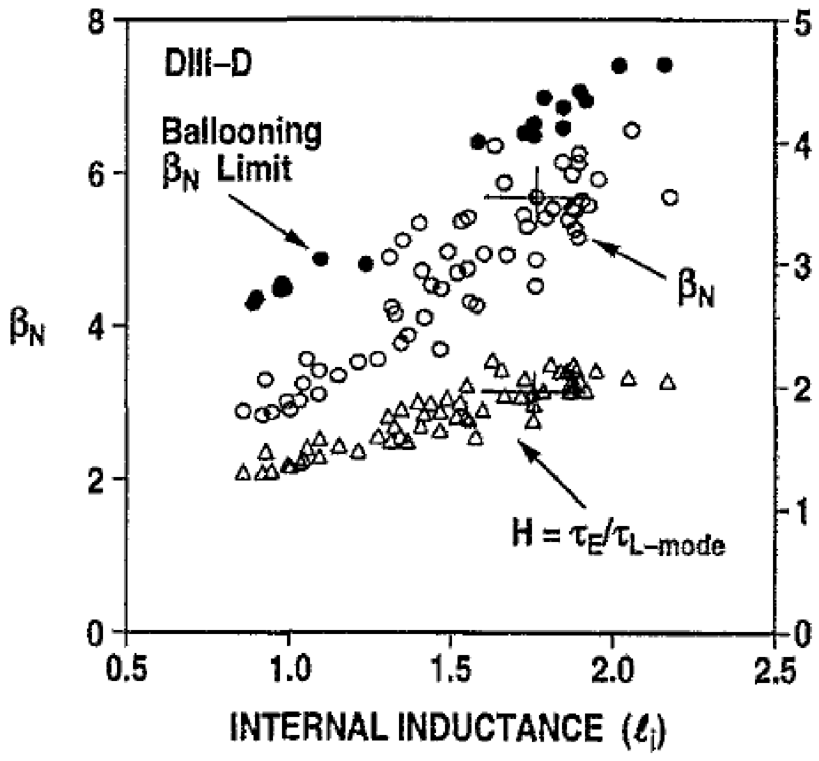
→ The column should be elongated vertically as much as possible.

- The experimental data for critical β are summarized by a simple empirical formulae

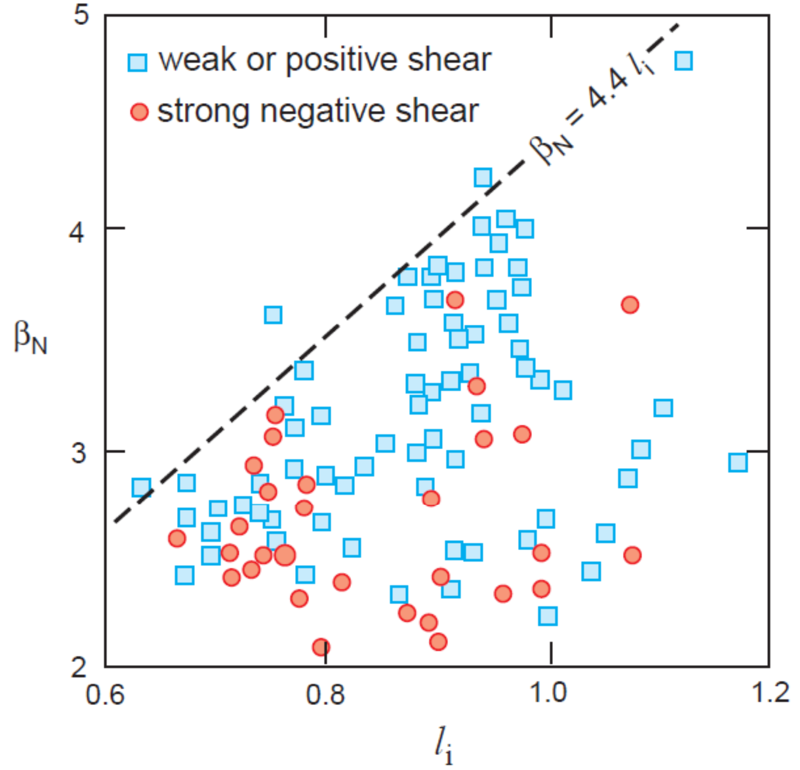
$$\beta_c = \beta_N\frac{I}{aB_T} = 4l_i\frac{I}{aB_T}$$

