(4) Stress – Stress Behavior (Stiffness) of Soils.

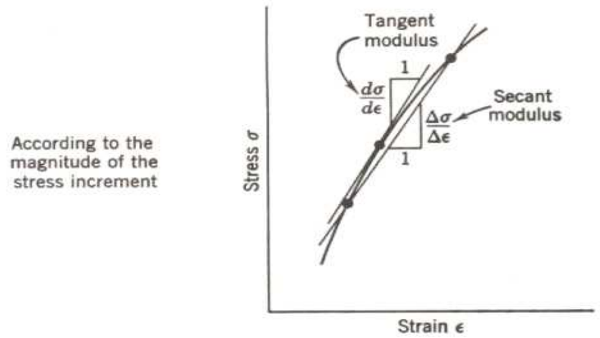
- **Stress-Strain Behavior**

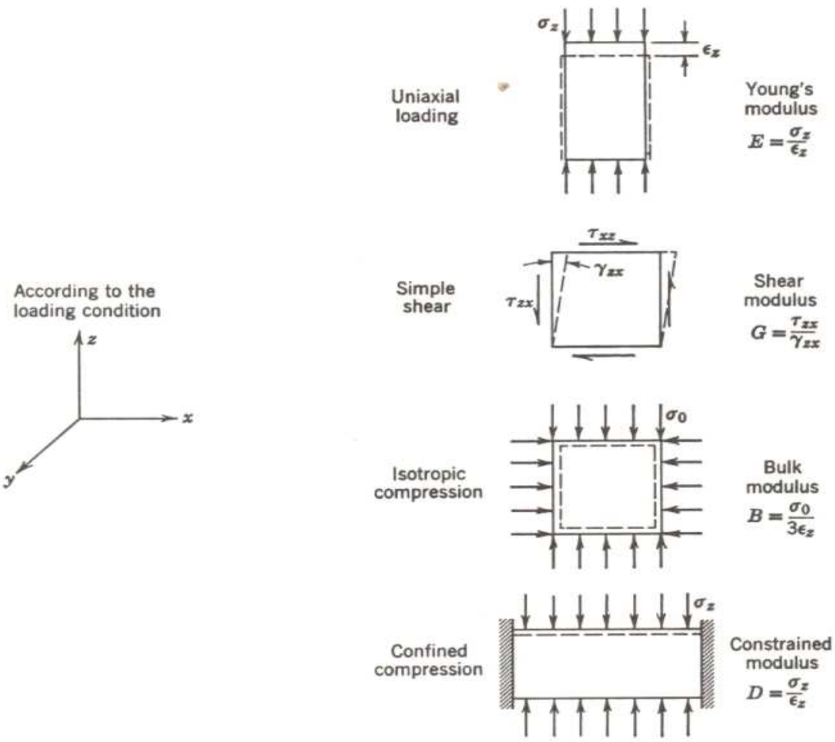
  → depends on the composition, void ratio, stress history of the soil, manner in which the stress applied, and so on.

  → describes based on the theory of elasticity; Nonlinear stress-strain curves of a solid are linearized , i.e., replaced by straight lines.

  → Modulus and Poisson's ratio are used for describing stress-strain behavior but they are not constant.

- **Concepts from the theory of elasticity**





ⅰ) For uniaxial case,

$$\varepsilon_z = \frac{\sigma_z}{E}$$

$$\varepsilon_x = \varepsilon_y = -\mu\varepsilon_z$$

(If shear stresses τ_{zx} are applied, then shear distortion $\gamma_{zx} = \frac{\tau_{zx}}{G}$ and $G = \frac{E}{2(1+\mu)}$)

ⅱ) For an elastic material with all stress components acting,

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \mu(\sigma_y + \sigma_z)] \quad \text{--------- Eq. (1)}$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \mu(\sigma_z + \sigma_x)] \quad \text{--------- Eq. (2)}$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \mu(\sigma_x + \sigma_y)] \quad \text{--------- Eq. (3)}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \quad \text{--------- Eq. (4)}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} \quad \text{--------- Eq. (5)}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G} \quad \text{--------- Eq. (6)}$$

- For isotropic compression ($\sigma_x = \sigma_y = \sigma_z = \sigma_0$),

$$\frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= \frac{3\sigma_0}{E}(1-2\mu)$$

