

*** Modification of stress path method for settlement computation of normally consolidated clays (Kim et al. (2011))**

- 1) Gradual change of Poisson's ratio during consolidation
 Ex) Circular footing under uniform vertical loading ($\nu_u=0.5 \rightarrow \nu_d=0.1$)

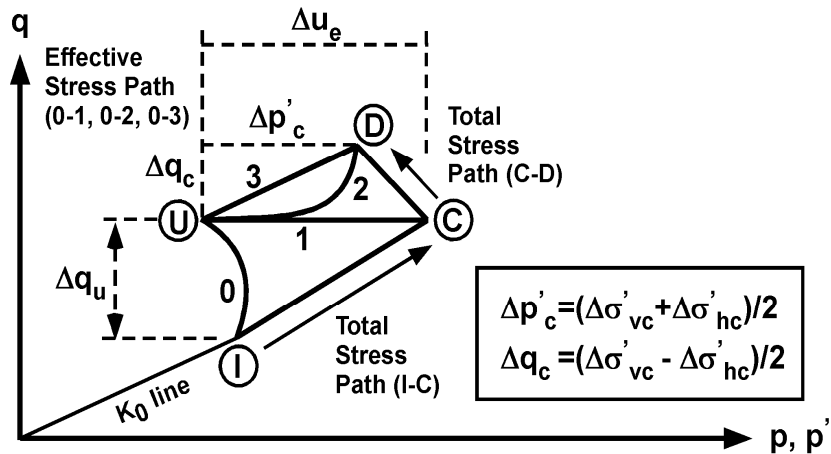


Fig. 2-26 Typical Stress Path of Soil Element

Based on the equation proposed by Poulos and Davis as below,

$$\Delta \sigma_v = p_0 \left[1 - \left\{ \frac{1}{1 + (a/z)^2} \right\}^{3/2} \right] \text{ and}$$

$$\Delta \sigma_h = \frac{p_0}{2} \left[(1 + 2\nu) - \frac{2(1 + \nu)}{(1 + (a/z)^2)^{1/2}} + \frac{1}{(1 + (a/z)^2)^{3/2}} \right] \quad (\text{Eq. 2-1})$$

Where, a is the radius of the circular footing
 p₀ is the pressure acting on the circular area
 z is the depth from the surface
 ν is the poisson's ration of the elastic half space

the paths C-D in Fig. 2-27 indicate the changes of total stresses. (significant in shallow depth)

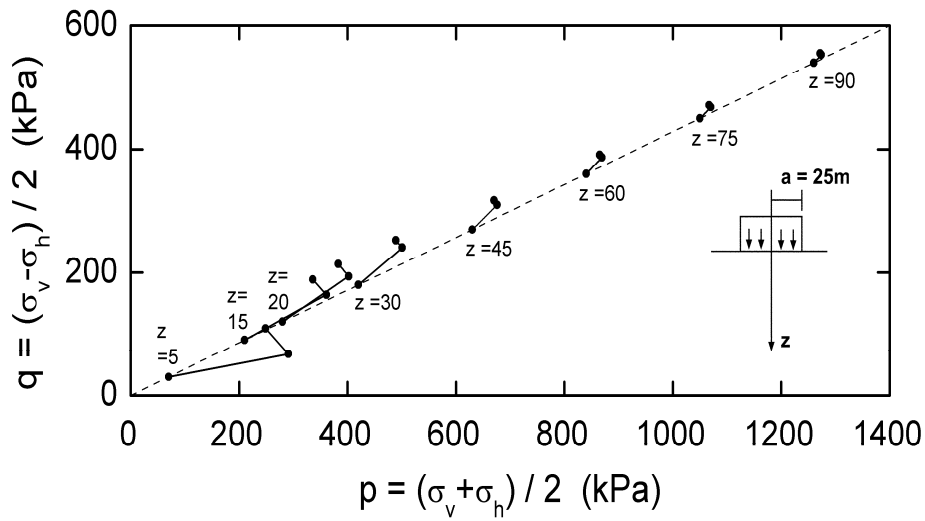


Fig. 2-27 Variation of Total Stress Paths for Various Depths of the Circular Footing

Figure 2-28 shows the ratio of the length of the stress path C-D to the length of the path I-C. (i.e. the relative magnitude of the total stress change during consolidation)

