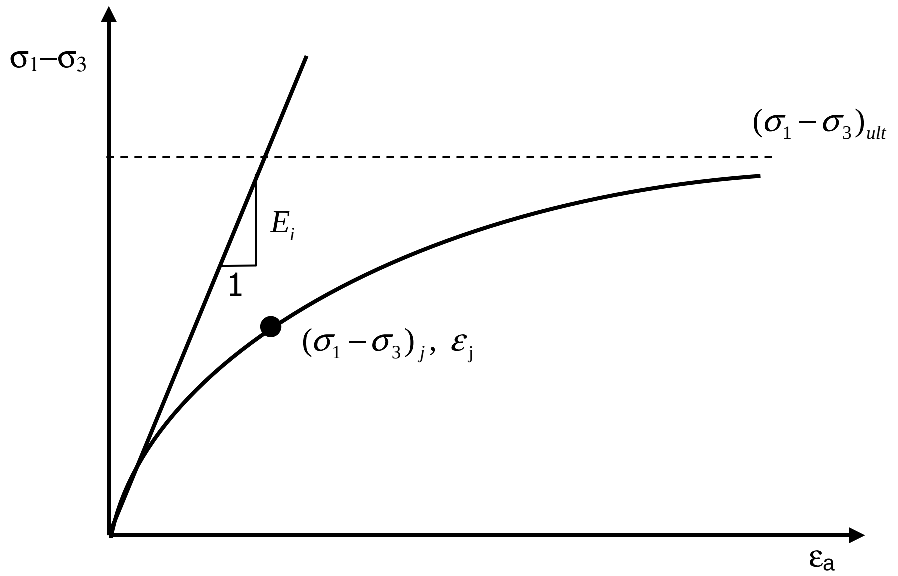
(5) Nonlinear Pseudo-Elastic Model

- **Duncan-Chang (1970)**

- **Incremental Stress-Strain Relations**

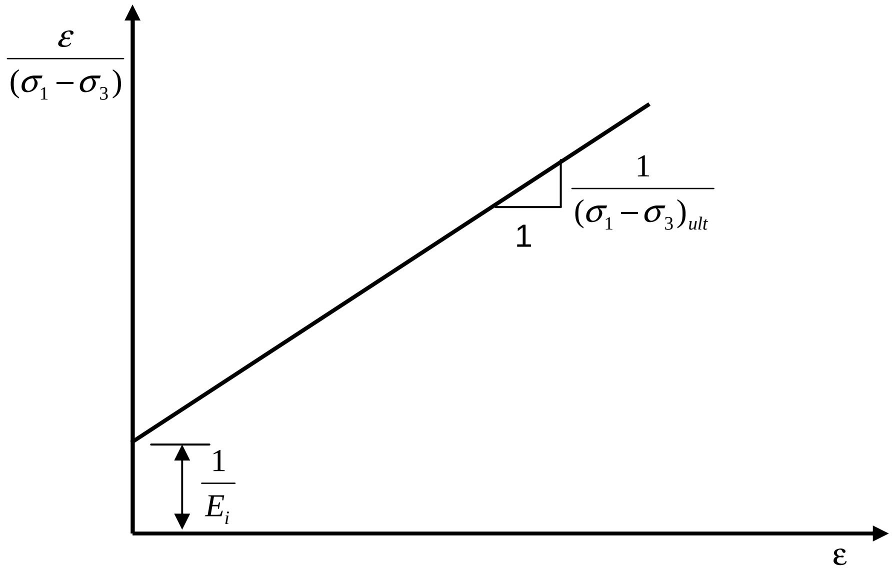
i) Nonlinearity

- Kondner and Zelasko (1963)
  - σ-ε relations are approximated by a hyperbola.



$$(\sigma_1-\sigma_3)_j = \frac{\varepsilon_j}{\dfrac{1}{E_i}+\dfrac{\varepsilon_j}{(\sigma_1-\sigma_3)_{ult}}} \qquad \text{---} \quad (1)$$

⇒ Replot σ-ε data on :



Transformed σ-ε relations :

$$\frac{\varepsilon}{(\sigma_1-\sigma_3)}=\frac{1}{E_i}+\frac{\varepsilon}{(\sigma_1-\sigma_3)_{ult}}$$

<u>Physical significance</u>

- Curve defined in terms of
  → E_i, Young's modulus.
  → Asymptotic value of principal stress difference (σ_1-σ_3) at failure.

ii) Stress-dependent nature of σ-ε behavior

For most cases : an increase of σ_3 results in increases in E_i & $(\sigma_1-\sigma_3)_{ult}$.

a) To account E_i dependence on stress (Janbu, 1964).