⇒ The bulk modulus,

$$B = \frac{\sigma_0}{\Delta V / V} = \frac{E}{3(1-2\mu)}$$

ⅲ) For confined compression ($\varepsilon_x = \varepsilon_y = 0$),

from Eqs.(1) ~(6), $\sigma_x = \sigma_y = \dfrac{\mu}{1-\mu}\sigma_z$

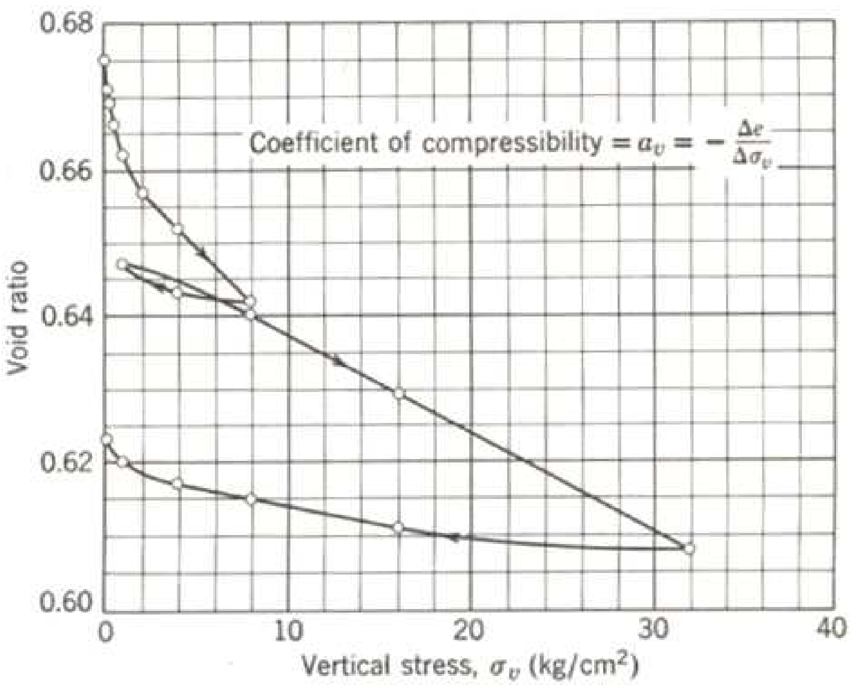
$$\varepsilon_z = \frac{\sigma_z}{E}\frac{(1+\mu)(1-2\mu)}{1-\mu}$$

and thus, $D = \dfrac{\sigma_z}{\varepsilon_z} = \dfrac{E(1-\mu)}{(1+\mu)(1-2\mu)}$

→ Alternate methods of portraying data for confined compression.

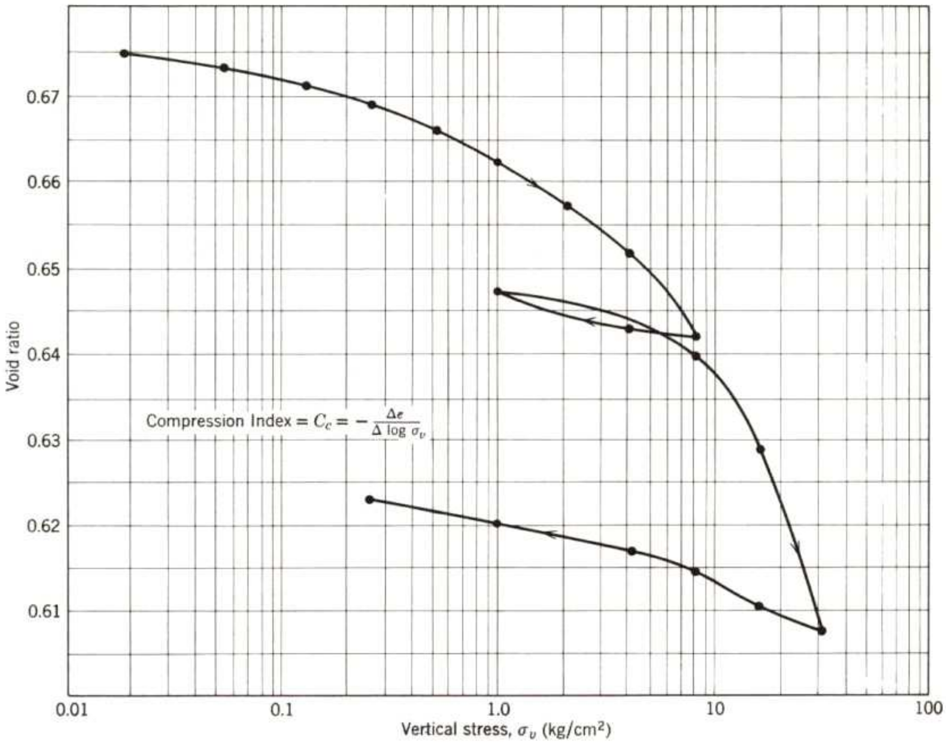
- the coefficient of compressibility, a_v :

$$a_v = -\frac{de}{d\sigma_v}$$



- the compression index, C_c:

$$C_c = -\frac{de}{d(\log \sigma_v)}$$



- the coefficient of volume change, m_v:

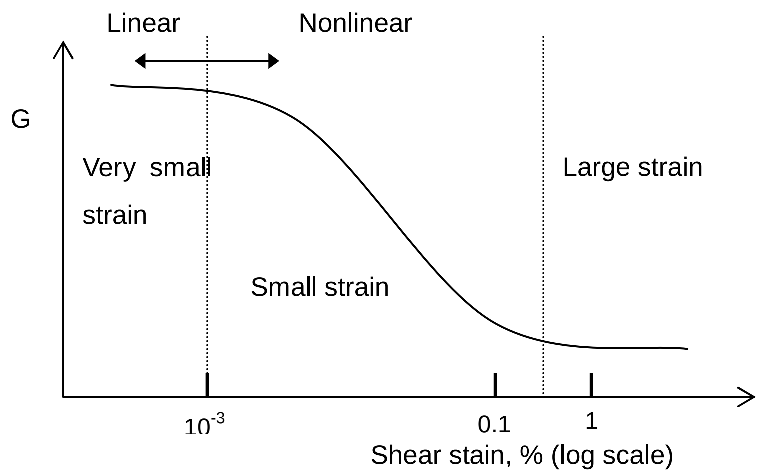
$$m_v = \frac{d\varepsilon_v}{d\sigma_v}$$