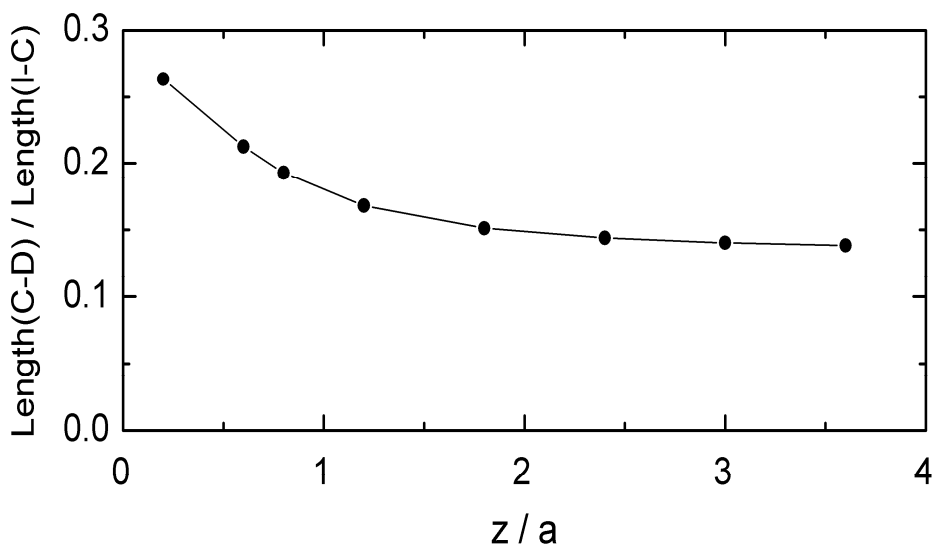


Fig. 2-28 Relative Length of Total Stress Path

2) New Testing concept for stress path method.

- Normalized stress-strain behavior by the initial stress state.
→ The normally consolidated clay exhibits the similar stress strain behavior when the stresses are normalized by the initial stress on the K_0 state. (Atkinson and Branshy (1978))

- Immediate settlements determined by deviator stress and not by mean normal stress.
→ Unique relationship between S_i and $\Delta\sigma_d$.

- Back pressure equalization.
 - Conventional dissipation process, initiated by opening a drainage valve. → provides isotropic stress path(U-C).
 - After undrained loading, we increase the back pressure as the same amount as the excess pore pressure and then open the drainage valve.

