Ship Stability

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Myung-Il Roh

Department of Naval Architecture and Ocean Engineering
Seoul National University

Ship Stability

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Ch. 2 Review of Fluid Mechanics

Introduction to Hydromechanics

Introduction to Hydromechanics

- Today, the branch of physics, which encompasses the theories and laws of the behavior of water and other liquids, is known as hydromechanics.

- Hydromechanics itself is subdivided into three fields:

(1) Hydrostatics, which deals with liquids at rest.
(2) Hydrodynamics, which studies liquids in motion.
(3) Hydraulics, dealing with the practical and engineering applications of hydrostatics and hydrodynamics.

Concept of Hydrostatics

Meaning of Hydrostatics

- **What is Hydrostatics?**

  Hydrostatics (from Greek *hydro*, meaning water, and *statics* meaning rest, or calm) describes the behavior of water in a state of rest.

  This science also studies the forces that apply to immersed and floating bodies, and the forces exerted by a fluid.

Definition of Pressure

- **Pressure***

Let a small pressure-sensing device be suspended inside a fluid-filled vessel.

We define the pressure on the piston from the fluid as the force divided by area, and it has units Newtons per sqaure meter called 'Pascal'.

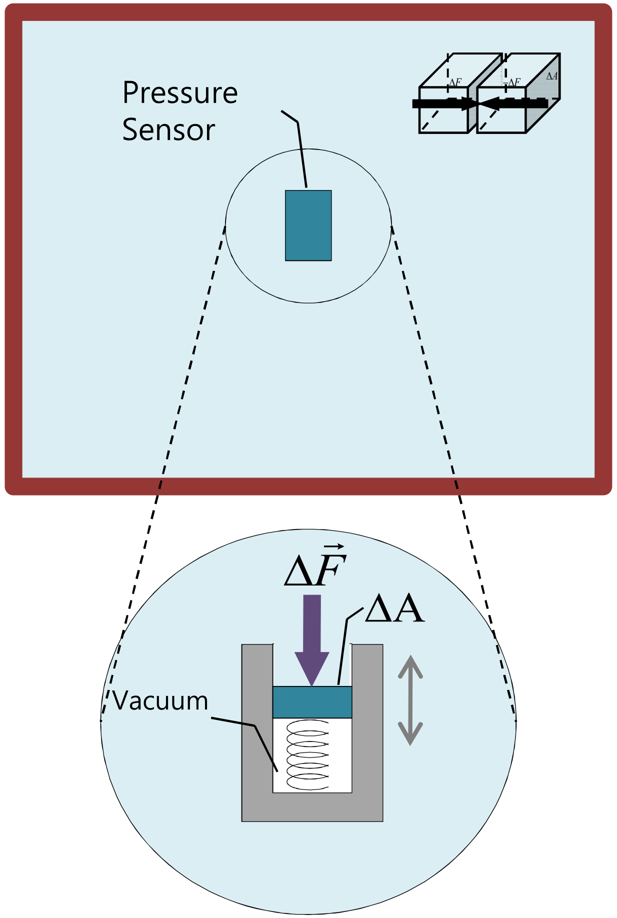
$$P = \frac{\Delta F}{\Delta A} \qquad (1\ \mathrm{Pa} = 1 N / m^2)$$

One newton per square meter is one Pascal.

We can find by experiment that at a given point in a fluid at rest, the pressure have the same value no matter how the pressure sensor is oriented.

Pressure is a scalar, having no directional properties, and force is a vector quantity.

But ΔF is only the magnitude of the force.



ΔF : Magnitude of normal force on area ΔA
ΔA : Surface area of the piston

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.361, 2004

Two Principles of Hydrostatics

Hydrostatics mainly consists of two principles.

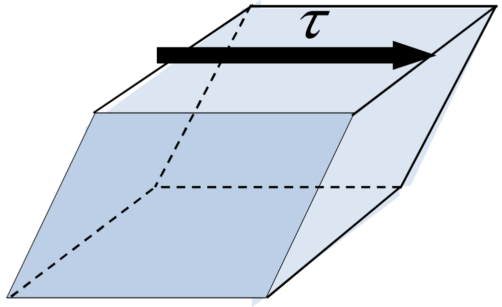
1. **Pascal's principle** says that the pressure applied to an enclosed fluid is transmitted undiminished.

2. **Archimedes' principle** states that the buoyant force on an immersed body has the same magnitude as the weight of the fluid which is displaced by the body.

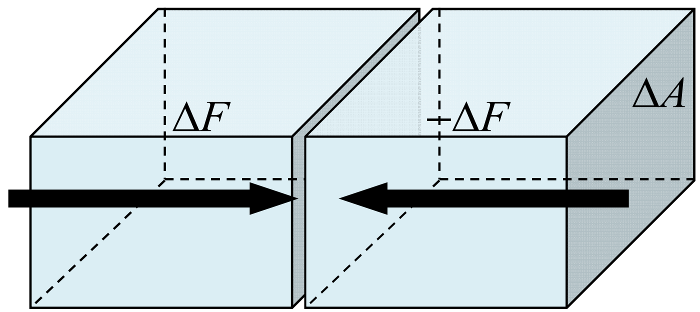
Definition of Fluid

- **What is a Fluid?***

A fluid, in contrast to a solid, is a substance that can flow, because it cannot withstand a shearing stress.



It can, however, exerts a force in the direction perpendicular to its surface.



ΔF : Magnitude of perpendicular force between the two cubes
ΔA : Area of one face of one of the cubes

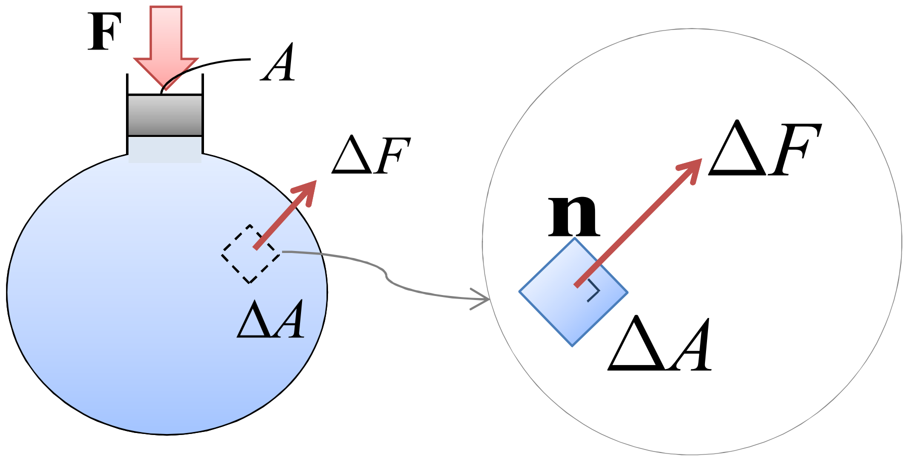
* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.360, 2004

Pascal's Principle

Pascal's Principle

- We will now consider a fluid element in static equilibrium in a closed container filled with a fluid which is either a gas or a liquid. The velocity of flow is everywhere zero.

- At first, we will neglect gravity. If a force F is applied on the cap of the container with an area A in this direction, then a pressure of F/A is applied.



ΔF : Magnitude of force

$$\lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = P$$

Pascal's Principle

Pascal's Principle

In the absence of gravity, the pressure is everywhere in this container **the same**.
That is what's called **Pascal's principle.**

A change in the pressure applied an enclosed fluid is **transmitted undiminished** to every portion of the fluid and to the walls of its container*.

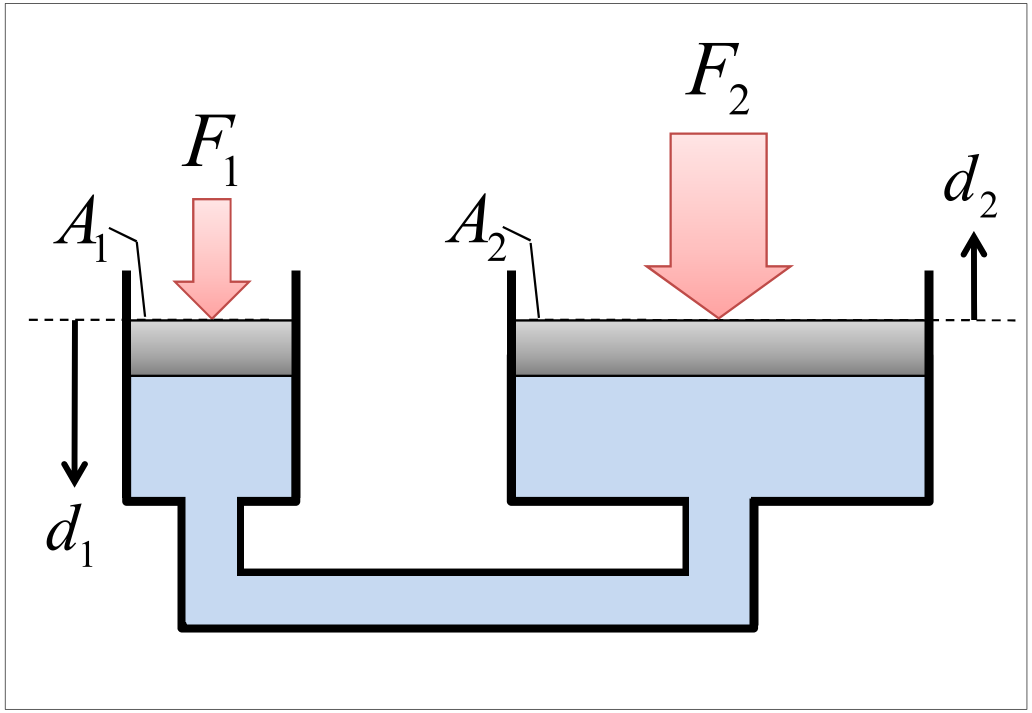
* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.366, 2004

Application of the Pascal's Principle

Pascal's Principle :
A change in the pressure applied an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of its container*.

• The idea of a Hydraulic jack

Pascal's Principle : $$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$



Consider a vessel with two pistons having area A_1 and area A_2. The vessel is filled with liquid everywhere.

Now a force F_1 on A_1 and a force F_2 on A_2 are applied. So the pressure on the left piston is F_1/A_1.

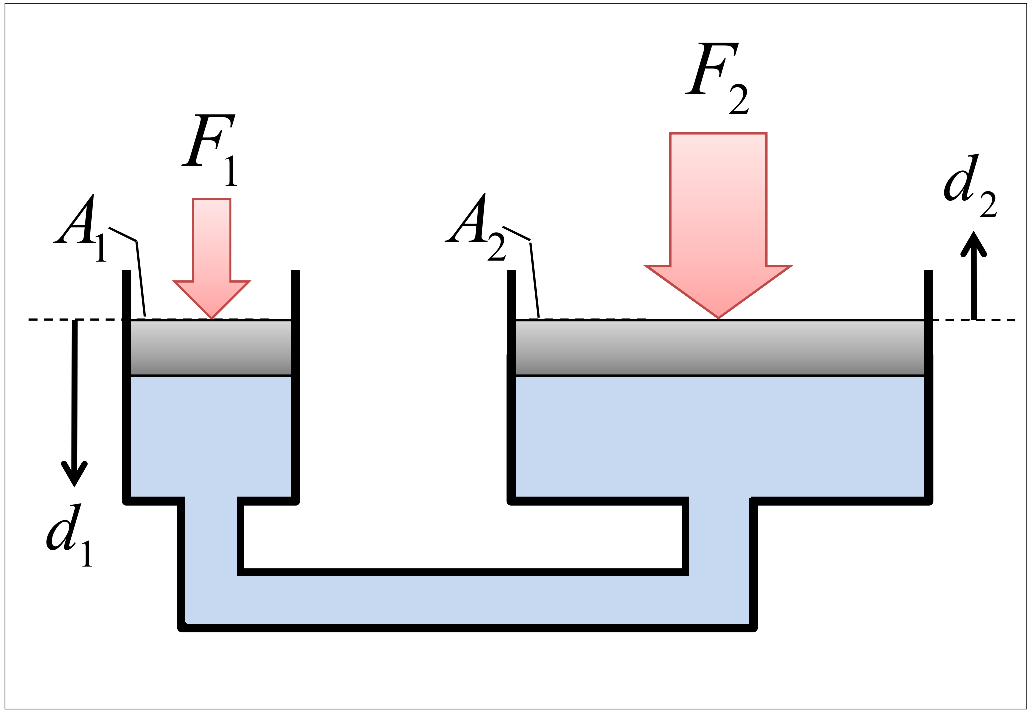
According to the Pascal's principle, everywhere in the fluid, the pressure must be the same. The pressure on the right piston, F_2/A_2 must be the same as the pressure F_1/A_1, if the liquid is not moving. The effect of gravity does not change the situation very significantly.

Example of Design of Hydraulic Jack (1/4)

Pascal's Principle : $$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Example) If $\frac{A_2}{A_1} = 100$, then $\frac{F_2}{F_1} = 100$.

(Pascal's Principle)

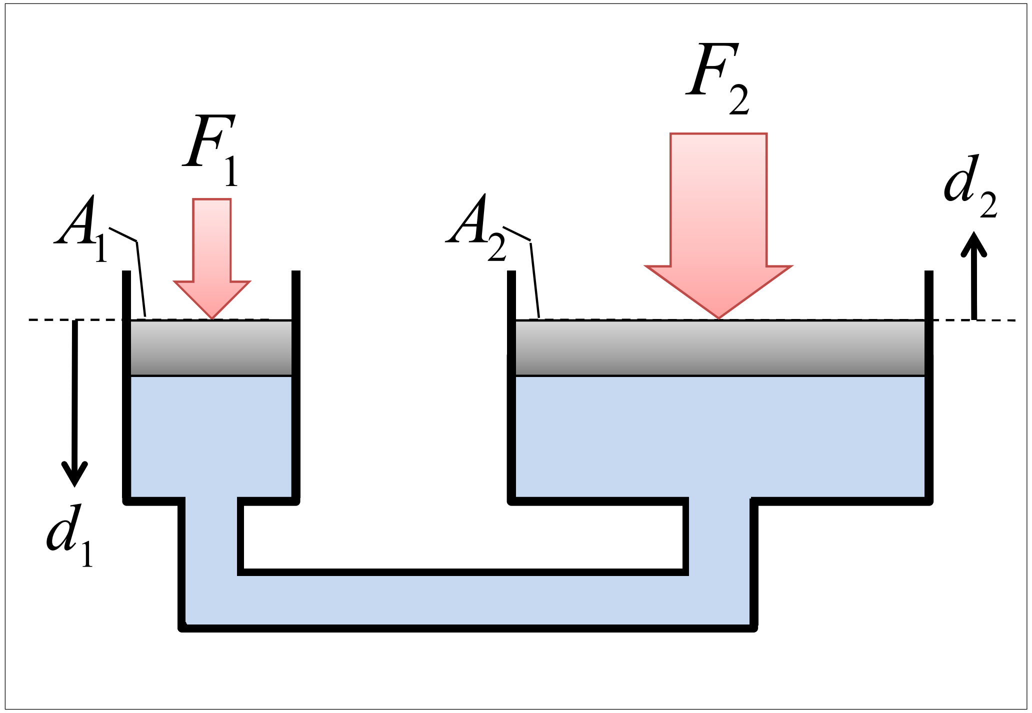


Example of Design of Hydraulic Jack (2/4)

Pascal's Principle : $$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Displaced Volume: $$A_1 d_1 = A_2 d_2$$

(Incompressible fluid)

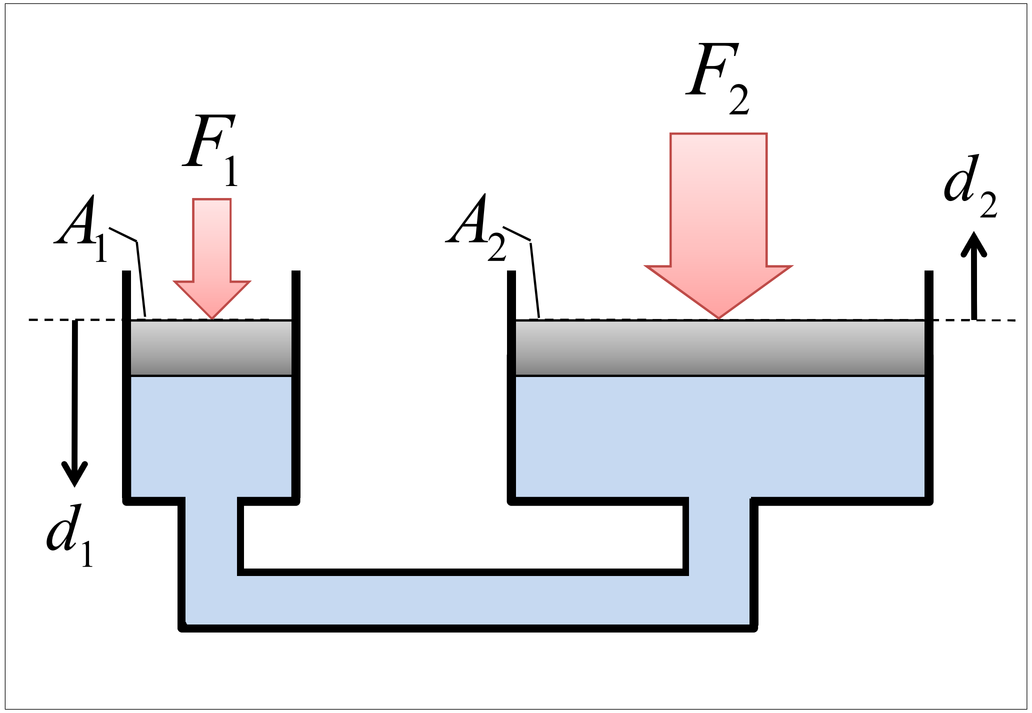


Example of Design of Hydraulic Jack (3/4)

Pascal's Principle : $$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Displaced Volume: $$A_1 d_1 = A_2 d_2$$

(Incompressible fluid)



Conservation of Energy:

$$F_1 d_1 = \underbrace{\left(\frac{A_1}{A_2} F_2\right)}_{F_1} \underbrace{\left(\frac{A_2}{A_1} d_2\right)}_{d_1} = F_2 d_2$$

(Pascal's Principle) **(Displaced Volume)**

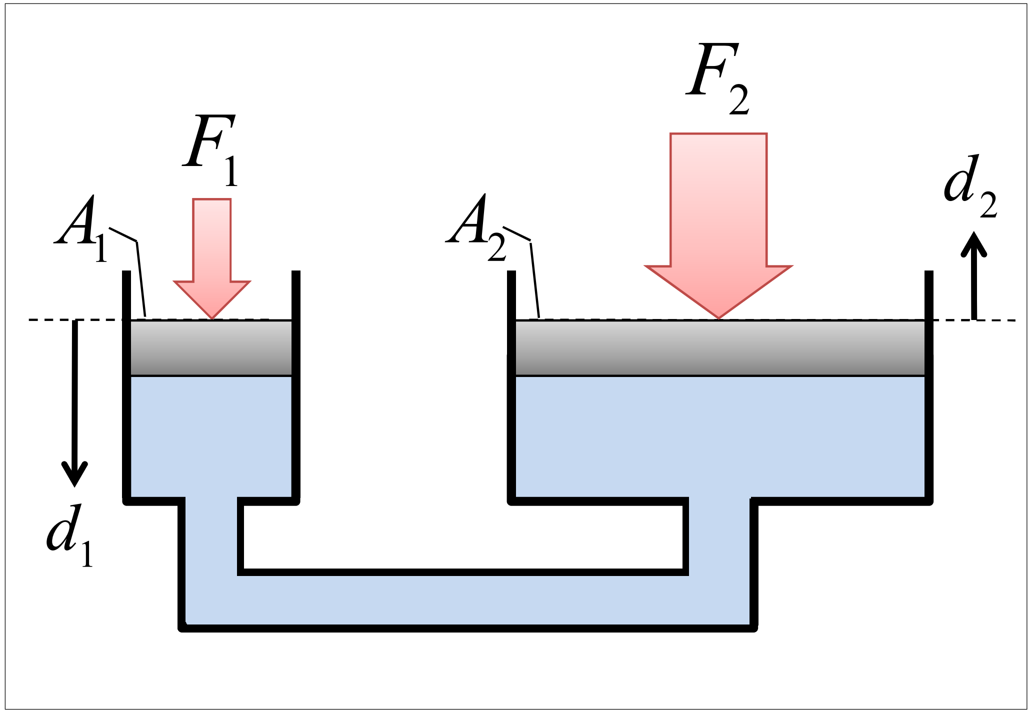
Example of Design of Hydraulic Jack (4/4)

Pascal's Principle : $$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Displaced Volume: $$A_1 d_1 = A_2 d_2$$

(Incompressible fluid)

Conservation of Energy: $$F_1 d_1 = F_2 d_2$$



Example) If $\frac{A_2}{A_1} = 100$ → $100F_1 = F_2$

(Pascal's Principle)

→ $d_1 = 100 d_2$

(Displaced Volume)

$$\Rightarrow F_1 d_1 = F_2 d_2$$

$\times 100$ $\times 100$

∴ **Conservation of Energy is satisfied.**

Hydrostatic Pressure

Hydrostatic Pressure (1/9)

- Hydrostatic Pressure

As every diver knows, the pressure increases with depth below the water.

As every mountaineer knows, the pressure decreases with altitude as one ascends into the atmosphere.

The pressure encountered by the diver and the mountaineer are usually called hydrostatic pressures, because they are due to fluids that are static (at rest).

Here we want to find an expression for hydrostatic pressure as a function of depth or altitude.

Hydrostatic Pressure (2/9)

Now, gravity, of course, has an effect on the pressure in the fluid.

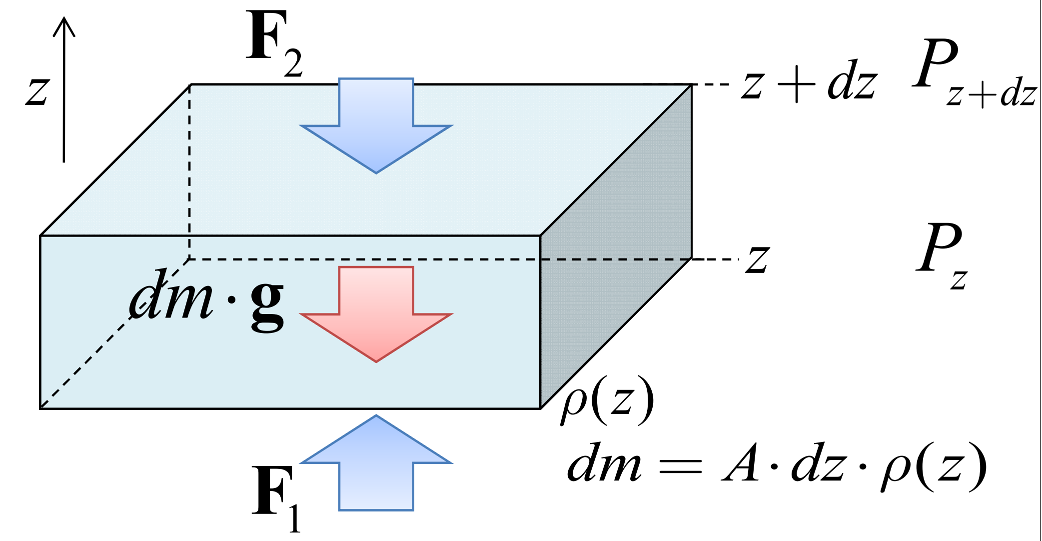
Hydrostatic pressure is due to fluids that are static (at rest).

Thus, there has to be static equilibrium.

Consider a fluid element in the fluid itself and assume the upward vertical direction as the positive z-coordinate.

The mass of the fluid element is the volume times the density, and the volume is face area times delta z, and then times the density, which may be a function of z.

Fluid Element



$\rho(z)$ = Density of the fluid element
dm = Mass of the fluid element
A = Horizontal base(or face) area
F_1 = Force that acts at the bottom surface(due to the water below the rectangular solid)
F_2 = Force that acts at the top surface(due to the water above the rectangular solid)
P_{z+dz} = Pressure at $z+dz$
P_z = Pressure at z

Hydrostatic Pressure (3/9)

Newton's 2nd Law : $\sum \mathbf{F} = m\ddot{\mathbf{z}}$

(Static Equilibrium : $\ddot{\mathbf{z}} = 0$**)**

$$\sum \mathbf{F} = 0$$

$$\mathbf{F}_1 - \mathbf{F}_2 - dm \cdot \mathbf{g} = 0$$

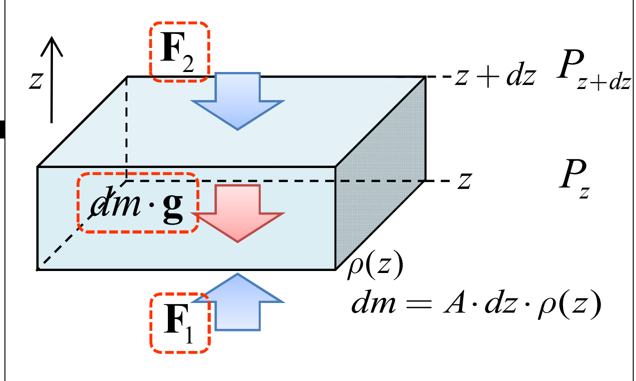
To describe the behavior of the fluid element, we apply the Newton's 2nd law to the free body diagram for the fluid element, as shown in the figure.

The gravitational force acting on the fluid element is delta m times g in the downward direction. The pressure force, which is always perpendicular to the surfaces, acting on the bottom surface is F_1 in the upward direction, whereas the pressure force acting on the top surface is F_2 in the downward direction.

We only consider forces in the vertical direction, because all forces in the horizontal direction will cancel, for obvious reasons. The fluid element is not going anywhere. It is just sitting still in the fluid. Thus, the fluid element is in static equilibrium.

