Chapter 7 Flow of a Real Fluid

7.1 Laminar Flow

7.2 Turbulent Flow and Eddy Viscosity

7.3 Fluid Flow Past Solid Boundaries

7.4 Characteristics of Boundary Layers

7.5 The Laminar Boundary Layer*

7.6 The Turbulent Boundary Layer*

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7.9 Flow Establishment

7.10 Shear Stress and Head Loss

7.11 Shear Stress Effects

7.12 Velocity Distribution

7.13 Separation – Internal Flow

7.14 Secondary Flow– Internal Flow

7.15 Navier-Stokes Equation for Two-Dimensional Flow*

7.16 Applications of the Navier-Stokes Equations*

Objectives:

- Introduce the concepts of laminar and turbulent flow
- Examine the condition under which laminar and turbulent flow occur
- Introduce influence of solid boundaries on qualitative views
- Derive Navier-Stokes equation for two-dimensional flow and discuss its applications

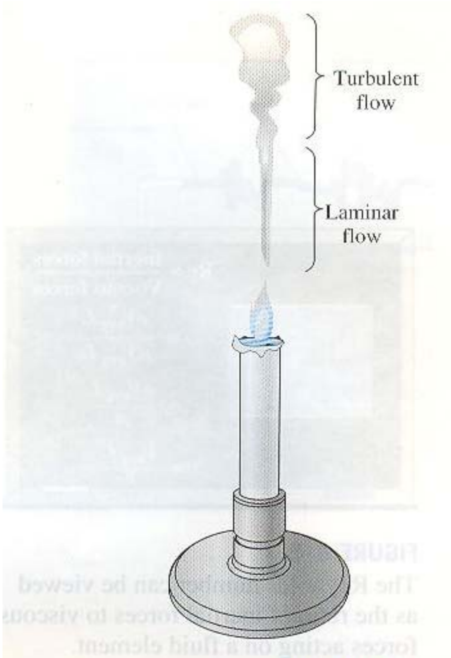


FIGURE 8–3
Laminar and turbulent flow regimes of candle smoke.

7.0 Introduction

• Ideal fluid

- In Chs.1 ~ 5, the flow of an ideal incompressible fluid was considered.
- Ideal fluid was defined to be inviscid, devoid of viscosity.
- There were no frictional effects between moving fluid layers or between the fluid and bounding walls.

• Real Fluid

- Viscosity introduces resistance to motion by causing shear or friction forces between fluid particles and between these and boundary walls.
- For flow to take place, work must be done against these resistance forces. In this process energy is converted into heat (mechanical energy loss).

	Ideal (inviscid) fluid	**Real (viscous) fluid**
Viscosity	inviscid	viscous
Velocity profile	Uniform (slip condition)	non-uniform(no-slip)
Eq. of motion	Euler's equation	Navier-Stokes equation (Nonlinear, 2nd-order P.D.E)

7.1 Laminar flow

• Laminar flow

- Agitation of fluid particles is a molecular nature only.

- Length scale ~ order of mean free path of the molecules

- Particles appear to be constrained to motion in parallel paths by the action of viscosity.

- Viscous action damps disturbances by wall roughness and other obstacles.

  → stable flow

- The shearing stress between adjacent layers is

$$\tau = \mu \frac{dv}{dy} \qquad (1.12)$$

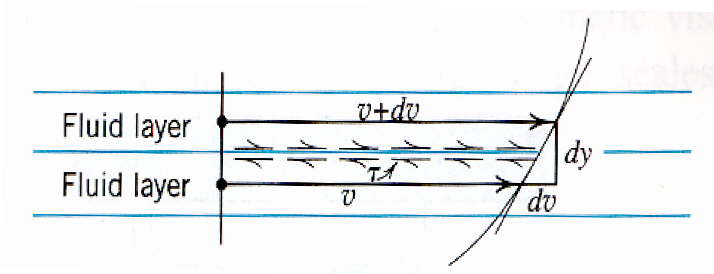


Fig. 7.1

• Turbulent flow

- Fluid particles do not retain in layers, but move in heterogeneous fashion through the flow.

- Particles are sliding past other particles and colliding with some in an entirely random manner.

- Rapid and continuous macroscopic mixing of the flowing fluid occurs.

- Length scale of motion >> molecular scales in laminar flow

• Two forces affecting motion

(i) Inertia forces, F_I

~ acceleration of motion

$$F_I = M\,a = \rho l^3 \left(\frac{V^2}{l} \right) = \rho V^2 l^2$$

(ii) Viscous forces, F_V

$$F_V = \tau\ A = \mu\ \frac{dV}{dy}\ l^2 = \frac{\mu V\, l^2}{l} = \mu V\, l$$

• Reynolds number R_e

$$R_e = \frac{F_I}{F_V} = \frac{\rho V^2 l^2}{\mu Vl} = \frac{\rho Vl}{\mu} = \frac{Vl}{\mu/\rho} = \frac{Vl}{\nu}$$

$$\mu = \text{dynamic viscosity} \qquad (\text{kg}\,\text{m}^{-1}\text{s}^{-1})$$

$$\nu = \frac{\mu}{\rho} = \text{kinematic viscosity} \qquad (\text{m}^2/\text{s})$$

• Inertia forces are dominant → turbulent flow

Viscous forces are dominant → laminar flow

• Reynolds dye stream experiments

low velocity → low Reynolds number → laminar flow

high velocity → high Reynolds number → turbulent flow

• Critical velocity

upper critical velocity: laminar → turbulent

lower critical velocity: turbulent → laminar

• Critical Reynolds number

(i) For pipe flow

$$R_e = \frac{Vd}{\nu}\ , \quad d = \text{pipe diameter} \tag{7.1}$$

$$\begin{cases} R_e < 2100 \rightarrow \text{laminar flow} \ \cdots \ \text{lower critical} \quad R_{c1} = 2100 \\ 2100 < R_e < 4000 \rightarrow \text{transition} \ \cdots \ \text{upper critical} \quad R_{c2} = 4000 \\ R_e \gg 4000 \rightarrow \text{turbulent flow} \end{cases}$$

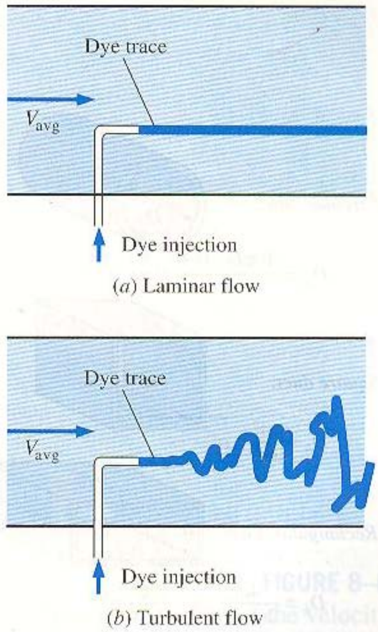


FIGURE 8–4
The behavior of colored fluid injected into the flow in laminar and turbulent flows in a pipe.

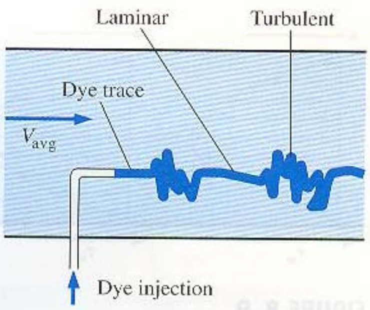
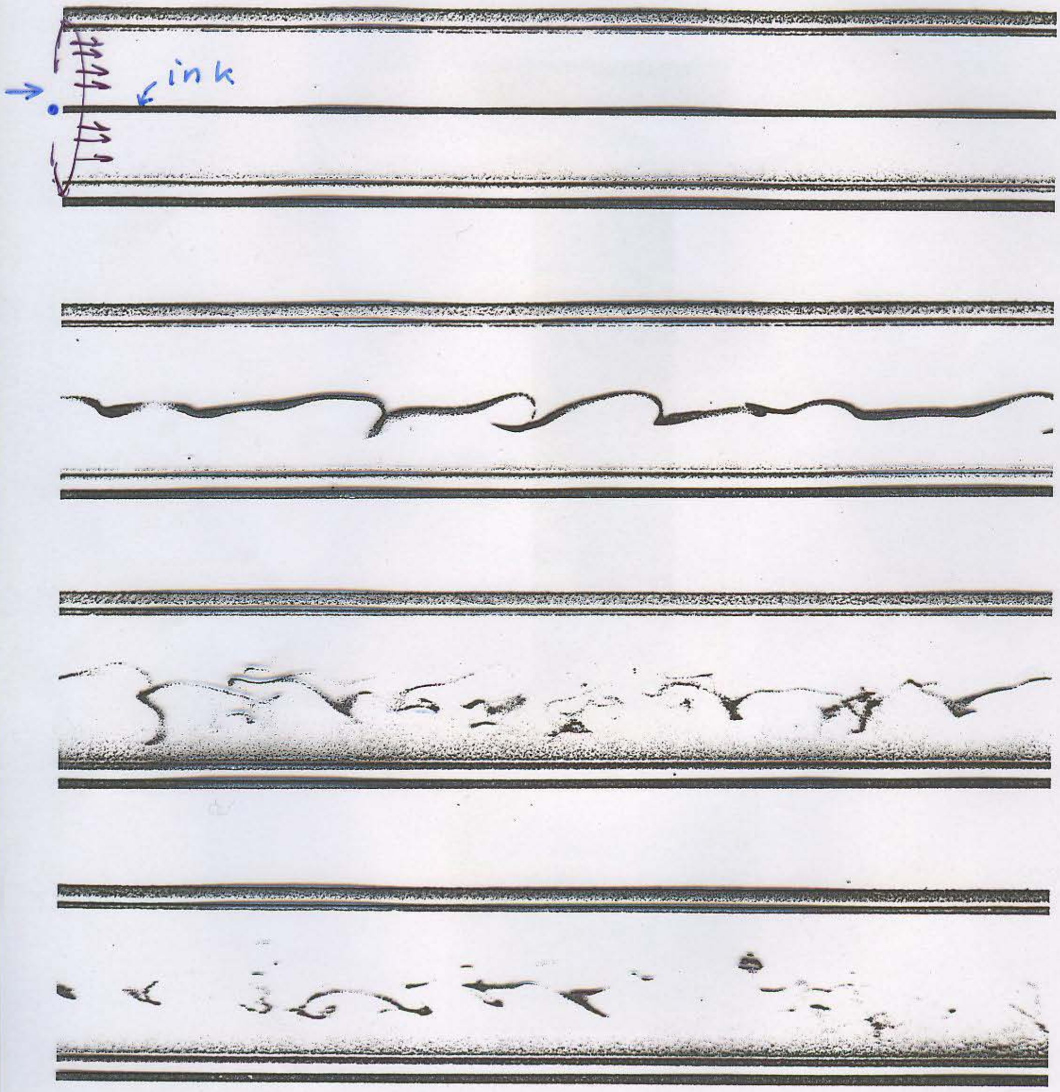


FIGURE 8–7
In the transitional flow region of $2300 \leq Re \leq 4000$, the flow switches between laminar and turbulent randomly.