Table 2-3 Relations between various stress-strain parameters for confined compression

	Constrained Modulus	Coefficient of volume change	Coefficient of compressibility	Compression index
Constrained Modulus	$D = \frac{\Delta\sigma_v}{\Delta\varepsilon_v}$	$D = \frac{1}{m_v}$	$D = \frac{1+e_0}{a_v}$	$D = \frac{(1+e_0)\sigma_{va}}{0.435C_c}$
Coefficient of volume change	$m_v = \frac{1}{D}$	$m_v = \frac{\Delta\varepsilon_v}{\Delta\sigma_v}$	$m_v = \frac{a_v}{1+e_0}$	$m_v = \frac{0.435C_c}{(1+e_0)\sigma_{va}}$
Coefficient of compressibility	$a_v = \frac{1+e_0}{D}$	$a_v = (1+e_0)m_v$	$a_v = -\frac{\Delta e}{\Delta\sigma_v}$	$a_v = \frac{0.435C_c}{\sigma_{va}}$
Compression index	$C_c = \frac{(1+e_0)\sigma_{va}}{0.435D}$	$C_c = \frac{(1+e_0)\sigma_{va}m_v}{0.435}$	$C_c = \frac{a_v\sigma_{va}}{0.435}$	$C_c = -\frac{\Delta e}{\Delta\log\sigma_v}$

Note. e_0 denotes the initial void ratio. σ_{va} denotes the average of the initial and final stresses.

- **Highly non-linear**
  (E or G is not constant but depends on strain level.)



- **Practices according to strain level**
  - Very small ($\varepsilon < 10^{-3}$ %): dynamics.
  - Small (10^{-3} %< ε < 0.1 – 1.0 %) : relevant to many designs under serviceable loads.
  - Large (>0.1 – 1.0%) : relevant to soil behavior near or correspondingly to failure.

- In the very small strain region (<10^{-3}%), stiffness is approximately constant (G_o, G_{max}, E_o, E_{max}).

- **Influence factors on stress – strain behavior.**

i) Soils

- Composition : Grading.
  Mineralogy.
  Particle shape.
  Texture.

- Fabric { Particle packing. (including density.)
  Layering.
  Discontinuity (joints, fissures, open cracks).

- Chemical alteration.

ii) In-situ or testing stress or strain condition

- Current stress state.
  { Mean effective stress level, p'.
  Stress difference, q.
  Principal stress direction.
- Aging (time at current stress state).
- Stress history.
- Stress path imposed by sedimentation and subsequent loading.
- Rate of stress (or strain) change.
- Drainage condition.
- Sample disturbance.