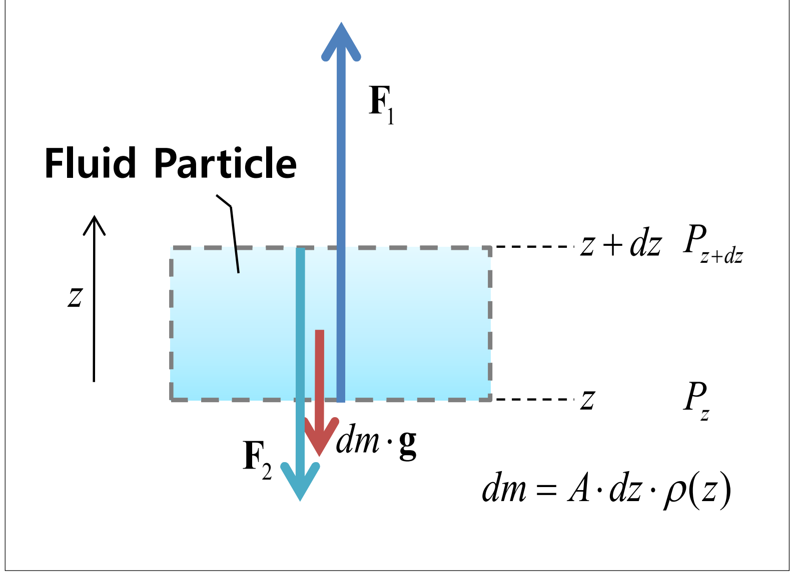
For the fluid element to be in static equilibrium, the upward force F_1 minus downward force F_2 minus delta mg must be zero.

Fluid Element



$\rho(z)$ = Density of the fluid element
dm = Mass of the fluid element
A = Horizontal base(or face) area
F_1 = Force that acts at the bottom surface(due to the water below the rectangular solid)
F_2 = Force that acts at the top surface(due to the water above the rectangular solid)
P_{z+dz} = Pressure at $z + dz$
P_z = Pressure at z

Free-body diagram for the fluid element



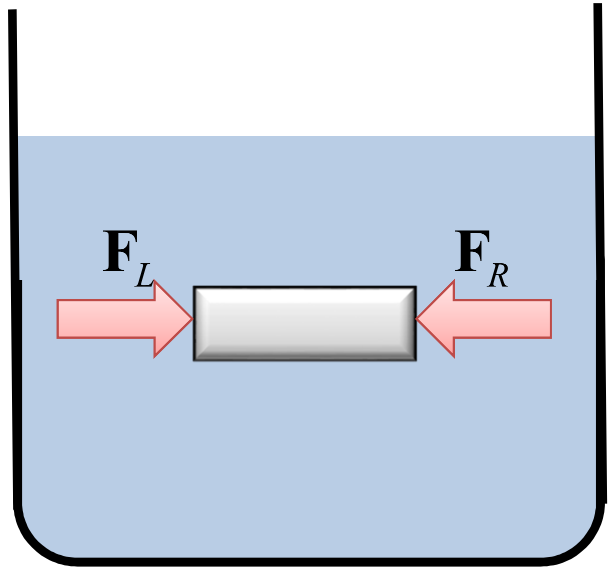
Hydrostatic Pressure (4/9)

Reference) Static Equilibrium

If a fluid is at rest in a container, all portions of the fluid must be in static equilibrium (at rest with respect to the observer).

Furthermore, all points at the same depth must be at the same pressure.

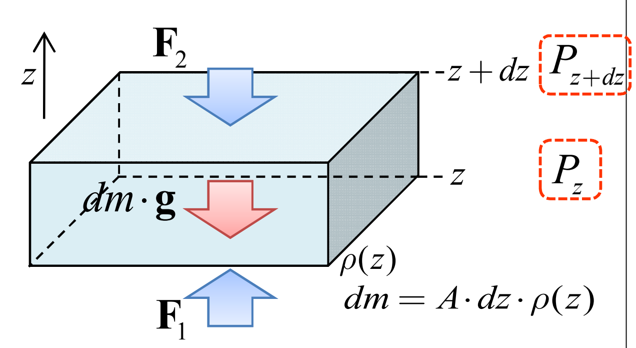
If this was not the case, a given portion of the fluid would not be in equilibrium.



For example, consider the small block of fluid. If the pressure were greater on the left side of the block than on the right, $\mathbf{F}_L$ would be greater than $\mathbf{F}_R$, and the block would accelerate and thus would not be in equilibrium.

Hydrostatic Pressure (5/9)

Fluid Element



$\rho(z)$ = Density of the fluid element
dm = Mass of the fluid element
A = horizontal base(or face) area
F_1 = Force that acts at the bottom surface(due to the water below the rectangular solid)
F_2 = Force that acts at the top surface(due to the water above the rectangular solid)
P_{z+dz} = Pressure at $z+dz$
P_z = Pressure at z

Newton's 2nd Law : $\sum \mathbf{F} = m\ddot{\mathbf{z}}$

$$\mathbf{F}_1 - \mathbf{F}_2 - dm \cdot \mathbf{g} = 0$$

Three forces act on vertically.
Thus we can consider <u>magnitude of vectors</u> only.

$$\Rightarrow P_z \cancel{A} - P_{z+dz} \cancel{A} - \cancel{A} \cdot dz \cdot \rho(z) \cdot g = 0$$

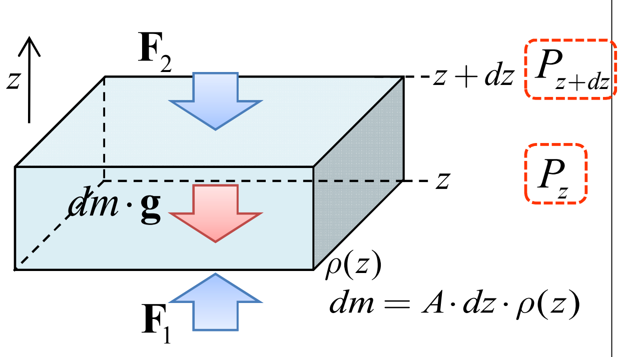
$$P_z - P_{z+dz} - dz \cdot \rho(z) \cdot g = 0$$

$$P_z - P_{z+dz} = dz \cdot \rho(z) \cdot g \qquad \times(-1)$$

$$P_{z+dz} - P_z = -dz \cdot \rho(z) \cdot g$$

Hydrostatic Pressure (6/9)

Fluid Element



$\rho(z)$ = Density of the fluid element
dm = Mass of the fluid element
A = horizontal base(or face) area
F_1 = Force that acts at the bottom surface(due to the water below the rectangular solid)
F_2 = Force that acts at the top surface(due to the water above the rectangular solid)
P_{z+dz} = Pressure at $z+dz$
P_z = Pressure at z

Newton's 2nd Law : $\sum \mathbf{F} = m\ddot{\mathbf{z}}$

$$\mathbf{F}_1 - \mathbf{F}_2 - dm \cdot \mathbf{g} = 0$$

$$P_{z+dz} - P_z = -dz \cdot \rho(z) \cdot g$$

$\div(dz)$

$$\frac{P_{z+dz} - P_z}{dz} = -\rho(z) \cdot g$$

$$\lim_{dz \to 0} \frac{P_{z+dz} - P_z}{dz} = -\rho(z) \cdot g = \frac{dP}{dz}$$

$$\therefore \frac{dP}{dz} = -\rho(z) \cdot g$$

: Change of Hydrostatic Pressure
(Due to gravity)

Hydrostatic Pressure (7/9)

$$\frac{dP}{dz} = -\rho(z) \cdot g$$: Change in Hydrostatic Pressure (**Due to gravity**)

Calculate the pressure difference between z_1 and z_2.

$$dP = -\rho(z) \cdot g \cdot dz$$

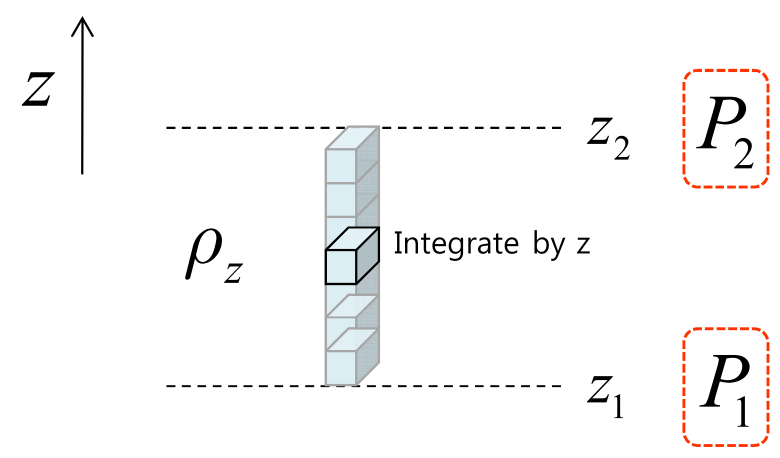
Integrate from z_1 to z_2.

$$\Rightarrow \int_{P_1}^{P_2} dP = -\int_{z_1}^{z_2} \rho(z) \cdot g \cdot dz$$

Most liquids are practically incompressible. In other words, the density of the liquid cannot really change. And so therefore, we could always use the constant density, ρ, instead of the varying density ρ(z). We will assume from now on that fluids are completely incompressible. We can, then, do a very simple integration.

We have now dP in the L.H.S, which we can integrate from some value P_1 to P_2. And that equals now minus rho g dz in the R.H.S, integrated from z_1 to z_2.

In the fluid



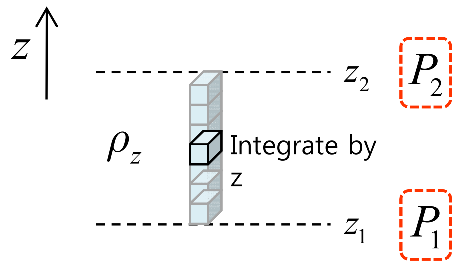
ρ_z = Density of a fluid
P_1 = Pressure at z_1
P_2 = Pressure at z_2

Hydrostatic Pressure (8/9)

Calculate the pressure difference between z_1 and z_2.

$$\int_{P_1}^{P_2} dP = -\int_{z_1}^{z_2} \rho(z) \cdot g \cdot dz$$

In the fluid



ρ_z = Density of a fluid
P_1 = Pressure at z_1
P_2 = Pressure at z_2

L.H.S: $\int_{P_1}^{P_2} dP = P_2 - P_1$

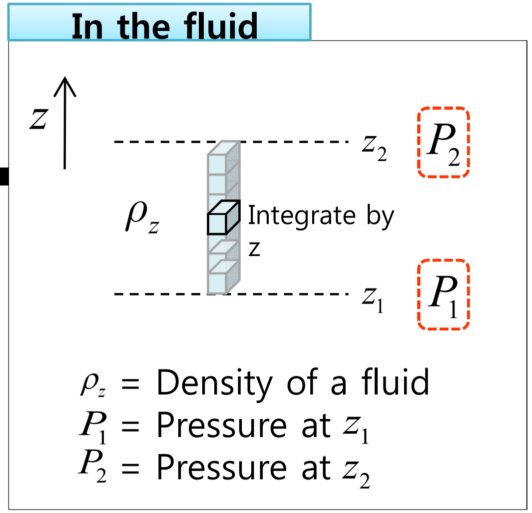
R.H.S: $-\int_{z_1}^{z_2} \boxed{\rho(z)} \cdot g \cdot dz = -\boxed{\rho} g \int_{z_1}^{z_2} dz = -\rho g (z_2 - z_1)$

Assume : Incompressible Fluid (ρ = constant)

L.H.S=R.H.S

$\therefore \boxed{P_2 - P_1 = -\rho g (z_2 - z_1)}$ **: Pascal's Law**

Hydrostatic Pressure (9/9)



$$P_2 - P_1 = -\rho g(z_2 - z_1)$$

$$\underline{P_1 - P_2 = \rho g(z_2 - z_1)}$$

We multiply a minus sign here, so we switch these around: ρ g times z_2 minus z_1.

What it means is we see immediately that if z_2 minus z_1 is positive, i.e. Z_2 is higher than Z_1, the pressure at P_1 is larger than the pressure at P_2.

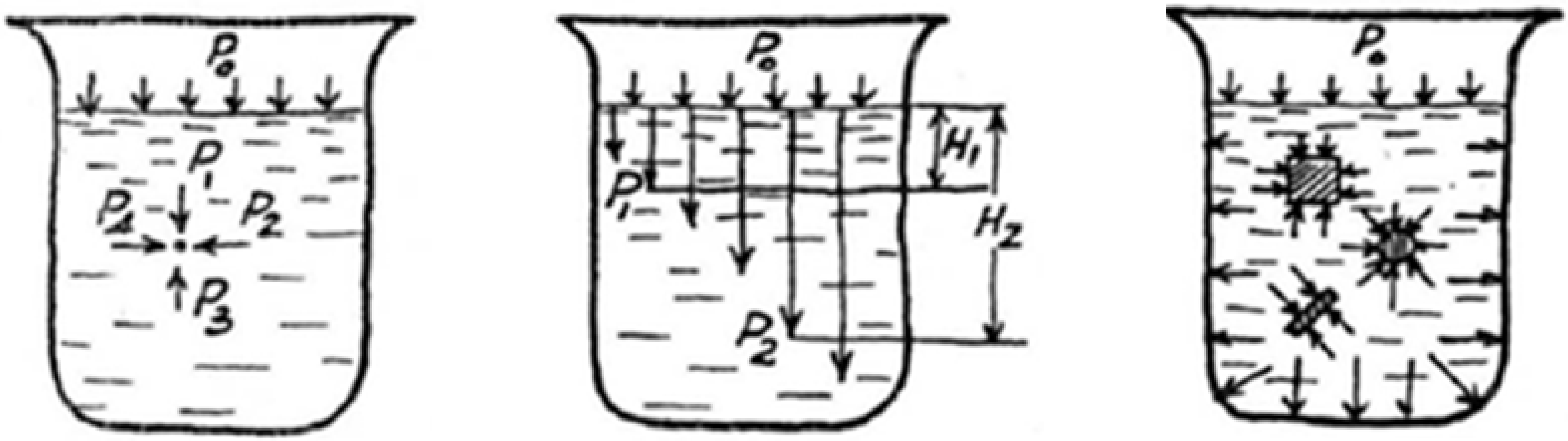
This is the hydrostatic pressure.

Hydrostatic Pressure (Incompressible fluid due to gravity)

The pressure at a point in a <u>fluid in static equilibrium</u> <u>depends on the depth</u> of that point, but not on any horizontal dimension of the fluid or its container.*

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.363, 2004

Three Basic Characteristics of Pressure in a Body of Fluid



$P_1 = P_2 = P_3 = P_4$	$P_2 > P_1$	$P \rightarrow \angle 90^\circ$
Hydrostatic pressure at any point in a body of water is equal in all directions.	Pressure in a body of water increases with depth of water.	Hydrostatic pressure is always applied perpendicular to any submerged body.

<Graphic presentation of the concept of hydrostatic pressure>

* Polevoy, S. L., Water Science and Engineering, Blackie Academic and Professional, pp.78, 1996

Archimedes’ Principle and Buoyant Force

Archimedes' Principle and Buoyant Force (1/4)

- **Static equilibrium of a rigid body in a fluid**

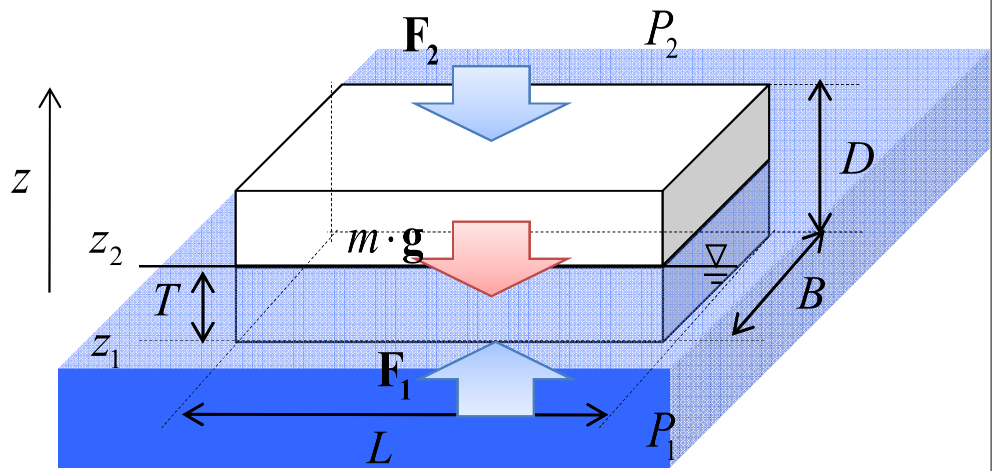
Consider a simple box shaped barge that floats in a fluid. That means the barge is <u>in static equilibrium</u>.

Thus, the gravitational force on the barge in the downward direction must be equal to a net upward force on it from the surrounding fluid, so called 'buoyant force'.

The length of the barge is L, the breadth is B, the depth is D, the immersed depth is T, its density is ρ_{obj}, and the density of the fluid is ρ_{sw}.

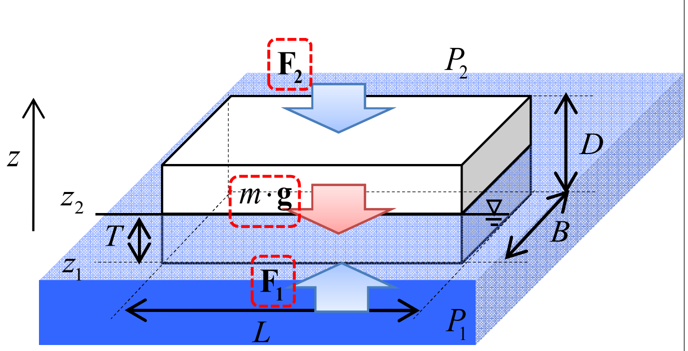
Let be the upward vertical direction as the positive z-coordinate. We define, then, the level of the bottom surface as z_1 and the level of the immersed depth as z_2.

On the top surface of the barge, there is the atmospheric pressure P_2, which is the same as it is on the fluid. And on the bottom surface we have a pressure P_1 in the fluid.



L = Length of the barge
B = Breadth of the barge
D = Depth of the barge
T = Draft of the barge($=z_2-z_1$)
ρ_{sw} = Density of the fluid
ρ_{obj} = Density of the barge

Archimedes' Principle and Buoyant Force (2/4)



L = Length of barge
B = Breadth of barge
D = Depth of barge
T = Draft of barge($=z_2-z_1$)
ρ_{sw} = Density of sea water
ρ_{obj} = Density of a barge

- **Static equilibrium of a barge in a fluid**

Newton's 2nd Law: $\sum \mathbf{F} = m \cdot \ddot{\mathbf{z}}$

(Static Equilibrium: $\ddot{\mathbf{z}} = 0$ **)**

$$\rightarrow \sum \mathbf{F} = 0$$

Assumption: Buoyant force of air is neglected.

$P_1 - P_2 = \rho_{sw} g T$ (Pascal's Law)

$$\boxed{\mathbf{F}_1 - \mathbf{F}_2} - m\mathbf{g} = 0$$

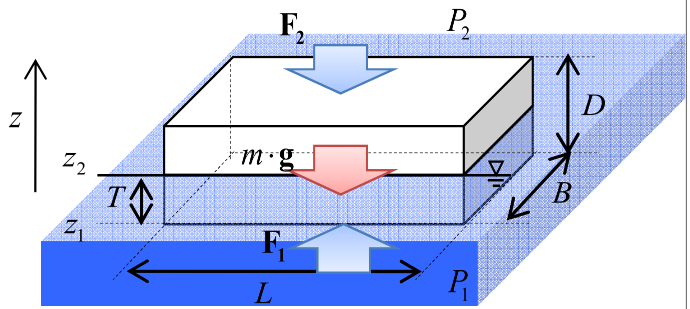
$\mathbf{F}_1 - \mathbf{F}_2$ **: Buoyant Force** $\mathbf{F}_B$

$\mathbf{F}_1$: Force which contains the hydrostatic pressure
$\mathbf{F}_2$: Force which contains the atmospheric pressure

To describe the behavior of the barge in the fluid, we apply the Newton's 2nd law to the barge as shown in the figure.
The gravitational force acting on the barge is mass, m, times g in the downward direction. The hydrostatic pressure force, which is always perpendicular to the surfaces, acting on the bottom surface is F_1 in the upward direction, whereas the atmospheric pressure force acting on the top surface is F_2 in the downward direction.
We only consider forces in the vertical direction, because all forces in the horizontal direction will cancel. If there were any net tangential component force, then the barge would start to move. The barge, however, is static, that means the barge is not moving anywhere. It is just sitting still in the fluid. Thus, the barge is in static equilibrium. For the barge to be in static equilibrium, the upward force F_1 minus downward force F_2 minus delta mg must be zero. Here the net upward hydrostatic pressure force, F_1-F_2, is so called the 'Buoyant force'.

Archimedes' Principle and Buoyant Force (3/4)

- **Buoyant force:** $F_B = F_1 - F_2$



L = Length of barge
B = Breadth of barge
D = Depth of barge
T = Draft of barge($=z_2-z_1$)
ρ_{sw} = Density of sea water
ρ_{obj} = Density of a barge

$$\rightarrow F_B = (L \cdot B) \cdot P_1 - (L \cdot B) \cdot P_2$$
$$= (L \cdot B) \cdot (P_1 - P_2)$$

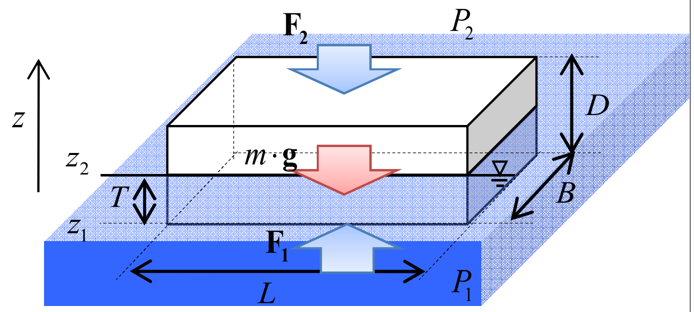
Assumption: Buoyant force of air is neglected.

Substitution: $P_1 - P_2 = \rho_{sw} g T$ (Pascal's Law)

$$\rightarrow F_B = (L \cdot B) \cdot \rho_{sw} g T$$

$$F_B = (L \cdot B \cdot T) \cdot \rho_{sw} \cdot g$$

Archimedes' Principle and Buoyant Force (4/4)



L = Length of barge
B = Breadth of barge
D = Depth of barge
T = Draft of barge($=z_2-z_1$)
ρ_{sw} = Density of sea water
ρ_{obj} = Density of a barge

$$\boxed{\therefore F_B = \underbrace{\underbrace{\underbrace{(L \cdot B \cdot T)}_{\text{Volume}} \cdot \rho_{sw}}_{\text{Mass}} g}}$$

Buoyant force is the weight of the displaced fluid.

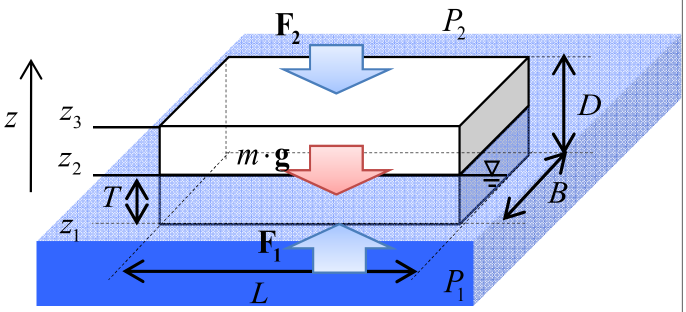
This is a very special case of a general principle which is called Archimedes' Principle.

Archimedes' Principle*

When a body is fully or partially submerged in a fluid, a buoyant force F_B from the surrounding fluid acts on the body. The force is directed upward and has a magnitude equal to the weight of the fluid which is displaced by the body.

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.368, 2004

Reference) Buoyant Force of Air



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D = Depth of barge
T = Draft of barge($=z_2 - z_1$)
ρ_{sw} = Density of sea water
ρ_{obj} = Density of a barge

● Static equilibrium of a barge

$$F_B = (L \cdot B) \cdot (P_1 - P_2)$$

Apply Pascal's Law: $\int_{P_1}^{P_2} dP = -\int_{z_1}^{z_3} \rho(z) \cdot g \cdot dz$

(Due to gravity)

(z_1~z_2: fluid, z_2~z_3: air)

L.H.S: $\int_{P_1}^{P_2} dP = P_2 - P_1$

R.H.S: $-\int_{z_1}^{z_3} \rho(z) \cdot g \cdot dz = -\int_{z_1}^{z_2} \rho_{sw} g dz - \int_{z_2}^{z_3} \rho_{air} g dz$

(Air, Sea water : incompressible)

$$= -\rho_{sw} g \int_{z_1}^{z_2} dz - \rho_{air} g \int_{z_2}^{z_3} dz$$

$$= -\rho_{sw} g (z_2 - z_1) - \rho_{air} g (z_3 - z_2)$$

L.H.S=R.H.S

$$\rightarrow P_1 - P_2 = \rho_{sw} g T + \rho_{air} g (D - T)$$

$$\Rightarrow P_1 - P_2 = \rho_{sw} g T$$

$\rho_{air} \simeq 1.2 kg/m^3, \rho_{sw} \simeq 1025 kg/m^3$

Ratio of ρ_{sw} to ρ_{air} is $\frac{1025}{1.2} \simeq 854$, $(\rho_{air} \ll \rho_{sw})$

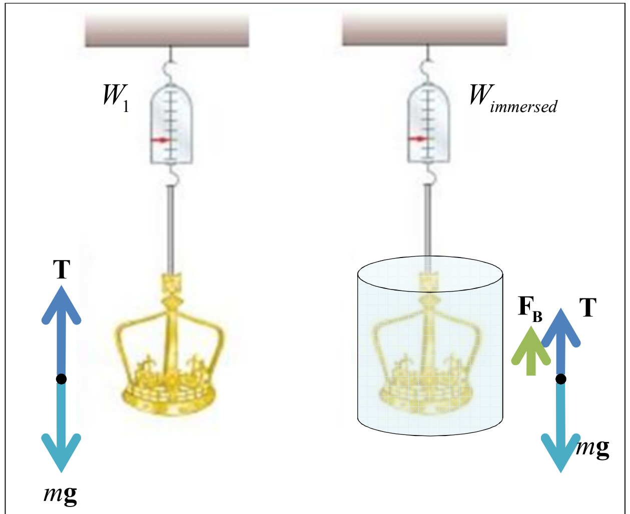
So buoyant force of air is negligible.

Archimedes' Principle and Buoyant Force
- Example: Archimedes and Crown Problem (1/2)

- **Apparent weight of a body in a fluid**

If we place a crown on a scale that is calibrated to measure weight then the reading on the scale is the crown's weight. However, if we do this underwater, the upward buoyant force on the crown from the water decreases the reading.

That reading is then an apparent weight. In general, an apparent weight is the actual weight of a body minus the buoyant force on the body.