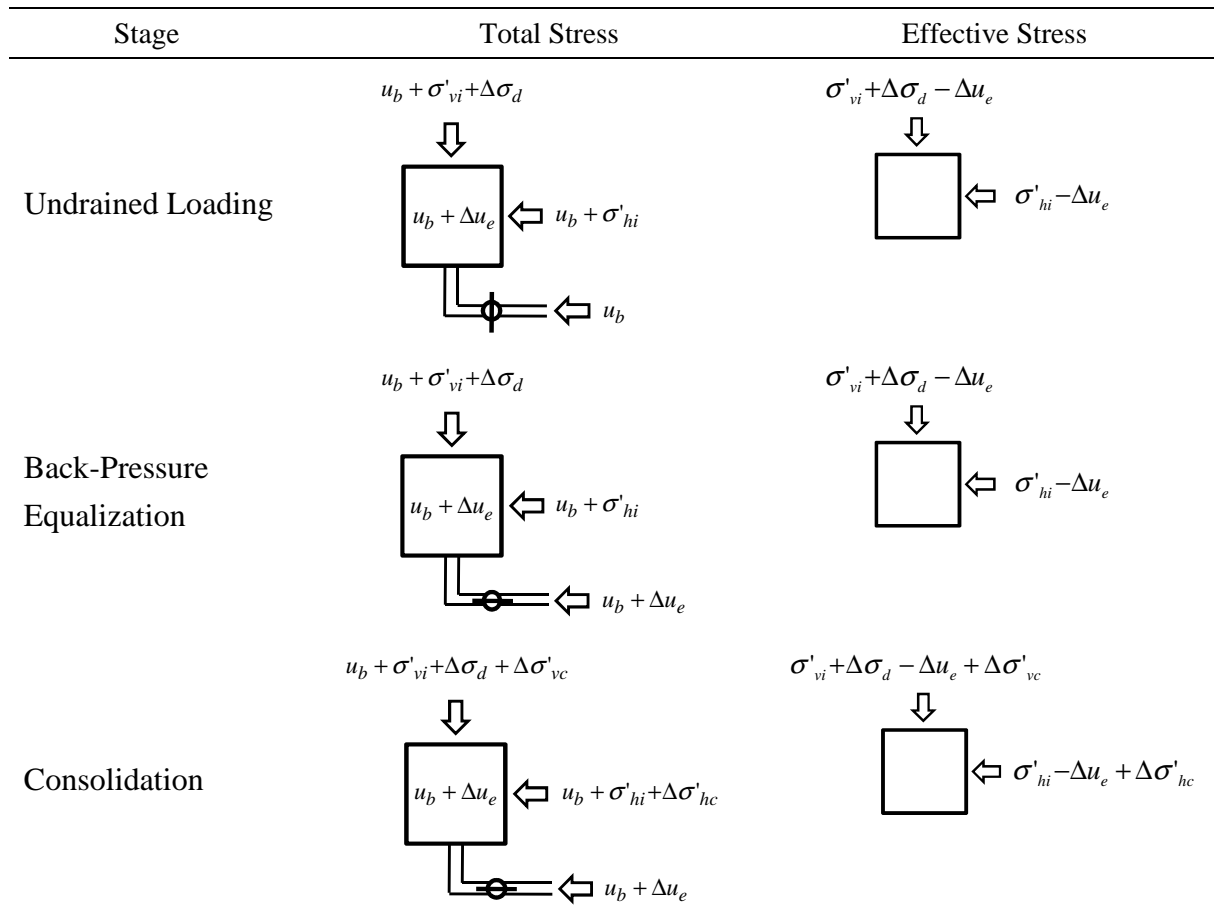


Fig. 2-29 Stresses of Soil Element During Back Pressure Equalization

- The back pressure equalization makes it possible to perform the test of the any stress paths including anisotropic stress path.
 - Additionally, it provides a methodology to measure effective stress-strain relationship at every stress points along the given consolidation stress path.
 - Charactering stress-strain behavior of clays.
 - The constitutive relationship in the triaxial condition;
- $$\begin{bmatrix} \varepsilon_{vc} \\ \varepsilon_{hc} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \Delta\sigma'_{vc} \\ \Delta\sigma'_{hc} \end{bmatrix} \tag{Eq. 2-2}$$
- To determine A, B, C, D, two sets of the stress-strain data with different $K_c (= \Delta\sigma'_{hc} / \Delta\sigma'_{vc})$ are required.

$$\begin{aligned} \begin{bmatrix} (\varepsilon_{vc})_I & (\varepsilon_{vc})_{II} \\ (\varepsilon_{hc})_I & (\varepsilon_{hc})_{II} \end{bmatrix} &= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} (\Delta\sigma'_{vc})_I & (\Delta\sigma'_{vc})_{II} \\ (\Delta\sigma'_{hc})_I & (\Delta\sigma'_{hc})_{II} \end{bmatrix} \\ &= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \frac{2}{1+(K_c)_I}(\Delta p'_c)_I & \frac{2}{1+(K_c)_{II}}(\Delta p'_c)_{II} \\ \frac{2(K_c)_I}{1+(K_c)_I}(\Delta p'_c)_I & \frac{2(K_c)_{II}}{1+(K_c)_{II}}(\Delta p'_c)_{II} \end{bmatrix} \end{aligned} \quad \text{(Eq. 2-3)}$$

- The compliance parameters, A, B, C, and D are not constant due to nonlinear stress-strain behavior of clays.

→ Iteration procedure with the constant strain energy concept is adopted.

$$\begin{aligned} (W = \frac{1}{2}(\Delta\sigma'_{vc} \varepsilon_{vc} + 2\Delta\sigma'_{hc} \varepsilon_{hc})) \\ = \left[A + (B + 2C)K_c + 2DK_c^2 \right] \frac{2(\Delta p'_c)^2}{(1 + K_c)^2} \end{aligned} \quad \text{(Eq. 2-4)}$$

→ The strains can be computed using the converged set of the parameters, A, B, C, and D.