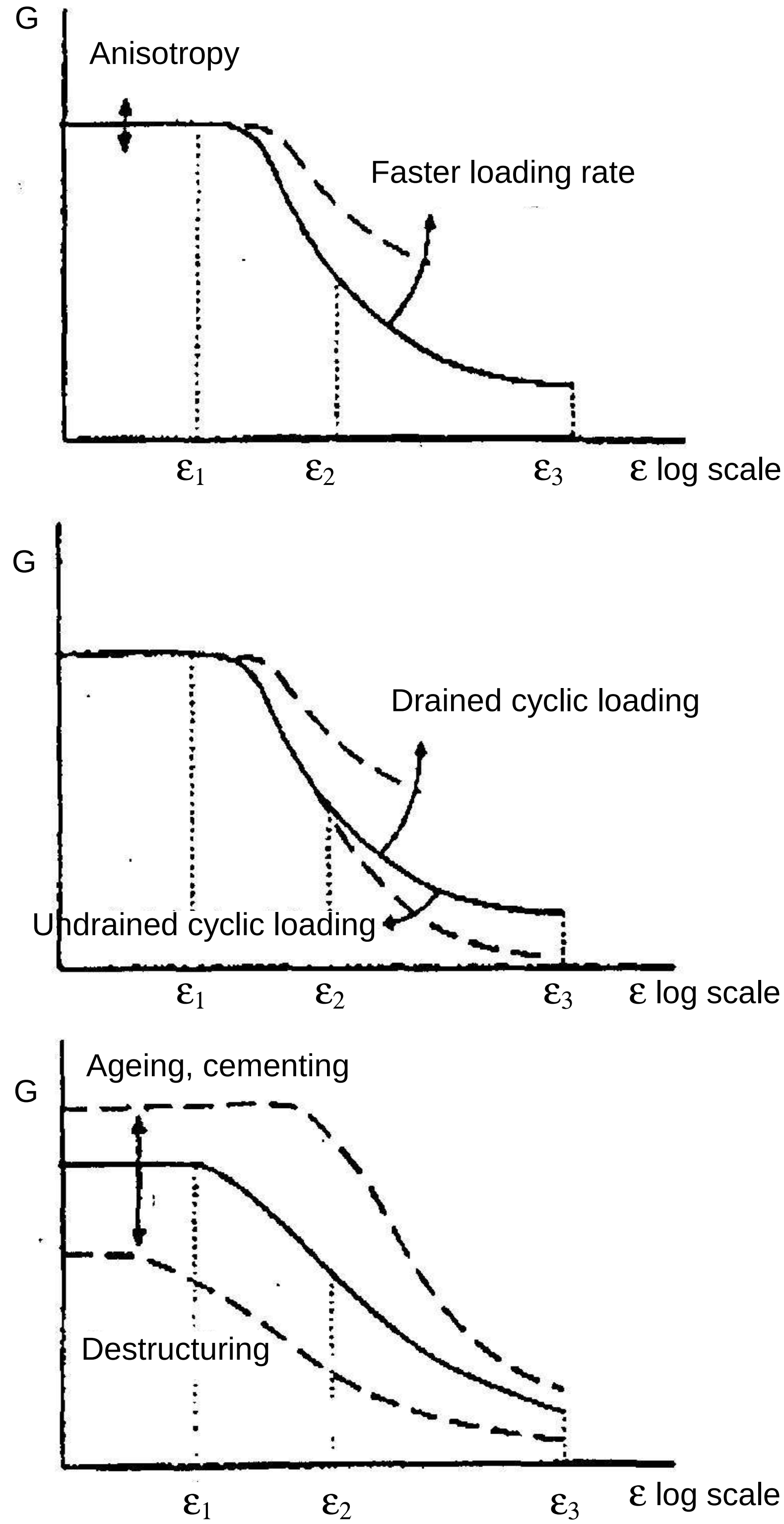


Fig. 2-15 Effects of loading rates, cyclic loading, and ageing on non-linearities of soil

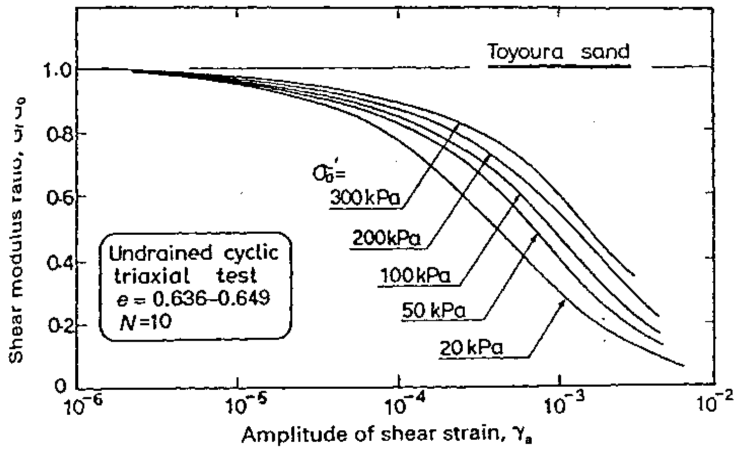


Fig 2-16. Effects of confining stress on the strain-dependent shear modulus (Kokusho, 1980)

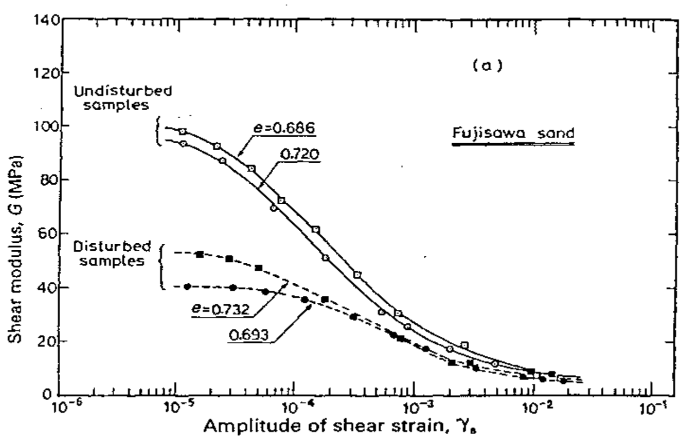


Fig. 2-17 Comparison of strain-dependent shear modulus of dense sand from undisturbed samples and from disturbed samples (Katayama *ea at.* 1986)

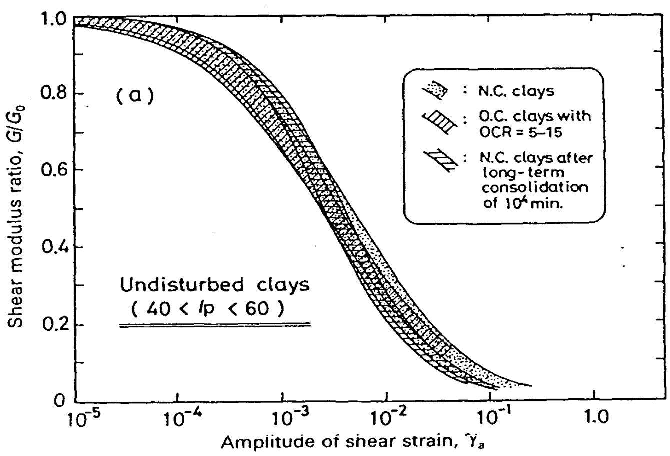


Fig. 2-18 Effects of consolidation histories on strain-dependent modulus (Kokusho *et al*, 1982)

- Test methods

i) Dynamic methods ($\gamma < 10^{-3}\%$)

- Based on direct measurements of shear wave velocity (v_s)

$$G = \rho v_s^2$$

① Bender Element Test in Triaxial apparatus
two thin piezoceramic plates which were bent by electric excitation, and of which bending changes the electric signal.