W_1 =Weight of the crown
V =Volume of the crown
ρ_{crown} =Density of the crown
$W_{immersed}$ =Weight immersed in water
ρ_w =Density of the water

$$\begin{pmatrix}\text{apparent}\\ \text{weight}\end{pmatrix} = \begin{pmatrix}\text{actual}\\ \text{weight}\end{pmatrix} - \begin{pmatrix}\text{magnitude of}\\ \text{buoyant force}\end{pmatrix}$$

Weight Loss

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.369, 2004

Archimedes' Principle and Buoyant Force
- Example: Archimedes and Crown Problem (2/2)

Question)

Is the crown made of pure gold?

Answer)

$$W_1 = V\rho_{crown}g$$

$$\rightarrow W_{immersed} = V\rho_{crown}g - \underline{V\rho_w g}$$

(Apparent Weight)

W_{Loss}: Weight Loss(Buoyant Force)

(Measure) (Measure) (Find & Compare) (Known)

$$\frac{W_1}{W_{Loss}} = \frac{V\rho_{crown}g}{V\rho_w g} = \frac{\rho_{crown}}{\rho_w}$$

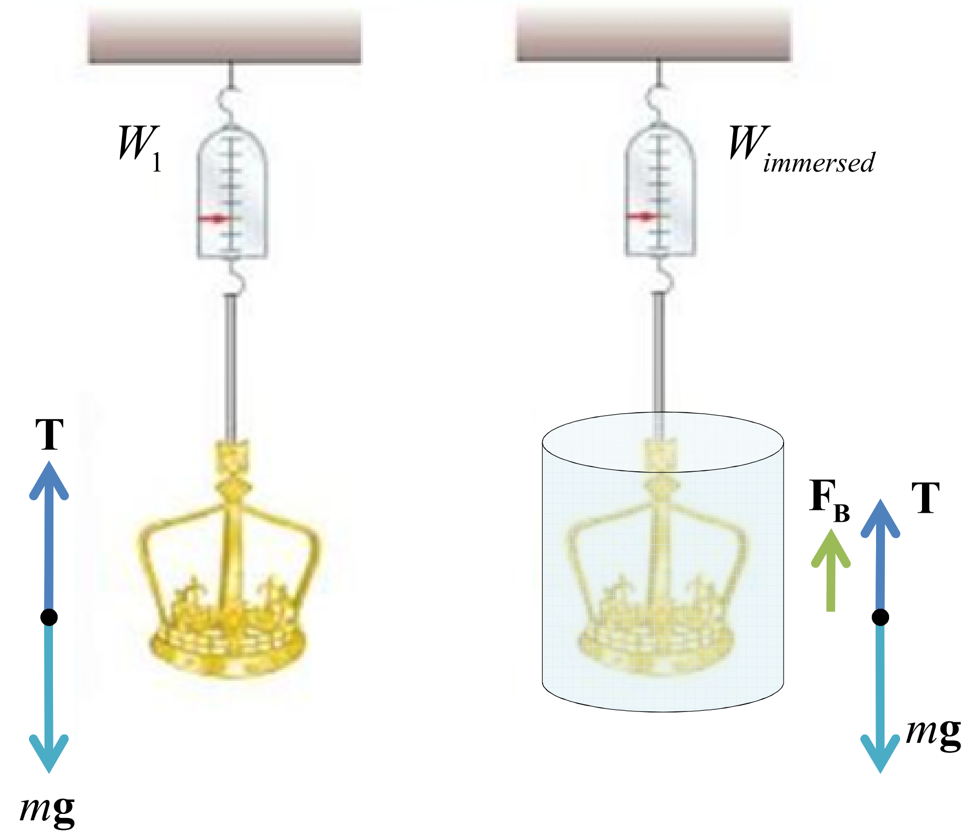
Archimedes lived in the third century B.C. Archimedes had been given the task to determine whether a crown was pure gold. He had the great vision to do the following: He takes the crown and he weighs it in a normal way.

So the weight of the crown - we call it W_1 - is the volume of the crown times the density of which it is made. If it is gold, it should be 19.3 gram per centimeter cube, and so volume of the crown x rho crown is the mass of the crown and multiplying mass by g is the weight of the crown.

Now he takes the crown and he immerses it in the water. And he has a spring balance, and he weighs it again. And he finds that the weight is less and so now he has the weight immersed in the water.

So what he gets is the weight of the crown minus the buoyant force, which is the weight of the displaced water. And the weight of the displaced water is the volume of the crown times the density of water times g. And so V x rho w x g is 'weight loss'.

And he takes W_1 and divides by the weight loss and it gives him rho of the crown divided by rho of the water. And he knows rho of the water, so he can find rho of the crown. It's an amazing idea; he was a genius.



W_1 =Weight of the crown

V =Volume of the crown

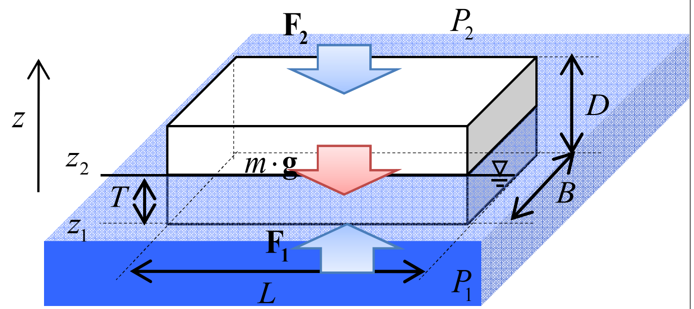
ρ_{crown} =Density of the crown

$W_{immersed}$ =Weight immersed in water

ρ_w =Density of the water

* Serway, R. A., College Physics, 8th Ed., Brooks/Cole, pp.287, 2009.

Archimedes' Principle and Buoyant Force
- Condition for floating



L = Length of barge
B = Breadth of barge
D = Depth of barge
T = Draft of barge
ρ_{sw} = Density of sea water
ρ_{obj} = Density of a barge

▪ **Condition for floating**

$$F_B = mg \quad (T < D)$$

▪ **For this barge to float, the buoyant force must be equal to gravitational force.**

$$(\cancel{L} \cdot \cancel{B} \cdot T) \cdot \rho_{sw} \cancel{g} = (\cancel{L} \cdot \cancel{B} \cdot D) \cdot \rho_{obj} \cancel{g}$$

→ $\underline{\rho_{sw} > \rho_{obj}}$: Float

Necessary condition for floating

$\rho_{sw} < \rho_{obj}$: Sink

Archimedes' Principle and Buoyant Force - Example: Floating Iceberg

Question)

What percentage of the volume of ice will be under the level of the water?

$$\rho_{ice} = 0.92\text{g/cm}^3, \rho_w = 1.0\text{g/cm}^3$$

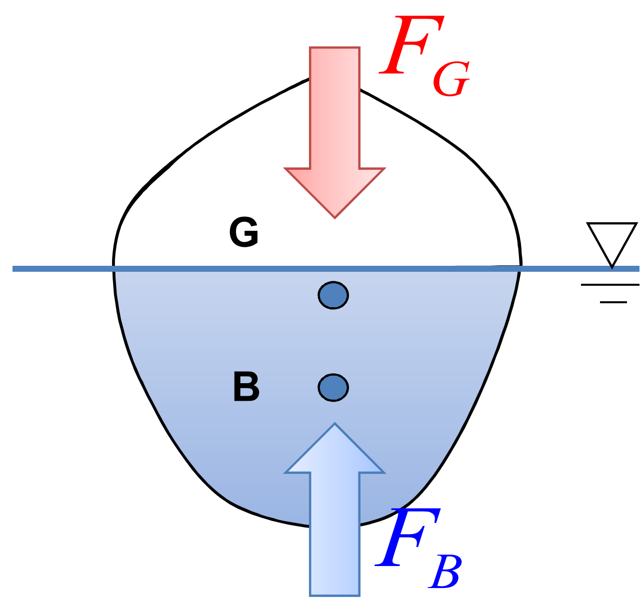
Answer)

$$Mg = V_{tot}\rho_{ice}\cancel{g} = V_{uw}\rho_w\cancel{g}$$

$$\rightarrow \frac{V_{uw}}{V_{tot}} = \frac{\rho_{ice}}{\rho_w}$$

$$\frac{\text{Underwater Volume}}{\text{Total Volume}} = \frac{V_{uw}}{V_{tot}} = \frac{\rho_{ice}}{\rho_w} = 0.92$$

$\therefore 92\%$ **of the iceberg is in underwater.**



M : Mass of iceberg
V_{tot} : Total volume of iceberg
V_{uw} : Volume underwater
ρ_{ice} : Density of the ice,
ρ_w : Density of underwater
g : Acceleration of gravity

* Ohanian, H. C., Physics, 2nd Ed., W. W. Norton & Company, pp.478, 1989

Archimedes' Principle and Buoyant Force
- Example: Waterline will change? (1/6)

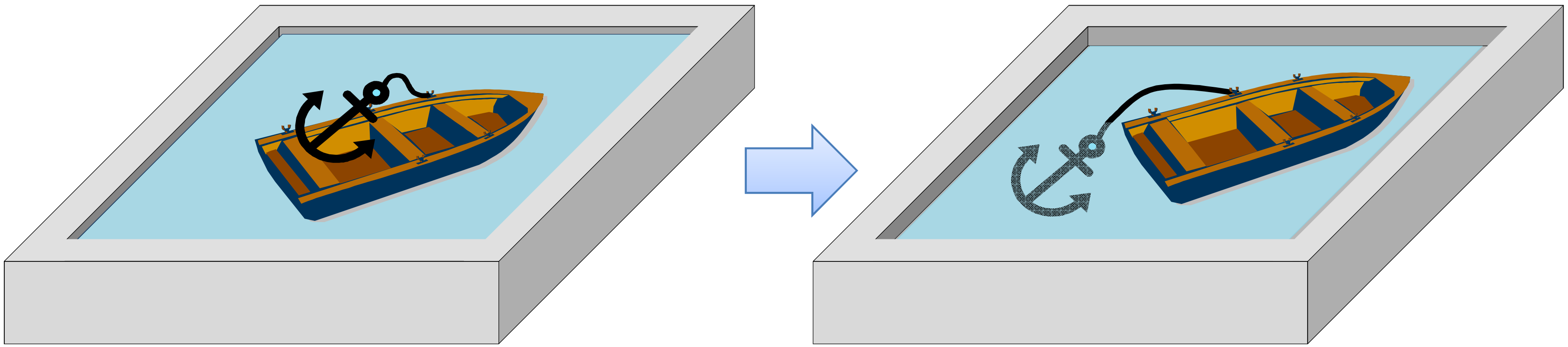
Question)

A boat with an anchor on board floats in a swimming pool that is somewhat wider than the boat. Does the pool water level move up, move down, or remain the same if the anchor is

(a) Dropped into the water or

(b) Thrown onto the surrounding ground?

(c) Does the water level in the pool move upward, move downward, or remain the same if, instead, a cork (or buoy) is dropped from the boat into the water, where it floats?

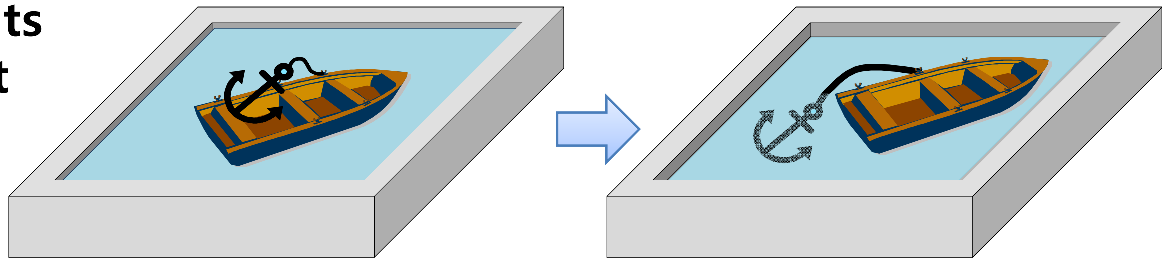


* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.377, 2004

Archimedes' Principle and Buoyant Force
- Example: Waterline will change? (2/6)

Question)

A boat with an anchor on board floats in a swimming pool that is somewhat wider than the boat. Does the pool water level move up, move down, or remain the same if the anchor is



(a) Dropped into the water

Answer)

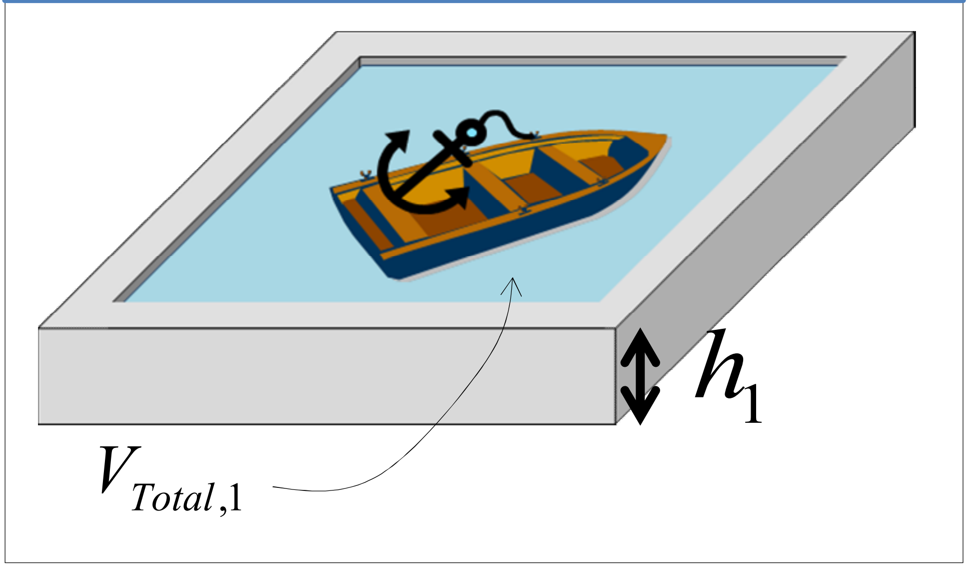
The volume under the water level is composed of the water and the volume displaced by the boat and anchor. After the anchor is dropped into the water, the buoyant force exerted on the anchor cannot compensate the weight of the anchor.

Thus the water level moves down.

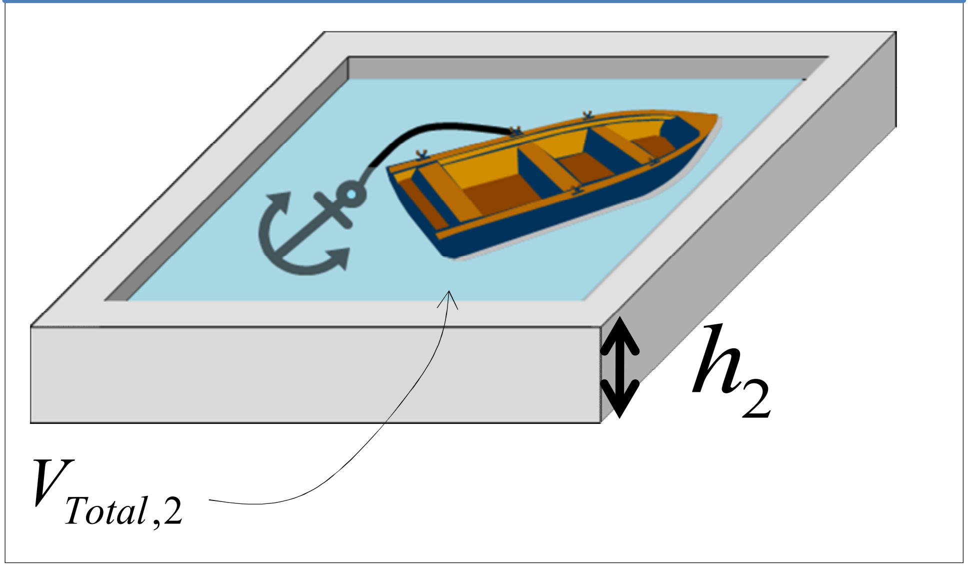
* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.377, 2004

Archimedes' Principle and Buoyant Force
- Example: Waterline will change? (3/6)

Stage1



Stage2



If the shape of water tanks are same, the waterline will be proportional to total volume (volume of water + volume displaced by the boat and the anchor).

$$h_1 = \frac{V_{Total,1}}{A} \qquad\qquad h_2 = \frac{V_{Total,2}}{A}$$

h : Waterline

A : Bottom area

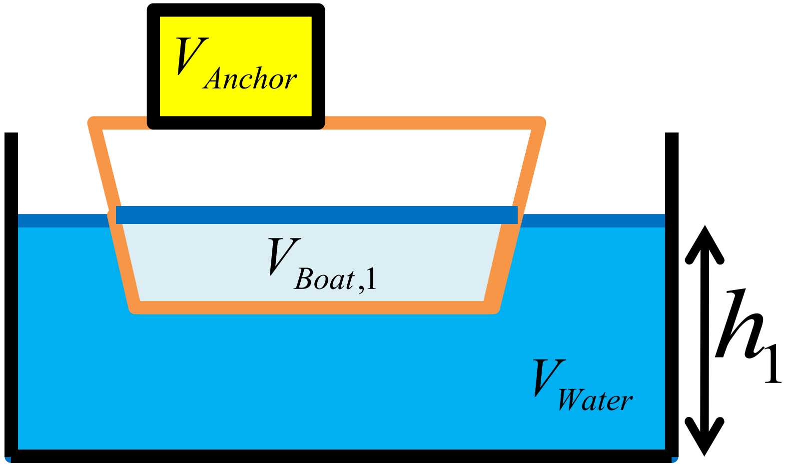
V_{Total} : **Total volume**

Example: Waterline will change? (4/6)
(a) Dropped into the water (1/2)

$$h = \frac{V_{Total}}{A}$$

$$V = \frac{M}{\rho} = \frac{Mg}{\rho g} = \frac{W}{\rho g}$$

Stage1

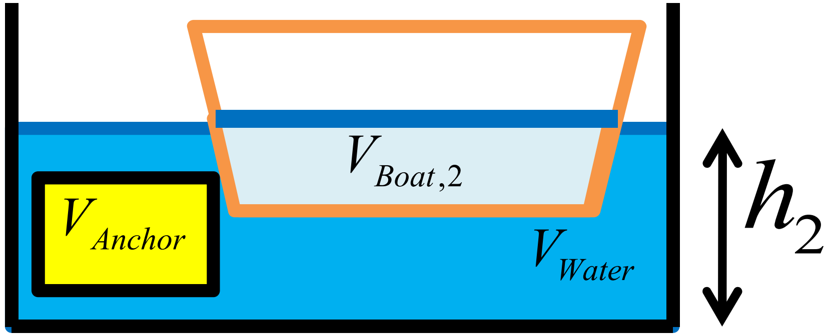


$$V_{Total,1} = V_{Boat,1} + V_{Water}$$

$$V_{Boat,1} = \frac{W_{Baot} + W_{Anchor}}{\rho_{Water} g}$$ (floating condition)

$$= \frac{W_{Boat} + W_{Anchor}}{\rho_{Water} g} + V_{Water}$$

Stage2



$$V_{Total,2} = V_{Boat,2} + V_{Anchor} + V_{Water}$$

$$V_{Boat,2} = \frac{W_{Boat}}{\rho_{Water} g}, \quad V_{Anchor} = \frac{W_{Anchor}}{\rho_{Anchor} g}$$

$$= \frac{W_{Boat}}{\rho_{Water} g} + \frac{W_{Anchor}}{\rho_{Anchor} g} + V_{Water}$$

h_1 : Height of the waterline in stage 1

h_2 : Height of the waterline in stage 2

W_{Baot} : Weight of the boat

W_{Anchor} : Weight of the anchor

ρ_{Water} : Density of the water

$V_{Boat,1}$: Displaced volume by the ship with the anchor

$V_{Boat,2}$: Displaced volume by the ship without the anchor

V_{Anchor} : Displaced volume by the anchor

V_{Water} : Volume of the water which is invariant

ρ_{Anchor} : Density of the anchor

$\rho_{Anchor} > \rho_{Water}$

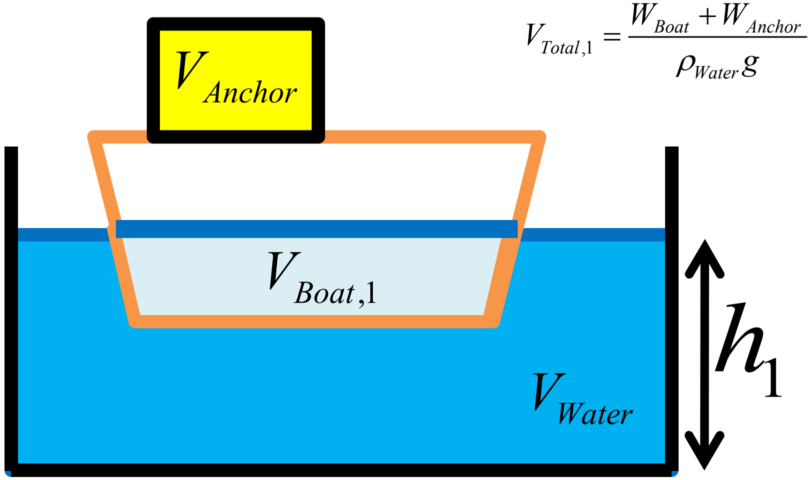
Example: Waterline will change? (4/6)
(a) Dropped into the water (2/2)

$$h = \frac{V_{Total}}{A}$$

$$V = \frac{M}{\rho} = \frac{Mg}{\rho g} = \frac{W}{\rho g}$$

Stage1

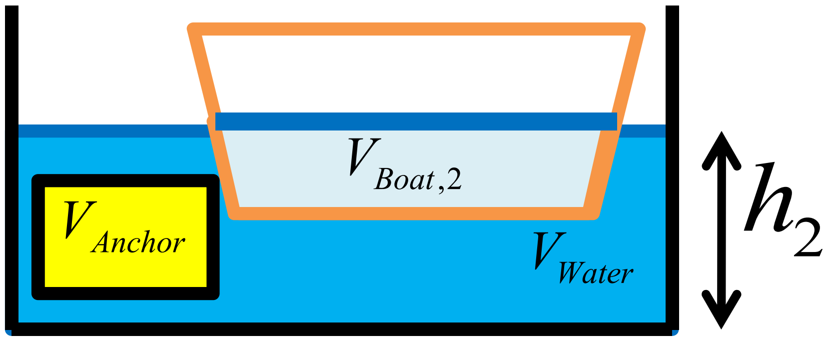
$$V_{Total,1} = \frac{W_{Boat} + W_{Anchor}}{\rho_{Water} g} + V_{Water}$$



Stage2

$\rho_{Luggage} > \rho_{Water}$

$$V_{Total,2} = \frac{W_{Boat}}{\rho_{Water} g} + \frac{W_{Anchor}}{\rho_{Anchor} g} + V_{Water}$$



$$V_{Total,1} - V_{Total,2} = \left(\cancel{\frac{W_{Boat}}{\rho_{Water} g}} + \frac{W_{Anchor}}{\rho_{Water} g} + \cancel{V_{Water}}\right) - \left(\cancel{\frac{W_{Boat}}{\rho_{Water} g}} + \frac{W_{Anchor}}{\rho_{Anchor} g} + \cancel{V_{Water}}\right)$$

$$= \left(\frac{W_{Anchor}}{\rho_{Water} g}\right) - \left(\frac{W_{Anchor}}{\rho_{Anchor} g}\right)$$

$$= \frac{W_{Anchor}}{g}\left(\frac{1}{\rho_{Water}} - \frac{1}{\rho_{Anchor}}\right) > 0 \qquad \left(\because \rho_{Anchor} > \rho_{Water}, \frac{1}{\rho_{Anchor}} < \frac{1}{\rho_{Water}}\right)$$

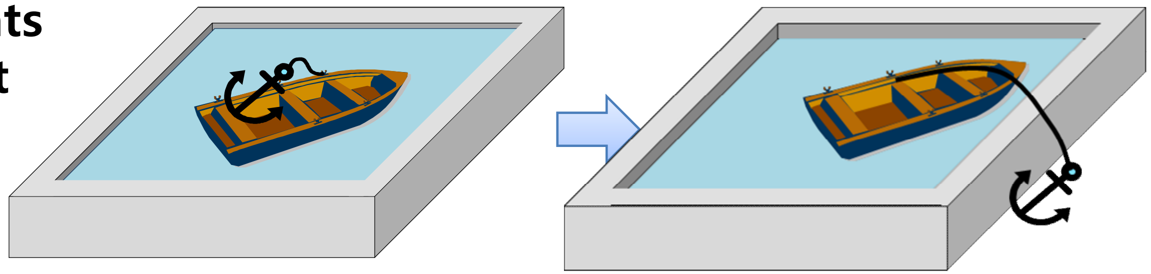
$$V_{Total,1} > V_{Total,2}, \quad \boxed{\therefore h_1 > h_2}$$

The waterline will go down!

Archimedes' Principle and Buoyant Force - Example: Waterline will change? (5/6)

Question)

A boat with an anchor on board floats in a swimming pool that is somewhat wider than the boat. Does the pool water level move up, move down, or remain the same if the anchor is



(b) Thrown onto the surrounding ground

Answer)

After the anchor is thrown onto the surrounding ground, the ground supports the weight of the anchor. So buoyant force exerted on the anchor is zero.

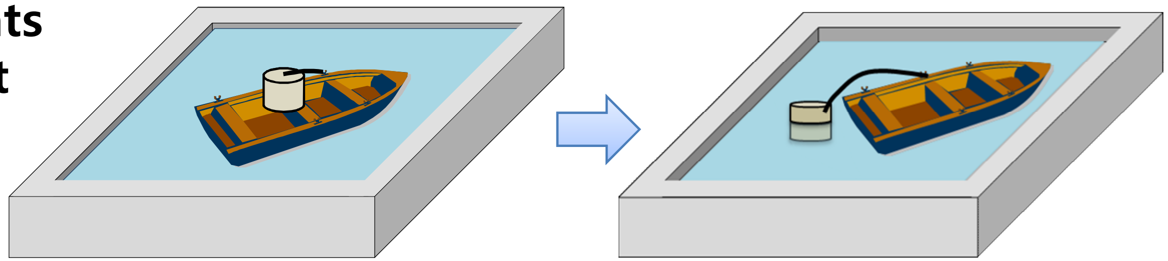
Thus the water level moves down.

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.377, 2004

Archimedes' Principle and Buoyant Force - Example: Waterline will change? (6/6)

Question)

A boat with an anchor on board floats in a swimming pool that is somewhat wider than the boat. Does the pool water level move up, move down, or remain the same if the anchor is



(c) If, instead, a cork is dropped from the boat into the water, where it floats, does the water level in the pool move upward, move downward, or remain the same?