103. Repetition of Reynolds' dye experiment. Osborne Reynolds' celebrated 1883 investigation of stability of flow in a tube was documented by sketches rather than photography. However the original apparatus has survived at the University of Manchester. Using it a century later, N. H. Johannesen and C. Lowe have taken this sequence of photographs. In laminar flow a filament of colored water introduced at a bell-shaped entry extends undisturbed the whole length of the glass tube. Transition is seen in the second of the photographs as the speed is increased; and the last two photographs show fully turbulent flow. Modern traffic in the streets of Manchester made the critical Reynolds number lower than the value 13,000 found by Reynolds.

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(ii) Open channel flow: $R_e < 500 \rightarrow$ laminar flow

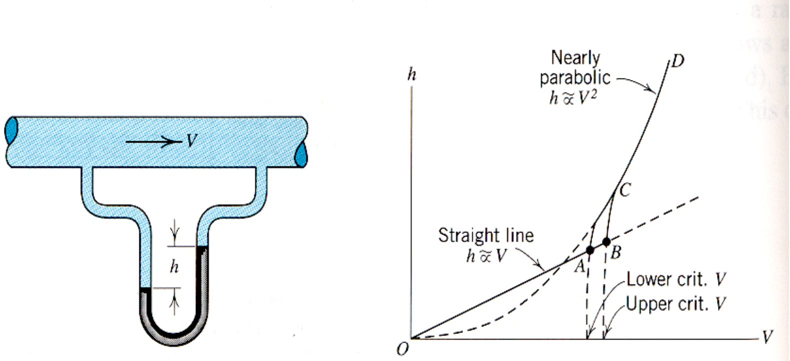
$$R_c = \frac{Vd}{\nu} = \frac{V(4R)}{\nu} = 2100 \qquad \therefore\ R_c = \frac{VR}{\nu} \cong 500$$

$$R = \text{hydraulic radius} = A = \frac{\pi d^2/4}{\pi d} = \frac{d}{4}$$

(iii) Flow about a sphere: $R_e < 1 \rightarrow$ laminar flow

where V = approach velocity; d = sphere diameter

• Experiment for two flow regimes



i) As V is increased → OABCD

OB (laminar flow) → BC (transition region) → CD (turbulent flow)

upper critical velocity

ii) As V is decreased → DCAO

DC (turbulent) → CA (transition) → AO (laminar flow)

lower critical velocity

[IP 7.1] Water at 15˚C flows in a cylindrical pipe of 30 mm diameter.

$$\nu = 1.339 \times 10^{-6}\ \mathrm{m^2/s} \quad \leftarrow \quad \text{water at } 15^{\circ}\mathrm{C} \quad \text{p.694} \quad \text{A. 2.4b}$$

Find largest flow rate for which laminar flow can be expected.

[Sol]

Take $R_c = 2100$ as the conservative upper limit for laminar flow

(a) For water

$$R_c = 2100 = \frac{Vd}{\nu} = \frac{V\left(30/10^3\right)}{1.139 \times 10^{-6}}$$

$$V_{water} = 0.080\ \mathrm{m/s}$$

$$Q_{water} = 0.0805\left(\frac{\pi}{4}(0.03)^2\right) = 5.69 \times 10^{-5}\ \mathrm{m^3/s} \qquad (4.4)$$

(b) For air

$$\nu_{air} = 1.46 \times 10^{-5}\ \mathrm{m^2/s} \quad \left(\mu_{air}/\rho \ = \ 1.8 \times 10^{-5}\ \mathrm{pa \cdot s} \,/\, 1.225\ \mathrm{kg/m^3}\right)$$

$$V_{air} = 1.022\ \mathrm{m/s} \qquad (7.1)$$

$$Q_{air} = 7.22 \times 10^{-4}\ \mathrm{m^3/s} \ \approx \ 13\, Q_{water} \qquad (4.4)$$

$$\mu_{air} < \mu_{water}$$

$$V_{c,air} > V_{c,water}$$

7.2 Turbulent Flow and Eddy Viscosity

• Turbulent flow

- Turbulence is found in the atmosphere, in the ocean, in most pipe flows, in rivers and estuaries, and in the flow about moving vehicles and aircrafts.

- Turbulence is generated primarily by friction effects at solid boundaries or by the interaction of fluid streams that are moving past each other with different velocities (shear flow).

wall turbulence

free turbulence

• Characteristics of turbulent flow (Tennekes & Lumley, 1972)

① Irregularity or randomness in time and space

② Diffusivity or rapid mixing → high rates of momentum and heat transfer

③ High Reynolds number

④ 3-D vorticity fluctuations → 3d nature of turbulence

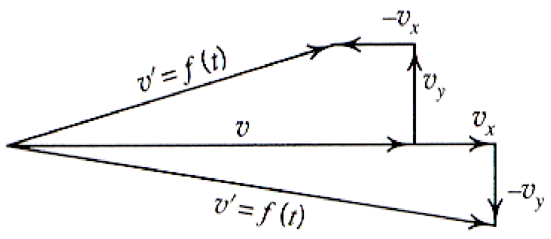
⑤ Dissipation of the kinetic energy of the turbulence by viscous shear stresses

[Energy cascade: energy supply from mean flow to turbulence]

⑥ Continuum phenomenon even at the smallest scales

⑦ Feature of fluid flows, not a property of fluids themselves

• Decomposition of turbulent flow



$$v_x^{'}(t) = \overline{v} + v_x$$

$$v_y^{'}(t) = \overline{v} + v_y$$

$v^{'}(t) =$ instantaneous turbulent velocity

$\overline{v} =$ time mean velocity $= \dfrac{1}{T}\int_0^T v^{'}(t)\,dt$

$v_x =$ turbulent fluctuation in *x*-direction

$v_y =$ turbulent fluctuation in *y*-direction

$$\overline{v_x} = \frac{1}{T}\int_0^T v_x dt = 0$$

$$\overline{v_y} = 0$$

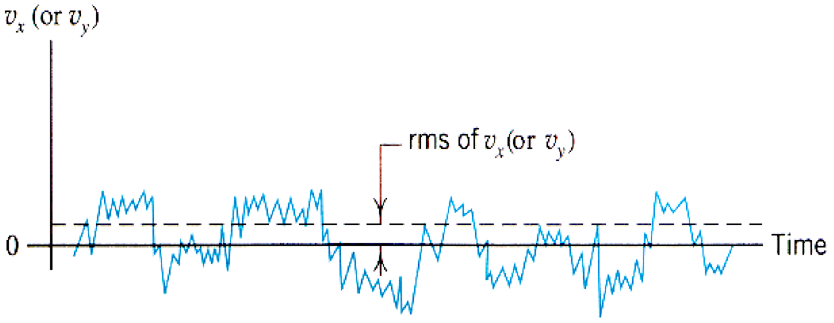


Fig. 7.4

- rms $= \sqrt{\overline{(v_x')^2}} = \left[\frac{1}{T} \int_0^T v_x'^2 dt \right]^{1/2}$

- Relative intensity of turbulence $= \dfrac{\sqrt{\overline{v_x'^2}}}{\overline{v}}$

- Mean time interval, T

T = times scale = meaningful time for turbulence fluctuations

- air flow: $10^{-1} \sim 10^{0}$ sec

- pipe flow: $10^{-1} \sim 10^{0}$ sec

- open flow: $10^{0} \sim 10^{1}$ sec

~ measure of the scale of the turbulence

~ size of the turbulent eddies $\propto$ size of boundary

~ order of (pipe radius, channel width or depth, boundary layer thickness)

→ The intensity of turbulence increases with velocity, and scale of turbulence increases with boundary dimensions.

[Re] Measurement of turbulence

(i) Hot-wire anemometer

~ use laws of convective heat transfer

~ Flow past the (hot) sensor cools it and decrease its resistance and output voltage.

~ record of random nature of turbulence

(ii) Laser Doppler Velocitymeter (LDV)

~ use Doppler effect

(iii) Acoustic Doppler Velocitymeter (ADV)

(iv) Particle Image Velocimetry (PIV)

Hot wire anemometer

Figure 15.21 Two forms of hot-wire anemometer probes:
(a) wire mounted normal to probe axis,
(b) wire mounted parallel to probe axis

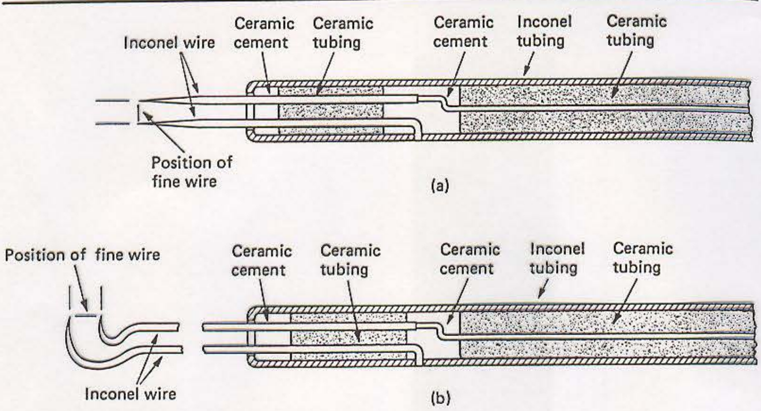
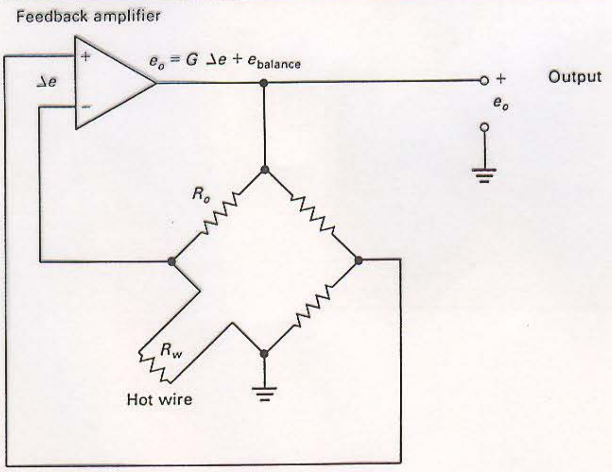


Figure 15.22 Constant-temperature-anemometer bridge circuit



Laser Doppler velocimetry

Figure 15.25 LDA transmitter and receiver packages (Courtesy of David Carr, Aerometrics Inc., Sunnyvale, CA)