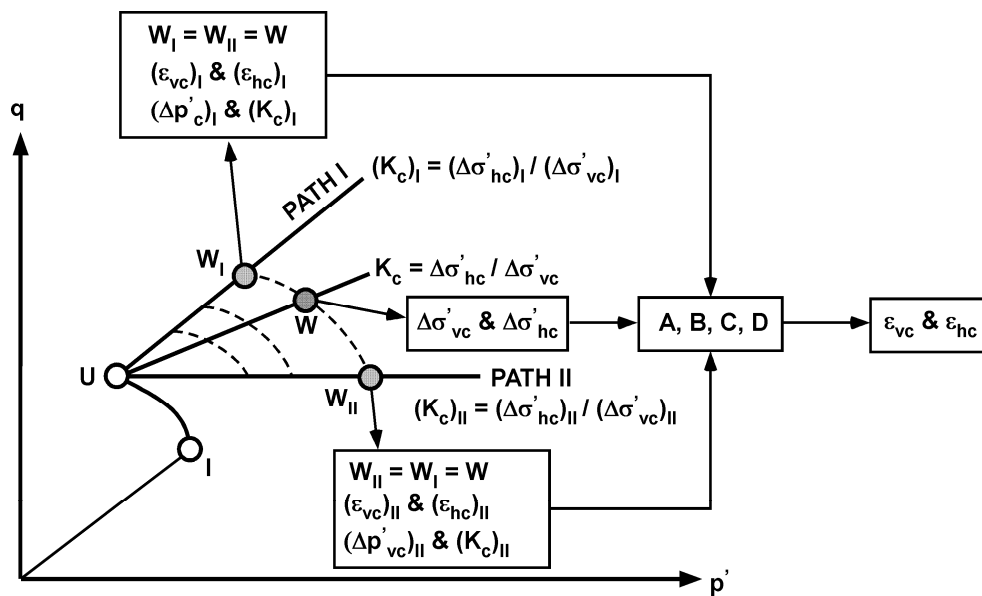


Fig. 2-30 Evaluating Strains For Consolidation Loading

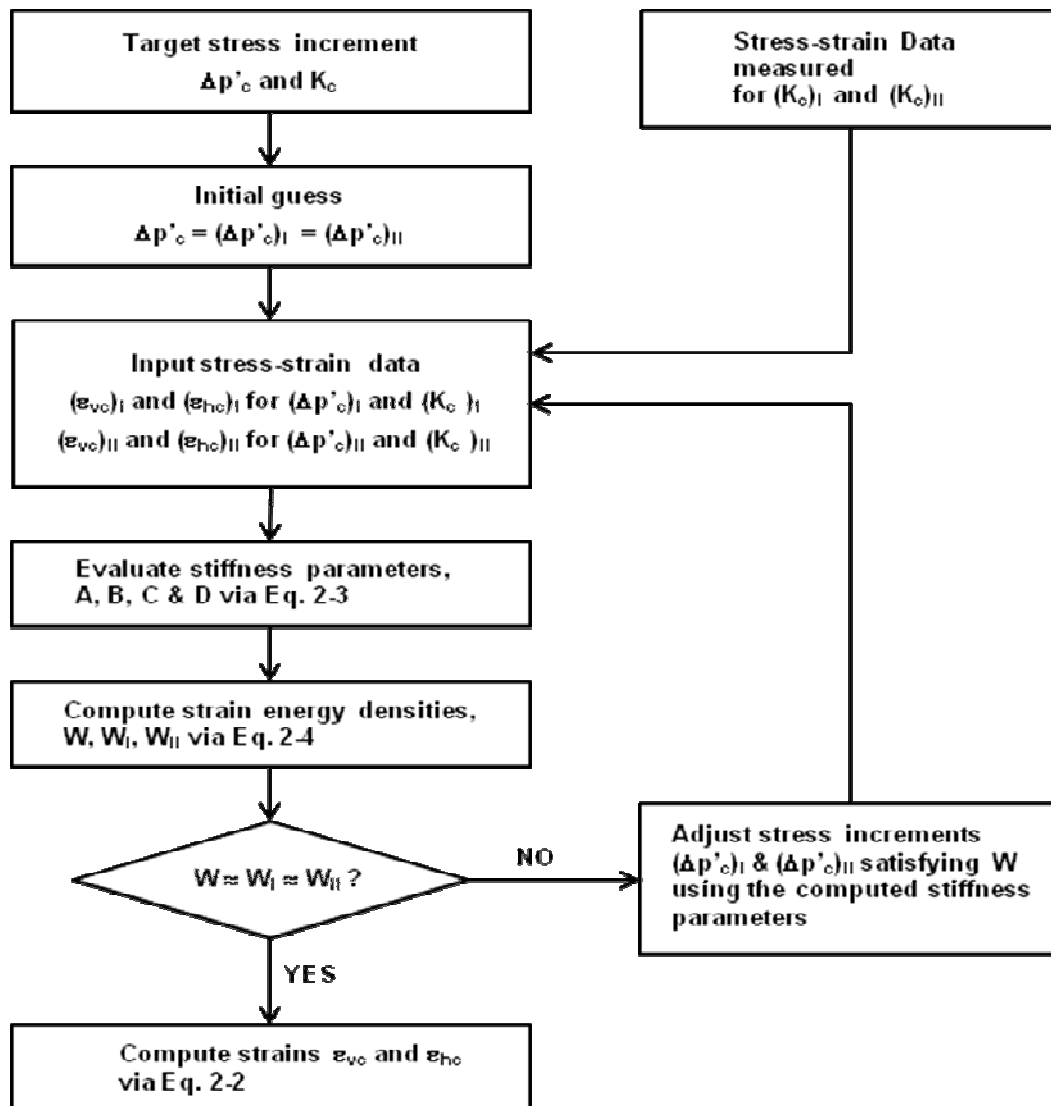


Fig. 2-31 Flow of the Proposed Stress Path Method

3) Experimental application.

- Soils → Reconstituted kaolinite specimens. (PI=23.3, CL)
- Stress path tests
 - Back pressure saturation
 - Reconsolidated under K_0 condition $\sigma'_{vc(max)} = 200 \text{ kPa}$ (twice larger than maximum past pressure)
 - Undrained loading followed back pressure equalization.
 - The axial stress was incrementally increased by 3.0 kPa.

- Then, consolidated with a selected stress path of K_c values.
 → Axial and horizontal stresses were applied at a rate of 0.8 kPa/h.
- Test summary

