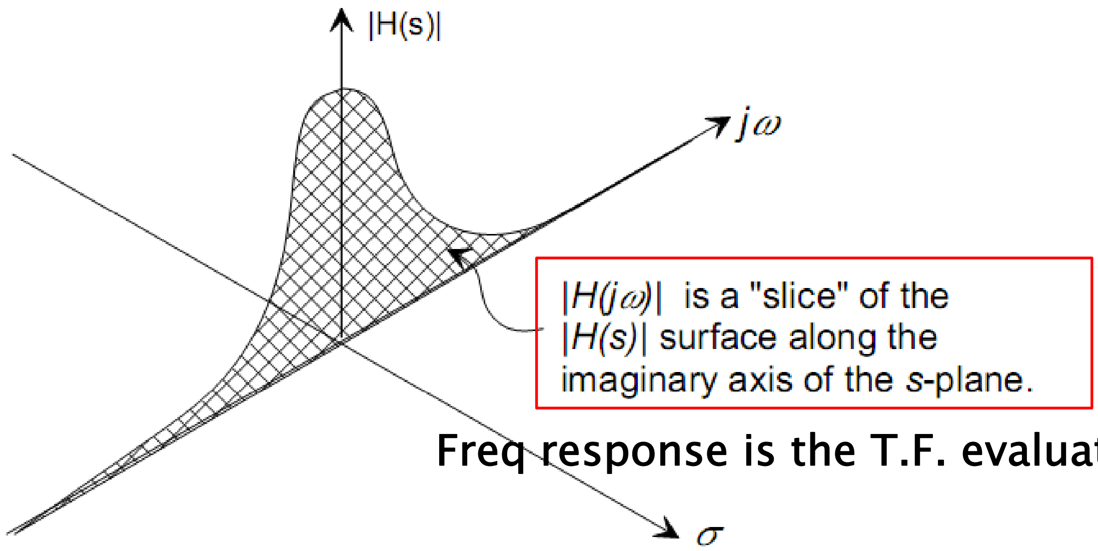
Frequency Domain Analysis III

Frequency Response

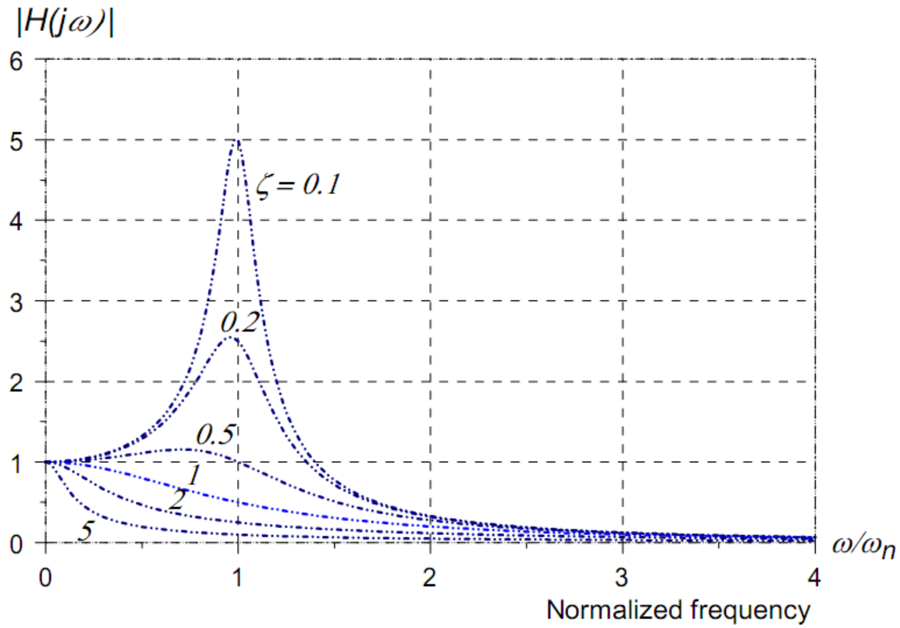
$u(t) = A \sin(\omega t + \theta)$ → **Linear System $H(s)$** → $y_{ss}(t) = A|H(j\omega)| \sin(\omega t + \theta + \angle H(j\omega))$