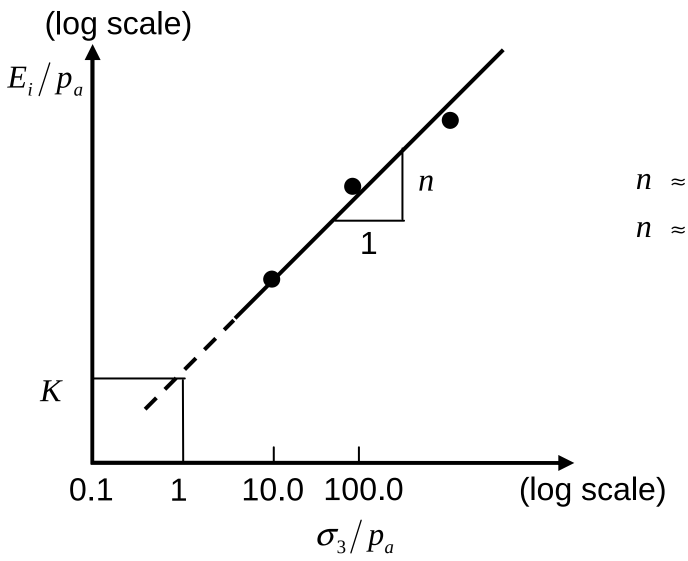
$$E_i = Kp_a\left(\frac{\sigma_3}{p_a}\right)^n \quad \text{---} \quad (2)$$

K ≡ modulus number

n ≡ modulus exponent

p_a ≡ atmospheric pressure

To evaluate K, n; perform 3 TX tests at different values of σ_3.



$n \approx 0.5$ for sands

$n \approx 0.0$ for NC saturated clays under UU tests

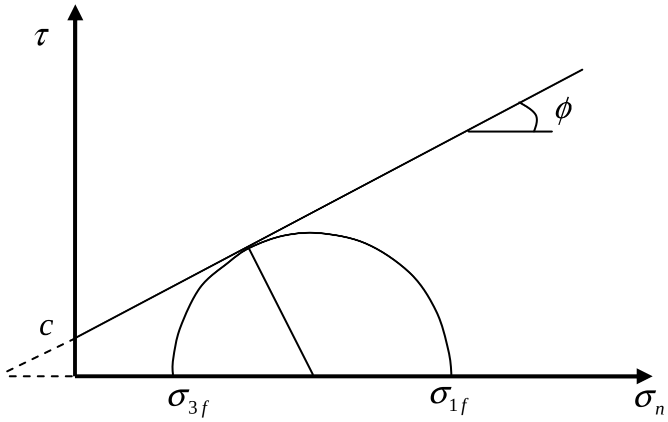
b) To account $(\sigma_1-\sigma_3)_{ult}$ dependence on σ_3.

- relate $(\sigma_1-\sigma_3)_{ult}$ to $(\sigma_1-\sigma_3)_f$.

$$R_f = \text{failure ratio} = \frac{(\sigma_1-\sigma_3)_f}{(\sigma_1-\sigma_3)_{ult}} \quad (0.5 \le R_f \le 0.95)$$

- define failure (i.e. $(\sigma_1-\sigma_3)_f$).

Based on Mohr-Coulomb failure criteria.



General expression (from geometry of circle)

$$(\sigma_1-\sigma_3)_f = \frac{2c\cos\phi + 2\sigma_3\sin\phi}{1-\sin\phi}$$

$$(\sigma_1-\sigma_3)_{ult} = \frac{2c\cos\phi + 2\sigma_3\sin\phi}{R_f(1-\sin\phi)} \qquad \text{------------------} \quad (3)$$

iii) Relate tangential modulus, E_t to stress level.