Operation Limits

$$\beta_N \approx 4l_i$$



E. J. Strait, Phys. Plasmas 1 1415 (1994)

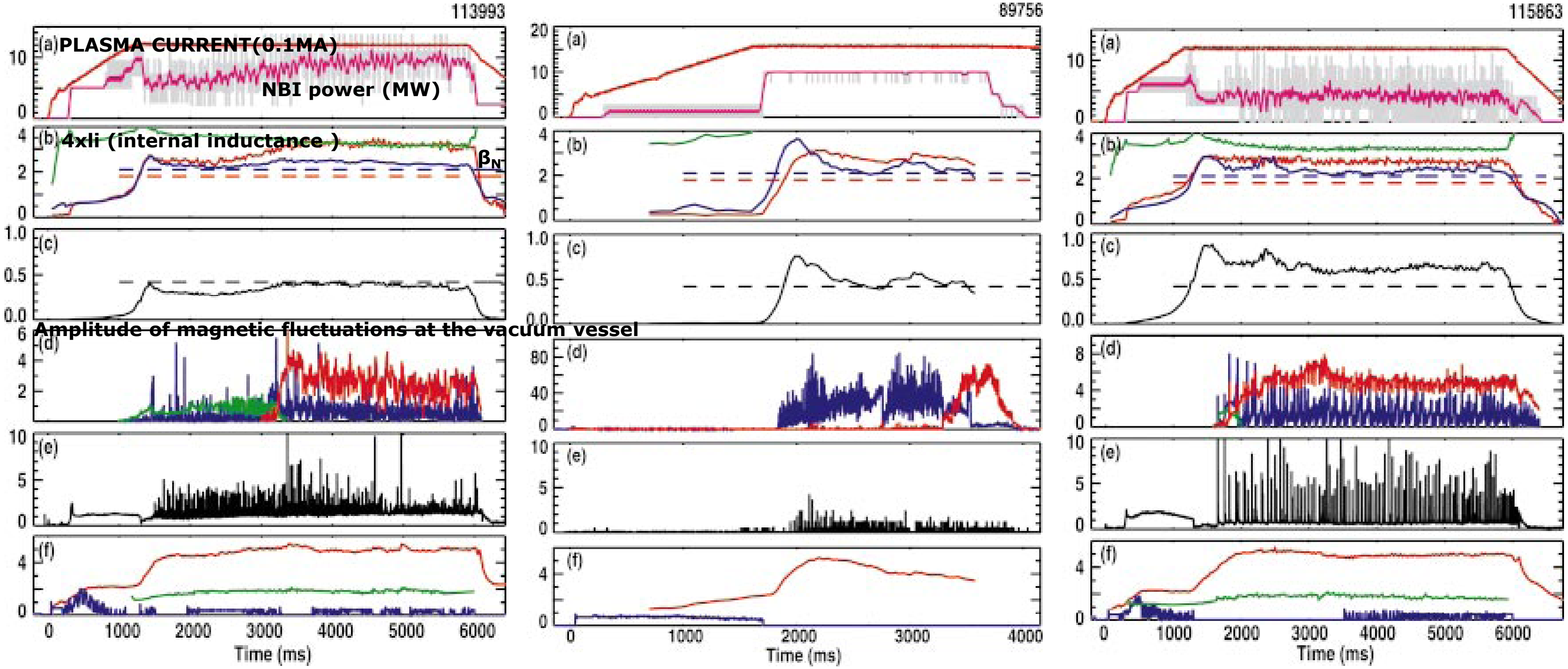


ITER Physics Basis, Nuclear Fusion **39** 2261 (1999)

Operation Limits

- **DIII-D hybrid modes**

$q_{95}>4$ without sawteeth　　$q_{95}=3.6$ without sawteeth　　$q_{95}=3.2$, $\beta_N=2.7$, $H_{89P}=2.3$



limited by tearing modes　　limited by fishbones　　limited by sawteeth