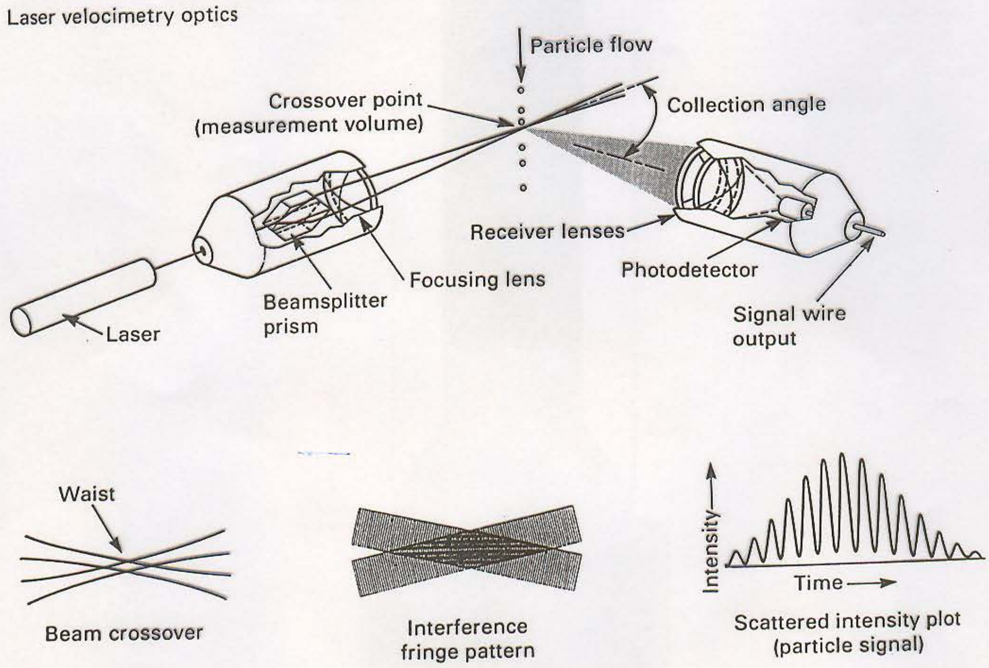
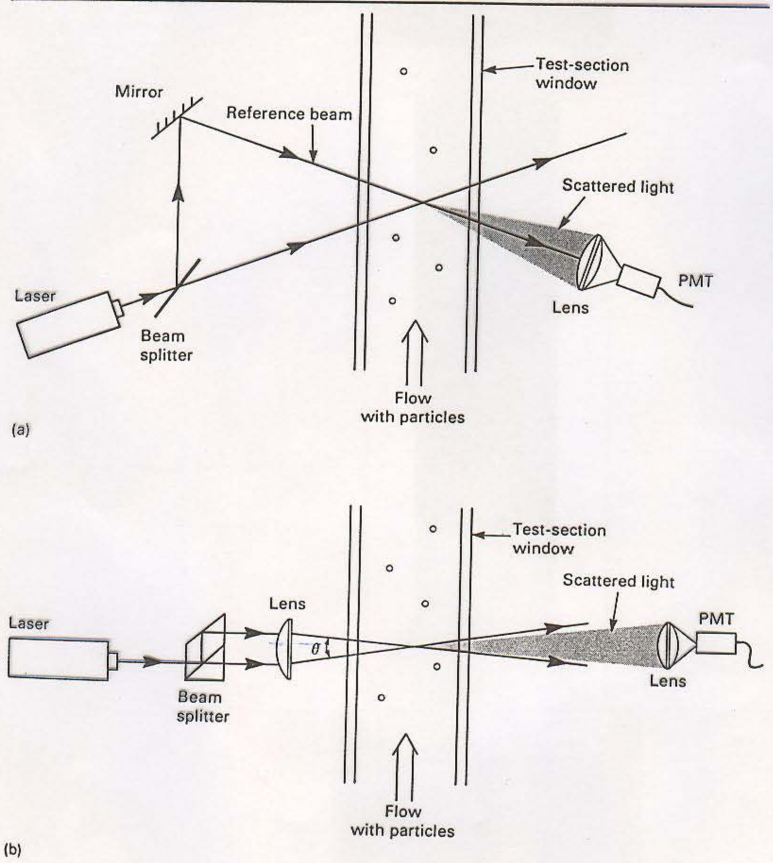


Figure 15.24 Laser-Doppler optical systems: (a) reference-beam arrangement, (b) differential-Doppler arrangement



fringe spacing:

$$f_D = \frac{V_x}{\delta} = \left(\frac{2V_x}{\lambda}\right) \sin\left(\frac{\theta}{2}\right), \qquad \textbf{(15.18)}$$

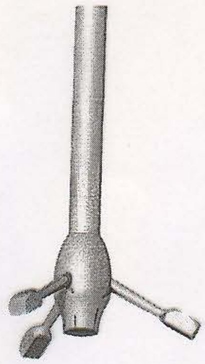
where

f_D = the Doppler-shift frequency,

V_x = the particle velocity in the direction normal to the fringes.

Acoustic Doppler Velocimeter

MicroADV



High Resolution Acoustic Doppler Velocimeter

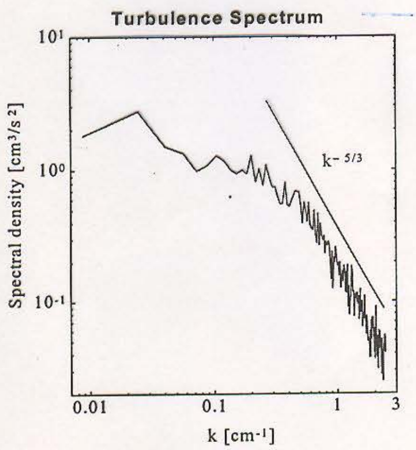
The SonTek MicroADV is the most significant breakthrough in current meter technology since the original SonTek ADV.

The original SonTek ADV has compared favorably with Laser systems costing ten times as much; the MicroADV fares even better.

The higher acoustical frequency of 16MHz makes the MicroADV the optimal instrument for laboratory work for several reasons:

- **Small sampling volume – less than 0.1 cm^3**
- **High sampling rates – up to 50 Hz**
- **Small optimal scatterer – excellent for low flows**

Like all SonTek instruments, the MicroADV is extremely simple to set up and use. Most users are taking high quality data within minutes of receiving the system.

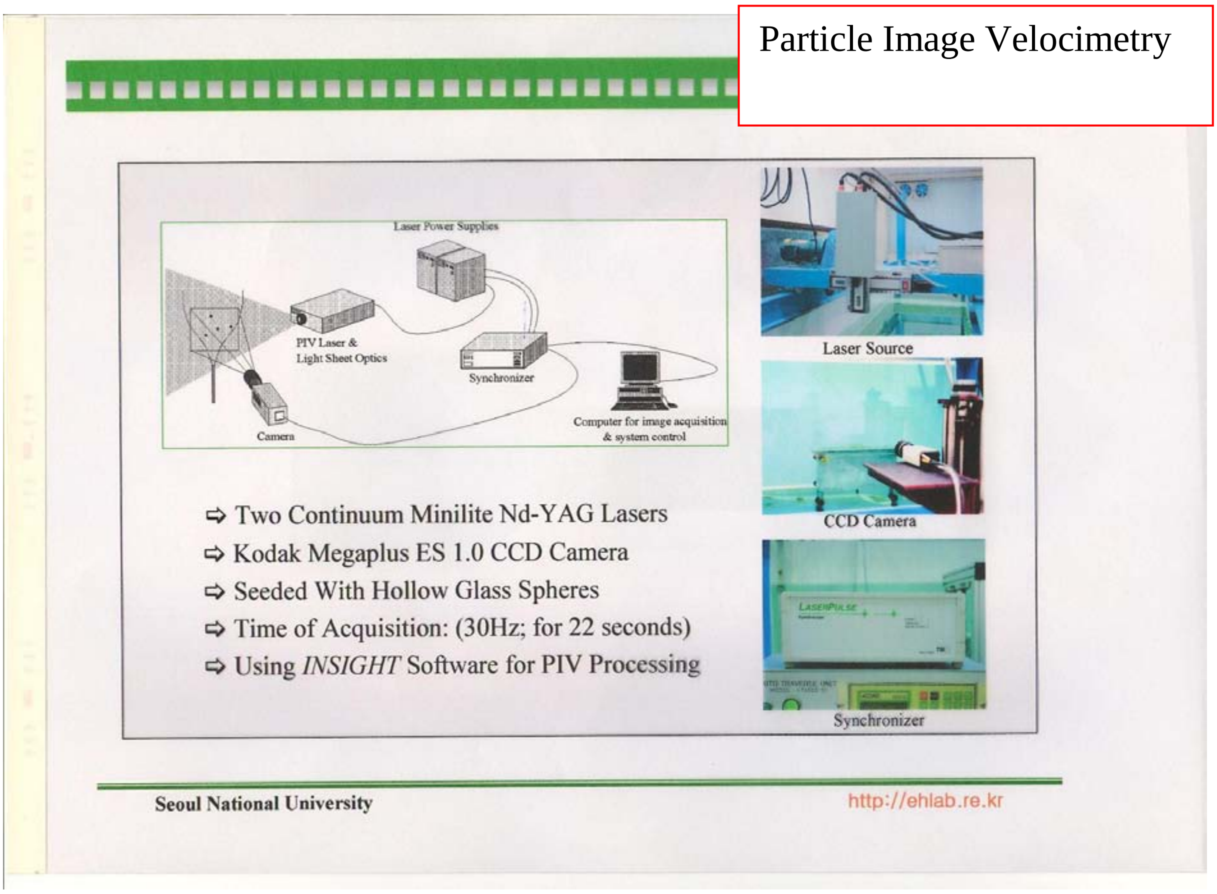


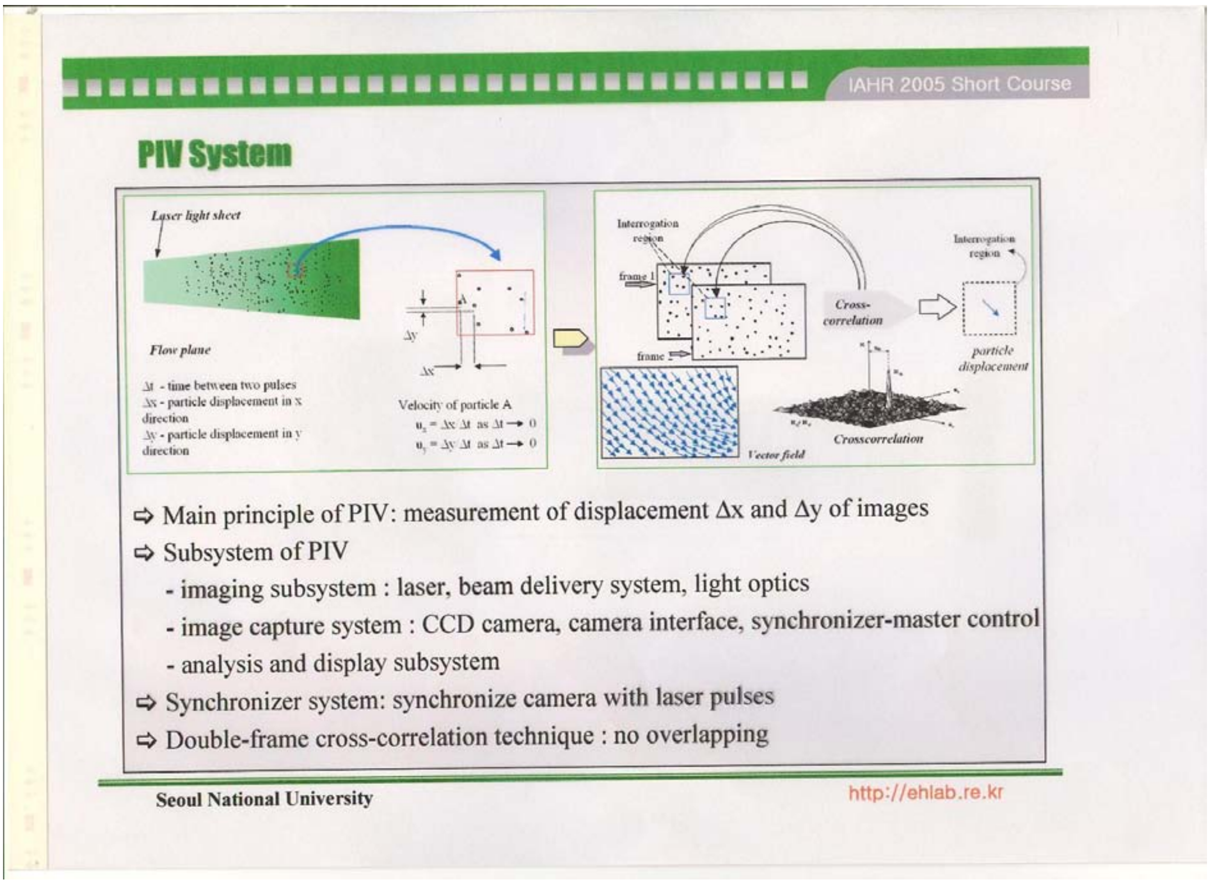
Additional Features:

- **Three axis velocity measurement**
- **High accuracy – 1% of measured**
- **Large velocity range – 1 mm/s - 2.5 m/s**
- **Excellent low flow performance**
- **No recalibration requirement**
- **Low price**
- **Comprehensive software**

The ADV, like all SonTek instruments, comes complete with our no-questions-asked warranty and our renowned customer service. The MicroADV is available from SonTek and authorized SonTek representatives worldwide.

Particle Image Velocimetry



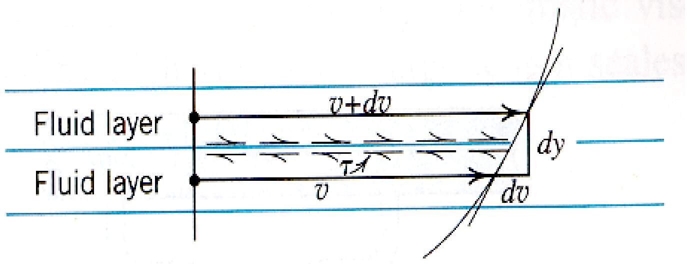


• Turbulence

- Because turbulence is an entirely chaotic motion of small fluid masses, motion of individual fluid particle is impossible to trace.

→ Mathematical relationships may be obtained by considering the average motion of aggregations of fluid particles or by statistical methods.

• Shearing stresses in turbulent flow



Let time mean velocity $v = \bar{v}$

Thus, velocity gradient is $\dfrac{dv}{dy}$

Now consider momentum exchange by fluid particles moved by turbulent fluctuation

Mass moved to the lower layer tends to speed up the slower layer (v)

Mass moved to the upper layer tends to slow down the faster layer $(v + \Delta v)$

→ This is the same process as if there were a shearing stress between two layers.

• Problem of useful and accurate expressions for turbulent shear stress in terms of mean velocity gradients and other flow properties

1) Boussinesq (1877)

~ suggest the similar equation to laminar flow equation

$$\tau = \varepsilon \frac{dv}{dy} \tag{7.2}$$

ε = eddy viscosity

= property of flow (not of the fluid alone)

$= f$ (structure of the turbulence, space)

$$\tau_{total} = (\mu + \varepsilon)\frac{dv}{dy}$$

where μ = viscosity action, ε = turbulence action

2) Reynolds (1895)

~ suggest the turbulent shear stress with time mean value of the product of $v_x v_y$

$$\tau = -\rho\, \overline{v_x\, v_y}$$ ~ Reynolds stress

v_x = fluctuating velocity along the direction of general mean motion

v_y = fluctuating velocity normal to the direction of general mean motion

$\overline{v_x v_y}$ = time mean value of the product of $v_x v_y$ $= \frac{1}{T}\int v_x v_y d$ [illegible]

3) Prandtl (1926)

~ propose that small aggregations of fluid particles are transported by turbulence a certain mean distance, *l*, from regions of one velocity to regions of another.

~ termed the distance *l* the mixing length → Prandtl's mixing length theory

$$\tau = \rho l^2 \left(\frac{dv}{dy} \right)^2 \tag{7.3}$$

where l = mixing length = $f(y)$

Comparing Eqs. (7.2) and (7.3) gives

$$\varepsilon = \rho l^2 \frac{dv}{dy} \tag{7.4}$$

• Flow near the boundary wall

~ turbulence is influenced by the wall = wall turbulence

$$l = \kappa y \tag{7.5}$$

where $\kappa =$ von Karman constant ≈ 0.4; $y =$ distance from wall

$$\tau = \rho \kappa^2 y^2 \left(\frac{dv}{dy} \right)^2 \tag{7.6}$$

[IP 7.2] Show that if laminar flow is parabolic velocity profile, the shear stress profile must be a straight line.

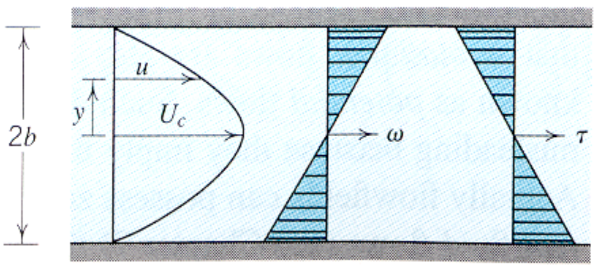
[Sol]

$$\tau = \mu \frac{dv}{dy} \tag{1.12}$$

$$v = C_1 y^2 + C_2 \quad \rightarrow \quad \text{parabolic}$$

$$\frac{dv}{dy} = 2\, C_1\, y$$

$$\therefore \quad \tau = 2\, C_1\, \mu\, y = C\, y \quad \rightarrow \quad \text{straight line}$$



[IP 7.3] A turbulent flow of water occurs in a pipe of 2 m diameter.

$$v = 10 + 0.8 \ln y \quad (\text{logarithmic})$$

$$\tau\Big|_{y=1/3m} = 103 \text{ Pa}$$

Calculate $\varepsilon, \; l, \; \kappa$

[Sol]

$$\tau = \varepsilon \frac{dv}{dy} \qquad \text{(a)}$$

$$= \rho l^2 \left(\frac{dv}{dy} \right)^2 \qquad \text{(b)}$$

$$= \rho \kappa^2 y^2 \left(\frac{dv}{dy} \right)^2 \qquad \text{(c)}$$

$$\frac{dv}{dy}\bigg|_{y=1/3m} = \frac{0.8}{y}\bigg|_{y=1/3m} = 2.4 \; s^{-1}$$

(a): $103 = \varepsilon (2.4)$ $\qquad \varepsilon = 42.9 \text{ Pa} \cdot \text{s}$

(b): $103 = 10^3 \; l^2 (2.4)^2$ $\qquad l = 0.134 \text{ m} \approx 10\%$ of pipe radius

(c): $103 = 10^3 \; \kappa^2 \left(\frac{1}{3} \right)^2 (2.4)^2$ $\qquad \kappa = 0.401$

7.3 Fluid Flow Past Solid Boundaries

• Flow phenomena near a solid boundary

- external flows: flow around an object immersed in the fluid
  (over a wing or a flat plate, etc.)
- internal flows: flow between solid boundaries
  (flow in pipes and channels)

(1) Laminar flow over smooth or rough boundaries

~ possesses essentially the same properties, the velocity being zero at the boundary surface and the shear stress throughout the flow

~ surface roughness has no effect on the flow as long as the roughness are small relative to the flow cross section size. → viscous effects dominates the whole flow

- smooth boundary layer
- rough boundary layer

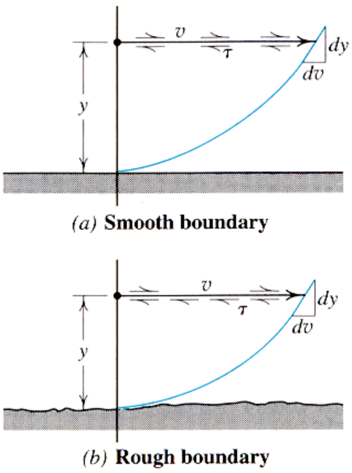


Fig. 7.6

(2) Turbulent flow over smooth or rough boundaries

- Flow over a smooth boundary is always separated from the boundary by a <u>sublayer</u> of viscosity-dominated flow (laminar flow).

[Re] Existence of laminar sublayer

Boundary will reduce the available mixing length for turbulence motion.

→ In a region very close to the boundary, the available mixing length is reduced to zero (i.e., the turbulence is completely extinguished).

→ A film of viscous flow over the boundary results.

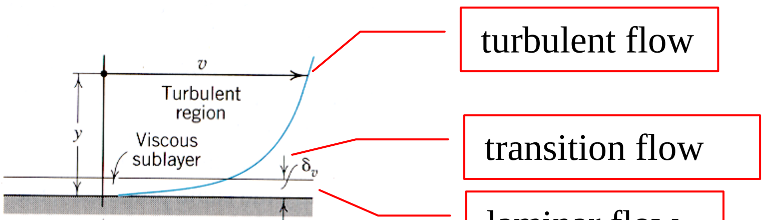
- Shear stress:

Inside the viscous sublayer: $\tau = \mu \dfrac{dv}{dy}$

Outside the viscous sublayer: $\tau = \rho l^2 \left(\dfrac{dv}{dy}\right)^2$

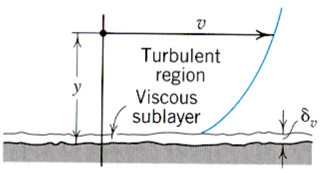
- Between the turbulent region and the viscous sublayer lies a transition zone in which shear stress results from a complex combination of both turbulent and viscous action.

→ The thickness of the viscous sublayer varies with time. The sublayer flow is unsteady.



(a) Smooth boundary

- Roughness of the boundary surface affects the



(b) Rough boundary

physical properties (velocity, shear, friction) of the fluid motion.

→ The effect of the roughness is dependent on the relative size of roughness and viscous sublayer.

- Classification of surfaces based on ratio of absolute roughness e to viscous sublayer thickness δ_v

i) Smooth surface: $\frac{e}{\delta_v} \le 0.3$

- Roughness projections are completely submerged in viscous sublayer.

→ They have no effect on the turbulence.

ii) Transition: $0.3 < \frac{e}{\delta_v} < 10$

iii) Rough surface: $10 \le \frac{e}{\delta_v}$

- However, the thickness of the viscous sublayer depends on certain properties of the flow.