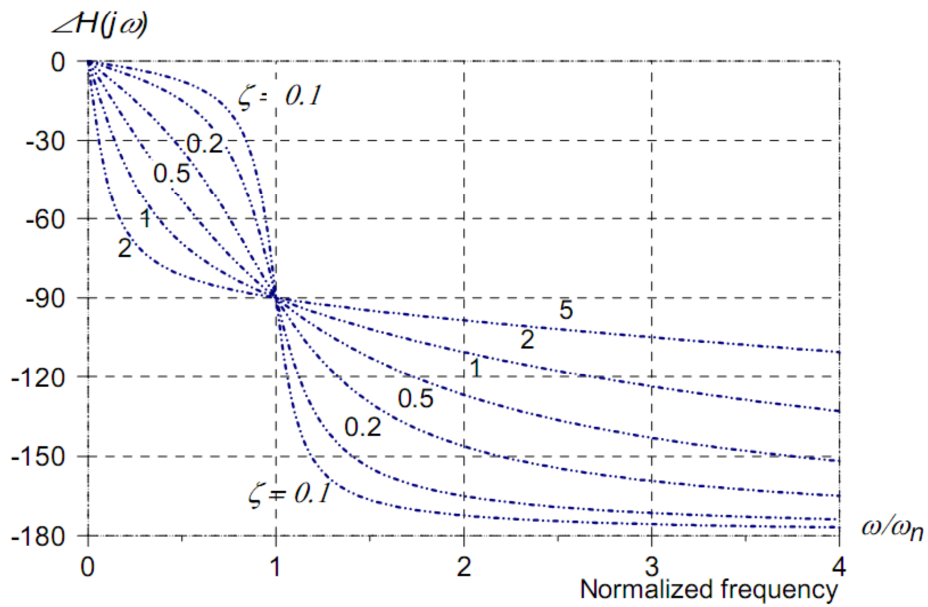
$$H(j\omega) = H(s)|_{s=j\omega}$$



Freq response is the T.F. evaluated at s=jw

Frequency Response and Damping ratio





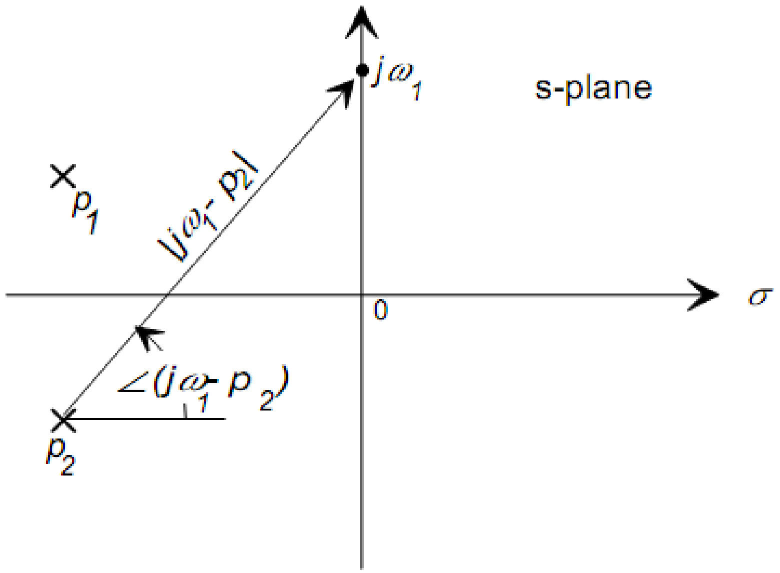
$$\omega_m = \omega_n \sqrt{1-2\zeta^2} \quad , \qquad M_m = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