Answer)

After the cork is dropped from the boat into the water, the cork floats in the water. So the buoyant force exerted on the cork has the same magnitude as that of the weight of the cork. Thus the volume displaced by the cork remains the same.

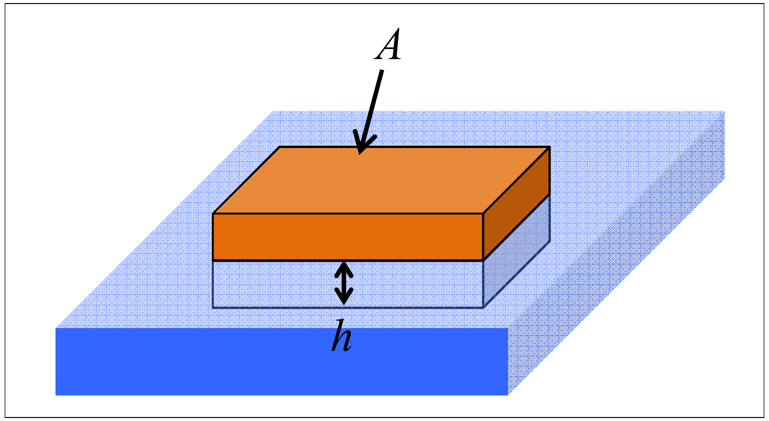
And the water level also remains the same.

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.377, 2004

Archimedes' Principle and Buoyant Force
- Example: Floating Down the River (1/2)

Question)*

A raft is constructed of wood having a density of 600 kg/m^3. Its surface area is 5.7 m^2, and its volume is 0.60 m^3. When the raft is placed in fresh water of density 1,000 kg/m^3, as in the figure, to what depth does the raft sink in the water?



<A raft partially submerged in water>

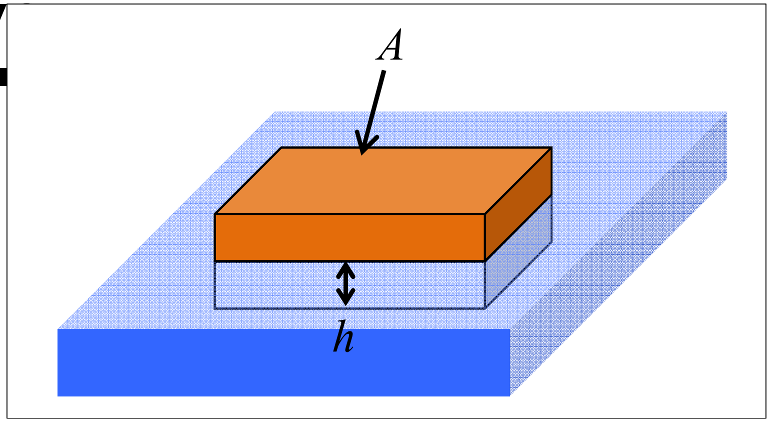
Hint)

The magnitude of the upward buoyant force acting on the raft must equal the weight of the raft if the raft is to float. In addition, from Archimedes' Principle the magnitude of the buoyant force is equal to the weight of the displaced water.

* Serway, R. A., College Physics, 8th Ed., Brooks/Cole, pp.273, 2009.

Archimedes' Principle and Buoyant Force
- Example: Floating Down the River (2/

Question)*



<A raft partially submerged in water>

Answer)

The magnitude of the upward buoyant force acting on the raft equals the weight of the displaced water, which in turn must equal the weight of the raft:

$$B = \rho_{water} g V_{water} = \rho_{water} g A h$$

Because the area *A* and density ρ_{water} are known, we can find the depth *h* to which the raft sinks in the water:

$$h = \frac{w_{raft}}{\rho_{water} g A} \quad \text{......... (1)}$$

The weight of the raft is

$$w_{raft} = \rho_{water} g V_{raft} = (600 kg/m^3)(9.8 m/s^2)(0.60 m^3) = 3.5 \times 10^3 N$$

Therefore, substitution into (1) gives

$$h = \frac{3.5 \times 10^5 \mathrm{N}}{(1000 kg/m^3)(9.8 m/s^2)(5.7 m^2)} = 0.060\mathrm{m}$$

* Serway, R. A., College Physics, 8th Ed., Brooks/Cole, pp.273, 2009.

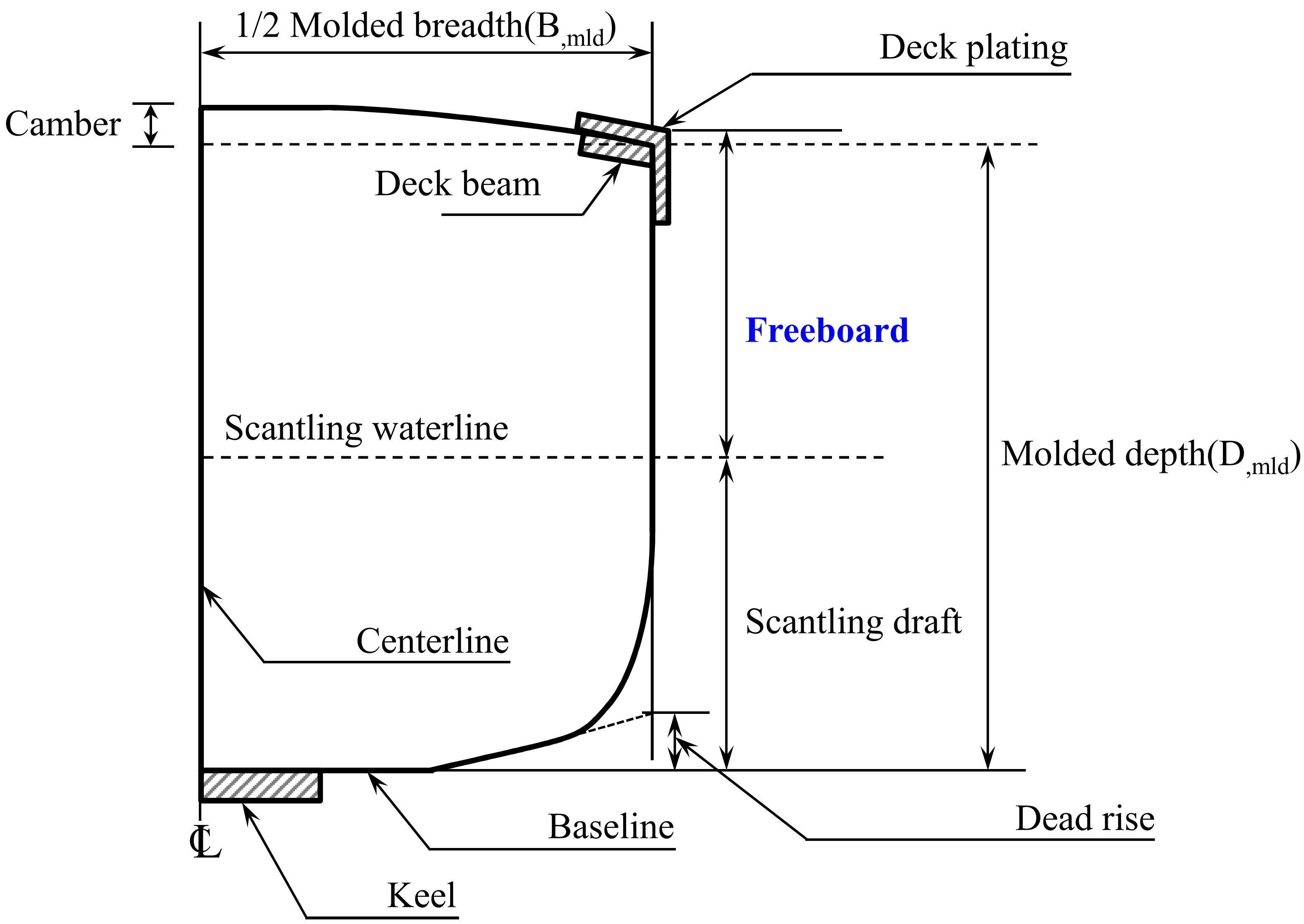
Question)

A 302,000DWT VLCC has a mass of 41,000 metric tons when empty and it can carry up to 302,000 metric tons of oil when fully loaded. Assume that the shape of its hull is approximately that of a rectangular parallelepiped 300m long, 60m wide, and 30m high.

(a) What is the draft of the empty tanker, that is, how deep is the hull submerged in the water?
Assume that the density of the sea water is 1.025Mg/m^3.

(b) What is the draft of the fully loaded tanker?

Archimedes' Principle and Buoyant Force - Freeboard (1/2)

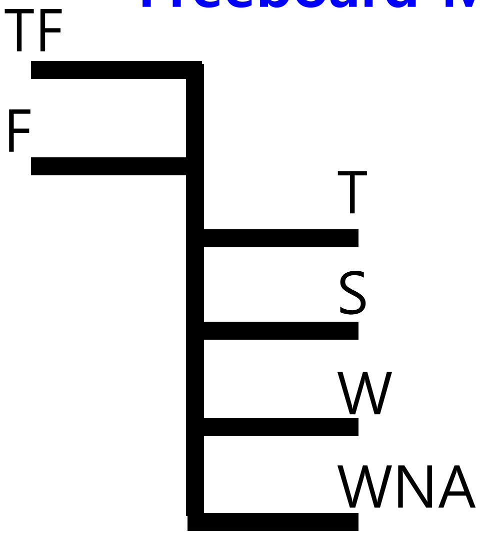


$$Freeboard = Depth(D_{mld}) - Draft(T) + t_{deckplating}$$

Archimedes' Principle and Buoyant Force - Freeboard (2/2)

$$W = F_B$$
$$= (L \cdot B \cdot T) \cdot \rho_{sw} g$$

Freeboard Mark



TF – Tropical Fresh Water

F – Fresh Water

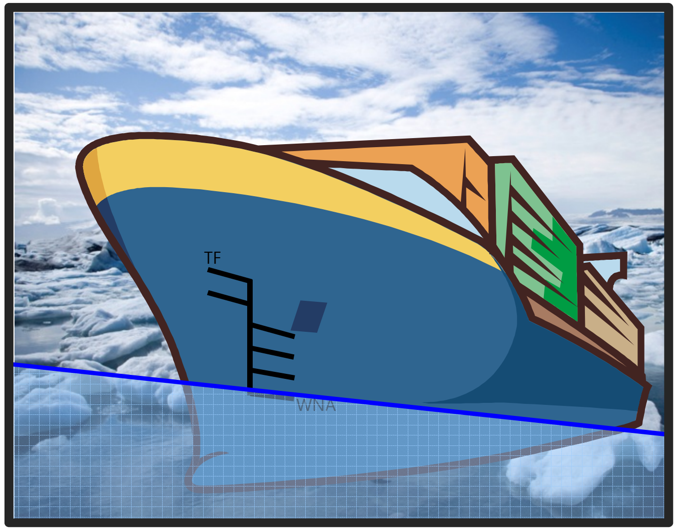
T – Tropical Sea Water

S – Summer Sea Water

W – Winter Sea Water

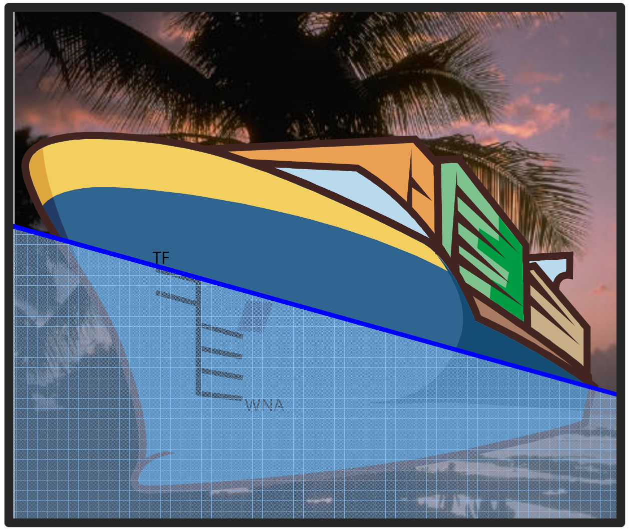
WNA – Winter North Atlantic

The heaviest water is in the North Atlantic in winter time. Ships there displace much less water than in other areas of the world ocean.



The density of water in the world ocean is 1.026 g/cm^3. The density of water in the North Atlantic is 1.028 g/cm^3.

Tropical fresh water is lightest. It occurs in tropical rivers(Amazon, Congo, and others). Some of these rivers are navigable by ocean steamers.



The density of water in navigable tropical rivers is 0.997 g/cm^3.

* Polevoy, S. L., Water Science and Engineering, Blackie Academic and Professional, p.93-97, 1996

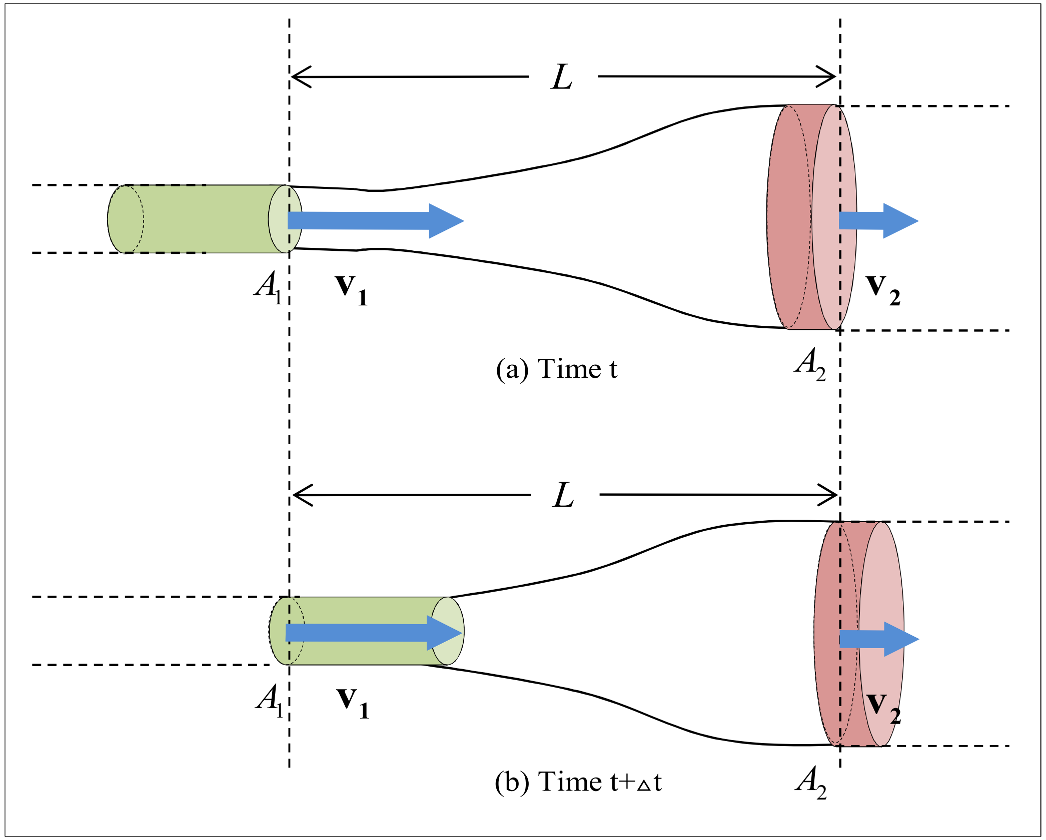
The Equation of Continuity and Bernoulli Equation

The Equation of Continuity* (1/3)

- **The Equation of Continuity**

The equation of continuity of flow is a mathematical expression of **the law of conservation of mass for flow.**

Here we wish to derive an expression that relates v and A for the steady flow of an ideal fluid through a tube with varying cross section.



* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.311, 2004

The Equation of Continuity* (2/3)

- **The Equation of Continuity**

The volume ΔV of fluid that has passed through the dashed line in that time interval Δt is

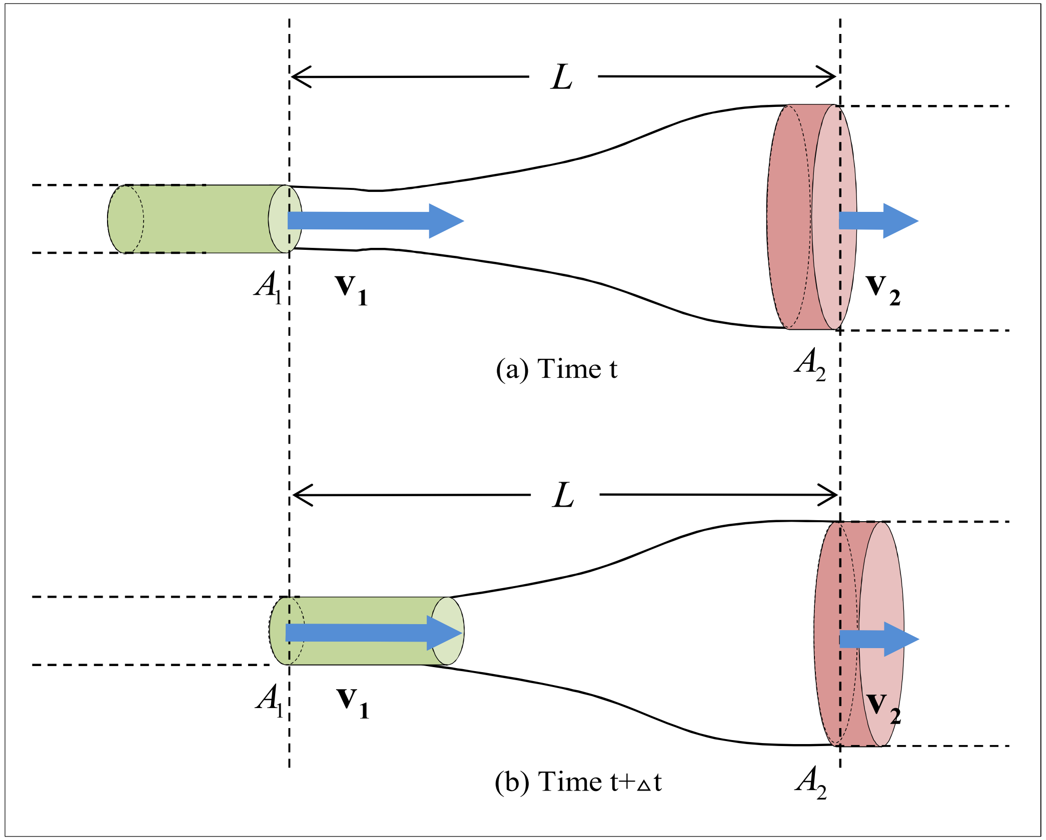
$$\Delta V = A \cdot \Delta x = A \cdot v \cdot \Delta t$$

Apply to both the left and right ends of the tube segment, we have

$$\Delta V = A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

$$\rightarrow \therefore \underline{A_1 v_1 = A_2 v_2}$$

: **Equation of Continuity** for the flow of an ideal fluid



* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.371, 2004

The Equation of Continuity* (3/3)

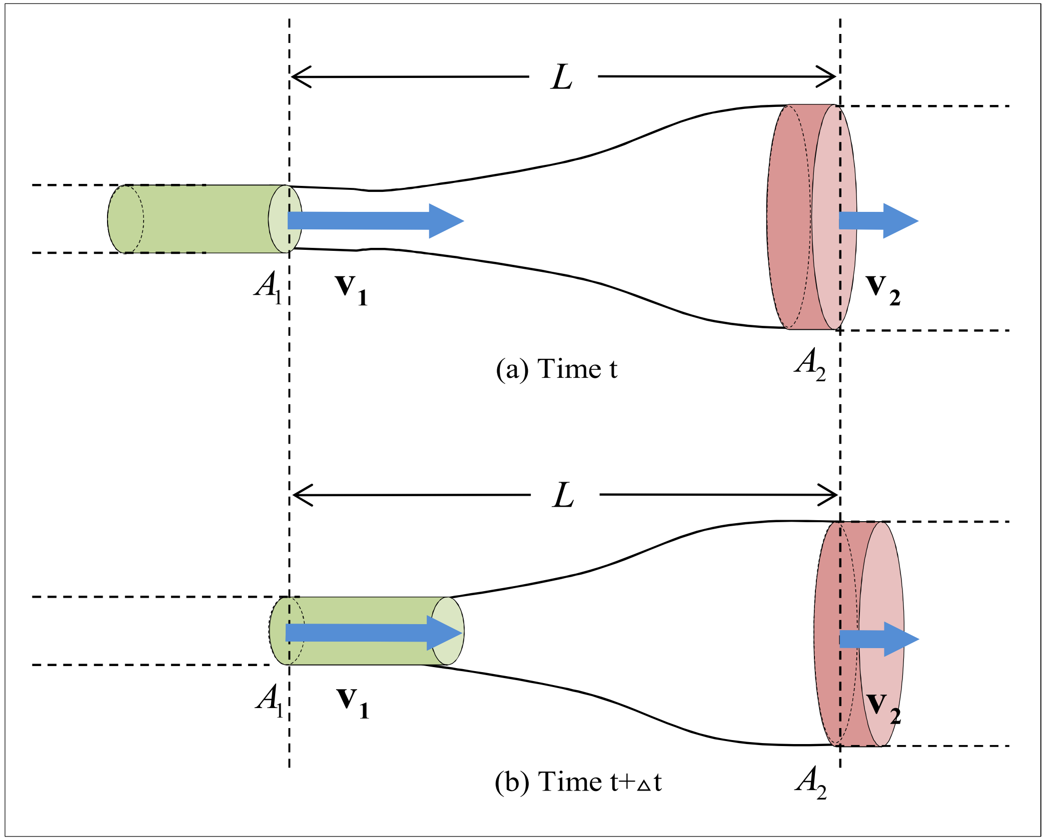
- **The Equation of Continuity**

$$A_1 v_1 = A_2 v_2$$

: **Equation of Continuity** for the flow of an ideal fluid

This relation between speed and cross-sectional area is called the equation of continuity for the flow of an ideal fluid.

The flow speed increases when we decrease the cross-sectional area through which the fluid flows.



* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.371, 2004

Bernoulli's Equation (1/9)

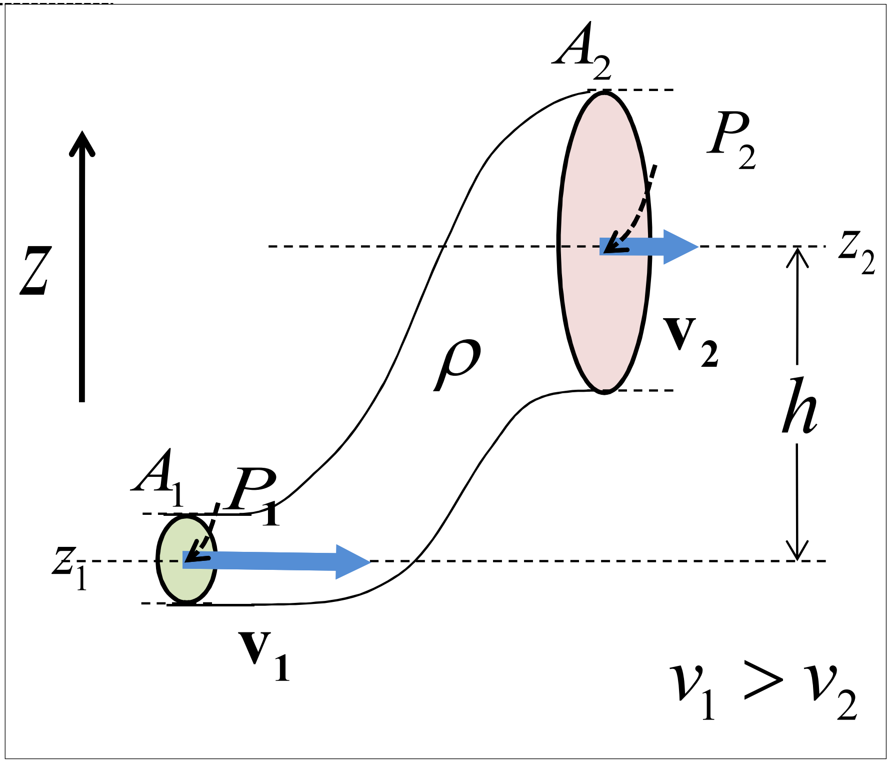
- **Bernoulli's Equation**

We can apply the principle of conservation of energy to the fluid.

Assumption: incompressible fluid (density is constant.)

(1) If this fluid is completely static, it seems that it is not moving.

$$P_1 - P_2 = \rho g(z_2 - z_1) = \rho gh$$ **: Pascal's Law**



Bernoulli's Equation (2/9)

- **Bernoulli's Equation**

We can apply the principle of conservation of energy to the fluid.

Assumption: incompressible fluid

---

(1) If this fluid is completely static, it seems that it is not moving.

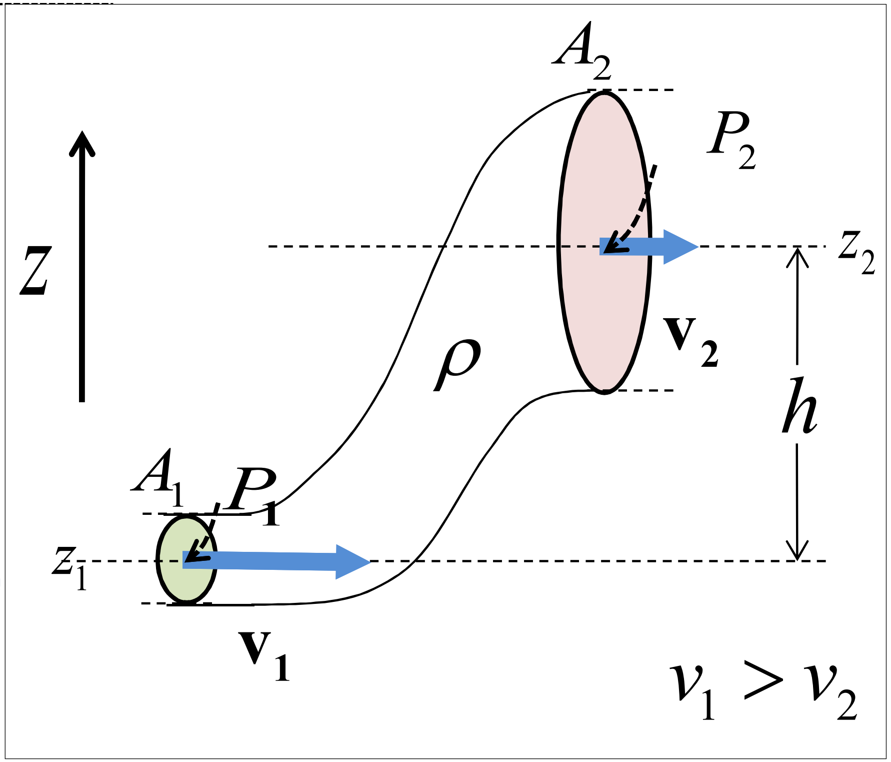
$$\underline{P_1 - P_2} = \rho g(z_2 - z_1) = \underline{\rho g h}$$ **: Pascal's Law**

Have the same dimension of $\dfrac{\text{Energy}}{\text{Volume}}$

mgh : Gravitational Potential Energy

$\dfrac{\text{Mass}}{\text{Volume}}$ =Density

$\rho g h = \dfrac{\text{Gravitational Potential Energy}}{\text{Volume}}$



Bernoulli's Equation (3/9)

- **Bernoulli's Equation**

We can apply the principle of conservation of energy to the fluid.