→ The same boundary surface behave as a smooth one or a rough one depending on the size of the Reynolds number and of the viscous sublayer.

$$\delta_v = f\left(1/R_e \right)$$

$$i)\, v \uparrow \; \rightarrow R_e \uparrow \; \rightarrow \; \delta_v \downarrow \; \rightarrow \text{rough surface}$$

$$ii)\, v \downarrow \; \rightarrow R_e \downarrow \rightarrow \; \delta_v \uparrow \; \rightarrow \text{smooth surface}$$

7.4 Characteristics of the Boundary Layers

• Boundary layer concept by Prandtl (1904)

- Inside boundary layer - frictional effects

- Outside boundary layer – frictionless (irrotational) flow

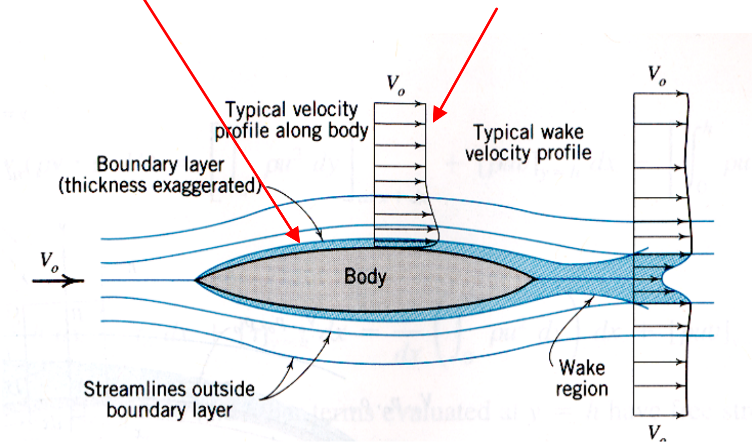


Fig. 7.9

• Mechanism of boundary layer growth

① Velocity of the particle at the body wall is zero.

② Velocity gradient $\left(dv/dy\right)$ in the vicinity of the boundary is high.

③ Large frictional (shear) stresses in the boundary layer $\left(\tau = \mu\left(dv/dy\right)\right)$ slow down successive fluid elements.

④ Boundary layers steadily thicken downstream along the body.

• Flow over a smooth flat plate

→ Boundary layer flow: laminar → transition → turbulent

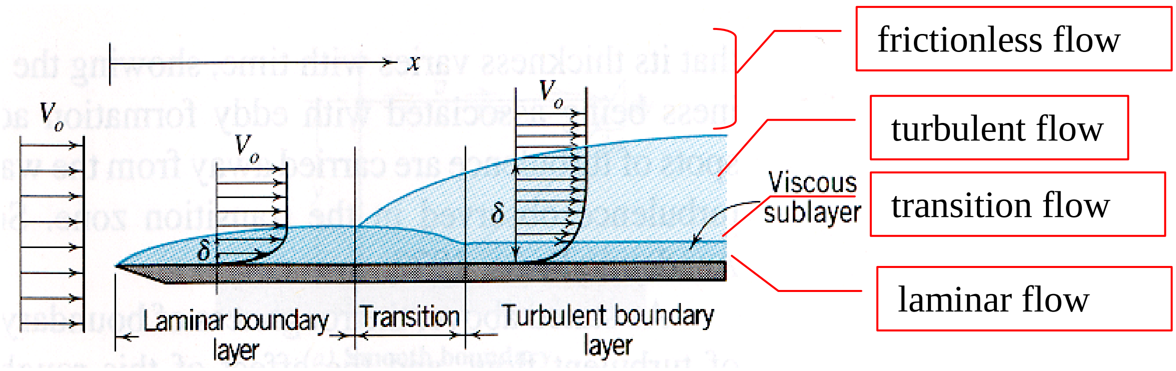


Fig. 7.8

- Laminar boundary layer

~ Viscous action is dominant.

$$R_x = \frac{V_0 x}{\nu} \qquad R_{x_c} = 500{,}000 \tag{7.7a}$$

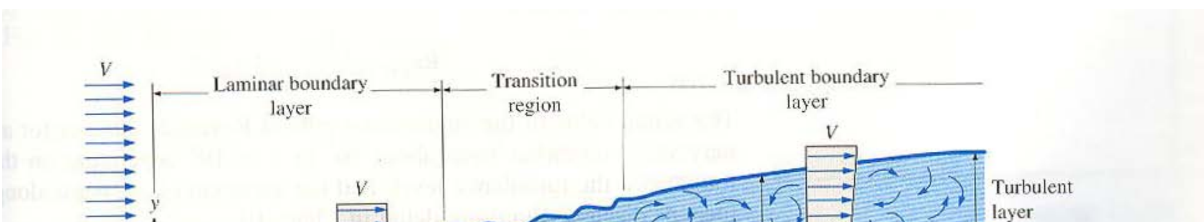
$$R_\delta = \frac{V_0 \delta}{\nu} \qquad R_{\delta_c} = 4{,}000 \tag{7.7b}$$

$R_x < 500{,}000$ or $R_\delta < 4{,}000$ → laminar boundary layer expected

- Turbulent boundary layer

~ Laminar sublayer exists.

$R_x > 500{,}000$ or $R_\delta > 4{,}000$



• Differences between Figs. 7.8 and 7.9

i) For the streamlined body, its surface has a curvature that may affect the boundary layer development either due to inertial effect or induced separation if the body is particularly blunt.

(ii) For the streamlined body, the velocity in the irrotational flow (no vorticity)

$$\xi = 0 \quad ; \quad \xi = \frac{d\Gamma}{dxdy} = 2\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

just outside the boundary layer, changes continuously along the body because of the disturbance to the overall flow offered by the body of finite width.

7.7 Separation

- Separation of moving fluid from boundary surfaces is important difference between ideal (inviscid) and real flow.

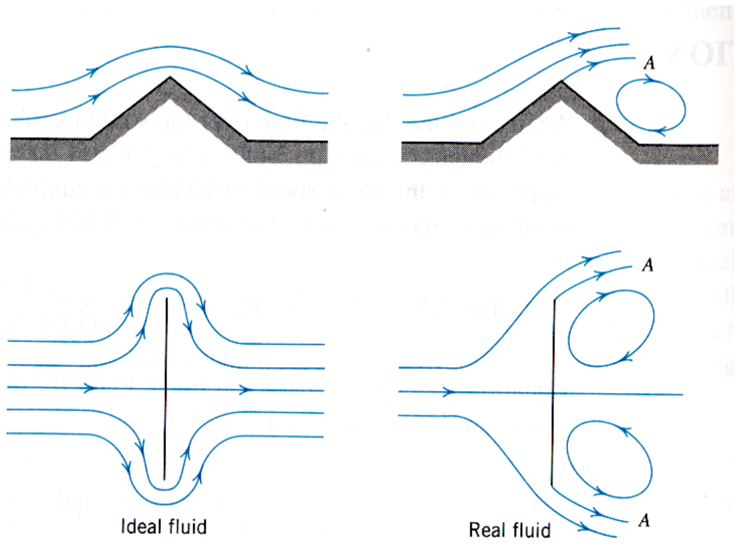


Fig. 7.13

• Ideal fluid flow: no separation

symmetrical streamline

• Flow of real fluid: separation, eddy, wake

asymmetric flowfields

[Re] Eddy:

- unsteady (time-varying)
- forming, being swept away, and re-forming
- absorbing energy from the mean flow and dissipation it into heat

• Surface of discontinuity divide the live stream from the adjacent and more moving eddies.

→ Across such surfaces there will be a high velocity gradient and high shear stress.

→ no discontinuity of pressure

→ tend to break up into smaller eddies (Fig. 7.14b)

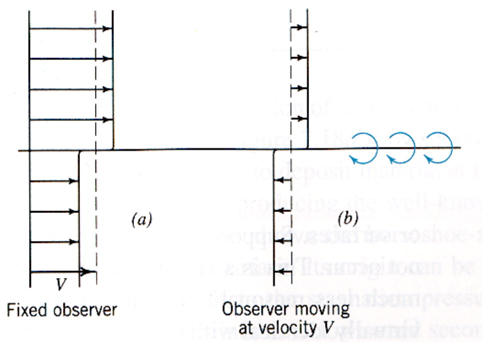
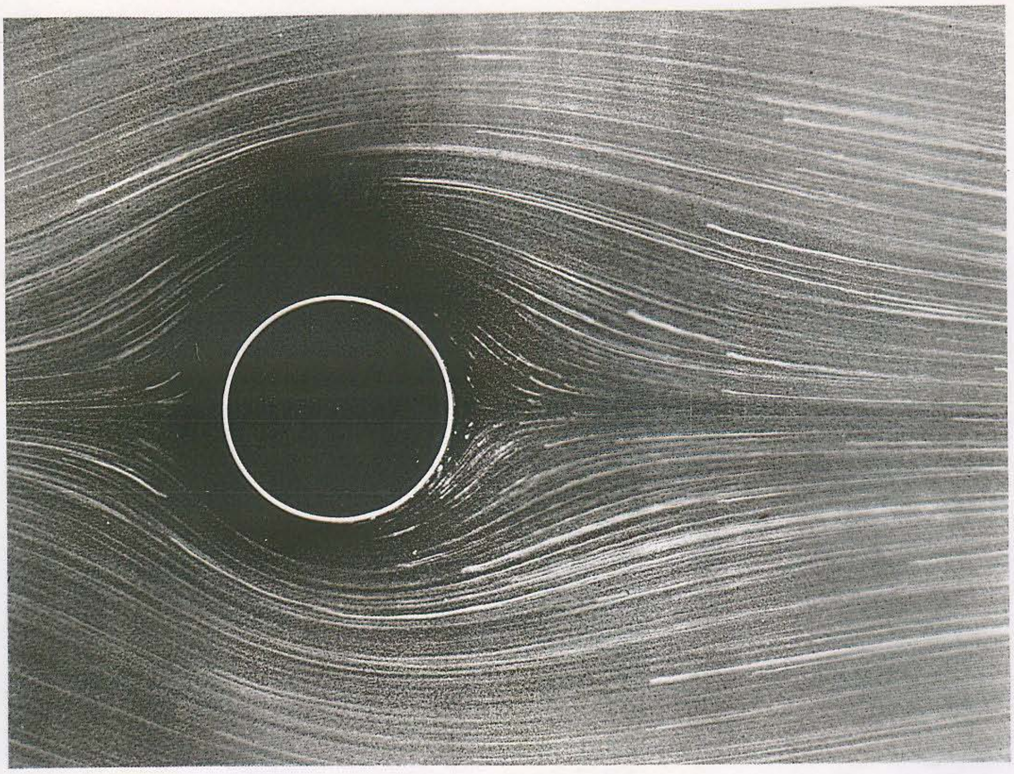
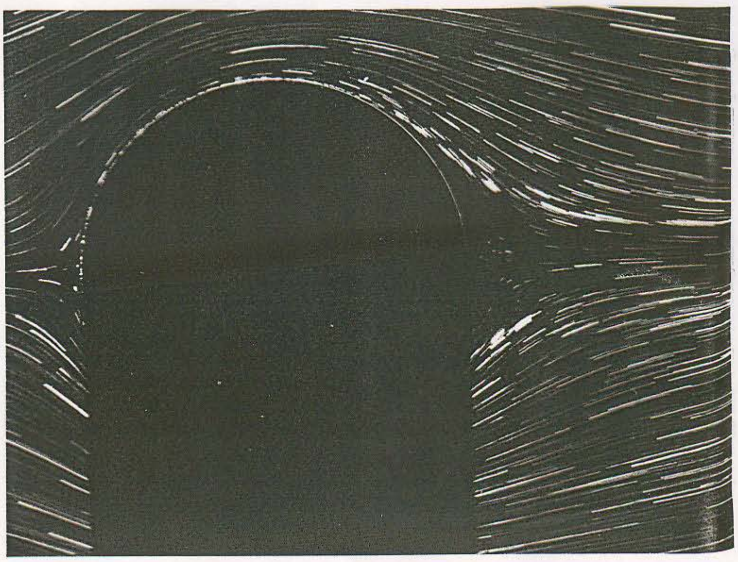