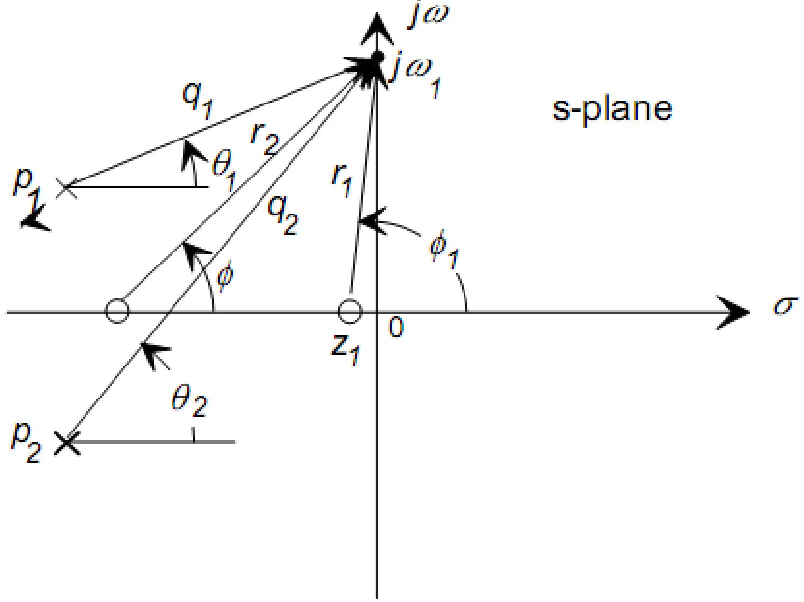
$$G(j\omega) = \frac{\omega_n^2}{\omega_n^2 - \omega^2 + 2\zeta\omega_n\omega j}$$

Frequency Response and Pole Zero Plot

$$H(j\omega) = K\frac{(j\omega - z_1)(j\omega - z_2)\ldots(j\omega - z_{m-1})(j\omega - z_m)}{(j\omega - p_1)(j\omega - p_2)\ldots(j\omega - p_{n-1})(j\omega - p_n)}$$

$$|j\omega - p_i| = \boxed{\sqrt{\sigma_i^2 + (\omega - \omega_i)^2},} \qquad \angle(s - p_i) = \boxed{\tan^{-1}\left(\frac{\omega - \omega_i}{-\sigma_i}\right)}$$





Frequency Response and Pole Zero Plot

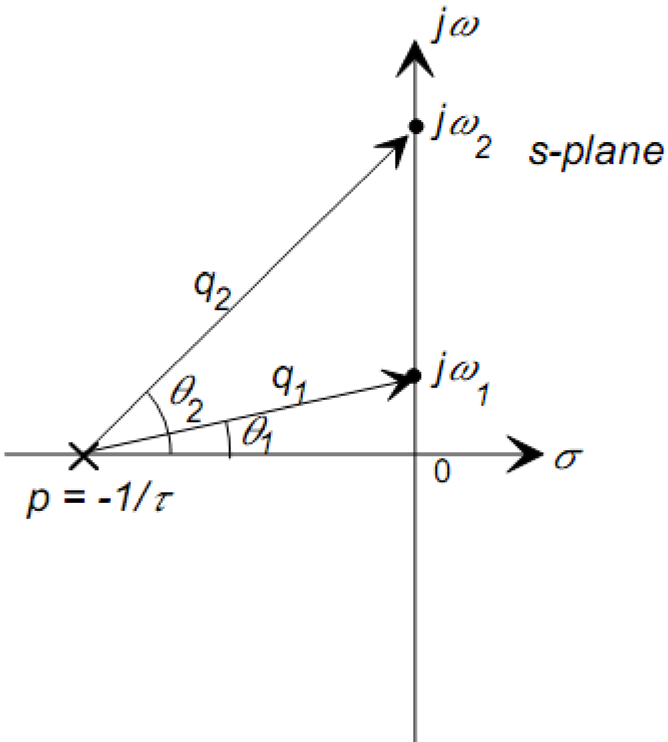
$$|H(j\omega)| = K\frac{\prod_{i=1}^{m}|(j\omega - z_i)|}{\prod_{i=1}^{n}|(j\omega - p_i)|} \qquad |H(j\omega)| = \boxed{K\frac{r_1 \ldots r_m}{q_1 \ldots q_n}}$$