Assumption: incompressible fluid

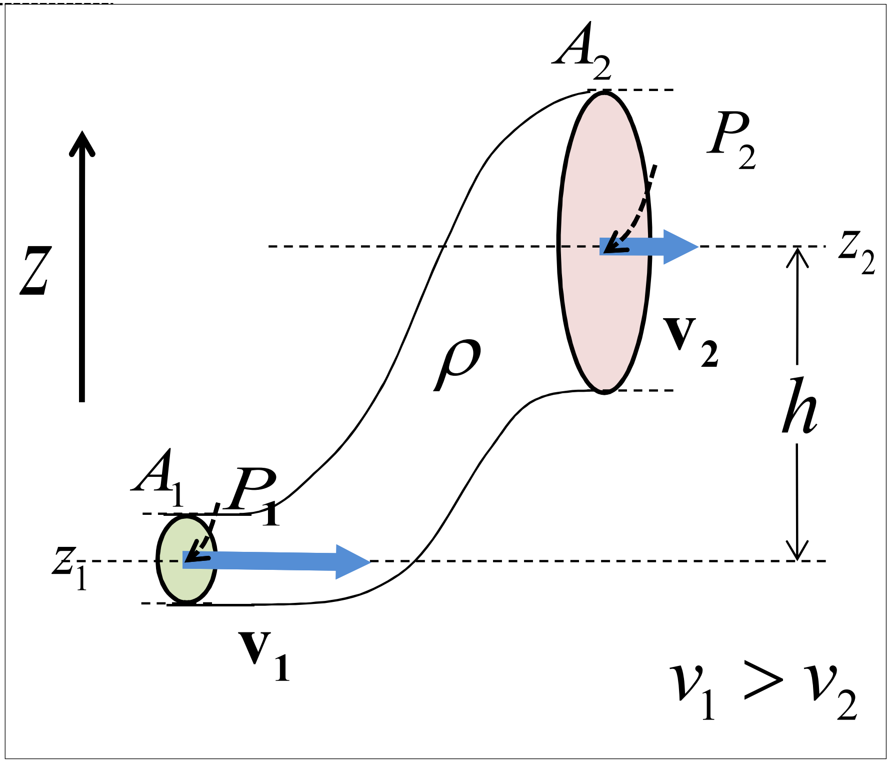
(2) If we now set this whole machine in motion, there are three players.

$$\frac{\text{Kinetic Energy}}{\text{Volume}} + \frac{\text{Gravitational Potential Rnergy}}{\text{Volume}} + P$$

Apply the Conservation of Energy

$$\boxed{\frac{1}{2}\rho v^2 + \rho g z + P_z = \text{Constant}}$$

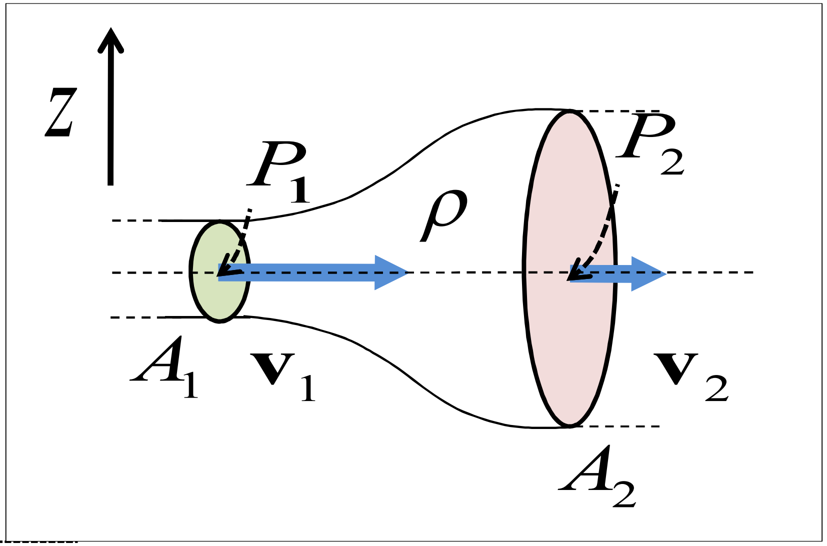
: Bernoulli's Equation



Bernoulli's Equation (4/9)

- **Example: Eliminate 'z'**

If we take z to be a constant, so that the fluid does not change elevation as it flows,



If we assume that $A_1 < A_2$,

By the **Equation of Continuity** (ideal fluid)

$$A_1 v_1 = A_2 v_2$$

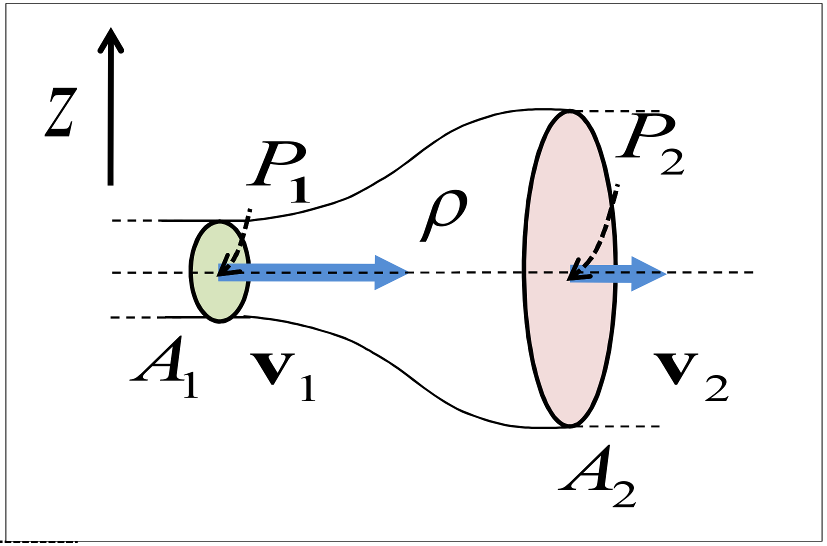
→ $A_1 < A_2 \rightarrow v_1 > v_2$

Bernoulli's Equation (5/9)

Bernoulli's Equation :

$$\frac{1}{2}\rho v^2 + \rho g z + P_z = \text{Constant}$$

- **Example: Eliminate 'z'**



If we take z to be a constant, so that the fluid does not change elevation as it flows,

Bernoulli' Equation becomes

$$\frac{1}{2}\rho v_1^{\ 2} + P_1 = \frac{1}{2}\rho v_2^{\ 2} + P_2$$

$$\rightarrow \quad v_1 > v_2 \rightarrow P_1 < P_2$$

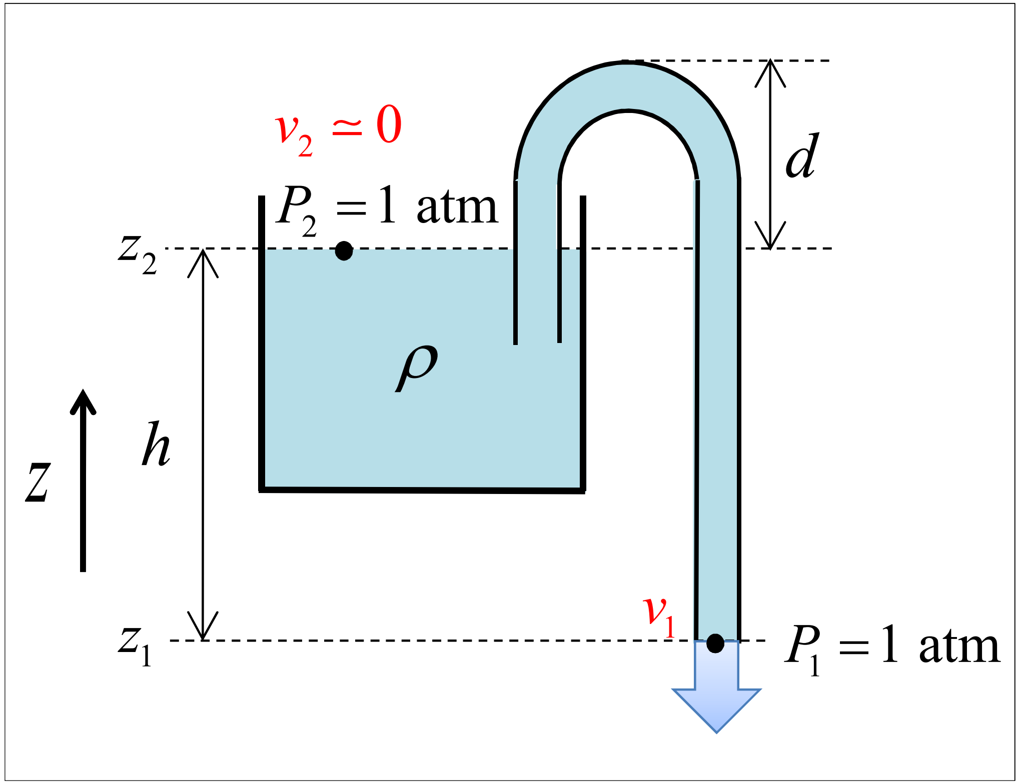
Which tell us that :

If the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.*

Bernoulli's Equation (6/9)
- Example : Siphon* (: Eliminate 'P') (1/3)

Figure on the right side shows a siphon, which a device for removing liquid from a container.

A tube must initially be filled, but once this has been done, liquid will flow through the tube until the liquid surface in the container is level with the tube opening at z_1. The liquid has density ρ and negligible viscosity.



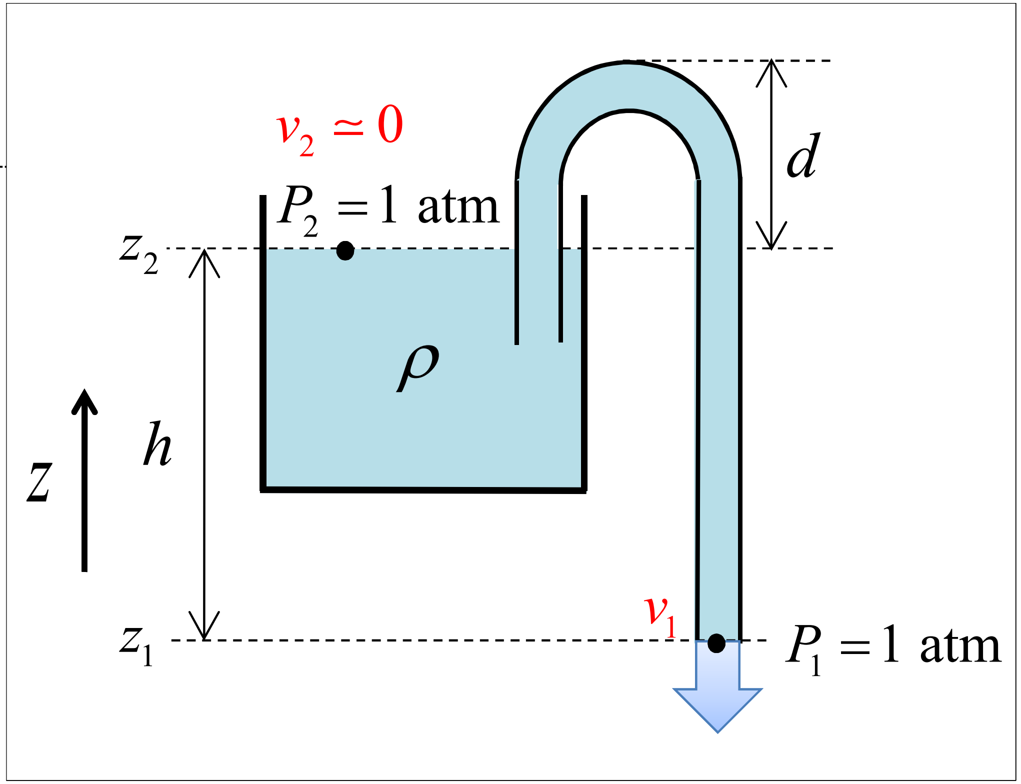
(a) With what speed does the liquid emerge from the tube at z_1?

(b) Theoretically, what is the greatest possible height d that a siphon can lift water?

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.383-384, 2004

Bernoulli's Equation (6/9)
- Example : Siphon* (: Eliminate 'P') (2/3)

(a) With what speed does the liquid emerge from the tube at z_1?



Bernoulli's Equation:

$$\frac{1}{2}\rho v^2 + \rho g z + P_z = \text{Constant}$$

$P_1 = P_2$ → **P term is eliminated.**

$$\rightarrow \frac{1}{2}\cancel{\rho} v_1^2 + \cancel{\rho} g z_1 = \cancel{\rho} g z_2$$

$$\frac{1}{2}v_1^2 + g z_1 = g z_2 \quad \frac{1}{2}v_1^2 = g(z_2 - z_1)$$

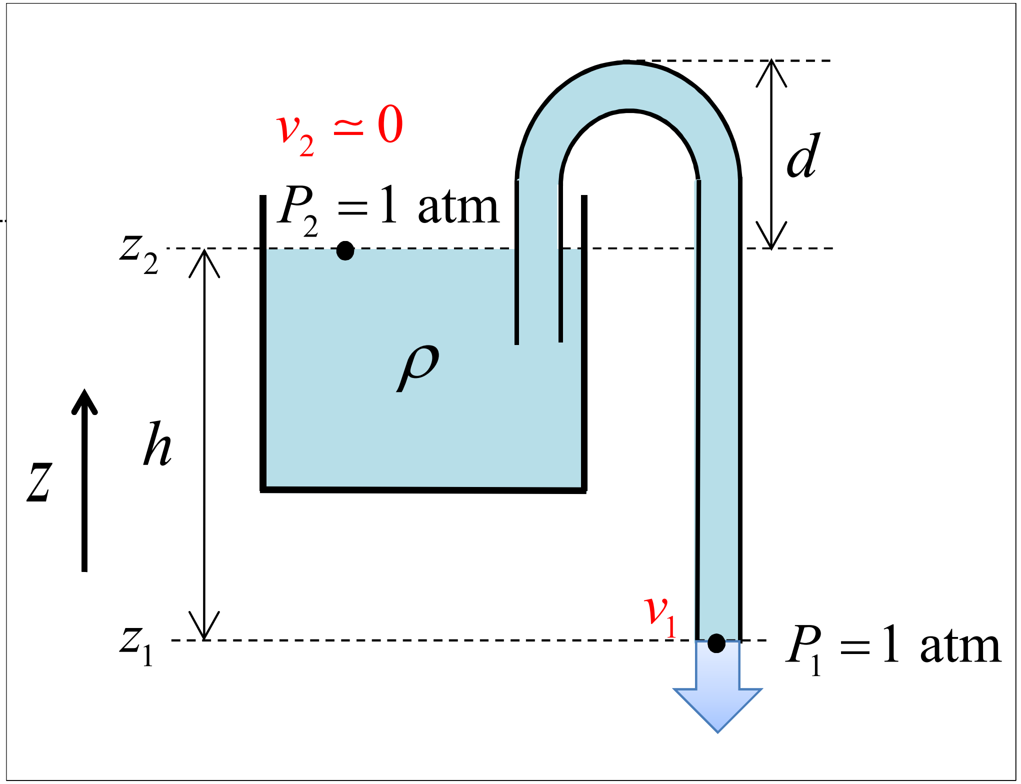
$$\rightarrow \frac{1}{2}v_1^2 = g(h)$$

$$\therefore v_1 = \sqrt{2gh}$$

Conversion of gravitational potential energy to kinetic energy

Bernoulli's Equation (6/9)
- Example : Siphon* (: Eliminate 'P') (3/3)

(b) Theoretically, what is the greatest possible height d that a siphon can lift water?



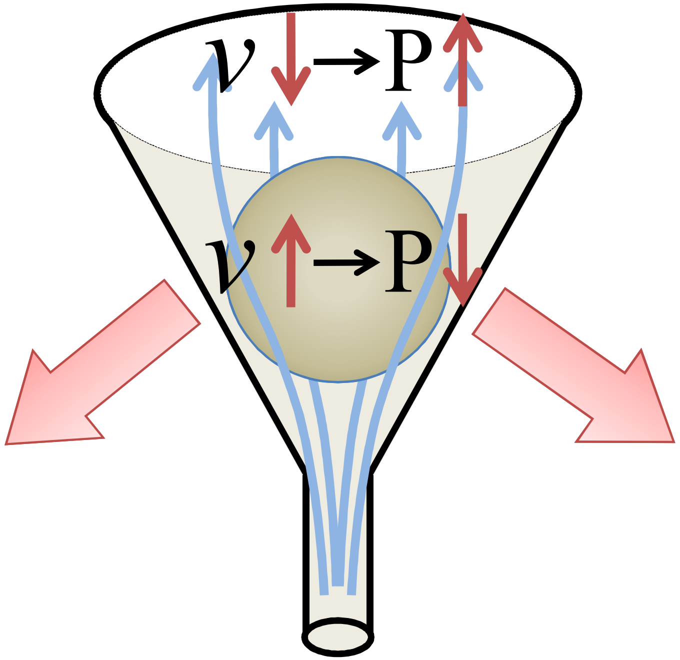
Barometric Pressure:

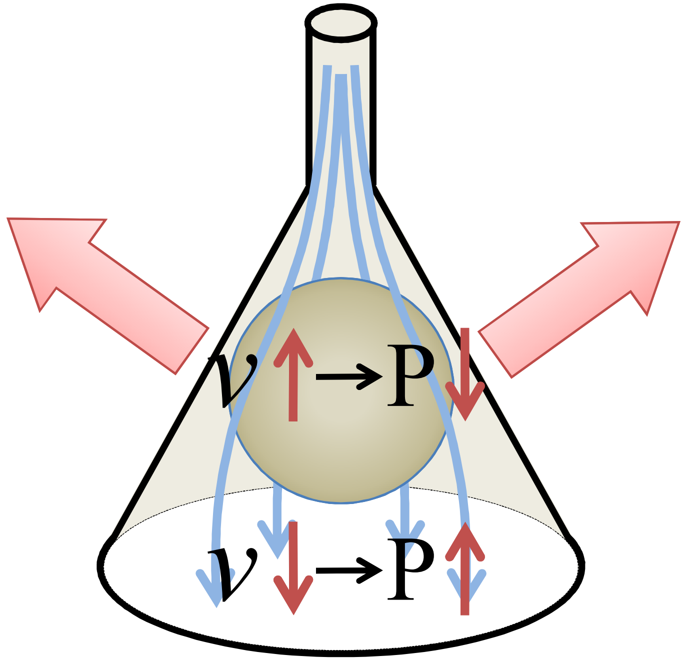
$$1 \text{ atm} = 1.01\times10^5 \text{Pa}$$
$$= 760 \text{torr}$$
$$\simeq 10 \text{m (Water)}$$

Therefore, This siphon would only work if d is less than 10m.

Bernoulli's Equation (7/9)

- **Example: Funnel with a Ping-Pong Ball**



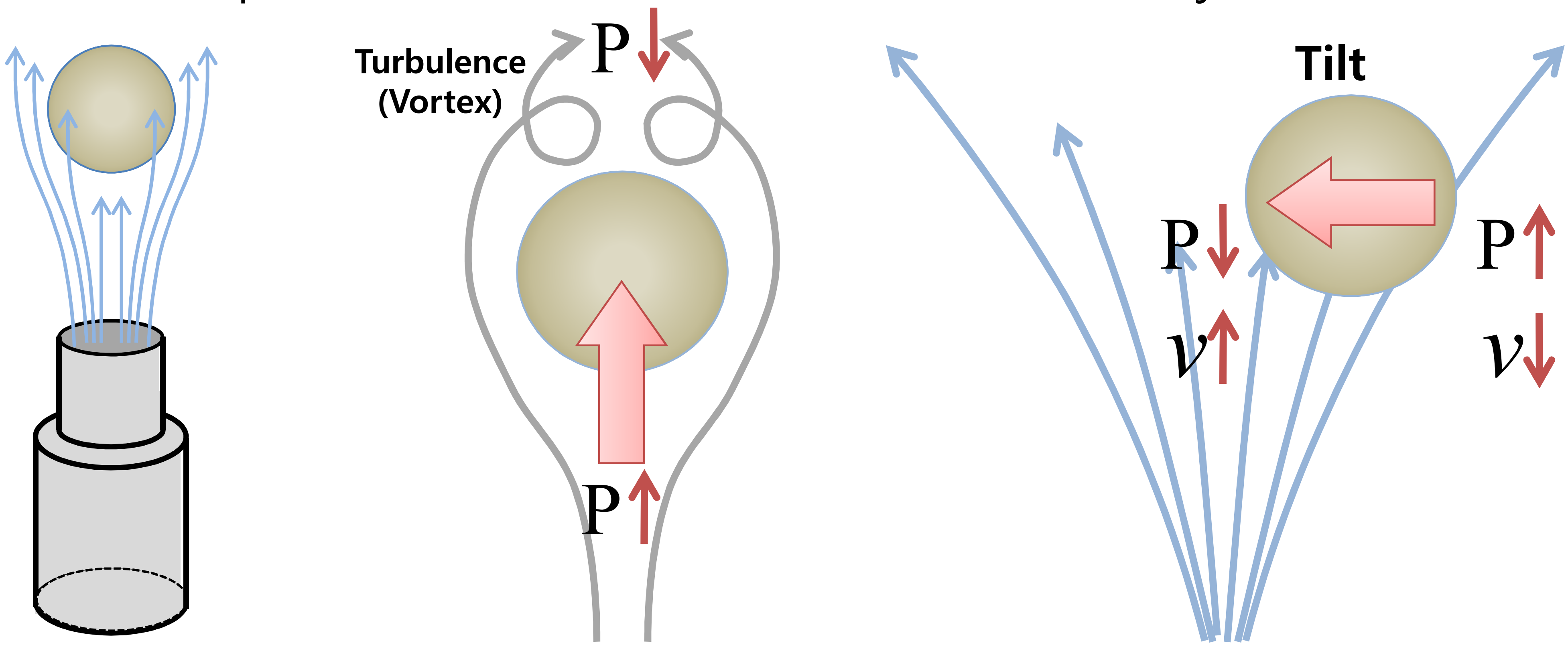


Movie Clip

Bernoulli's Equation (8/9)

- ## Example: Ping-Pong Ball in the jet of air*

If you place a ping-pong ball in the jet of air from a vacuum cleaner hose aimed vertically upward, the ping-pong ball will be held in stable equilibrium with this jet. Explain this by means of Bernoulli's equation. (Hint: The speed of air is maximum at the center of the jet.)



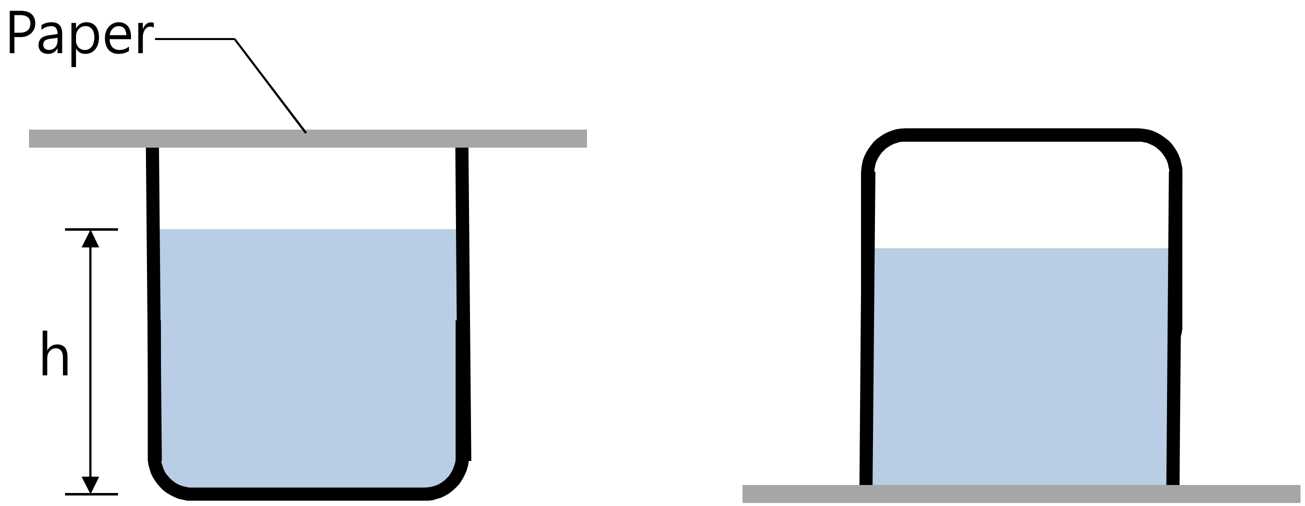
* Ohanian, H. C., Physics, 2nd Ed., W W. Norton & Company, p.354-355, pp.486, 1989

Bernoulli's Equation (9/9)

- **Example: A Glass Filled with Water***

Partially fill a tall drinking glass with water to depth h. Cut a square of sturdy paper somewhat wider than the mouth of the glass. Place the paper over the mouth. Spread the fingers of your left hand over the paper, pressing it against the mouth of the glass.

Grab the glass with your right hand and then as rapidly as you can, invert it with your left hand and then as rapidly as you can, invert it with your left hand still pressing the paper against the rim. Chances are you can then remove your left hand without the water pouring out. If h=11.0cm, what is the gauge pressure of the air now trapped in the above the water?



* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.385, 2004

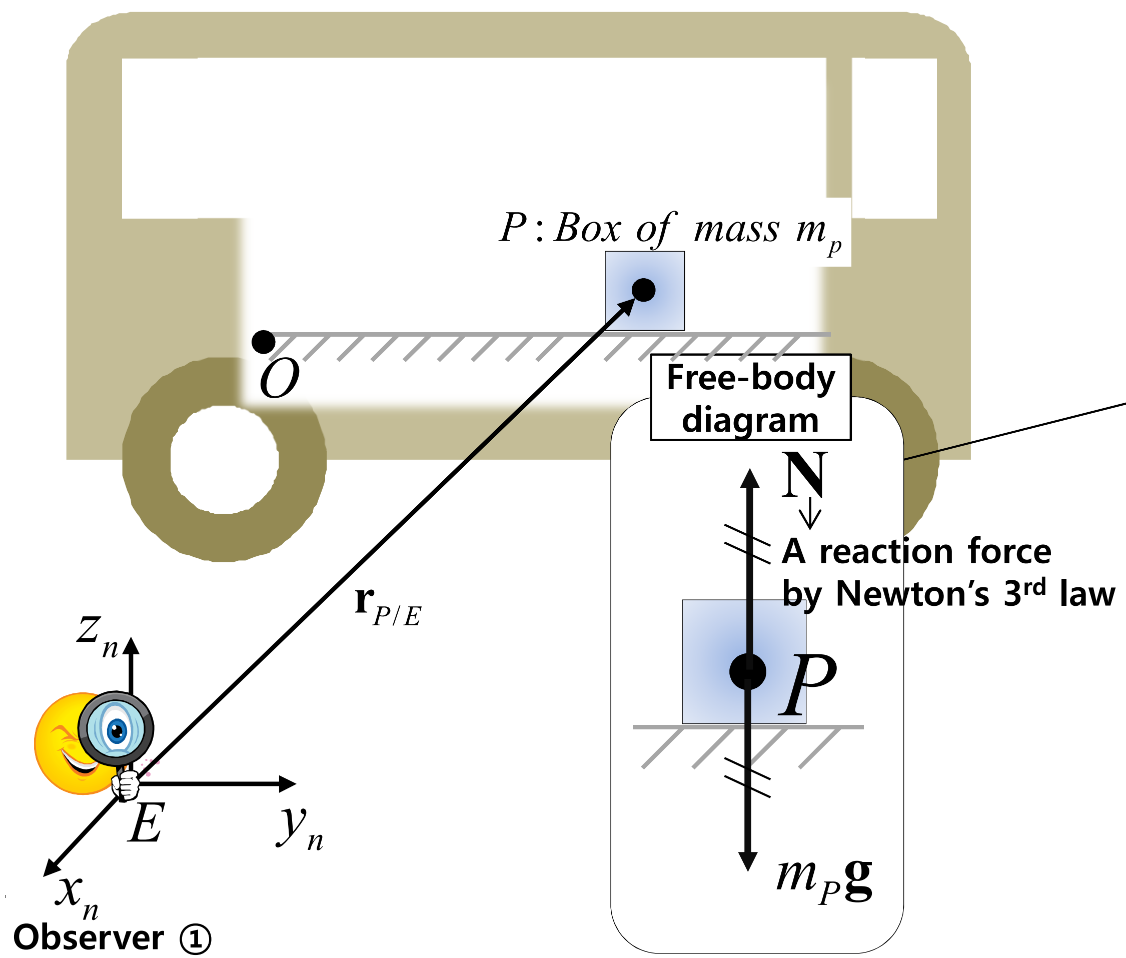
Reference Slides

Example of 'Perceived Gravity'
- Relative Motion -

Relative Motion
- Examples of a Bus (1/9)

Case #1

- A box is fixed on a bus which is **at rest**.
- Find the forces exerted on the box.



1. At first, we consider the forces exerted on the box in **vertical direction**. Newton's 2nd law is applied to the box in the bus.

$$m_P \ddot{\mathbf{r}}_{P/E} = \mathbf{F}$$

$$= m_P \mathbf{g} + \mathbf{N}$$

Since the box is **at rest, it is in static equilibrium**.

$$\ddot{\mathbf{r}}_{P/E} = 0$$

$$0 = m_P \mathbf{g} + \mathbf{N}$$

$$\Downarrow$$

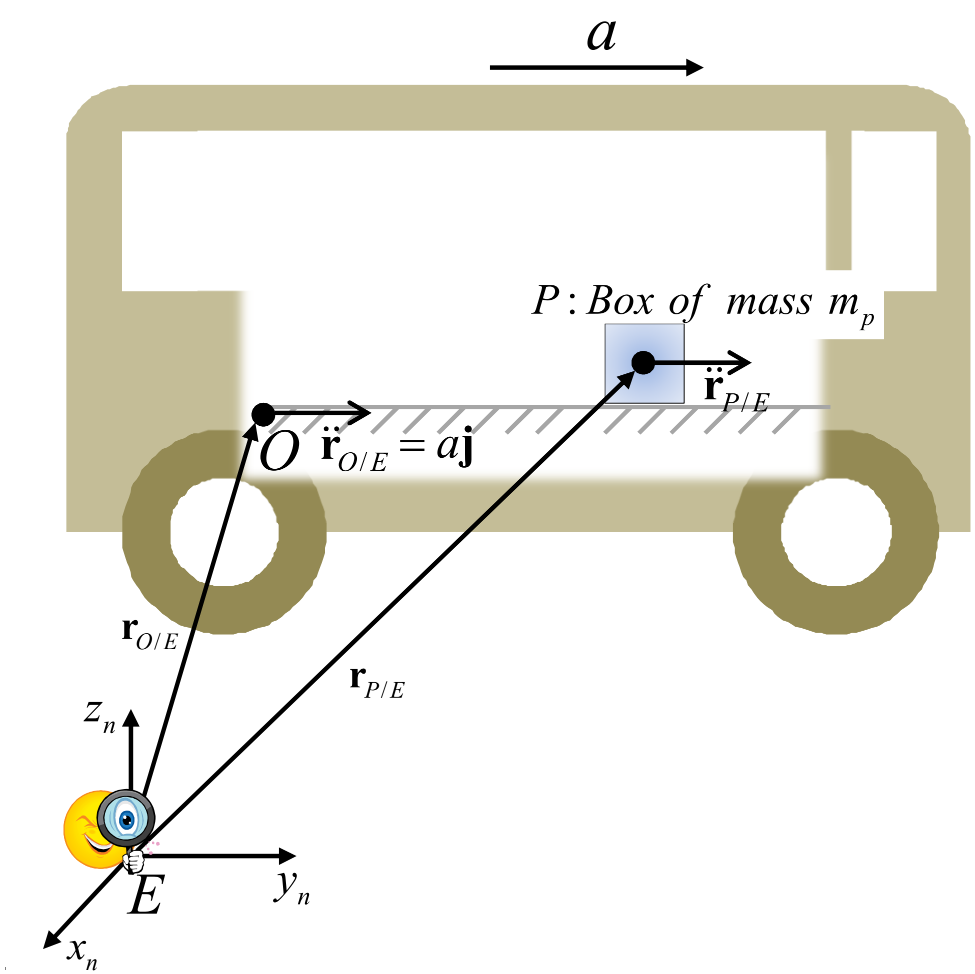
$$\boxed{\mathbf{N} = -m_P \mathbf{g}}$$

2. There is no force in horizontal direction.

Relative Motion
- Examples of a Bus (2/9)

Case #2

- A box is fixed on a bus which is moving with an acceleration of a in horizontal direction.
- Find the force exerted on the box in horizontal direction.



Observer ①

We apply Newton's 2[nd] law to the box in the bus.

$$m_P \, \ddot{\mathbf{r}}_{P/E} = \mathbf{F}_P$$

$$\ddot{\mathbf{r}}_{P/E} = a\mathbf{j}$$

$$m_P \, a\mathbf{j} = \mathbf{F}_P$$

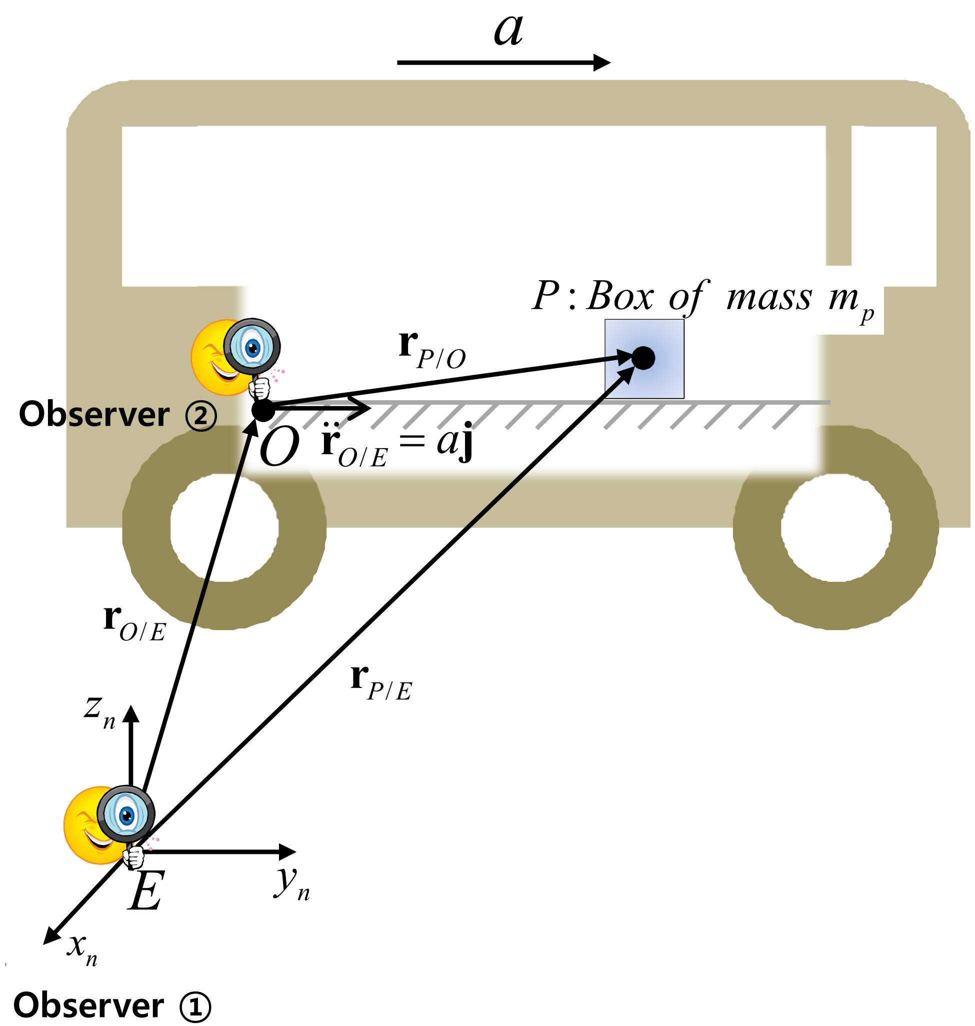
➡ The force exerted on the box is $m_P \, a$ in horizontal direction.

Relative Motion
- Examples of a Bus (3/9)

$$m_P\ddot{\mathbf{r}}_{P/O} = \underbrace{\mathbf{F}_P}_{\text{External Force}} \underbrace{- m_P\ddot{\mathbf{r}}_{O/E}}_{\text{Inertial force}}$$

Case #2

- A box is fixed on a bus which is moving with an acceleration of a in horizontal direction.
- Find the force exerted on the box in horizontal direction.



An observer ② in the bus describes the force exerted on the box.

The observer ② is located at the origin of the non-inertial reference frame which moves with an acceleration of a.

So, the inertial force should be considered.

$$m_P\ddot{\mathbf{r}}_{P/O} = \mathbf{F}_P \underbrace{- m_P\ddot{\mathbf{r}}_{O/E}}_{\text{inertial force}}$$

$$\mathbf{F}_P = m_P a\mathbf{j}$$
$$\ddot{\mathbf{r}}_{O/E} = a\mathbf{j}$$

$$= m_P a\mathbf{j} - m_P a\mathbf{j}$$

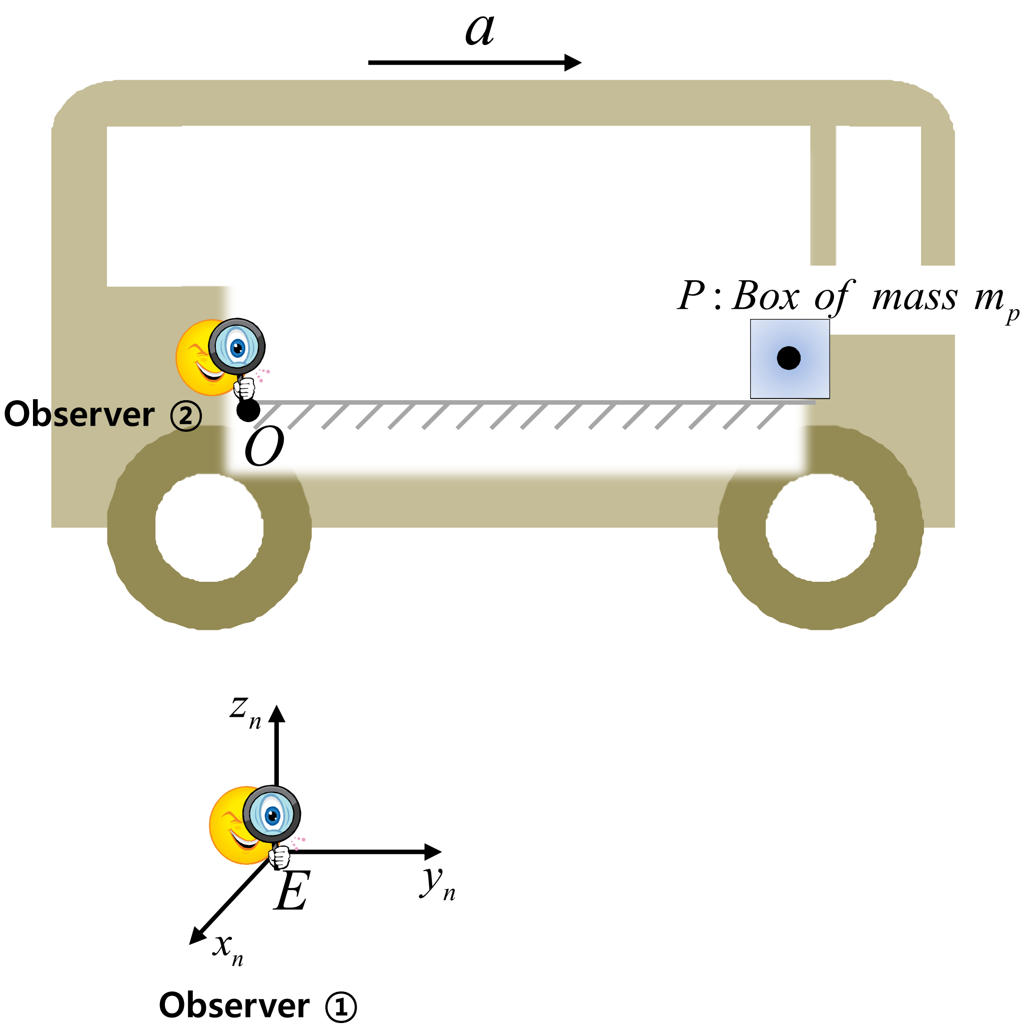
$$= 0\mathbf{j}$$

The observer ② recognizes that no force is exerted on the box.

Relative Motion
- Examples of a Bus (4/9)

Case #3

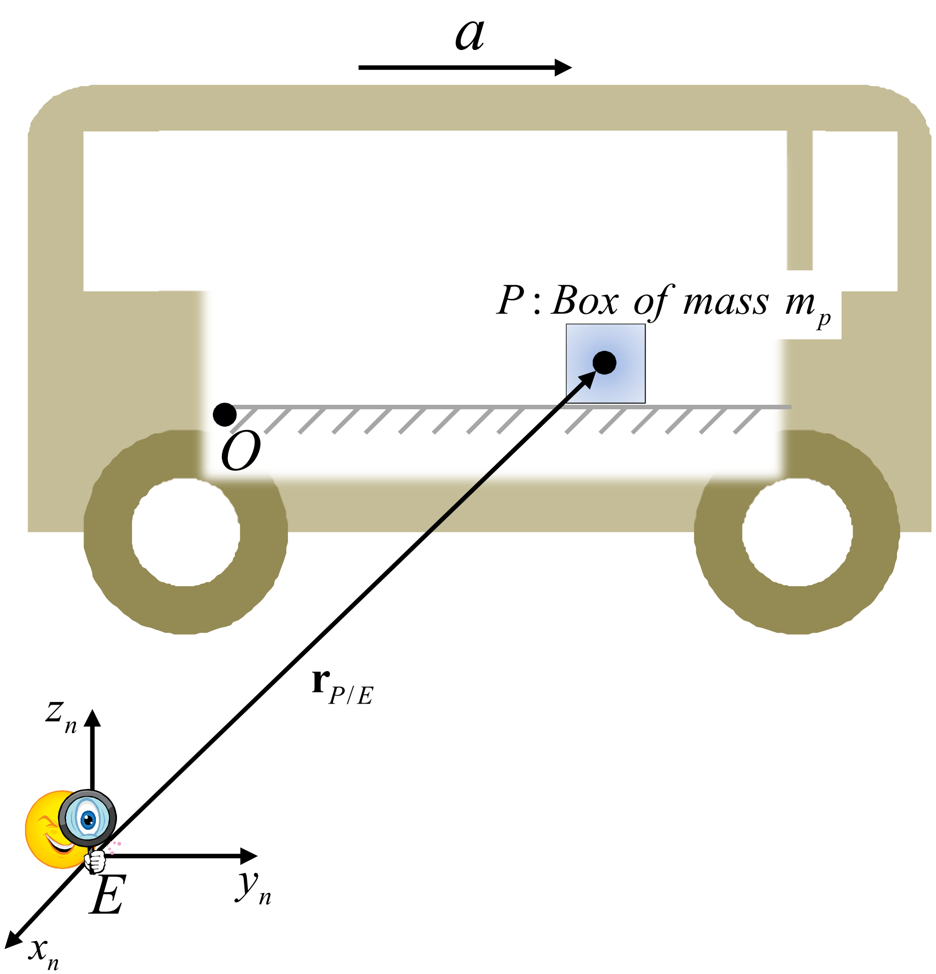
- The box is **not fixed** and there is **no friction** btw the box and the bus.
- The bus is **moving with an acceleration of a in horizontal direction.**
- Find the force exerted on the box in horizontal direction.



Relative Motion
- Examples of a Bus (5/9)

Case #3

- The box is **not fixed** and there is **no friction** btw the box and the bus.
- The bus is **moving with acceleration of a in horizontal direction.**
- Find the force exerted on the box in horizontal direction.



We apply Newton's 2nd law to the box in the bus.

$$m_P\,\ddot{\mathbf{r}}_{P/E} = \mathbf{F}_P$$

$$0 = \mathbf{F}_P$$

$$\ddot{\mathbf{r}}_{P/E} = 0$$

➡ The force exerted on the box is zero in horizontal direction.

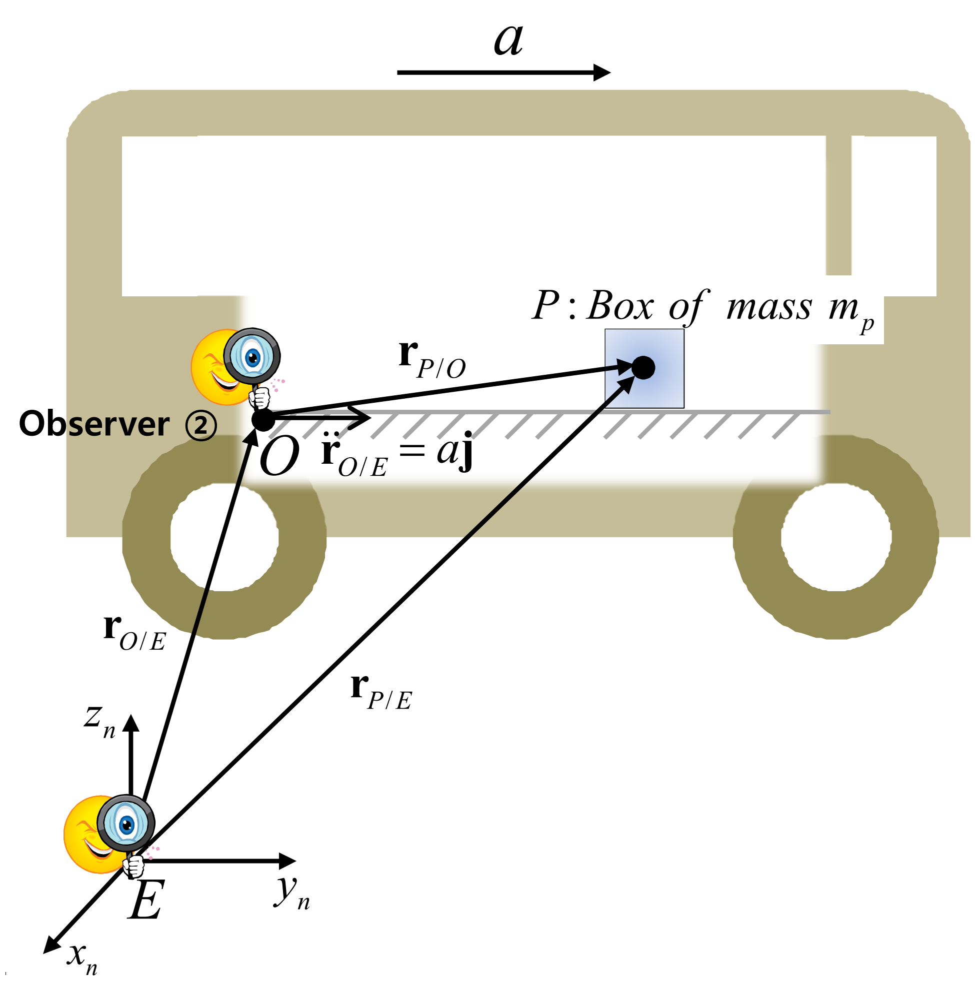
Observer ①

Relative Motion
- Examples of a Bus (6/9)

$$m_P \ddot{\mathbf{r}}_{P/O} = \underbrace{\mathbf{F}_P}_{\text{External Force}} \underbrace{- m_P \ddot{\mathbf{r}}_{O/E}}_{\text{Inertial force}}$$

Case #3

- The box is not fixed and there is no friction btw the box and the bus.
- The bus is moving with acceleration of a in horizontal direction.
- Find the force exerted on the box in horizontal direction.



An observer ② in the bus describes the force exerted on the box.

The observer ② is located at the origin of the non-inertial reference frame which moves with an acceleration of a.

So, the inertial force should be considered.

$$m_P \ddot{\mathbf{r}}_{P/O} = \mathbf{F}_P \underbrace{- m_P \ddot{\mathbf{r}}_{O/E}}_{\text{inertial force}} \qquad \mathbf{F}_P = 0\mathbf{j}$$

$$= 0\mathbf{j} - m_P\, a\mathbf{j} \qquad \ddot{\mathbf{r}}_{O/E} = a\mathbf{j}$$

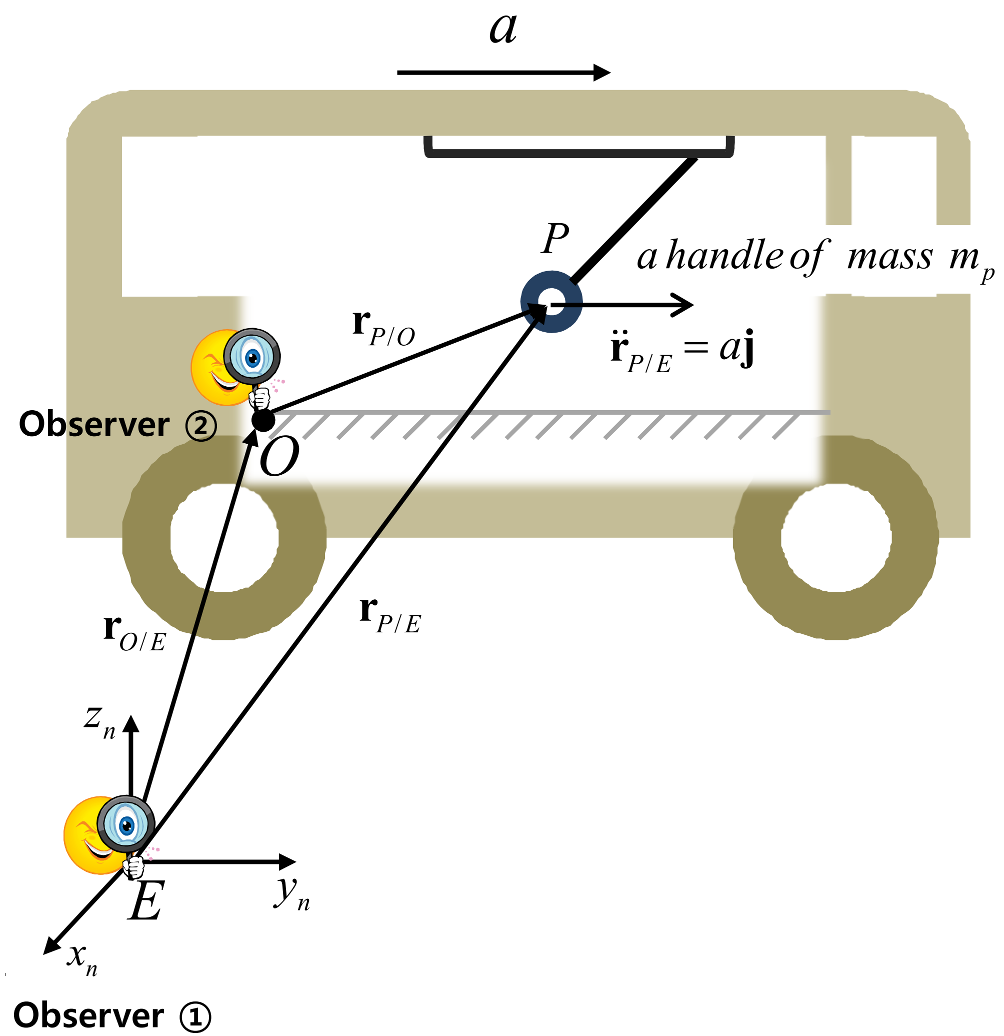
$$= -m_P\, a\mathbf{j}$$

The observer ② recognizes that the negative force $-m_p a$ is exerted on the box.

Relative Motion
- Examples of a Bus (7/9)

Case #4

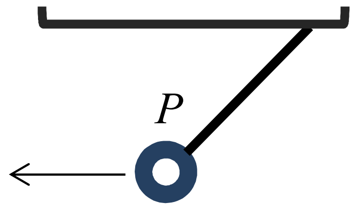
- The bus is **moving with an acceleration of a in horizontal direction.**
- The handle is connected to the top of the bus by the strap.
- Find the tension of the strap.