Fig. 7.14



24. Circular cylinder at $R=1.54$. At this Reynolds number the streamline pattern has clearly lost the fore-and-aft symmetry of figure 6. However, the flow has not yet separated at the rear. That begins at about $R=5$, though the value is not known accurately. Streamlines are made visible by aluminum powder in water. *Photograph by Sadatoshi Taneda*



25. Sphere at $R=9.8$. Here too, with wall effects negligible, the streamline pattern is distinctly asymmetric, in contrast to the creeping flow of figure 8. The fluid is evidently moving very slowly at the rear, making it difficult to estimate the onset of separation. The flow is presumably attached here, because separation is believed to begin above $R=20$. Streamlines are shown by magnesium cuttings illuminated in water. *Photograph by Madeleine Coutanceau and Michele Payard*

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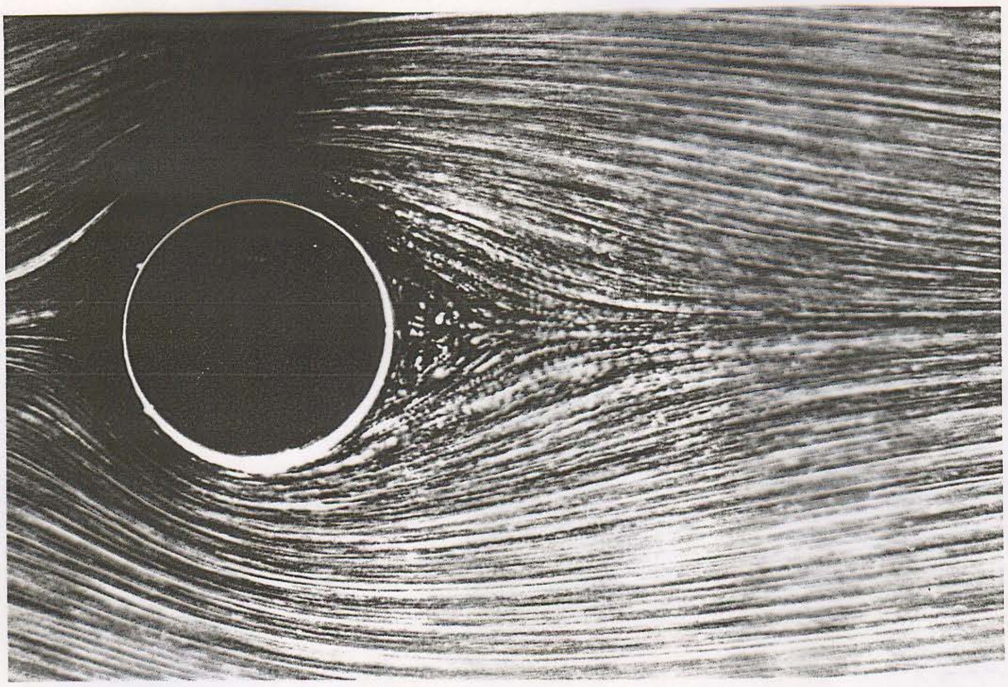
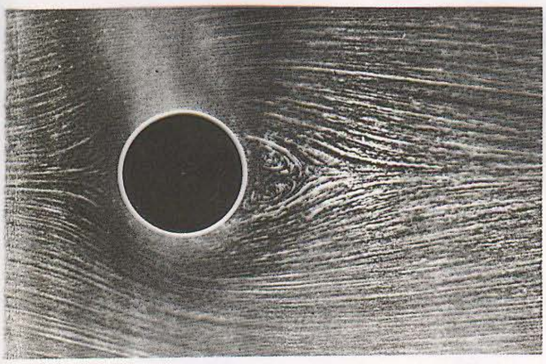
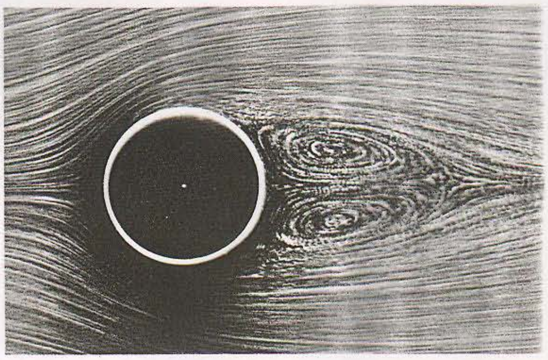


40. Circular cylinder at R=9.6. Here, in contrast to figure 24, the flow has clearly separated to form a pair of recirculating eddies. The cylinder is moving through a tank of water containing aluminum powder, and is illuminated by a sheet of light below the free surface. Extrapolation of such experiments to unbounded flow suggests separation at R=4 or 5, whereas most numerical computations give R=5 to 7. *Photograph by Sadatoshi Taneda*





41. Circular cylinder at R=13.1. The standing eddies become elongated in the flow direction as the speed increases. Their length is found to increase linearly with Reynolds number until the flow becomes unstable above R=40. *Taneda 1956a*

42. Circular cylinder at R=26. The downstream distance to the cores of the eddies also increases linearly with Reynolds number. However, the lateral distance between the cores appears to grow more nearly as the square root. *Photograph by Sadatoshi Taneda*

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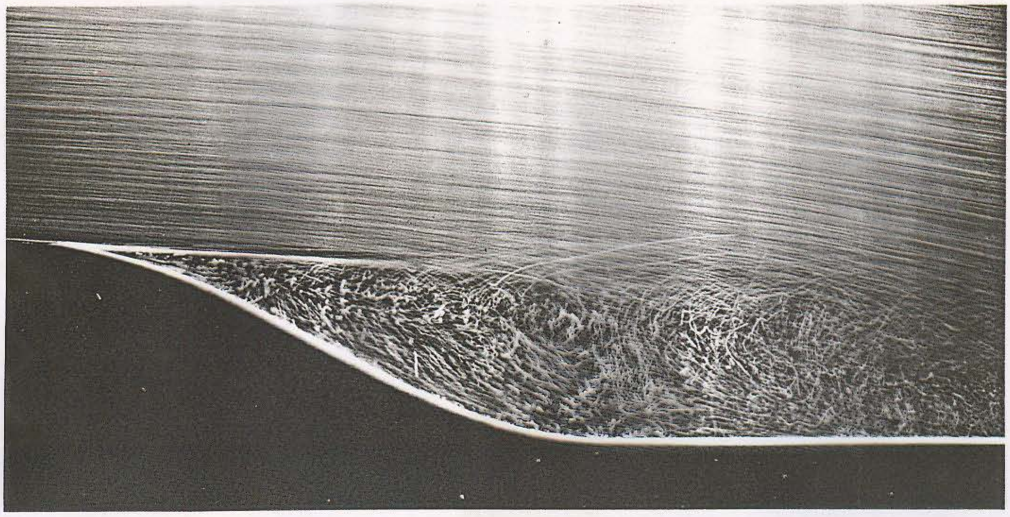
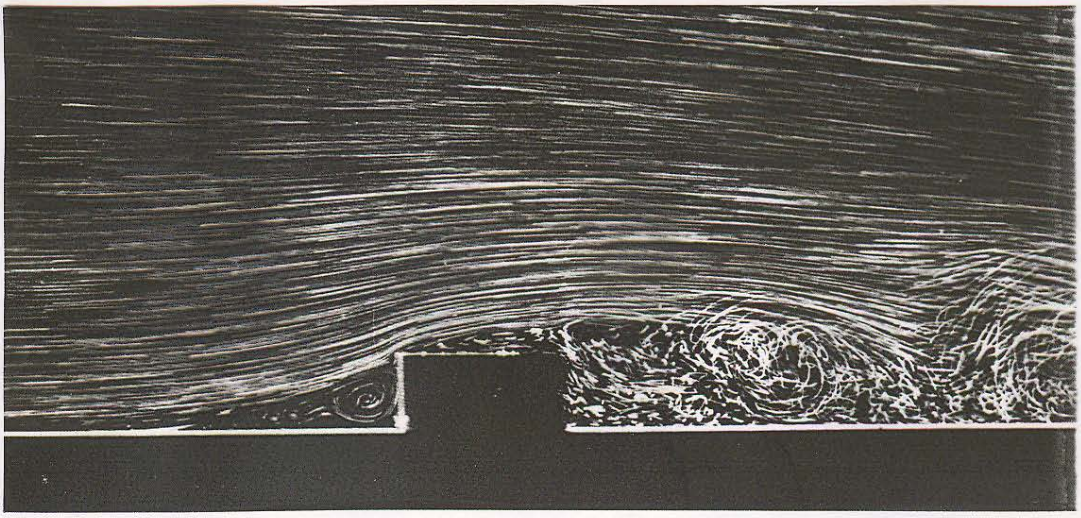


38. Laminar separation from a curved wall. Air bubbles in water show the separation of a laminar boundary layer whose Reynolds number is 20,000 based on distance from the leading edge (not shown). Because it is free of bubbles, the boundary layer appears as a thin dark line at the left. It separates tangentially near the start of the convex surface, remaining laminar for the distance to which the dark line persists, and then becomes unstable and turbulent. *ONERA photograph, Werlé 1974*



39. Turbulent separation over a rectangular block on a plate. The step height is large compared with the thickness of the oncoming laminar boundary layer. The flow is effectively plane, so that the recirculating region ahead of the step is closed, whereas in the corresponding three-dimensional flow of figure 92 it is open and drains around the sides. *ONERA photograph, Werlé 1974*

27

• Free streamline:

Under special circumstances the streamlines of a surface of discontinuity become free streamline.

Along the free streamline, the pressure is constant.

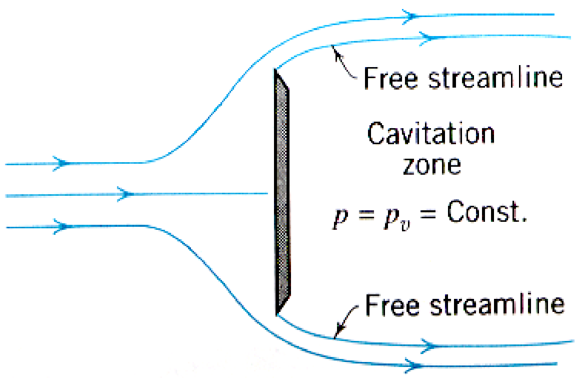


Fig. 7.15

• Separation point

The prediction of separation may be simple for sharp-cornered obstructions.

However, prediction is more complex matter for gently curved (streamlined) objects or surfaces.

→ The analytical prediction of separation point location is an exceedingly difficult problem.