Differentiate eq. (1) with respect to ε.

$$\frac{d(\sigma_1-\sigma_3)}{d\varepsilon}=\frac{d\left[\dfrac{\varepsilon}{1/E_i+\varepsilon/(\sigma_1-\sigma_3)_{ult}}\right]}{d\varepsilon}$$

Let $a=\dfrac{1}{E_i}$, $b=\dfrac{1}{(\sigma_1-\sigma_3)_{ult}}$

$$\frac{d(\sigma_1-\sigma_3)}{d\varepsilon}=\frac{d\left[\dfrac{\varepsilon}{a+b\varepsilon}\right]}{d\varepsilon}$$

$$=\frac{a+\cancel{b\varepsilon}-\cancel{b\varepsilon}}{(a+b\varepsilon)^2}$$

$$=\frac{a}{(a+b\varepsilon)^2} \quad \text{---} \quad \text{(i)}$$

Rewrite eq. (1) in terms of strain.

$$\varepsilon=\frac{a(\sigma_1-\sigma_3)}{1-b(\sigma_1-\sigma_3)} \quad \text{---} \quad \text{(ii)}$$

Sub. (ii) into (i).

$$E_t=\frac{d(\sigma_1-\sigma_3)}{d\varepsilon}=\frac{a}{\left[a+\dfrac{ab(\sigma_1-\sigma_3)}{1-b(\sigma_1-\sigma_3)}\right]^2}$$

$$=\frac{[1-b(\sigma_1-\sigma_3)]^2}{a}$$

$$=\left[1-\frac{\sigma_1-\sigma_3}{(\sigma_1-\sigma_3)_{ult}}\right]^2 E_i \quad \text{-----------------------------------} \quad \text{(iii)}$$

Put eq. (2), (3) into (iii).

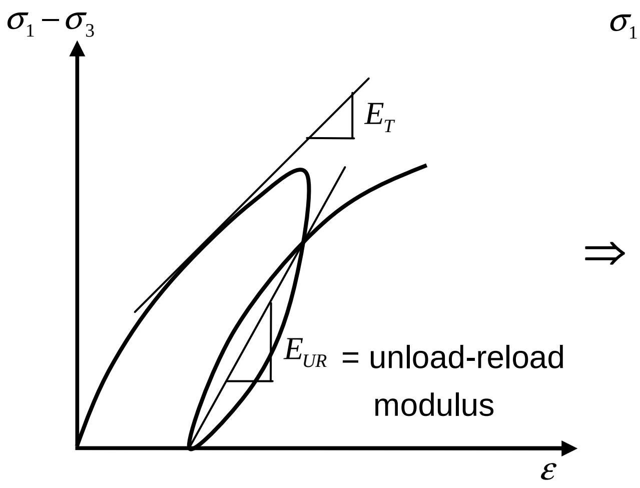
$$E_t = \left[1 - \frac{R_f(1-\sin\phi)(\sigma_1-\sigma_3)}{2c\cos\phi + 2\sigma_3\sin\phi}\right]^2 Kp_a(\frac{\sigma_3}{p_a})^n \qquad \text{----------------------} \quad (4)$$

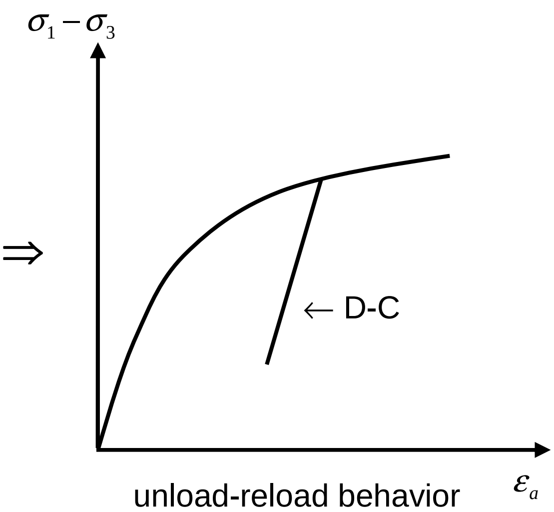
$\Rightarrow$ E_t is a $f(\phi,\ c,\ K,\ n,\ R_f)$

$\leftarrow (\sigma_1-\sigma_3)_{ult}$

iv) Cyclic loading behavior

- Use different E values for loading and unloading-reloading.
  $E_{UR} > E_T \quad (E_{UR} > E_i)$