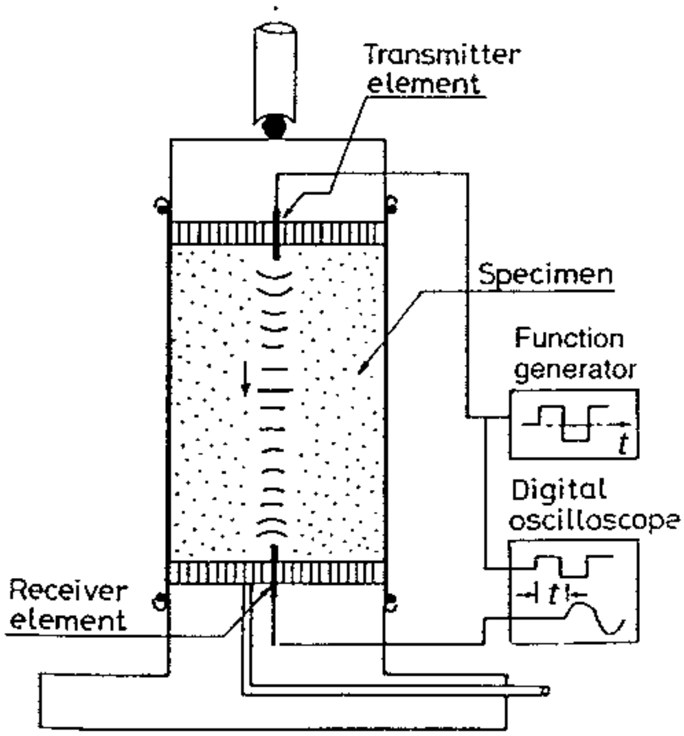


Fig. 2-19 Use of bender elements in the triaxial test appartus

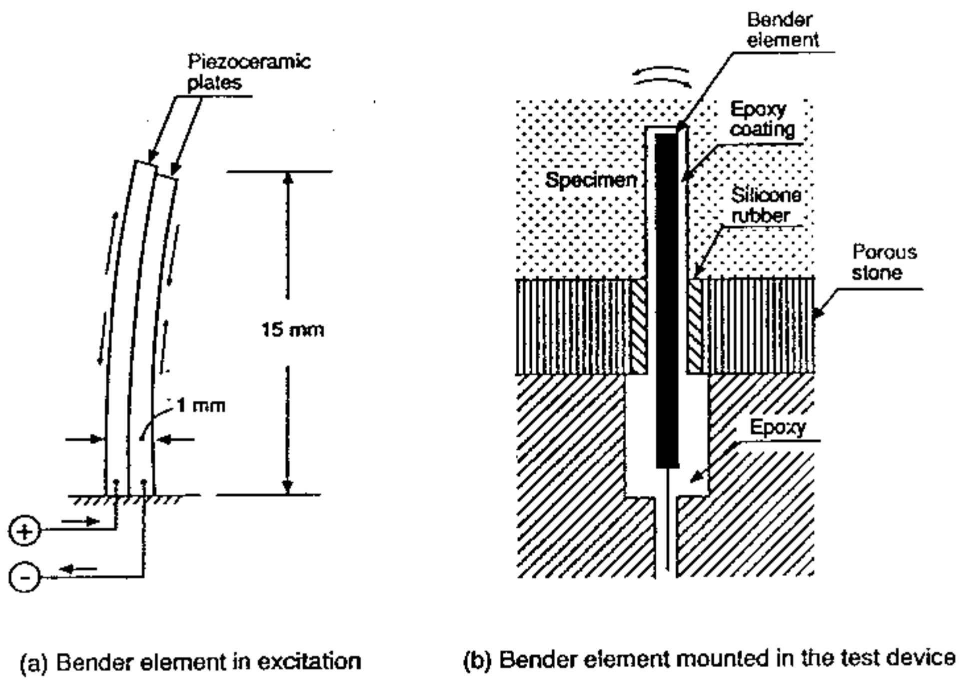


Fig. 2-20 Details of the bender element(Dyvik and Madshus, 1985)

② Geophysical methods with elastic wave.