→ Thus, it is usually obtained more reliably from experiments.

• For small ratio of thickness to length → no separation (Fig. 7.13)

For large ratio of thickness to length → separation (Fig. 7.14)

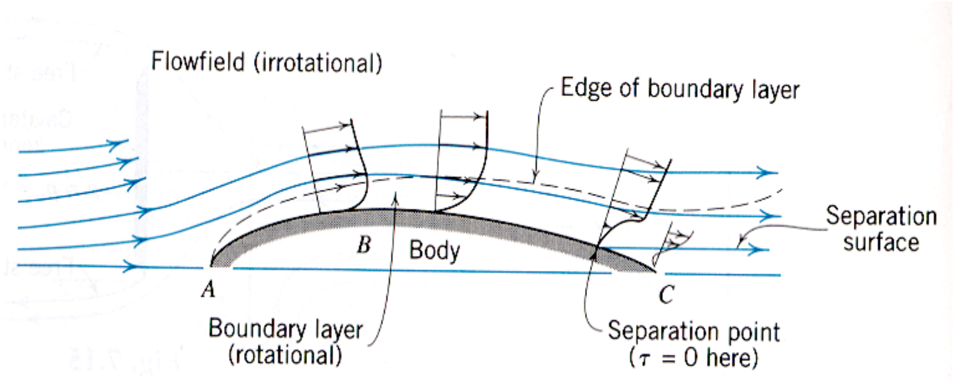


Fig. 7.17

Proceeding from A to B, the pressure falls because the flow is accelerating.

→ This produces a favorable pressure gradient which strengthens the boundary layer.

From B to C, the pressure rises as the flow decelerates because the body is thinning.

→ This produces a adverse (unfavorable) pressure gradient which weakens the boundary layer sufficiently to cause separation.

• Acceleration of real fluids tends to be an efficient process, deceleration an inefficient one.

Accelerated motion: stabilize the boundary layer, minimize energy dissipation

Decelerated motion: promote separation, instability, eddy formation, and large energy dissipation

7.8 Secondary Flow

Another consequence of wall friction is the creation of a flow within a flow, a secondary flow superposed on the main primary flow.

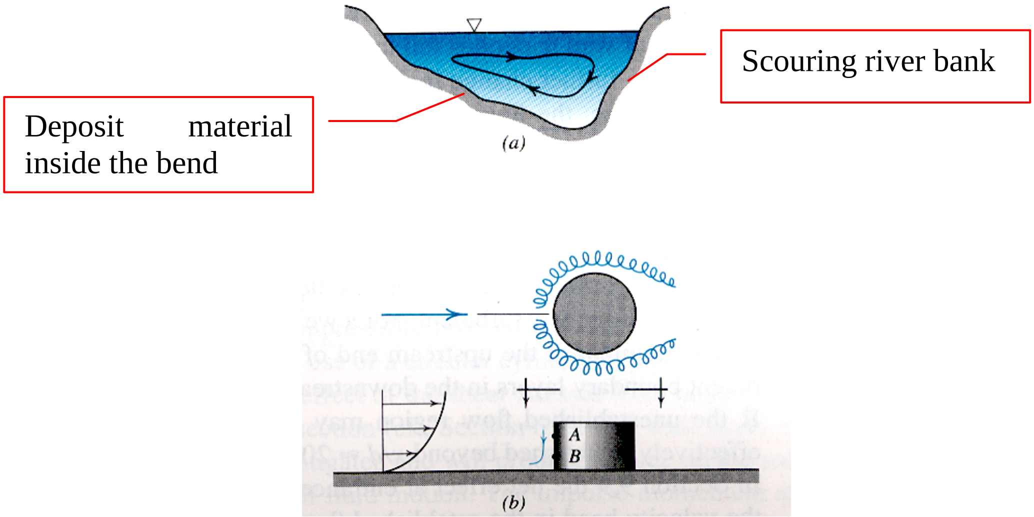


Fig. 7.18

1) Secondary flow occurring at the cross section of the meandering river: Fig. 7.18a)

2) Horseshoe-shaped vortex around the bridge pier: Fig. 7.18b)

~ Downward secondary flow from *A* to *B* induces a vortex type of motion, the core of the vortex being swept downstream around the sides of the pier.

~ This principle is used on the wings of some jet aircraft, vortex generators being used to draw higher energy fluid down to the wing surface to forestall large-scale separation.

7.9 Flow Establishment - Boundary layers

- At the entrance to a pipe, viscous effects begin their influence to lead a growth of the boundary layer.

- Unestablished flow zone:

~ dominated by the growth of boundary layers accompanied by diminishing core of irrotational fluid at the center of the pipe

- Established flow zone:

~ Influence of wall friction is felt throughout the flow field.

~ There is no further changes in the velocity profiles.

~ Flow is everywhere rotational.

- Flow in a boundary layer may be laminar if $R_e\left(=\frac{Vd}{\nu}\right) < 2100$ or turbulent if $Re \geq$ 2100.

1) Laminar flow

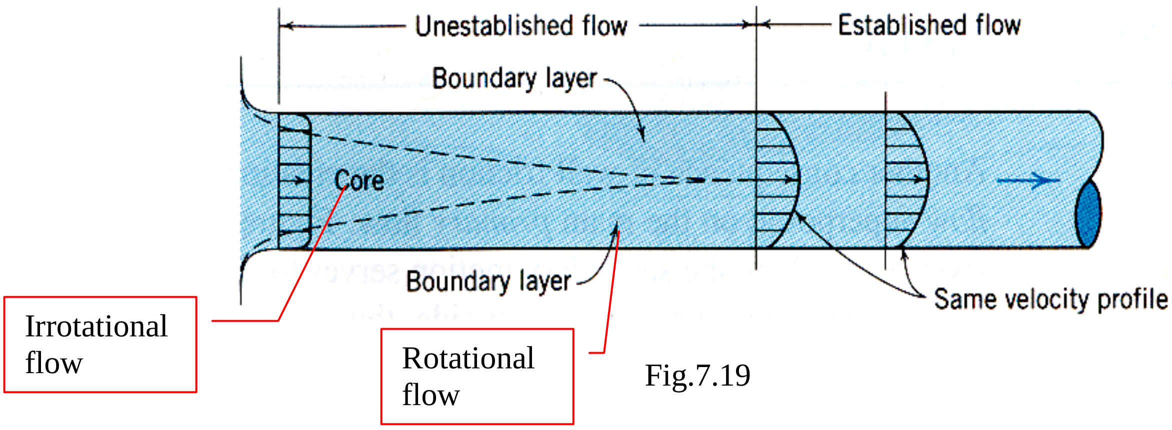


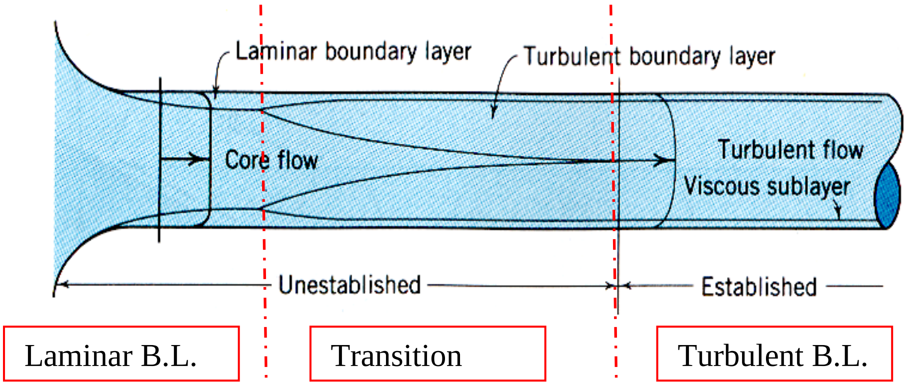
Fig.7.19

$x =$ length of unestablished flow zone

$$\frac{x}{d} \approx \frac{R_e}{20} \quad \left(\approx \frac{2100}{20} \approx 100 \right)$$

Thus, $x < 100\, d$ → unestablished flow

2) Turbulent flow



• Comparison of flat plate boundary layers (Fig. 7.9) with those of the pipe entrance (Fig. 7.16): For the pipe entrance:

(1) The plate has been rolled into a cylinder so it is not flat.

(2) The core velocity steadily <u>increases downstream</u> whereas the corresponding free stream velocity of Fig. 7.9 remains essentially constant.

(3) The pressure in the fluid diminishes in a downstream direction whereas for the flat plate there is no such pressure variation.

7.10 Shear Stress and Head Loss

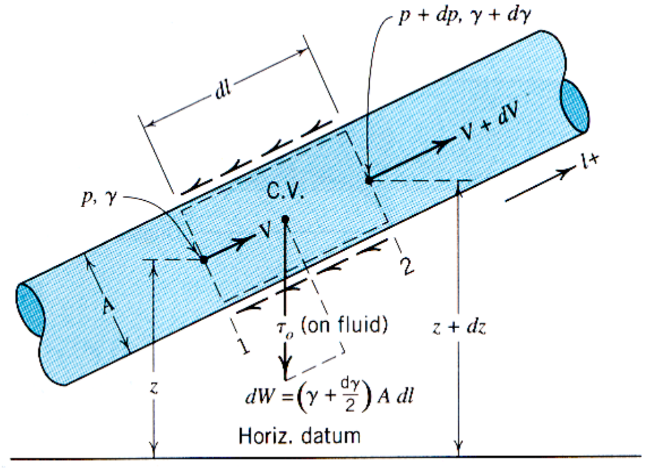


Fig. 7.21

What is the effect of the friction forces on the boundary of a control volume, such as the inside of a pipe?

→ The Impulse-momentum equation provides a clear answer.

• Wall shear stress τ_0 is a basic resistance to flow.