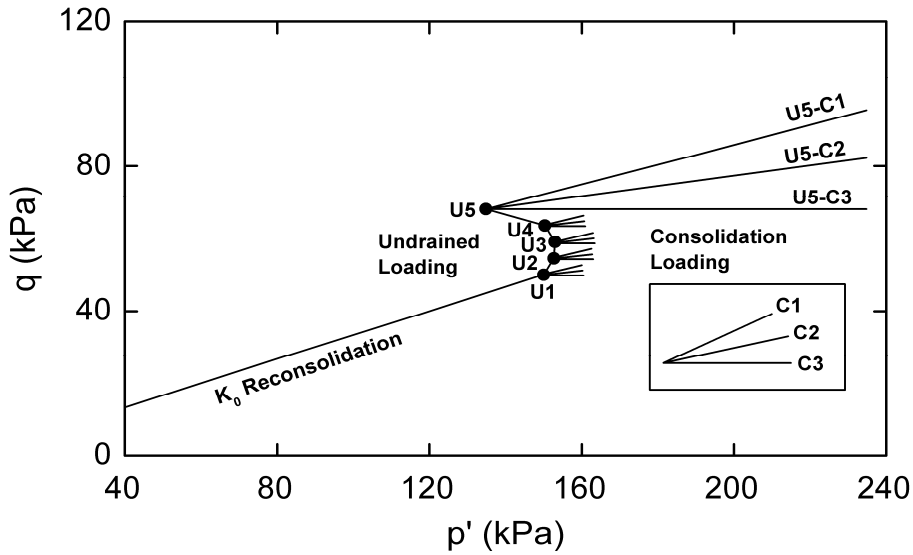


Fig. 2-32 Consolidation and Stress Probing Paths

→ 5 undrained loadings under 3 stress path tests with $K_c = 0.57, 0.75, 1.00$ were performed.