Empirically, $E_{UR} = K_{UR}\,p_a(\frac{\sigma_3}{p_a})^n$

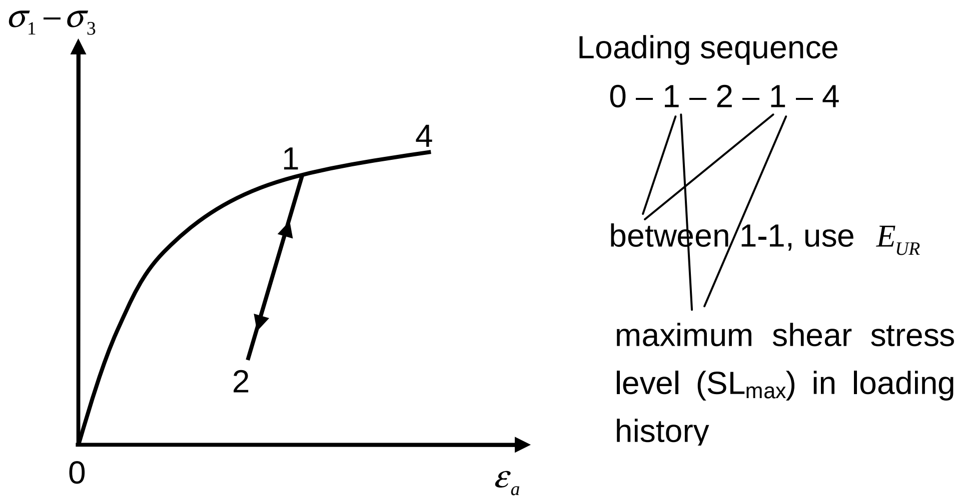
Assume that n is same for unloading-reloading as for primary loading.

K_{UR} $\equiv$ unload-reload modulus number.

$K_{UR} > K$ for dense sand (20% greater)

for very loose sand (as much as 3 times greater)

- How long does soil stay "elastic" during unloading-reloading?



- Notes

Define stress level (SL)

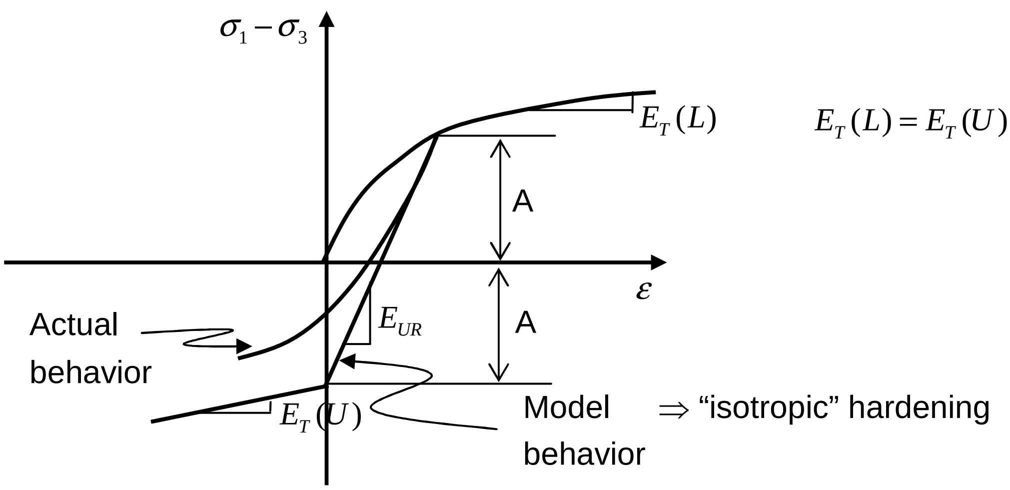
$$SL = \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_f}$$

$$0 \le SL \le 1.0$$

↑ Isotropic Loading ↑ failure

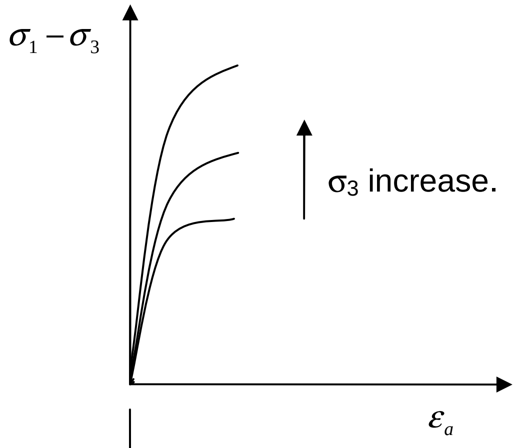
If $SL < SL_{\max}$, then use elastic (E_{UR}) for unloading-reloading.