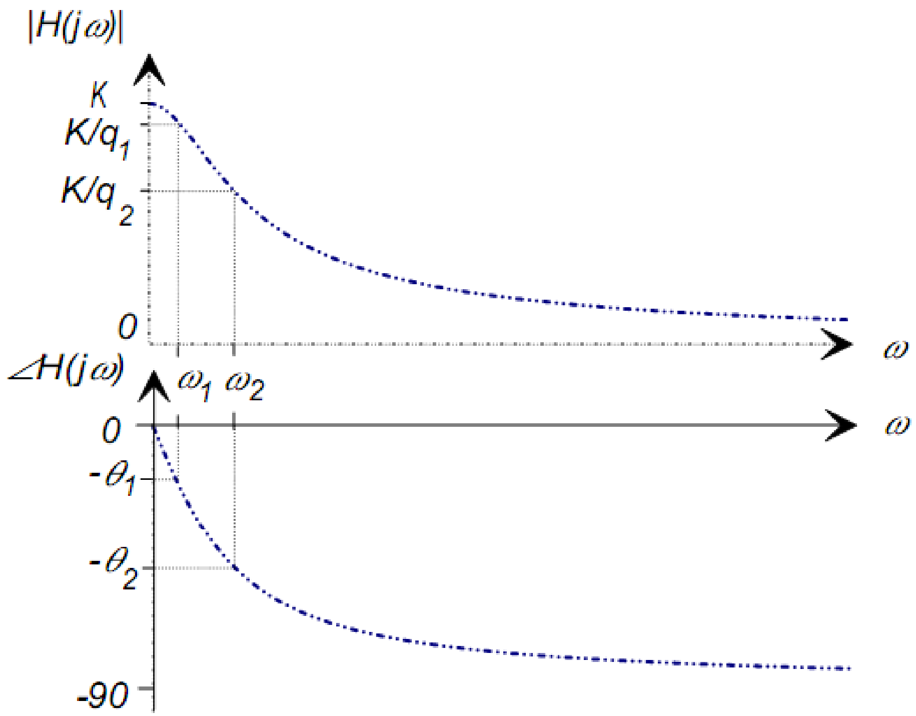
$$\angle H(j\omega) = \sum_{i=1}^{m}\angle(j\omega - z_i) - \sum_{i=1}^{n}\angle(j\omega - p_i)$$

$$\angle H(j\omega) = \boxed{(\phi_1 + \ldots + \phi_m) - (\theta_1 + \ldots + \theta_n)}$$

Frequency Response of a First Order System

A system with pole at s=−1/t





At low freq → the gain is finite. Phase has a lag.

As freq increases → gain decreases, phase lag increases

At ver high freq → gain 0, phase angle approaches pi/2

High Frequency Response

Magnitude Response

$$|H(j\omega)| = K\frac{r_1 \ldots r_m}{q_1 \ldots q_n}$$

$$\lim_{\omega\to\infty} |H(j\omega)| = \boxed{K\frac{1}{\omega^{n-m}}}$$

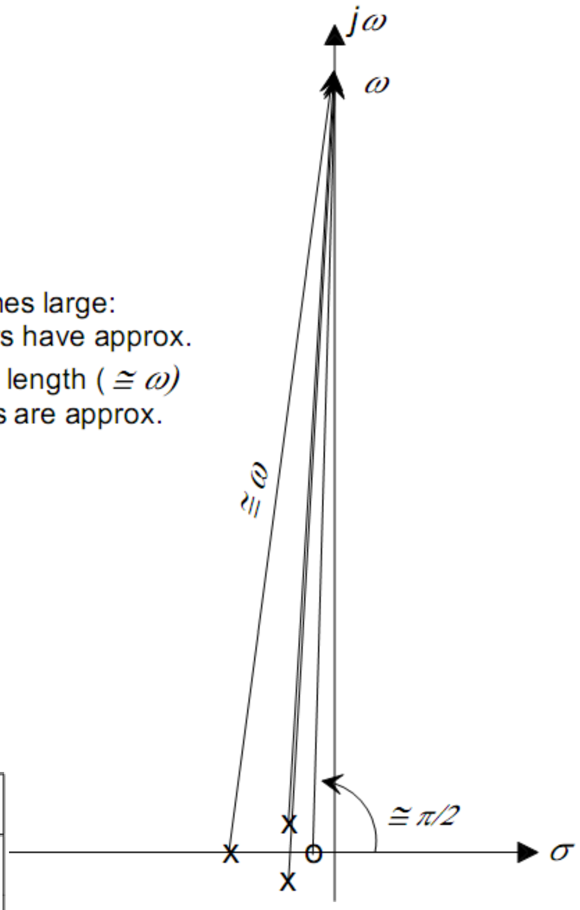
Phase Response

$$\angle H(j\omega) = (\phi_1 + \ldots + \phi_m) - (\theta_1 + \ldots + \theta_n)$$