~ acting on the periphery of the streamtube <u>opposing the direction of the fluid motion</u>

~ cause energy dissipation (energy loss $= h_L$)

Now, apply impulse-momentum equation between ① & ② along the direction of streamtube

$$\sum \vec{F} = Q\rho\left(\vec{V}_2 - \vec{V}_1\right)$$

Pressure force

Gravity force

$$pA - (p + dp)A - \tau_0 P dl - \left(\gamma + \frac{d\gamma}{2}\right) A dl \frac{dz}{dl}$$

Shear force is included for real fluid.

$$= (V + dV)^2 A(\rho + d\rho) - V^2 A\rho$$

in which $P =$ perimeter of the streamtube

Assume momentum correction factor, $\beta_1 = \beta_2 = 1$

Neglect smaller terms containing products of differential quantities

$$-dpA - \tau_0 P dl - \gamma A dz = 2A\rho V dV + AV^2 d\rho$$

$$= A\left\{\rho d\left(V^2\right) + V^2 d\rho\right\} = Ad\left(\rho V^2\right)$$

Divide by $A\gamma$

$$\frac{dp}{\gamma} + \frac{V}{g} dV + dz = -\frac{\tau_0 dl}{\gamma}\frac{P}{A}$$

$$\frac{dp}{\gamma} + d\left(\frac{V^2}{2g}\right) + dz = -\frac{\tau_0 dl}{\gamma R_h} \qquad (7.33)$$

where $R_h = \dfrac{A}{P} =$ hydraulic radius

For established incompressible flow, γ is constant; $d(1/\gamma) = 0$

$$d\left(\frac{p}{\gamma}+\frac{V^2}{2g}+z\right)=-\left(\frac{\tau_0 dl}{\gamma R_h}\right)$$

Integrating this between points 1 and 2 yields

$$\left(\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1\right)-\left(\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2\right)=\frac{\tau_0\left(l_2-l_1\right)}{\gamma R_h} \qquad (7.34)$$

Now, note that the difference between total heads is the drop in the energy line between points 1 and 2. Thus, Eq. (7.34) can be rewritten as

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_{L_{1-2}} \qquad (7.35)$$

→ Work-energy equation for real fluid flow

Comparing (7.34) and (7.35) gives

$$h_{L_{1-2}}=\frac{\tau_0\left(l_2-l_1\right)}{\gamma R_h} \qquad (7.36)$$

Head loss

Resistance to flow

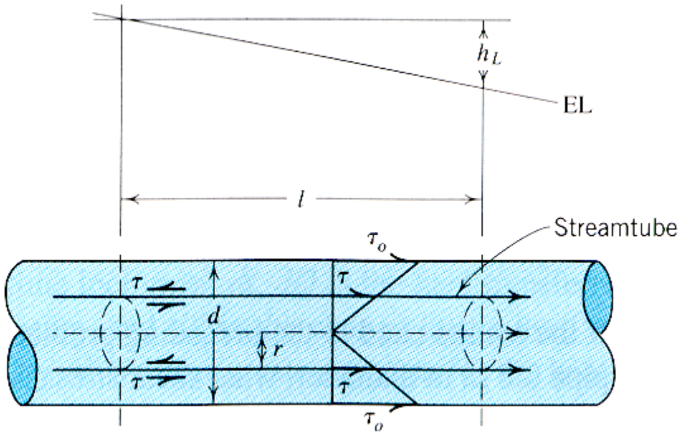
Solving for shear stress gives

$$\tau_0=\frac{\gamma R_h h_L}{l}=\gamma R_h S_f$$

where S_f = enegy slope = $\dfrac{h_L}{l}$

[Re] Head loss is attributed to a "rise in the internal energy of the fluid caused by the viscous shear stresses."

▪ Distribution of shear stress in the pipe flow



Consider the streamtube of radius r

$$\tau_0 \rightarrow \tau$$

$$R_h \rightarrow \frac{r}{2} \qquad \left(R_h = \frac{P}{A} = \frac{\pi r^2}{2\pi r} = \frac{r}{2} \right)$$

$$h_{L_{1-2}} \rightarrow h_L$$

$$l_2 - l_1 \rightarrow l$$

Substituting these into (7.36) gives

$$\tau = \left(\frac{\gamma h_L}{2l} \right) r \qquad (7.37)$$

→ The shear stress in the fluid varies linearly with distance from the centerline of the pipe.

~ applicable to both laminar and turbulent flow in pipes

[IP 7.6]

Water flows in a 0.9 m by 0.6 m rectangular conduit.

$$\Delta l = 60 \text{ m}$$

$$\Delta h_L = 10 \text{ m}$$

Calculate the resistance stress exerted between fluid and conduit walls.

[Sol]

$$\tau_0 = \frac{\gamma R_h}{\Delta l} \Delta h_L$$

$$R_h = \frac{A}{P} = \frac{0.9 \times 0.6}{2\left(0.9 + 0.6\right)} = \frac{0.54}{3} = 0.18 \text{ m}$$

$$\therefore \quad \tau_0 = \frac{9.8 \times 10^3 \times 0.18}{60} \cdot 10 = 0.29 \text{ kPa}$$

~ Flow is not axisymmetric

→ τ_0 is mean shear stress on the perimeter

[IP 7.7]

Water flows in a cylindrical pipe of 0.6 m in diameter.

$$\tau_0 = \frac{\gamma h_L}{2\Delta l} R = \frac{\left(9.8 \times 10^3\right)}{2(60)} \frac{0.6}{2} = 0.25 \text{ kPa}$$

$$\tau = \tau_0 \frac{r}{R}$$

τ in the fluid at a point 200 mm from the wall:

$$\tau\Big|_{r=100\,\text{mm}} = \tau_0 \frac{(0.3 - 0.2)}{0.3} = \frac{1}{3}\left(0.25\right) = 0.083 \text{ kPa}$$

7.11 The First Law of Thermodynamic and Shear Stress Effects

Relationship between shear stresses and energy dissipation

$$\frac{dQ}{dt}+\frac{dW}{dt}=\frac{dE}{dt} \tag{7.39}$$

where $dQ=$ heat transferred to the system

$dW=$ work done on the system

$dE=$ change in the total energy of the system

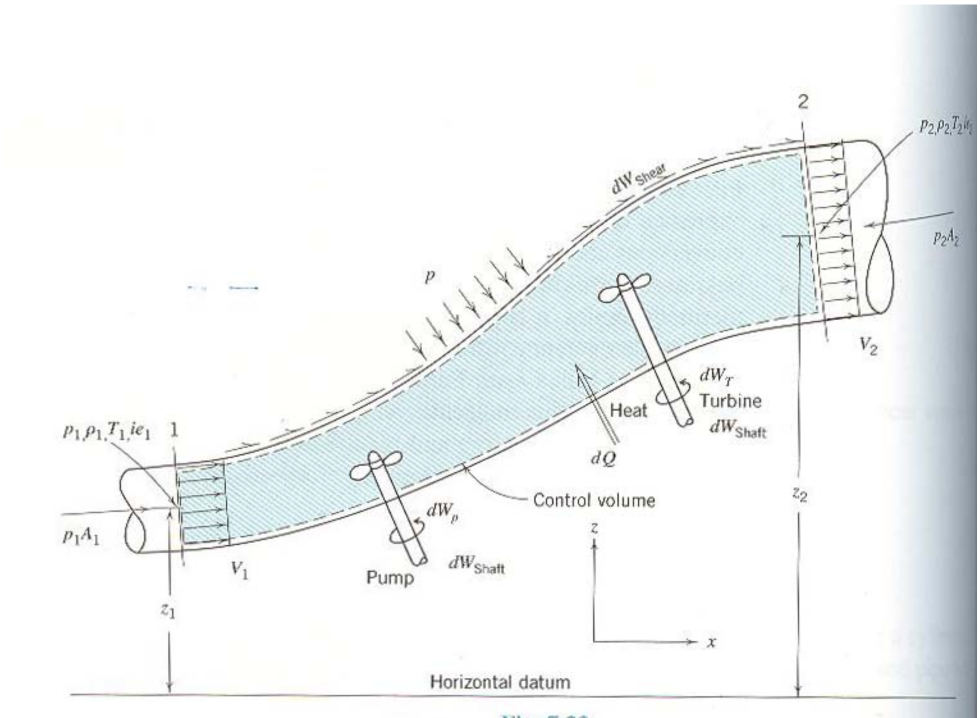


Fig. 7.23

Fig. 7.23 compared to Fig. 5.8

$$dW_{shear} \neq 0$$

• General energy equation for steady incompressible flow

$$\left(\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1\right)+E_p=\left(\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2\right)+E_T+h_{L_{1-2}} \quad (7.46)$$

[Cf] Eq.(5.47): work-energy eq. for ideal fluid

where $h_{L_{1-2}}=\frac{\tau_0\left(l_2-l_1\right)}{\gamma R_h}=\frac{1}{g}\left(ie_2-ie_1-q_H\right)$

$ie=$ internal energy per mass

$q_H=\frac{1}{\dot{m}}\frac{dQ}{dt}=$ heat added to the fluid per unit of mass

→ head loss is a conversion of energy into heat

7.12 Velocity Distribution and its significance

In a real fluid flow, the shearing stresses produce velocity distributions.

→ nonuniform velocity distribution

[Cf] Uniform distribution for ideal fluid flow

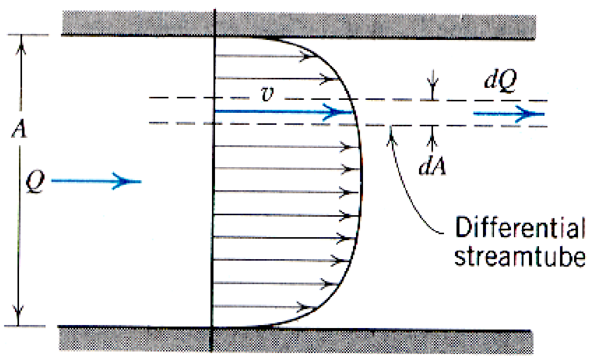


Fig. 7.24

Total kinetic energy flux $(J/s) = \frac{\rho}{2}\iint v^3 dA$ (7.47)

Total momentum flux(N) $= \rho\iint_A v^2 dA$ (7.48)

[Re] $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}\rho vol\, v^2$

$$K.E./time = \frac{1}{2}\rho\frac{vol}{t}v^2 = \frac{1}{2}\rho Q v^2 = \frac{1}{2}\rho A v v^2 = \frac{1}{2}\rho A v^3$$

momentum flux $= \frac{mv}{t} = \rho\frac{vol}{t}v = \rho Q v = \rho A v^2$

Use mean velocity V and total flow rate Q

$$\text{Total kinetic energy} = \alpha Q \gamma \frac{V^2}{2g} = \frac{\gamma}{2g} Q V^2 \alpha = \frac{\rho}{2} Q V^2 \alpha \qquad (7.49)$$

$$\text{Momentum flux } = \beta Q \rho V \qquad (7.50)$$

where α, β = correction factors

(1) Energy correction factor

Combine (7.47) and (7.49)

$$\frac{\rho}{2} \alpha Q V^2 = \frac{\rho}{2} \int_A v^3 dA$$

$$\alpha = \frac{1}{V^2} \frac{\int_A v^3 dA}{Q} = \frac{1}{V^2} \frac{\int_A v^3 dA}{\int_A v dA}$$

where $Q = \int_A v dA$

(2) Momentum correction factor

Combine (7.48) and (7.50)

$$\beta Q \rho V = \rho \int_A v^2 dA$$

$$\beta = \frac{1}{V} \frac{\int_A v^2 dA}{Q} = \frac{1}{V} \frac{\int_A v^2 dA}{\int_A v dA}$$

[Ex] $\alpha = \beta = 1$ for uniform velocity distribution

$\alpha = 1.54$, $\beta = 1.20$ for parabolic velocity distribution (laminar flow)

$$v = v_c \left(1 - r^2 / R^2\right)$$

$\alpha = 1.1$, $\beta = 1.05$ for turbulent flow

• Correction in the Bernoulli equation in real fluid flow

→ nonuniform velocity distribution

→ bundle of energy lines

→ use single effective energy line of aggregation of streamlines $= \alpha \dfrac{V^2}{2g}$

$$\alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + h_{L_{1-2}} \qquad (7.53)$$

where $h_{L_{1-2}} =$ head loss between sections 1 and 2

Homework Assignment # 7

Due: 1 week from today

Prob. 7.1

Prob. 7.7

Prob. 7.12

Prob. 7.17

Prob. 7.52

Prob. 7.58

Prob. 7.69