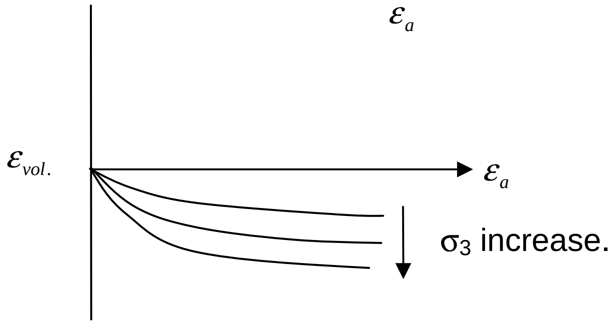
- What does SL criteria imply about complete cycles of loading?



v) Nonlinear volume change

- No expansion, only compression
- Typical triaxial compression test results



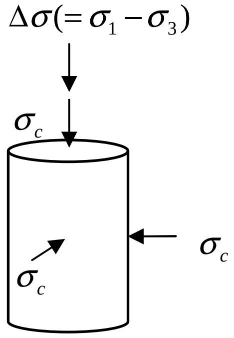


K_B = Bulk modulus= $\dfrac{\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3}{3\varepsilon_{vol}} \rightarrow \varepsilon_{vol} = \dfrac{\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3}{3K_B}$

K_B is a function of σ_3 and it is independent of stress level $(\sigma_1 - \sigma_3)$. K_B is not changed by increase of loading.

Vol. strains are a function of changes in mean normal stress.

Triaxial test



$$K_B = \frac{(\sigma_1 - \sigma_3)}{3\varepsilon_v}$$

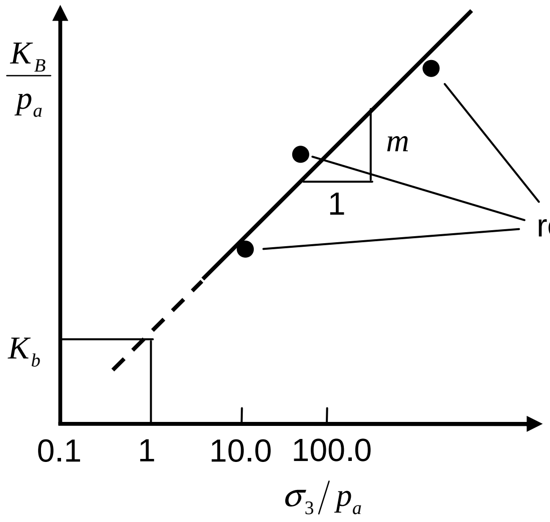
$$\rightarrow \varepsilon_v = \frac{(\sigma_1 - \sigma_3)}{3K_B}$$

Use Janbu relation.

$$K_B = K_b p_a (\frac{\sigma_3}{p_a})^m$$

$K_b \equiv$ bulk modulus number

$m \equiv$ bulk modulus exponent



Typical soils : 0 < m < 1

m < 0 ⇒ K_B decreases as σ_3 increase.

i.e. : quick clay

<u>Restrictions on K_B</u>

0 < ν < 0.5

$$\nu = \frac{1}{2} - \frac{E_t}{6K_B} \qquad (\Leftarrow \ K_B = \frac{E_t}{3(1-2\nu)})$$

for ν =0, $K_B = E_t/3$

ν =0.49, $K_B = 16.7E_t$

$$E_t/3 < K_B < 16.7E_t$$

Summary

From i) ~ v) & 8 parameters,

Symbol	Name	Purpose
K K_{UR} n	modulus # unload-reload modulus # modulus exponent	relate E_i, E_{UR} to σ_3.
c, ϕ	Mohr-Coulomb parameters	relate $(\sigma_1-\sigma_3)_f$ to σ_3.
R_f	failure ratio	relate $(\sigma_1-\sigma_3)_f$ to $(\sigma_1-\sigma_3)_{ult}$.
K_b m	bulk modulus number bulk modulus exponent	relate K_B to σ_3.