- Downhole test

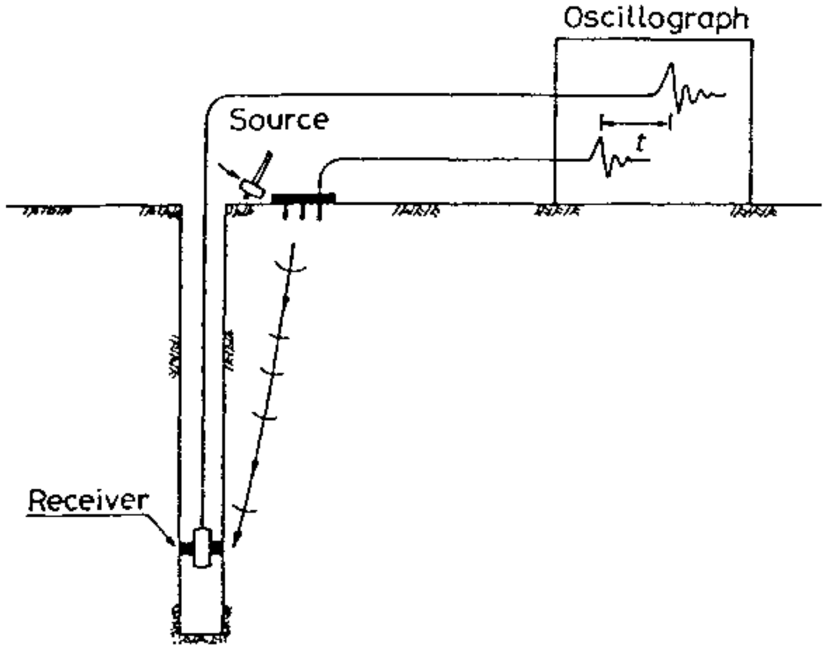


Fig. 2-21 Velocity logging by the downhole method

- Crosshole test

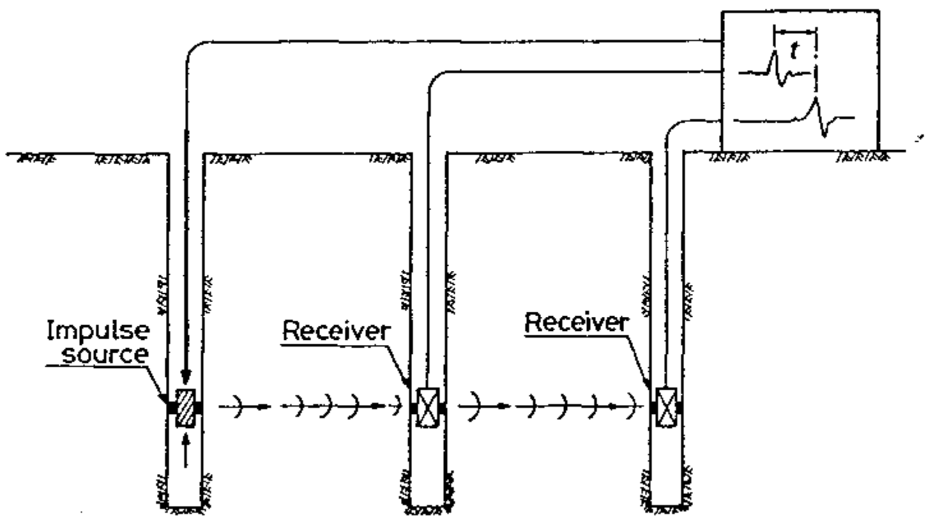
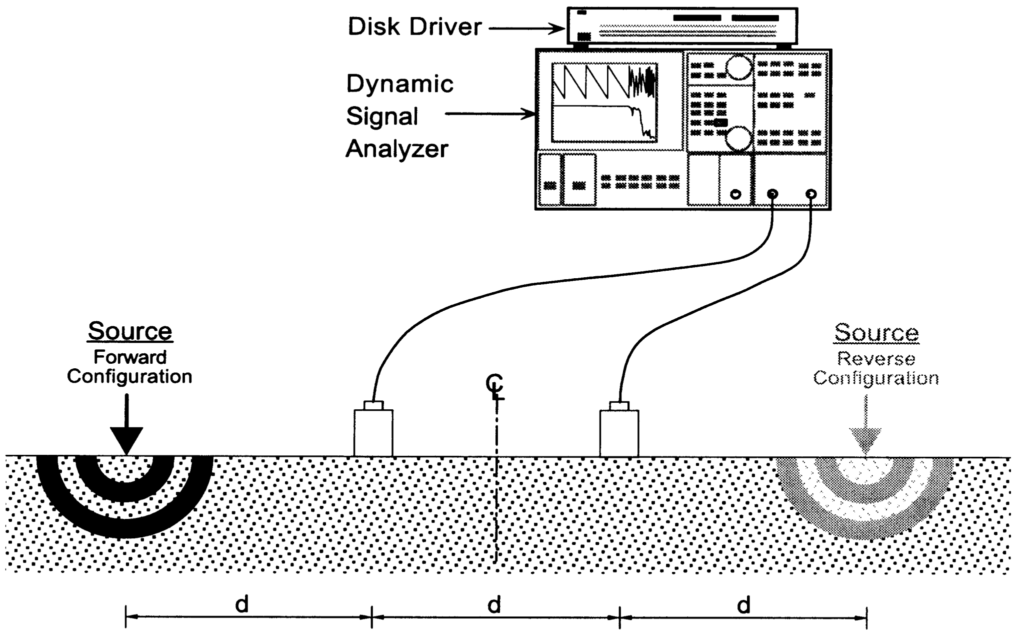


Fig. 2-22 Velocity logging by the crosshole method

- SASW test


Disk Driver
Dynamic
Signal
Analyzer
Source
Forward
Configuration
Source
Reverse
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ii) Resonant column test ($10^{-5}\% < \gamma < 10^{-1}\%$)

- Measure the resonant frequency of soils with varying amplitude of strain.
  - ⇒ Determine shear wave velocity.
  - ⇒ Give G.

iii) Cyclic TX with internal deformation measurement (inner cell measurements).