- Test results
 - ① Undrained loading results

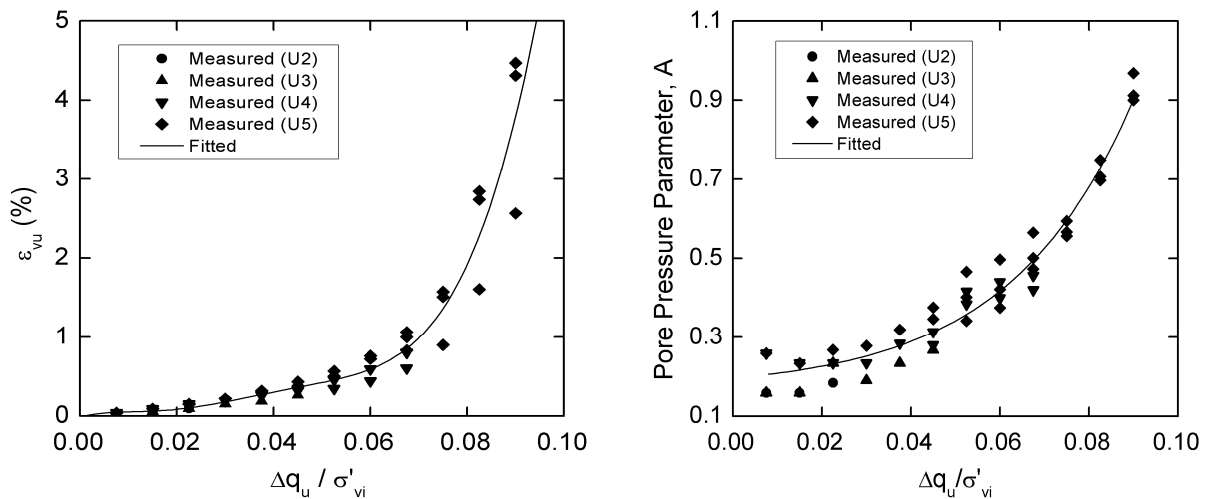


Fig. 2-33 Vertical Strain and Pore Pressure Parameter During Undrained Loading

② Stress path loading results

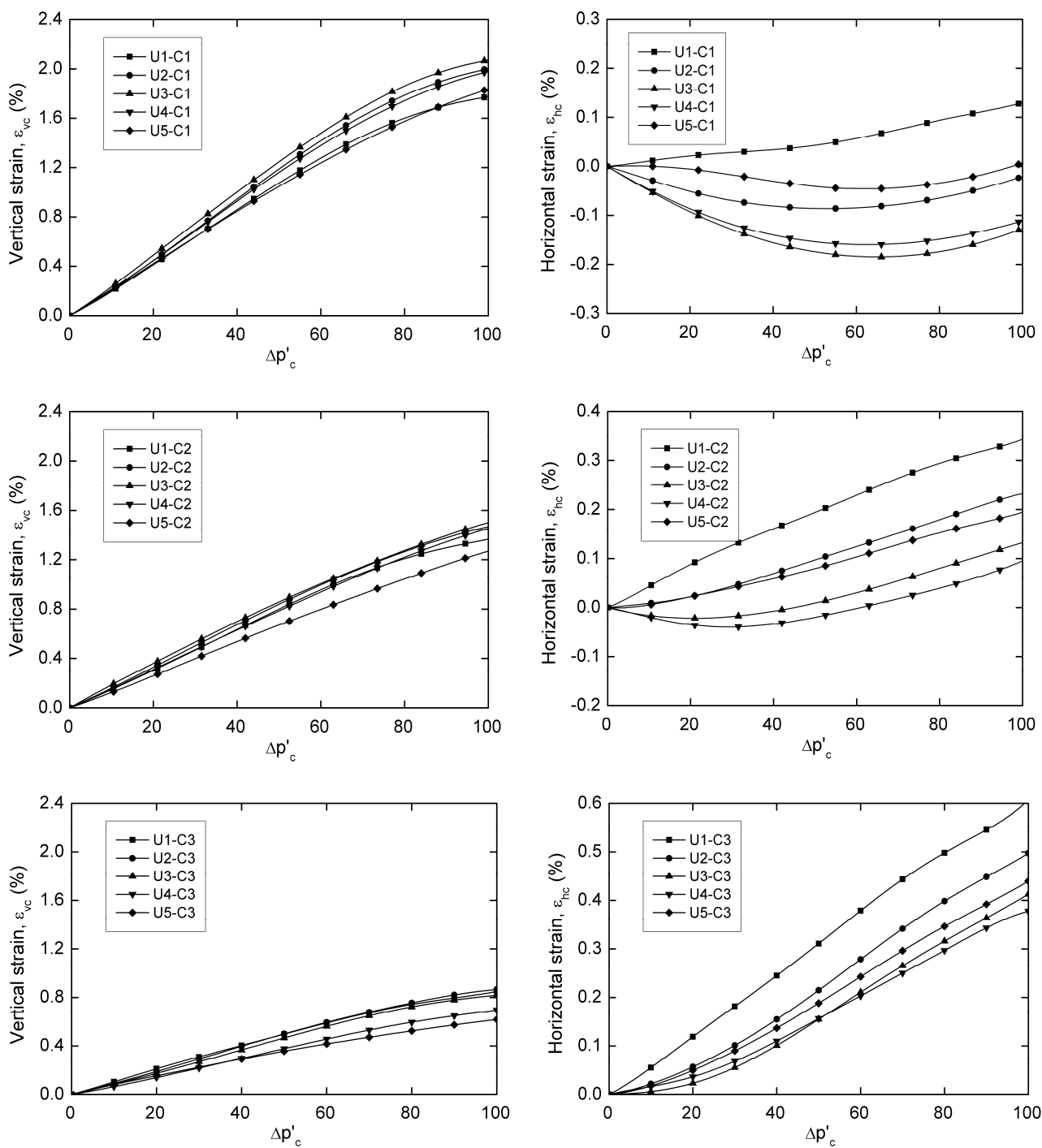


Fig. 2-34 Stress-Strain Responses During Consolidation

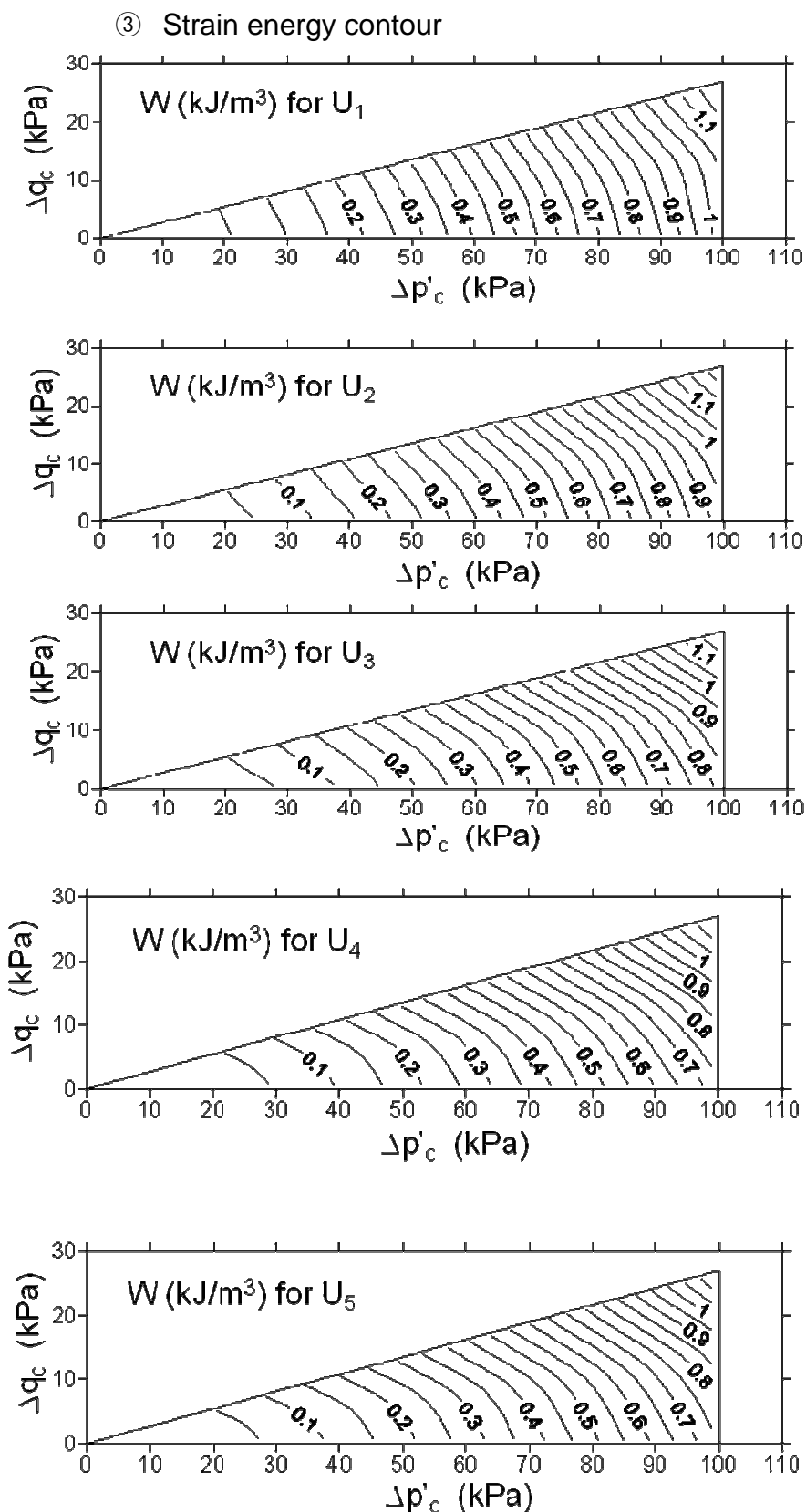
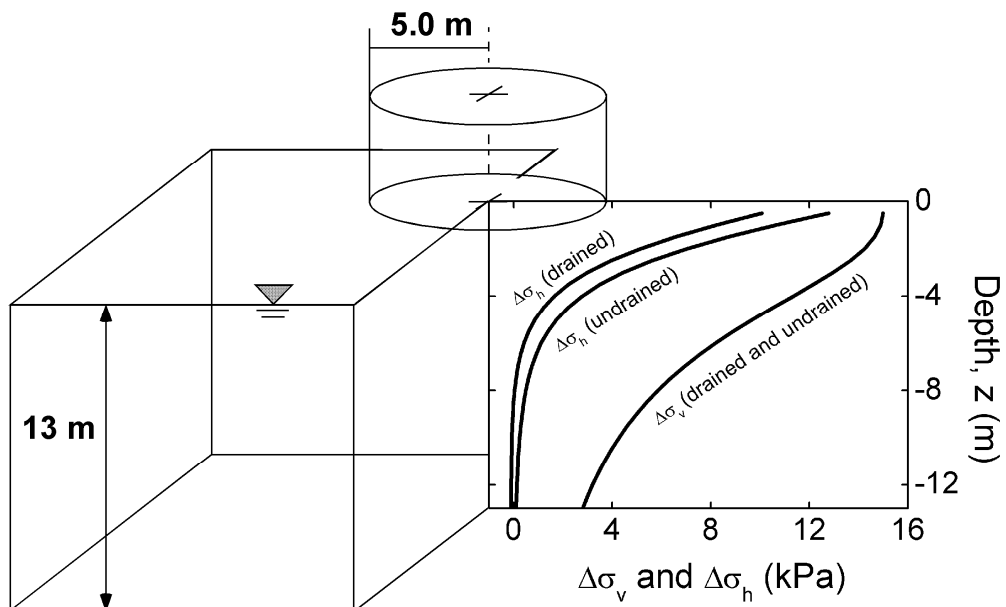


Fig. 2-35 Contour of the Strain Energy Density During Consolidation

4) An example of evaluating consolidation settlement

- Given conditions



- Circular footing with $r=5\text{m}$, and applied pressure, $p_0 = 15\text{ kPa}$
- 13m deep clay stratum (fully saturated)

Table 2-4 Summary of index properties and consolidation characteristics

Unit weight (kN/m^3)	17.0
Specific Gravity	2.59
Liquid Limit (%)	44.4
Plasticity Index (%)	23.3
Percentage passing through #200 sieve (%)	98.0
Unified Classification	CL