Comments

i) Limitations

a) Useful for predicting movements in stable earth masses (no plastic considerations).

b) Does not include volume changes caused by shear-induced dilatancy (only volumetric compression is allowed for monotonically-increasing loads).

c) Parameters are not fundamentally soil properties, but represent empirical parameters for a limited set of loading conditions.

d) Full stress reversal must occur before nonlinear (inelastic) strains develop for cyclic loads or load reversals.

ii) Advantages

a) Parameters come from simple TX tests.

b) Effective stress and total stress relationships are the same, but parameters different.

c) General model ; both sands (+) clays can be handled.

d) Values of parameters have been calculated for more than 150 soils.

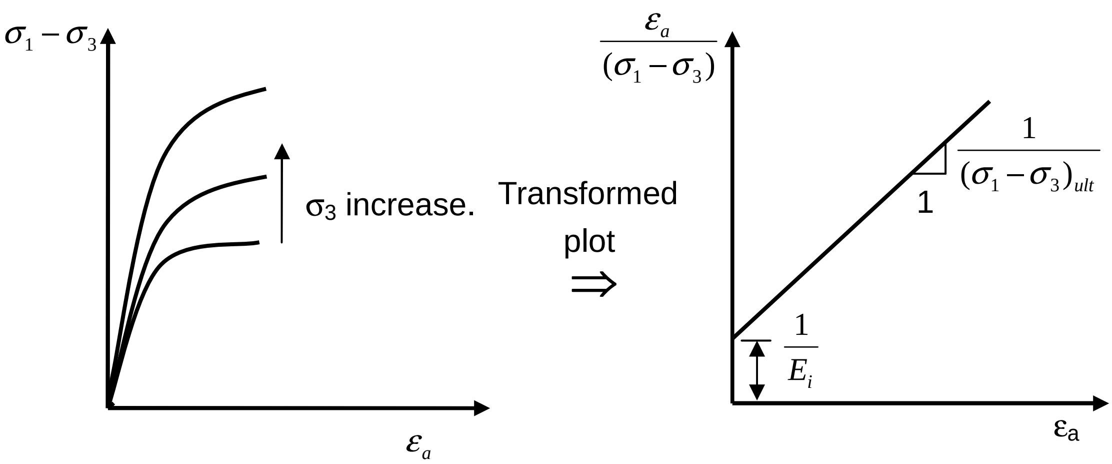
e) Can handle anisotropic soils with plane strain or true TX test results.

- **Parameter Identification**

Drained conditions : 3-CID TXC tests with volume change measurements.

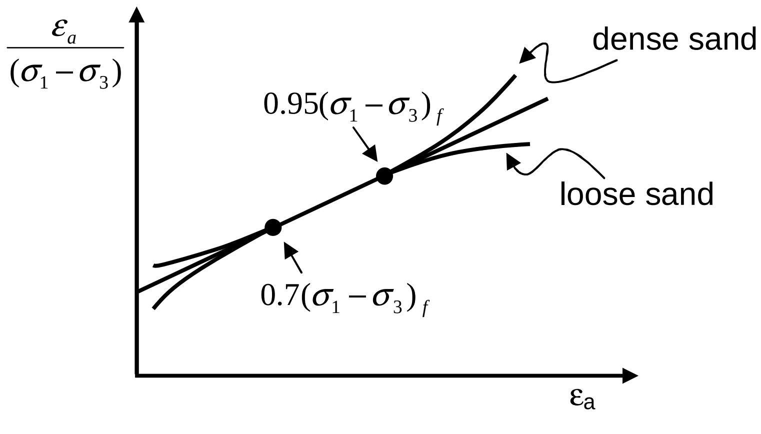
Undrained conditions : 3-CIU TXC tests.

① Plot σ-ε data.



3 curves are plotted.