$10^{-3}\% < \varepsilon < 10^{-1}\%$

iv) Static with external deformation measurement.

$10^{-1}\% < \varepsilon$

- Shear modulus (G_{max}) for small strain. (ε < 0.001%)

For sands,

$$G_{max} = AF(e)(\sigma_o')^n$$

where , A ≡ non-dimensional constant.
σ_o' ≡ mean normal effective stress.
n = 0.5

For Toyoura sand,

$$G_{max} = 8400\frac{(2.17-e)^2}{(1+e)}(\sigma_o')^{0.5}$$

where, G_o and σ_o' are in terms of kPa.

For Clays (Hardin and Black (1968)),

$$G_{max} = AF(e)(OCR)^{K_s}(\sigma_o')^n$$

where, K_s = 0 for PI < 40
1 for PI > 40.

For remolded kaolinate clay (PI = 21),

$$G_{max} = 3300\frac{(2.97-e)}{1+e}(\sigma_o')^{0.5}$$

- G - γ relationship

  - Ramberg – Osgood Model.

$$\gamma = (\tau / G_{\max}) + C(\tau / G_{\max})^R,$$

C, R ≡ non-dimensional constant.

Tangential Modulus, $G_t = \dfrac{\partial\tau}{\partial\gamma} = \dfrac{G_{\max}}{1 + R \cdot C \cdot \left|\dfrac{\tau}{G_{\max}}\right|^{R-1}}$

- **Initial modulus, E for <u>large strain. (> 10^{-1} %)</u>**

- Can be important for soft clays (E_u).
- Is evaluated from static laboratory tests (Triaxial tests).

i) Consolidation stress.

$E_u = f(\sigma_c')$

$\rightarrow$ E_u / σ_c', (or E_u / s_u) = constant. $\rightarrow$ Fig.2-23 in page 56

$\rightarrow$ Janbu, $E = KP_a(\frac{\sigma_3}{P_a})^n$, (n ≈ 0.5 for sands.)

ii) Sample disturbance.

Disturbance $\rightarrow$ underestimates E_u ($E_{field} = (2 \sim 10)E_{lab}$).

a) use a modulus during a second cycle of loading.

b) use the undisturbed block sample.

c) SHANSEP.

iii) Rate of loading. (or strain rate.)

$E_{fast} > E_{slow}$.

($E_{dynamic} \approx (1.5 \sim 2.0)\ E_{static}$).

iv) Load cycle.

$E_{initial\ loading} < E_{subsequent\ cycle}$.

v) Initial stress condition.

$\rightarrow$ Initial shear stress (depending on OCR). $\rightarrow$ Fig.2-24 in page 57

$\rightarrow$ Isotropic consolidation vs. K_o consolidation.

vi) Stress path for loading. (Fig. 2-24 in page 57)

vii) Aging.

Aging $\rightarrow$ increase density $\rightarrow$ increase E.

Recommendation for testing

$\rightarrow$ Allow at least 1(log t) cycle of creep at σ_c' prior to starting the shearing phase of the test.

viii) Strain (or stress) level.