Natural Water Content (%)	37.6 ~ 40.2
Max. Past Pressure (kPa)	98 ~ 116
Compression Index	0.253 ~ 0.286
Recompression Index	0.110 ~ 0.126
Permeability (10^{-9} m/s)	5.24 ~ 29.3

- Initial stresses at the depth of 6.5m

$$\sigma'_{vi} = 46.7 \text{ kPa}, \sigma'_{hi} = 23.4 \text{ kPa}$$

- Elastic solutions at the depth of 6.5m

$$\rightarrow \text{With } \nu = 0.5, (\Delta\sigma_v)_u = 7.55 \text{ kPa and } (\Delta\sigma_h)_u = 0.90 \text{ kPa}$$

$$\rightarrow \text{With } \nu = 0.3, (\Delta\sigma_v)_d = 7.55 \text{ kPa and } (\Delta\sigma_h)_d = 0.30 \text{ kPa.}$$

- The settlement computation

$$\Delta q_u / \sigma'_{vi} = ((\Delta\sigma_v)_u - (\Delta\sigma_h)_u) / 2\sigma'_{vi} = 0.071$$

$$\rightarrow \text{Fig. 2-33 gives } \varepsilon_{vu} = 1.12\%, A = 0.54.$$

$$\rightarrow \Delta u_e = A((\Delta\sigma_v)_u - (\Delta\sigma_h)_u) + (\Delta\sigma_h)_u = 4.49 \text{ kPa}$$