Ref.) D-C

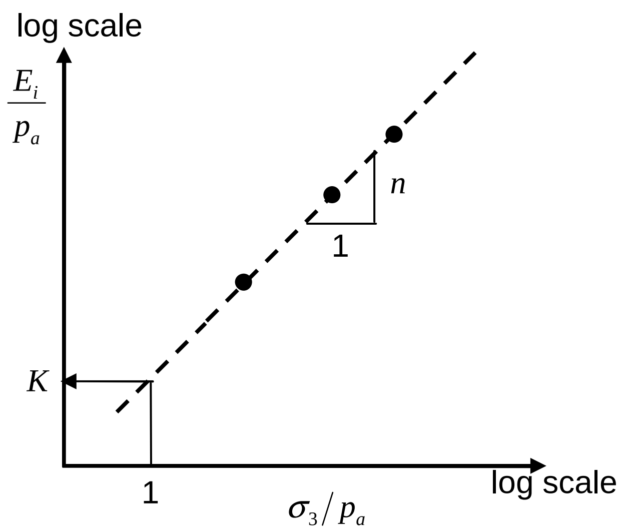


⇒ U.C. Berkeley's experience indicated using these pts to define straight line gives best fit of all responses.

⇒ We find $\frac{1}{E_i}$, $\frac{1}{(\sigma_1 - \sigma_3)_{ult}}$.

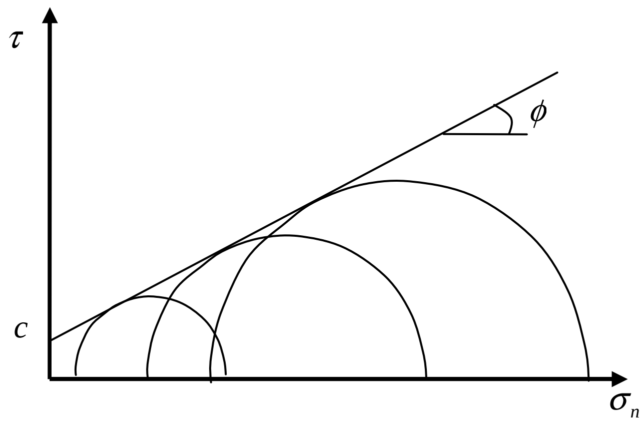
Each test gives 1 set of values ⇒ For R_f , use average value

② E_i .



We find, K , n .

③ Plot Mohr circles at failure.



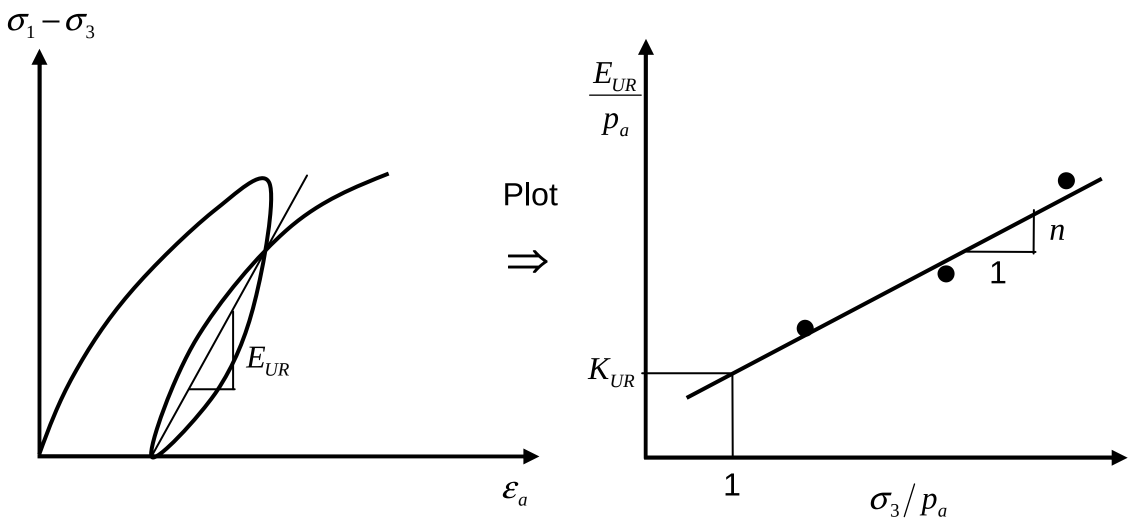
c , ϕ $\rightarrow$ $(\sigma_1 - \sigma_3)_f$

Then, we can compute $R_f = \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{ult}} = \frac{2c\cos\phi + 2\sigma_3 \sin\phi}{(\sigma_1 - \sigma_3)_{ult}(1 - \sin\phi)}$

↖

Use average value among values with 3 sets of σ_3 and $(\sigma_1 - \sigma_3)_{ult}$

④ To find unload-reload parameters (ideally need 1 unload-reload cycle at each of 3 confining stresses).