Since the handle is moving with the same speed of the bus, the acceleration of the handle with respect to observer ① is given by

$$\ddot{\mathbf{r}}_{P/E} = a\mathbf{j}$$

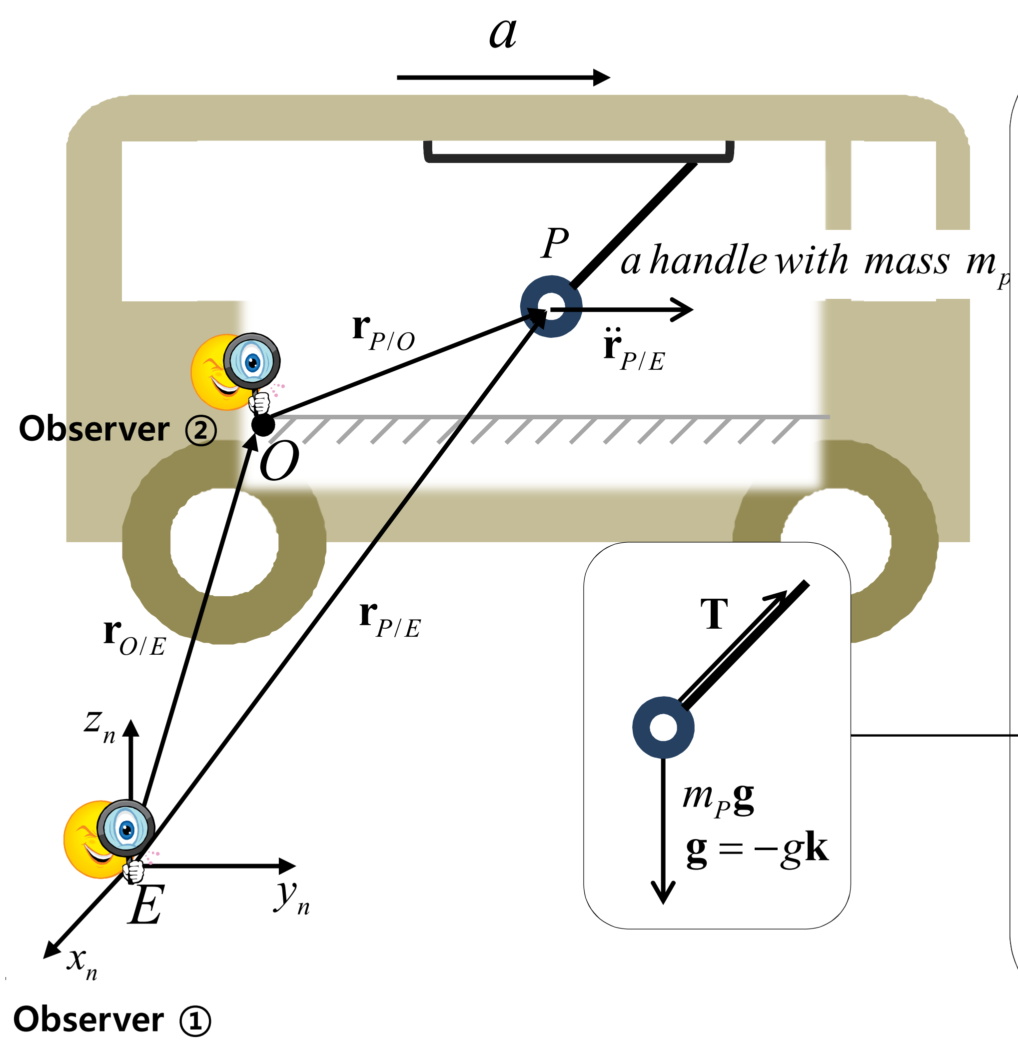
And, the handle is dragged to backward direction.



Relative Motion
- Examples of a Bus (8/9)

Case #4

- The bus is moving with an acceleration of a in horizontal direction.
- The handle is connected to the top of the bus by the strap.
- Find the tension of the strap.



We apply Newton's 2nd law to the box in the bus.

$$m_P \ddot{\mathbf{r}}_{P/E} = \mathbf{F}_P$$

$$= \mathbf{T} + m_P\mathbf{g}$$

$$\mathbf{F}_P = \mathbf{T} + m_P\mathbf{g}$$

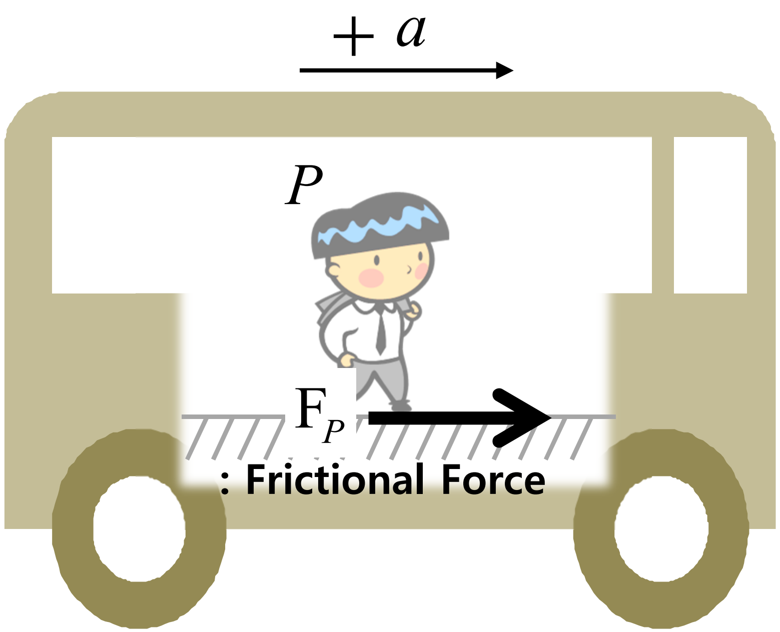
$$\mathbf{T} = m_P \ddot{\mathbf{r}}_{P/E} - m_P\mathbf{g}$$

$$\ddot{\mathbf{r}}_{P/E} = a\mathbf{j}, \quad \mathbf{g} = -g\mathbf{k}$$

$$= m_P a\mathbf{j} + m_P g\mathbf{k}$$

Relative Motion
- Examples of a Bus (9/9)
Case 5: Person in a Bus: <u>Inertial Force</u> (1/2)

A bus is moving with acceleration of a in horizontal direction. and the person "P" is standing on the bus and moves with the same acceleration *a* with the bus.



Consider the horizontal motion.

1. In inertial frame

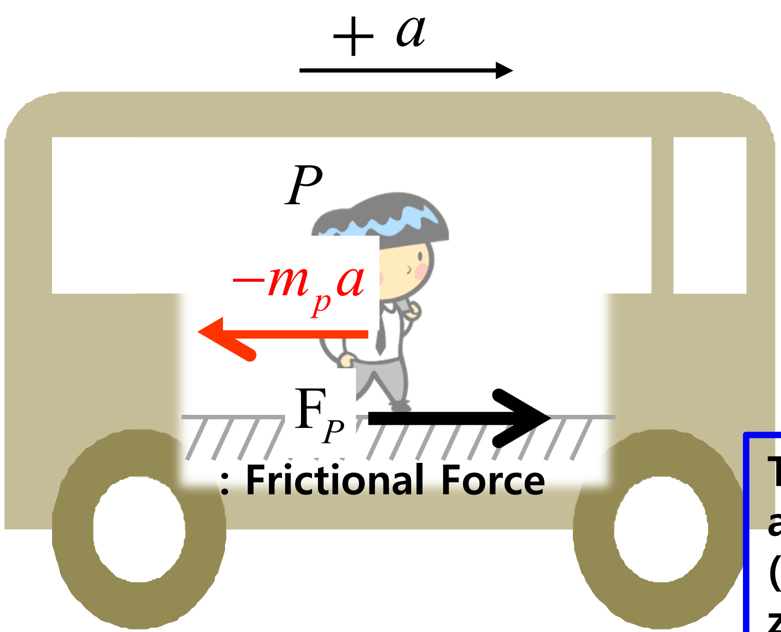
$$\underline{m_P a} = \underline{F_P} \quad \cdots \text{(1)}$$

The person "P" is accelerated in forward direction with an acceleration "a".

The external force exerted on the person "P" is the frictional force between base and feet of the person "P".

Relative Motion
- Examples of a Bus (9/9)
Case 5: Person in a Bus: Inertial Force (2/2)

A bus is moving with acceleration of a in horizontal direction.
and the person "P" is standing on the bus and moves with the same acceleration a with the bus.



Consider the horizontal motion.

1. In inertial frame

$$m_P a = F_P \quad \cdots \text{(1)}$$

2. In non-inertial frame (According to D'Alembert Principle)

$$0 = F_P - m_P a \quad \cdots \text{(2)}$$

The person "P" is not accelerated (the acceleration is zero).

Dynamic Equilibrium

➡ The force "$-m_p a$" is an inertial force.

The person "P" perceives the friction force "F_p" and an additional force "$m_p a$" in backward direction.
i.e. The person perceives the moment caused by the friction force "F_p" and an additional force "$-m_p a$".

3. What is the magnitude of the inertial force "$-m_p a$"?

➡ From the equation (1), $-m_p a = -F_P$

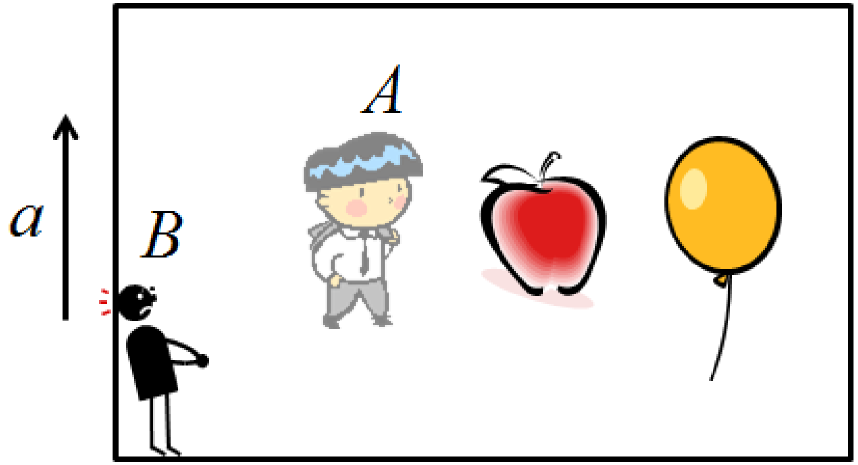
Example of an Astronaut, an Apple, and a Helium-filled Balloon in a Spacecraft

An astronaut "A", an apple and a helium-filled balloon are in a spacecraft.

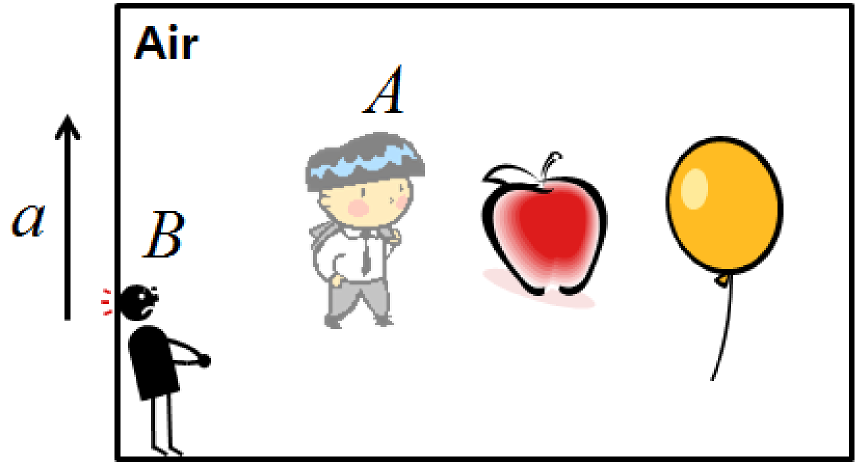
There is no gravity, so they are all floating in space.

The spacecraft is going to accelerate in the upper direction with an acceleration "*a*". And an astronaut "B" who is moving with the same acceleration "*a*" observes the motion of the astronaut "A", apple, and balloon in the spacecraft.

$\left(\rho_{person} = 1030 \text{ kg/m}^3,\ \rho_{apple} = 760 \text{ kg/m}^3,\ \rho_{balloon} = 0.18 \text{ kg/m}^3,\ \rho_{air} = 1.23 \text{ kg/m}^3\right)$



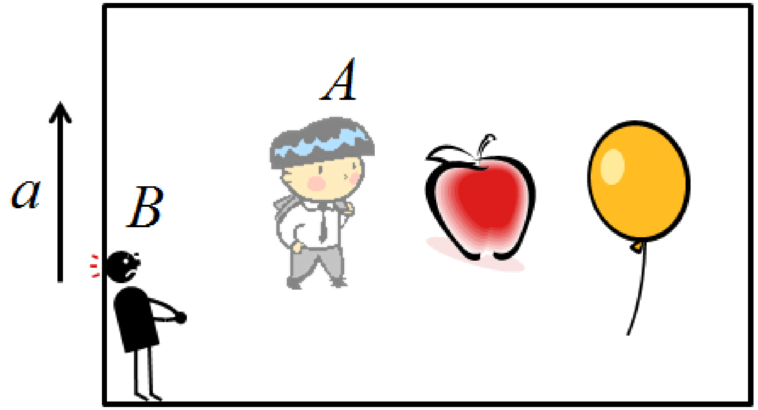
Case 1: There is no air inside of the spacecraft



Case 2: Air is filled in the spacecraft

Example of an Astronaut, an Apple, and
a Helium-filled Balloon in a Spacecraft

Case 1: there is no air inside of the spacecraft (1/2)



(1) What will be the motion of the astronaut "A", apple and balloon observed by the astronaut "B"? Do they go downward or upward?

In the case of the astronaut "A":

1. In inertial frame

$$m_A a_A = F_A \cdots \text{(1)}$$

$$\rightarrow 0 = 0$$

$$\left(\text{R.H.S: } F_A = \underbrace{F_{A,Body}}_{=0} + \underbrace{F_{A,Surface}}_{=0} \right)$$

(Because there is no gravity and no air inside of the spacecraft.)

2. In non-inertial frame (observed by the astronaut "B")

$$m_A a_A - m_A a = F_A - m_A a \cdots \text{(2)}$$

$$\rightarrow m_A \left(\underline{-a} \right) = \underline{-m_A a}$$

The astronaut "A" goes downward (the acceleration is -a)

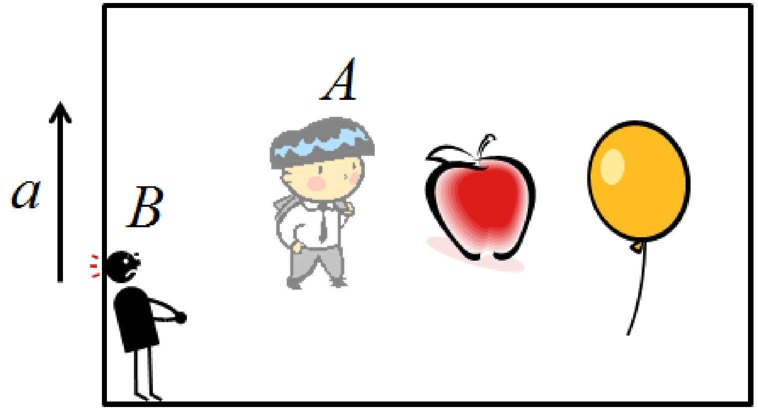
→ The force "$-m_A a$" is an inertial force.

When the astronaut "B" observes the astronaut "A", an additional force "$-m_A a$" is acting on the astronaut "A" in downward direction

⇨ **The astronaut "A", the apple, and the balloon will go downward.**

Example of an Astronaut, an Apple, and a Helium-filled Balloon in a Spacecraft

Case 1: there is no air inside of the spacecraft (2/2)



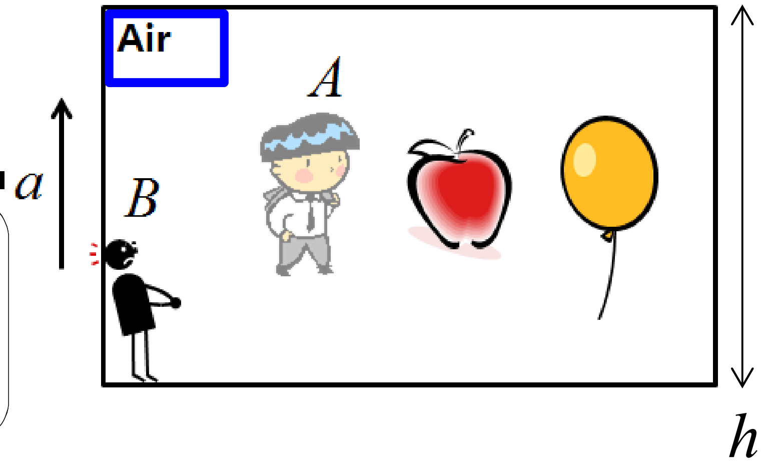
(1) What will be the motion of the astronaut "A", apple and balloon observed by the astronaut "B"? Do they go downward or upward?

⇨ The astronaut "A", the apple, and the balloon will go downward.

(2) What will be the "relative motion" of the astronaut "A", apple and balloon?

⇨ Their motion will be same.

Case 2: Air is filled in the spacecraft (1/3)



(1) The height of the spacecraft is "h". Then what will be the difference in the air pressure between the pressure at the bottom and at the ceiling?

$$\Delta P = \rho_{air} a h$$

2. Assume that air is filled in the spacecraft.

(2) What will be the motion of the astronaut "A", apple and balloon observed by the astronaut "B"? Do they go downward or upward?

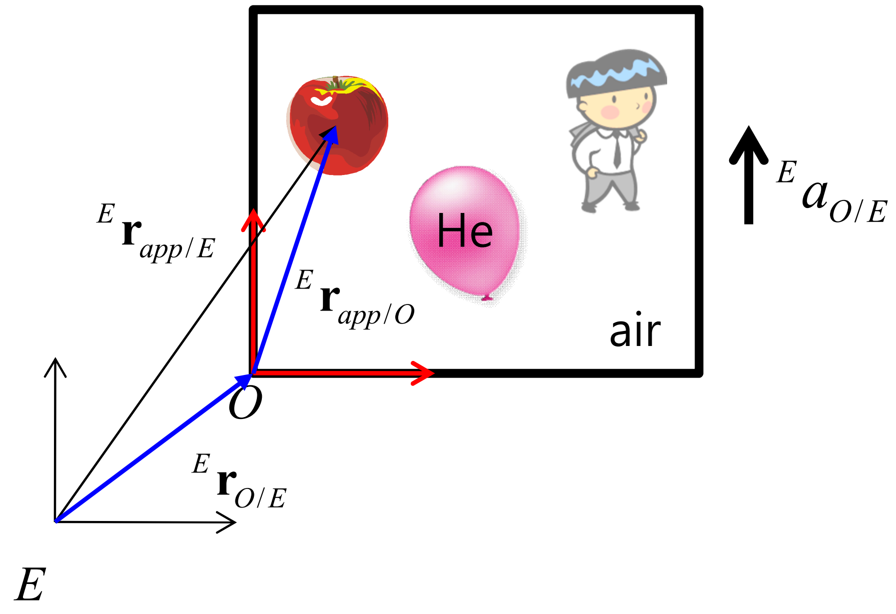
(3) What will be the "relative motion" of the astronaut "A", apple and balloon?

Example of an Astronaut, an Apple, and a Helium-filled Balloon in a Spacecraft

Case 2: Air is filled in the spacecraft (2/3)

According to Newton's Second Law, an apple is

$$m_{app}\ {}^{E}a_{app/E} = \sum {}^{E}\mathbf{F}_{app} \cdots (1)$$



L.H.S

$$m_{app}\ {}^{E}a_{app/E} = m_{app}\ {}^{E}\ddot{r}_{app/E} \cdots (2)$$

$$\left(\begin{aligned} & {}^{E}r_{app/E} = {}^{E}r_{O/E} + {}^{E}r_{app/O} \\ \rightarrow\ & {}^{E}\ddot{r}_{app/E} = {}^{E}\ddot{r}_{O/E} + {}^{E}\ddot{r}_{app/O} \cdots (3) \end{aligned} \right)$$

Substitute (3) into (2),

$$m_{app}\ {}^{E}a_{app/E} = m_{app}\ {}^{E}\ddot{r}_{app/E} = m_{app}\left({}^{E}\ddot{r}_{O/E} + {}^{E}\ddot{r}_{app/O} \right) = m_{app}\ {}^{E}\ddot{r}_{O/E} + m_{app}\ {}^{E}\ddot{r}_{app/O} \cdots (4)$$

R.H.S

$$\sum {}^{E}\mathbf{F}_{app} = \underbrace{{}^{E}\mathbf{F}_{app,body}}_{=0} + \underbrace{{}^{E}\mathbf{F}_{app,surface}}_{= V_{app}\rho_{air}\ {}^{E}a_{O/E}}$$

because there's no gravity

Case 2: Air is filled in the spacecraft (3/3)

So, (1) become

$$m_{app}\,{}^{E}\ddot{r}_{O/E} + m_{app}\,{}^{E}\ddot{r}_{app/O} = V_{app}\rho_{air}\,{}^{E}a_{O/E}$$

$$\rightarrow m_{app}\,{}^{E}\ddot{r}_{app/O} = -m_{app}\,{}^{E}\ddot{r}_{O/E} + V_{app}\rho_{air}\,{}^{E}a_{O/E}$$

$$= -V_{app}\rho_{app}\,{}^{E}a_{O/E} + V_{app}\rho_{air}\,{}^{E}a_{O/E}$$

$$= V_{app}\,{}^{E}a_{O/E}\left(-\rho_{app} + \rho_{air}\right)$$

$$\therefore m_{app}\,{}^{E}\ddot{r}_{app/O} = V_{app}\,{}^{E}a_{O/E}\left(-\rho_{app} + \rho_{air}\right)$$

$$\therefore m_{app}\,{}^{E}\ddot{r}_{app/O} = V_{app}\,{}^{E}a_{O/E}\left(-\rho_{app} + \rho_{air}\right)$$

If $\rho_{app} > \rho_{air}$, ${}^{E}\ddot{r}_{app/O} < 0$.
So the apple will fall.

In the similar way,

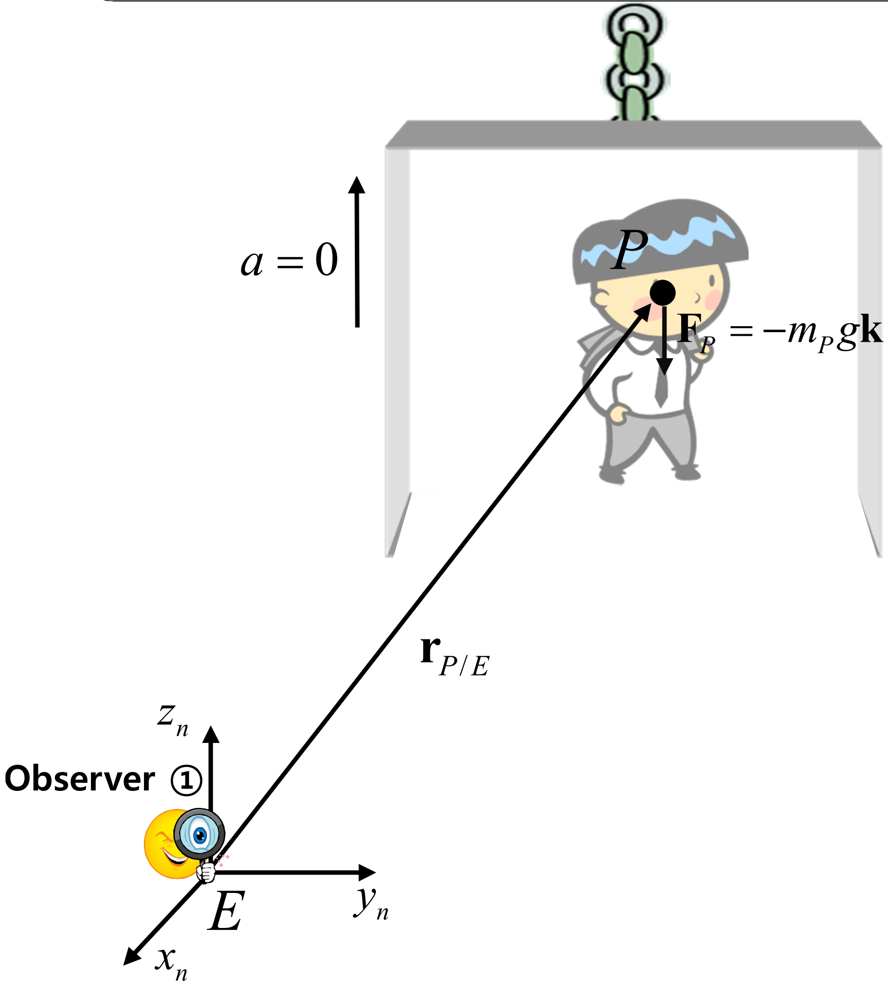
a helium-filled balloon

$$\therefore m_{bal}\,{}^{E}\ddot{r}_{bal/O} = V_{bal}\,{}^{E}a_{O/E}\left(-\rho_{bal} + \rho_{air}\right)$$

If $\rho_{bal} < \rho_{air}$, ${}^{E}\ddot{r}_{bal/O} > 0$.
So the balloon will rise.

Relative Motion
- Examples of an Elevator (1/5)

Case #1
- A person stands in an elevator which is at rest $(a=0)$, and the bottom of the elevator is not attached.
- What will happen?