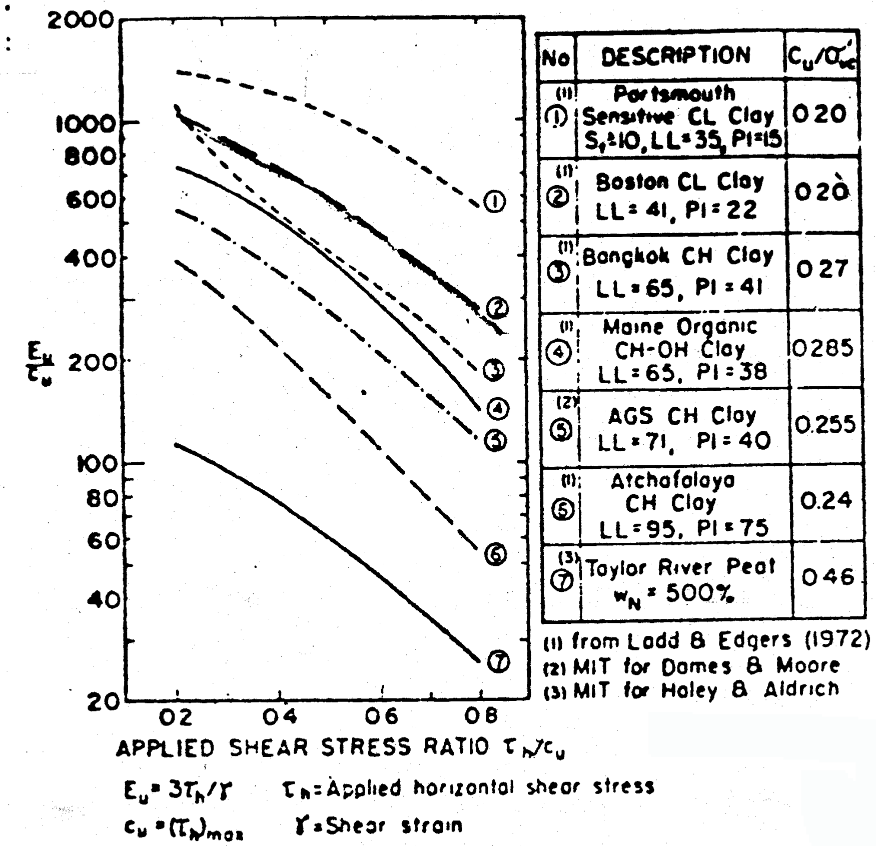

Fig. 2-23 Normalized Young's modulus vs stress level from CK_0U direct simple shear tests on seven normally consolidated soils

*See Mitchell (1976) for a review of methods of fabric measurement and typical results.

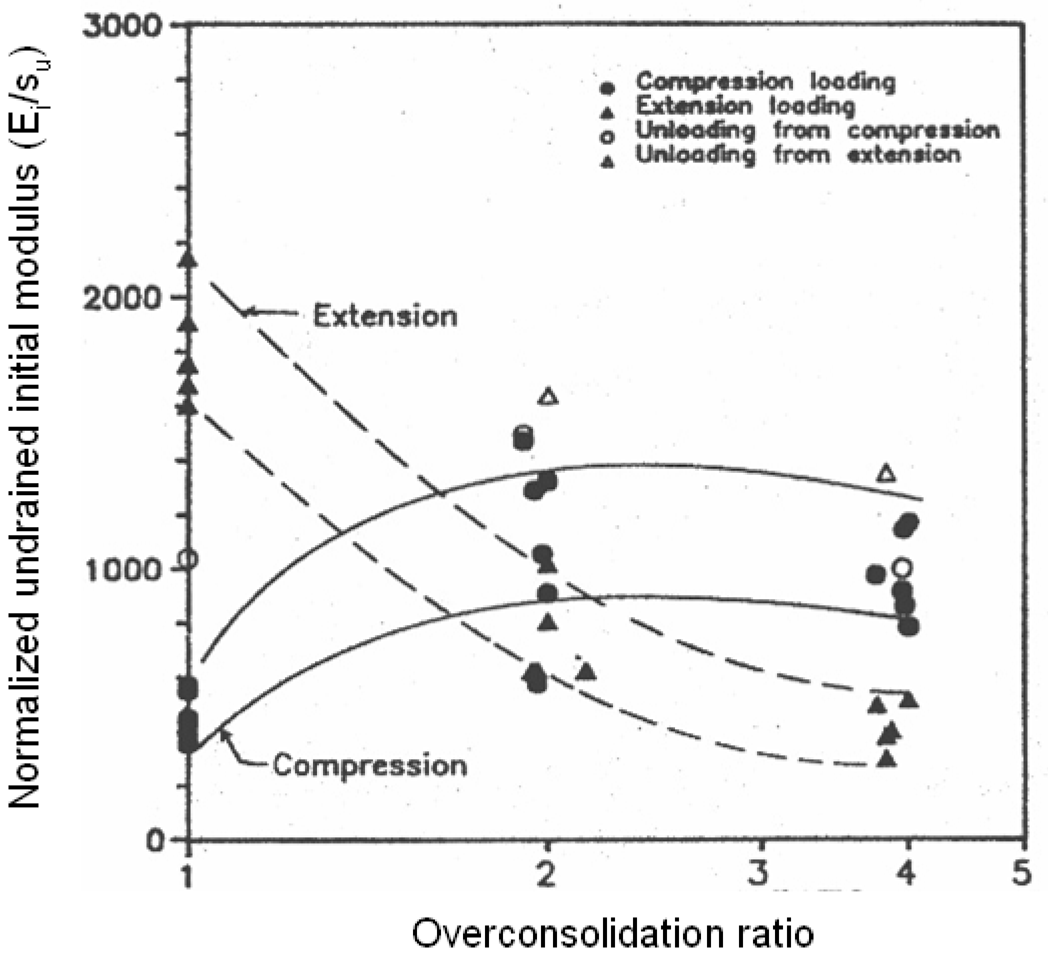

Fig. 2-24 Normalized Undrained Moduli (estimated at 0.1% axial strain)

- Ways to estimate E_u.

1. Correlations, $E_u = (200 \sim 2000)s_u$.

2. Run CIU/CKoU triaxial tests based on recompression approach, obtain E_u from σ-ε plot and multiply result by (2 to 5).

3. To reduce sample disturbance effect, run triaxial test based on SHANSEP.

4. Stress path method. (same as 2 or 3, but run test according to stress path method principles.)