Assume slope of line is same as for primary loading.

⑤ For volume change parameters (only for drained tests).

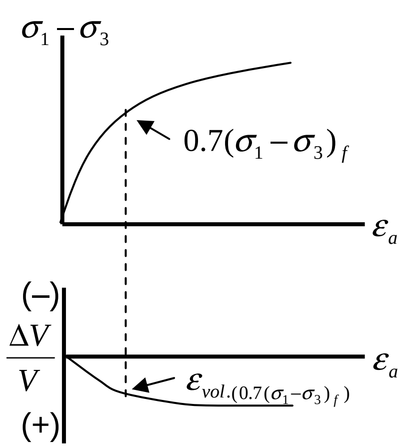
- Recall : $K_B = \dfrac{\Delta\sigma_0}{\Delta\varepsilon_{vol.}}$

  for triaxial shearing, $K_B = \dfrac{\sigma_1 - \sigma_3}{3\varepsilon_{vol.}}$

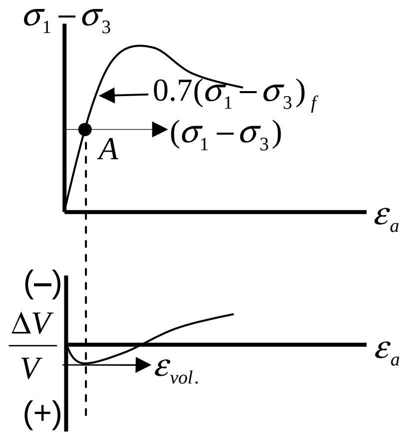
- To select K_B :

  Two possibilities; (based on $0.7(\sigma_1 - \sigma_3)_f$)

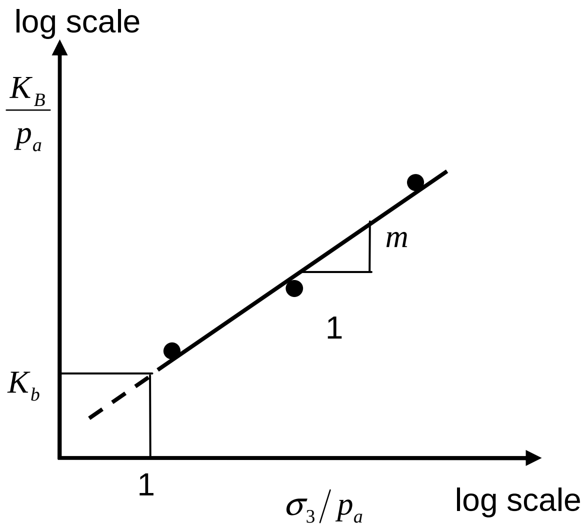
  1) If the slope of volume change curve does not reach 0 before the pt. that corresponds to $0.7(\sigma_1 - \sigma_3)_f$, then use $0.7(\sigma_1 - \sigma_3)_f$ and corresponding $\varepsilon_{vol.}$ to K_B.



2) If the slope of the volume change curve = 0 before $0.7(\sigma_1-\sigma_3)_f$ stress level is reached, then use the $(\sigma_1-\sigma_3)$ and $\varepsilon_{vol.}$ at that point to compute KB.



$$\Rightarrow \; K_B = \frac{(\sigma_1-\sigma_3)}{3\varepsilon_{vol.}}$$



- <u>Eliminating inconsistencies from data</u>

1) Select data appropriate for problem under consideration.
   - Natural soils : Need to run tests on high quality undisturbed samples.
   - Fills : Use specimens compacted to water content and dry density (with same compaction energy) as the fill was in field.
   - Use drained conditions that are expected in field loading.

2) Smooth the data.
   - Stress controlled loading.