- The change of the effective stresses during consolidation

$$\Delta\sigma'_{vc} = (\Delta\sigma_v)_d - (\Delta\sigma_v)_u + \Delta u_e = 7.55 - 7.55 + 4.49 = 4.49 \text{ kPa}$$

$$\Delta\sigma'_{hc} = (\Delta\sigma_h)_d - (\Delta\sigma_h)_u + \Delta u_e = 0.30 - 0.90 + 4.49 = 3.89 \text{ kPa}$$

$$\Rightarrow \Delta p'_c = (\Delta\sigma'_{vc} + \Delta\sigma'_{hc}) / 2 = 4.19 \text{ kPa}$$

$$K_c = \Delta\sigma'_{hc} / \Delta\sigma'_{vc} = 0.87$$

- For $K_c = 0.87$ and $\Delta p'_c / \sigma'_{vi} = 4.19 / 46.7 = 0.090$, ε_{vc} , in the middle of the stratum is estimated 0.22%, based on iteration procedure,

Table 2-5 Iteration procedure in computing consolidation strain

Iteration	Input stress		Normalized stress		Computed strain (%)			
	increment		increment					
1 st	$(\Delta p'_c)_I$	$(\Delta p'_c)_{II}$	$\frac{(\Delta p'_c)_I}{\sigma'_{vi}}$	$\frac{(\Delta p'_c)_{II}}{\sigma'_{vi}}$	$(\epsilon_{vc})_I$	$(\epsilon_{hc})_I$	$(\epsilon_{vc})_{II}$	$(\epsilon_{hc})_{II}$
	4.188	4.188	0.090	0.090	0.406	-0.067	0.123	0.034
	Stiffness parameters (MPa ⁻¹)				Strain energy density (kJ/m ³)			
	A	B	C	D	W	W _I	W _{II}	
1.381	-1.234	-0.397	0.438	0.283	0.725	0.200		
2 nd	$(\Delta p'_c)_I$	$(\Delta p'_c)_{II}$	$\frac{(\Delta p'_c)_I}{\sigma'_{vi}}$	$\frac{(\Delta p'_c)_{II}}{\sigma'_{vi}}$	$(\epsilon_{vc})_I$	$(\epsilon_{hc})_I$	$(\epsilon_{vc})_{II}$	$(\epsilon_{hc})_{II}$
	2.617	4.982	0.056	0.107	0.254	-0.043	0.149	0.043
	Stiffness parameters (MPa ⁻¹)				Strain energy density (kJ/m ³)			
	A	B	C	D	W	W _I	W _{II}	
1.378	-0.736	-0.416	0.603	0.931	0.487	1.260		
10 th	$(\Delta p'_c)_I$	$(\Delta p'_c)_{II}$	$\frac{(\Delta p'_c)_I}{\sigma'_{vi}}$	$\frac{(\Delta p'_c)_{II}}{\sigma'_{vi}}$	$(\epsilon_{vc})_I$	$(\epsilon_{hc})_I$	$(\epsilon_{vc})_{II}$	$(\epsilon_{hc})_{II}$
	3.190	4.469	0.068	0.096	0.310	-0.052	0.132	0.037
	Stiffness parameters (MPa ⁻¹)				Strain energy density (kJ/m ³)			
	A	B	C	D	W	W _I	W _{II}	
1.380	-1.015	-0.408	0.510	0.563	0.568	0.552		
20 th	$(\Delta p'_c)_I$	$(\Delta p'_c)_{II}$	$\frac{(\Delta p'_c)_I}{\sigma'_{vi}}$	$\frac{(\Delta p'_c)_{II}}{\sigma'_{vi}}$	$(\epsilon_{vc})_I$	$(\epsilon_{hc})_I$	$(\epsilon_{vc})_{II}$	$(\epsilon_{hc})_{II}$
	3.211	4.458	0.069	0.095	0.312	-0.052	0.132	0.037
	Stiffness parameters (MPa ⁻¹)				Strain energy density (kJ/m ³)			
	A	B	C	D	W	W _I	W _{II}	
1.380	-1.022	-0.408	0.508	0.555	0.554	0.555		
Final	$\epsilon_{vc} = A\Delta\sigma_{vc} + B\Delta\sigma_{hc} = 1.380\text{MPa}^{-1} \times 4.49\text{kPa} - 1.022\text{MPa}^{-1} \times 3.89\text{kPa} = 0.22\%$							

strains $\epsilon_{hc} = C\Delta\sigma'_v + D\Delta\sigma'_h = -0.408\text{MPa}^{-1} \times 4.49\text{kPa} + 0.508\text{MPa}^{-1} \times 3.89\text{kPa} = 0.01\%$

- The settlement,

$$\rho = \int_0^H \epsilon_v dz \approx (\epsilon_{vu} + \epsilon_{vc})H = (1.12\% + 0.22\%) \times 13 \text{ m} = 17 \text{ cm}$$

- For further precise computation, the clay stratum could be divided into several layers.