$$\lim_{\omega\to\infty} \angle H(j\omega) = \boxed{-(n-m)\frac{\pi}{2}}$$

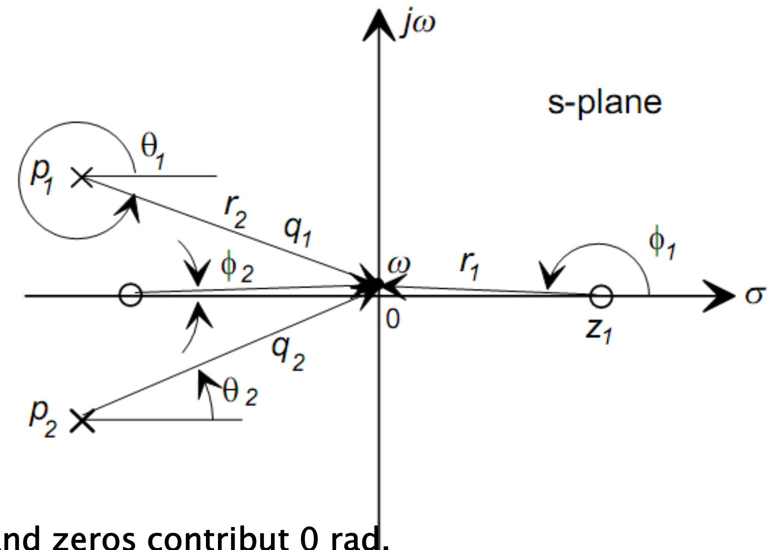
	$n > m$	$n = m$	$n < m$
$\lim_{\omega\to\infty} \lvert H(j\omega)\rvert$	0	K	∞
$\lim_{\omega\to\infty} \angle H(j\omega)$	$-(n-m)\pi/2$	0	$(m-n)\pi/2$

As ω becomes large:
1) All vectors have approx. the same length ($\cong \omega$)
2) All angles are approx. $\pi/2$



Low Frequency Response

Magnitude Response



$$|H(j\omega)| = K\frac{r_1 \dots r_m}{q_1 \dots q_n}$$

$$\lim_{\omega\to 0} |H(j\omega)| = \boxed{0} \quad \text{if there are zeros at the origin}$$

$$\lim_{\omega\to 0} |H(j\omega)| = \boxed{\infty} \quad \text{if there are poles at the origin}$$

$$\lim_{\omega\to 0} |H(j\omega)| = \boxed{K\frac{r_1 \dots r_m}{q_1 \dots q_n}} \quad \text{otherwise}$$

All real axis lhp poles and zeros contribut 0 rad.

Each complex conjugate pole or zero contriobute a total of 2pi rad.

A pole at the origin contributes -pi/2 rad

A zero at the origin contributes pi/2 rad

A r.h.p. real zero contributes pi rad

Phase Response

$$\angle H(j\omega) = (\phi_1 + \dots + \phi_m) - (\theta_1 + \dots + \theta_n)$$

$$\lim_{\omega\to 0} \angle H(j\omega) = \boxed{-(N-M)\frac{\pi}{2} + L\pi \text{ rad}}$$

N, M number of poles and zeros at the origin.

L: number of zeros at the r.h.p. real axis.

Frequency Response of poles and zeros close to imaginary axis

$$|H(j\omega)| = \boxed{\frac{K}{q_1 q_2}} \quad \angle H(j\omega) = \boxed{-(\theta_1 + \theta_2)}$$

The magnitude has apeak close to the freq near the pole.