→ **The person will fall down.**

- To understand this phenomena, we will apply Newton's 2nd law to the person in the elevator.

$$m_P \ddot{\mathbf{r}}_{P/E} = \mathbf{F}_P$$

$$m_P \ddot{\mathbf{r}}_{P/E} = -m_P g\,\mathbf{k}$$

$$\ddot{\mathbf{r}}_{P/E} = -g\,\mathbf{k}$$

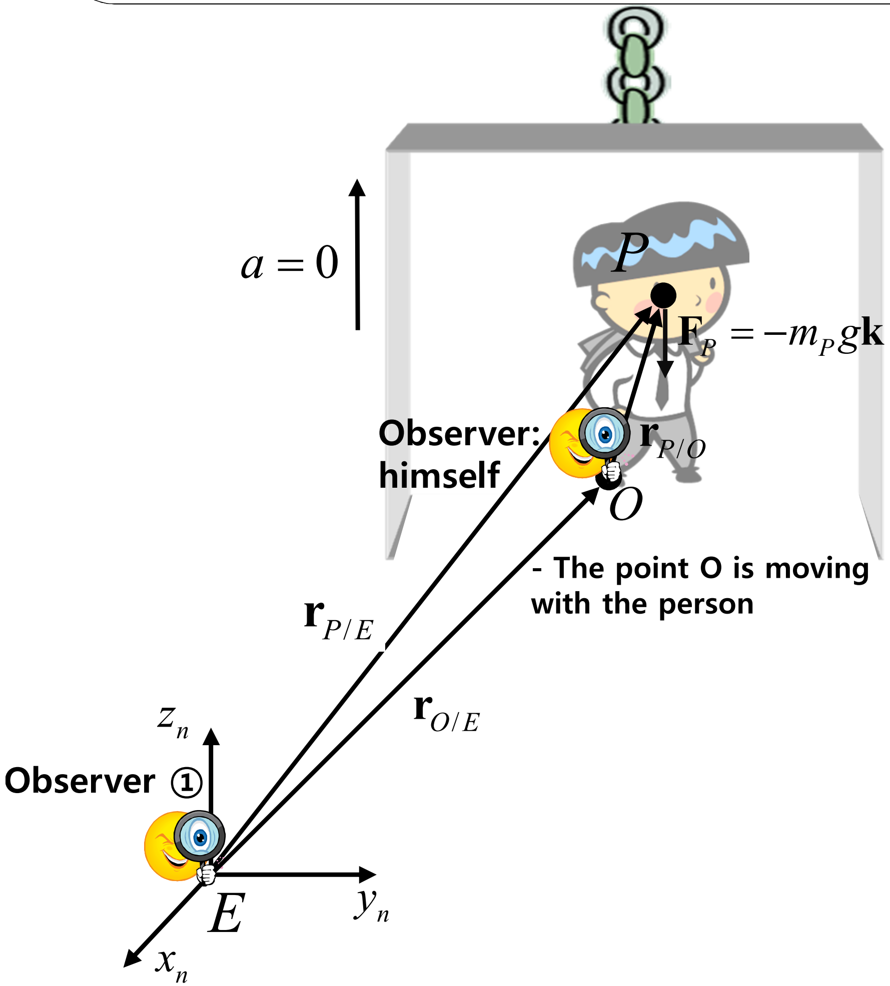
The person is moving with acceleration g in downward direction

The person feels that he is weightless.

Relative Motion
- Examples of an Elevator (2/5)

$$m_P\ddot{\mathbf{r}}_{P/O} = \underbrace{\mathbf{F}_P}_{\text{External Force}} \underbrace{- m_P\ddot{\mathbf{r}}_{O/E}}_{\text{Inertial force}}$$

Case #1
- A person stands in an elevator which is **at rest** $(a = 0)$, and the bottom of the elevator is **not attached**.
- What will happen?



→ The person will fall down.

When he observes himself, because he is moving with acceleration $-g$, the inertial force should be considered.

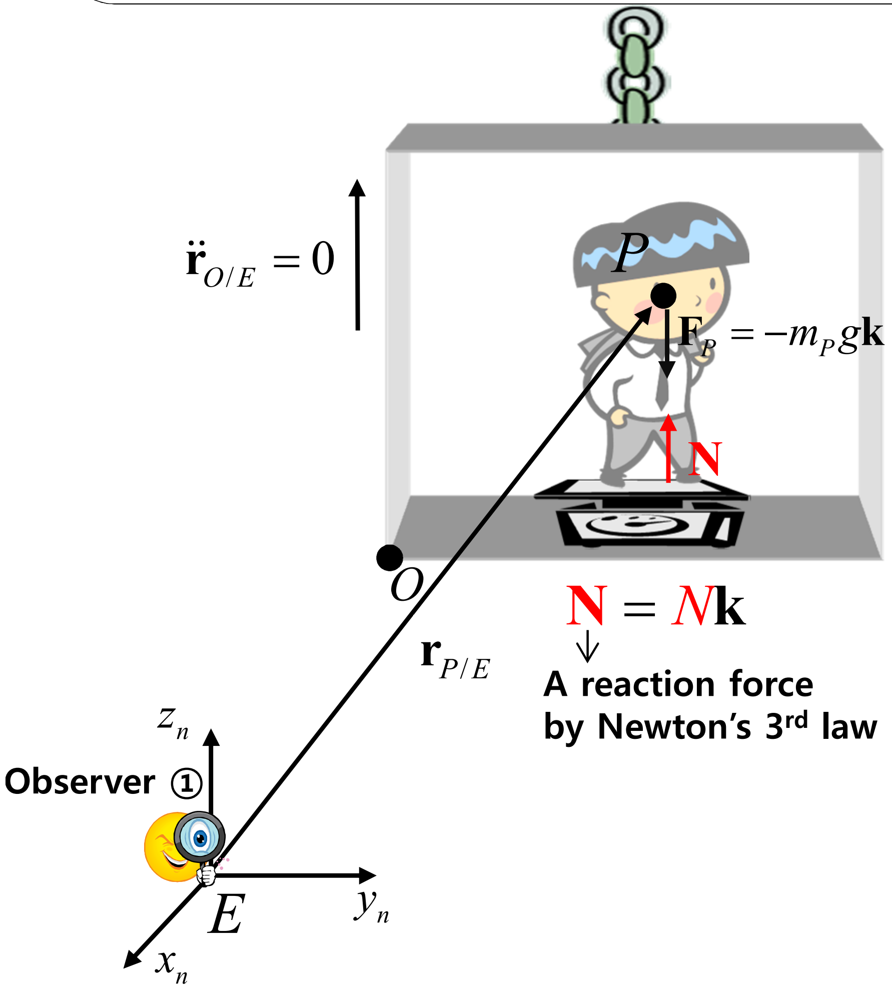
$$\begin{aligned} m_P\ddot{\mathbf{r}}_{P/O} &= \mathbf{F}_P \underbrace{- m_P\ddot{\mathbf{r}}_{O/E}}^{\text{inertial force}} \\ &= -m_P g\mathbf{k} + m_P g\mathbf{k} \\ &= 0 \end{aligned} \qquad \left.\begin{aligned} \mathbf{F}_P &= -m_P g\mathbf{k} \\ \ddot{\mathbf{r}}_{O/E} &= -g\mathbf{k} \end{aligned}\right.$$

Therefore the person feels that **he is weightless**.

Relative Motion
- Examples of an Elevator (3/5)

Case #2

- A person stands in an elevator which is **at rest** $(a=0)$, and the bottom of the elevator is **attached**.
- How much weight dose a bathroom scale indicate?



→ **The person is at rest.**

- We apply Newton's 2nd law to the person in the elevator.

$$m_P \ddot{\mathbf{r}}_{P/E} = \mathbf{F}_P$$
$$= -m_P g\,\mathbf{k} + N\mathbf{k}$$

Since the person is **at rest, static equilibrium**, $\ddot{\mathbf{r}}_{P/E} = 0$

$$0 = -m_P g\,\mathbf{k} + N\mathbf{k}$$

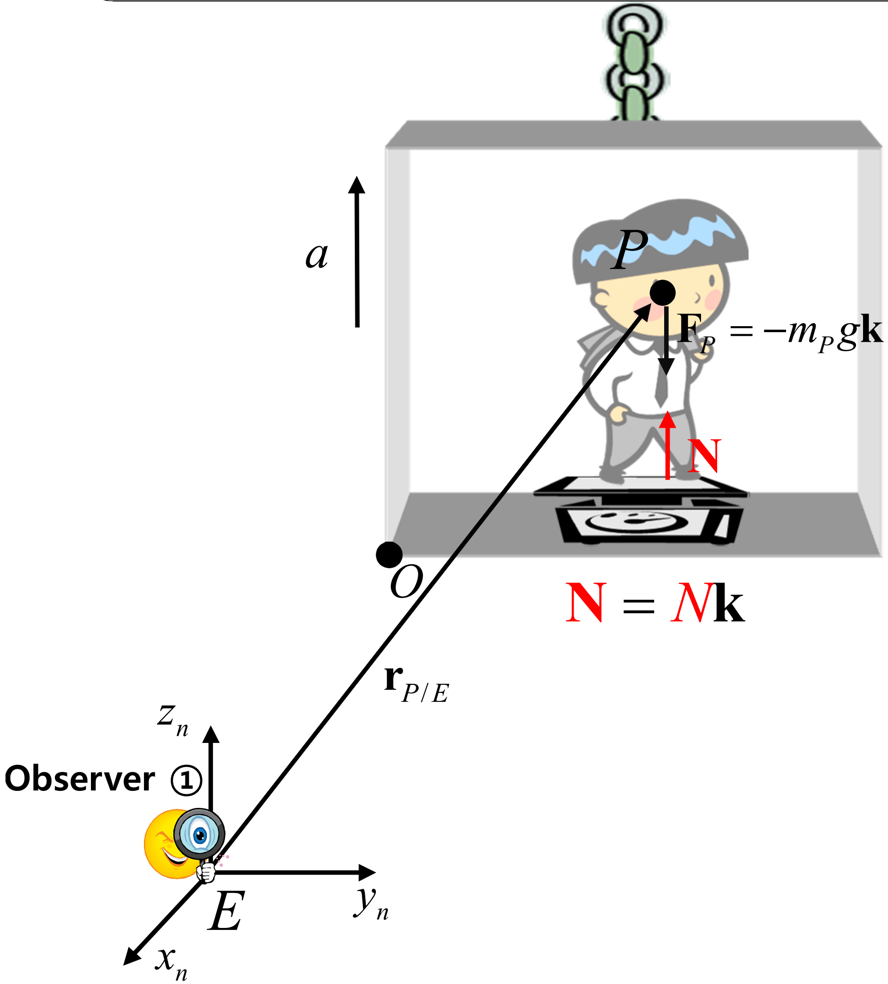
⇩

$$\boxed{N = m_P g}$$

The bathroom scale indicates $m_P g$

Relative Motion
- Examples of an Elevator (4/5)

Case #3
- A person stands in an elevator which **is moving upward** with an acceleration of a.
- How much weight dose a bathroom scale indicate?



→ The person is moving with the elevator.

- We apply Newton's 2nd law to the person in the elevator.

$$m_P \ddot{\mathbf{r}}_{P/E} = \mathbf{F}_P$$

$$= -m_P g\,\mathbf{k} + N\mathbf{k}$$

$$\ddot{\mathbf{r}}_{P/E} = a\mathbf{k}$$

$$m_P a\mathbf{k} = -m_P g\,\mathbf{k} + N\mathbf{k}$$

$$\Downarrow$$

$$N = m_P\left(g + a\right)$$

- The bathroom scale indicates $m_p(g+a)$
- The person feels additional force -$m_p a$

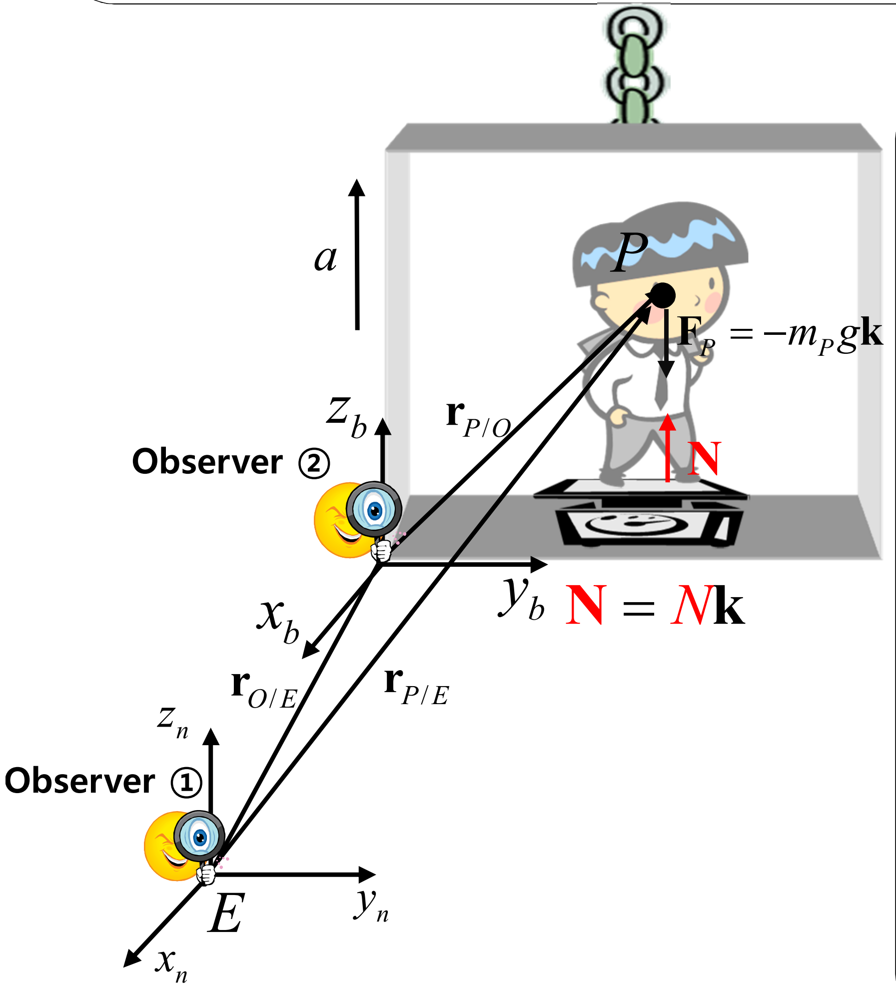
Relative Motion
- Examples of an Elevator (5/5)

$$m_P \ddot{\mathbf{r}}_{P/O} = \underbrace{\mathbf{F}_P}_{\text{External Force}} \underbrace{- m_P \ddot{\mathbf{r}}_{O/E}}_{\text{Inertial force}}$$

Case #3

- A person stands in an elevator which is moving upward with an acceleration of a.

- Find the exerted force on the person.



An observer② in the elevator describes the force exerted on the person.

The observer② is located at the origin of the non-inertial reference frame which moves with an acceleration of a.

So, the inertial force should be considered.

$$m_P \ddot{\mathbf{r}}_{P/O} = \mathbf{F}_P \overbrace{\boxed{- m_P \ddot{\mathbf{r}}_{O/E}}}^{\text{inertial force}}$$

$$= -m_P g\,\mathbf{k} + N\mathbf{k} \overbrace{\boxed{- m_P\, a\mathbf{k}}}^{\text{inertial force}}$$

$$= 0\mathbf{k}$$

$\mathbf{F}_P = -m_P g\,\mathbf{k} + N\mathbf{k}$

$\ddot{\mathbf{r}}_{O/E} = a\mathbf{j}$

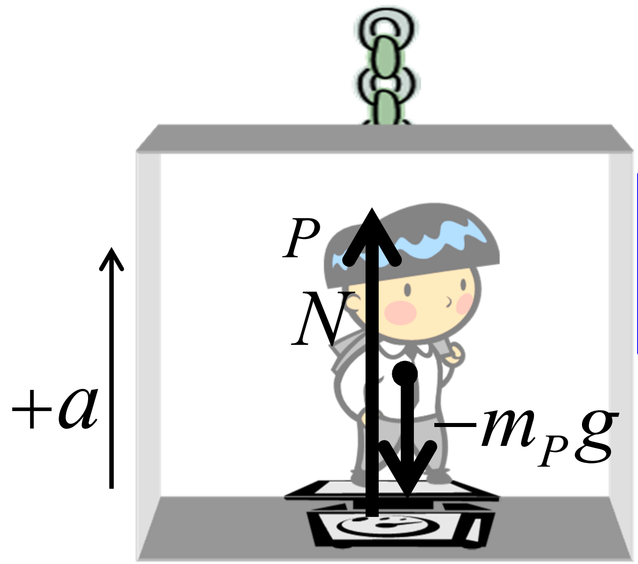
$N = m_P(g + a)$

- The observer② recognizes that the inertial force is exerted on the person.
- The person feels additional force $-m_p a$

Examples of a Person in an Elevator Cab (1/2)

Suppose that a person "P" is standing in an elevator.

The elevator has an upward acceleration a.



1. In inertial frame

$$\underline{m_P a} = \underline{N - m_P g} \cdots \quad \textbf{(1)} \qquad N = m_P(g+a)$$

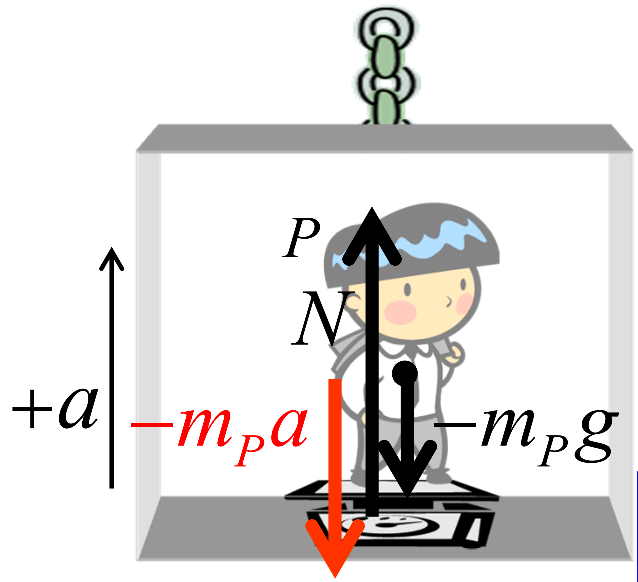
The person "P" is accelerated in upward direction with an acceleration "a"

The external force exerted on the person "P" is "(N-$m_p g$)"

Examples of a Person in an Elevator Cab (2/2)

Suppose that a person "P" is standing in an elevator.

The elevator has an upward acceleration a.



1. In inertial frame

$$m_P a = N - m_P g \cdots \text{ (1)} \qquad N = m_P(g+a)$$

2. In non-inertial frame (According to D'Alembert principle)

$$\underline{0} = \underline{(N - m_P g) - m_P a} \cdots \text{ (2)}$$

The person "P" is not accelerated (the acceleration is zero)

Dynamic Equilibrium

The force "$-m_p a$" is an inertial force

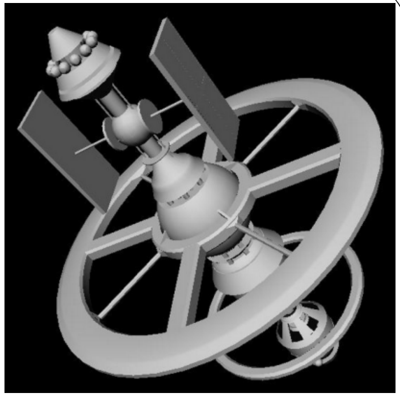
The person "P" perceives the external force (N-$m_p g$) and an additional force "$m_p a$" in downward direction.
→ The person perceives that his leg is compressed by the external force "(N-$m_p g$)" and the additional force "$-m_p a$"

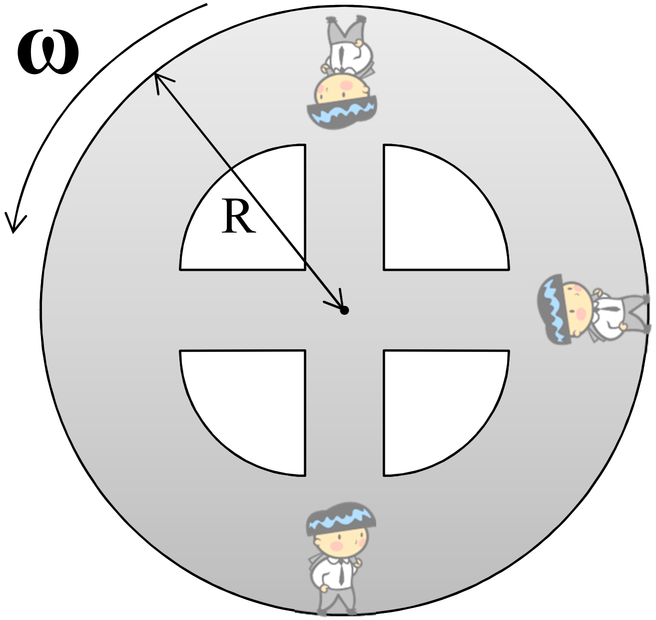
3. What is the magnitude of the inertial force "$-m_p a$"?

➡ From the equation (1), $-m_p a = -(N - m_p g)$

Inhabitant in a Space Station (1/3)

The generation of gravity by means of acceleration will play an important role in the design of the space stations of the future. This figure shows an example of proposed space station in the shape of a large spinning wheel which is designed to rotate in order to provide simulated gravity for their inhabitants.



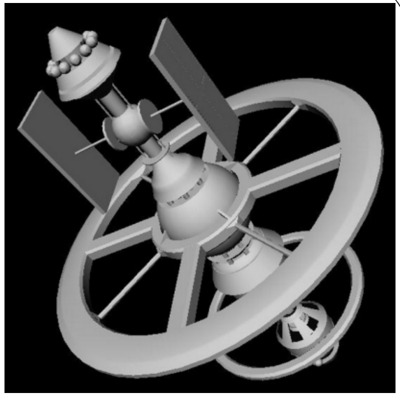


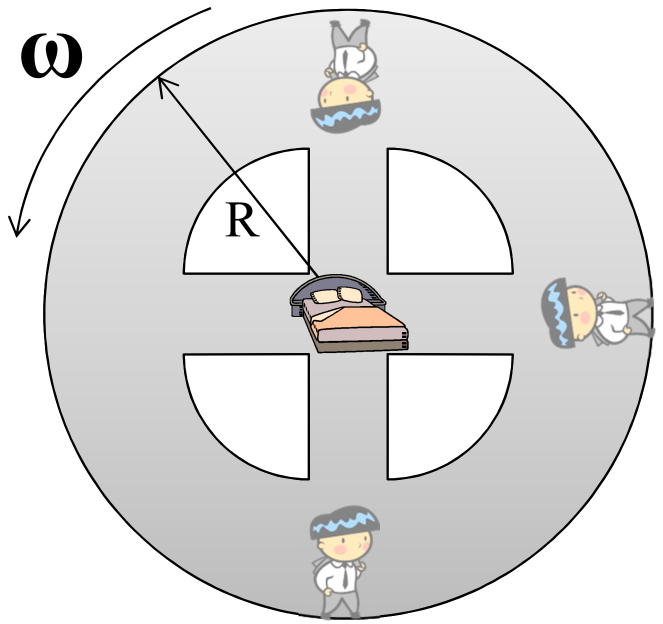
(a) If the distance from the axis of rotation of the station to the occupied outer wheel is R=100m, what rotation rate is necessary for the inhabitants to perceive the same amount of earth's gravity?

Centrifugal Acceleration: $$N = m \cdot a_c = m \cdot (\omega^2 R)$$
$$= m \cdot g$$
$$\Rightarrow \omega^2 R = g, \omega = \sqrt{g / R}$$
$$= \sqrt{9.81 \text{ m/s}^2 / 100\text{m}} = 0.313 \text{ rad/s}$$

Inhabitant in a Space Station (2/3)

The generation of gravity by means of acceleration will play an important role in the design of the space stations of the future. This figure shows an example of proposed space station in the shape of a large spinning wheel which is designed to rotate in order to provide simulated gravity for their inhabitants.



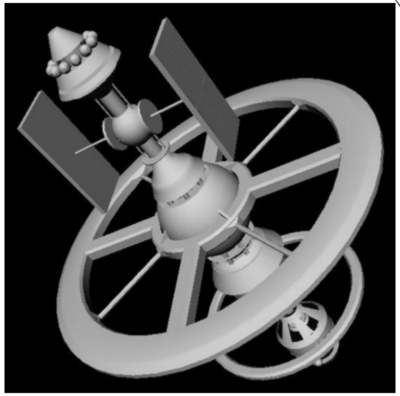


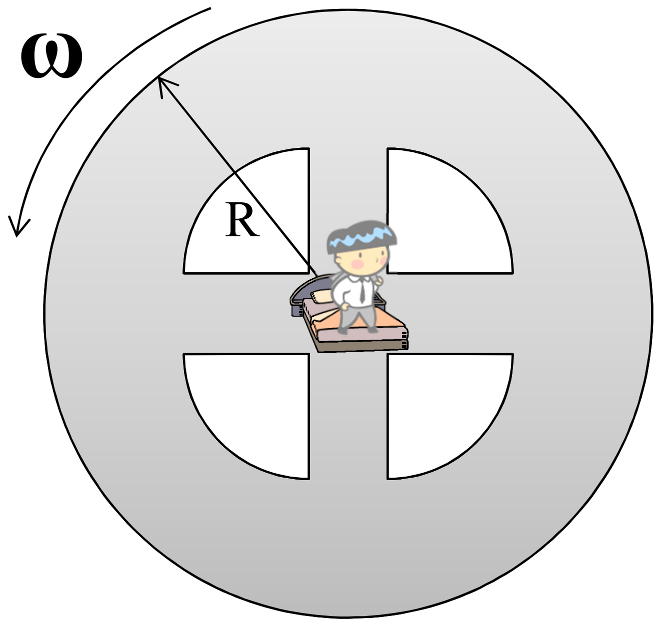
(b) Sleeping quarter is located in the center of this space station. If there is no staircase or elevator, can he walk towards the sleeping quarter?

He cannot walk up against gravity.
If there is a staircase, then he can go towards the sleeping center because of the reaction force from the stairs.

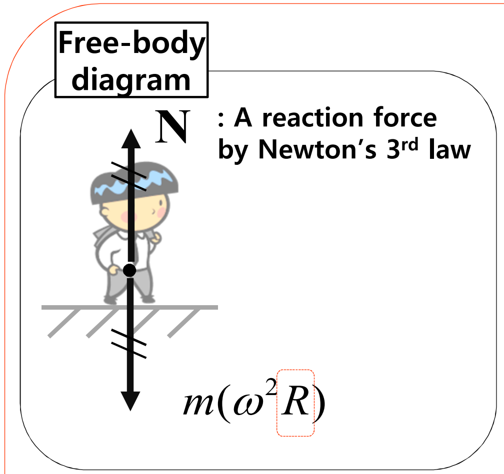
Inhabitant in a Space Station (3/3)

The generation of gravity by means of acceleration will play an important role in the design of the space stations of the future. This figure shows an example of proposed space station in the shape of a large spinning wheel which is designed to rotate in order to provide simulated gravity for their inhabitants.





(c) The person wakes up in the morning and decided to go back to the rim of the wheel. What will happen if the person is going into the corridor and starts moving?

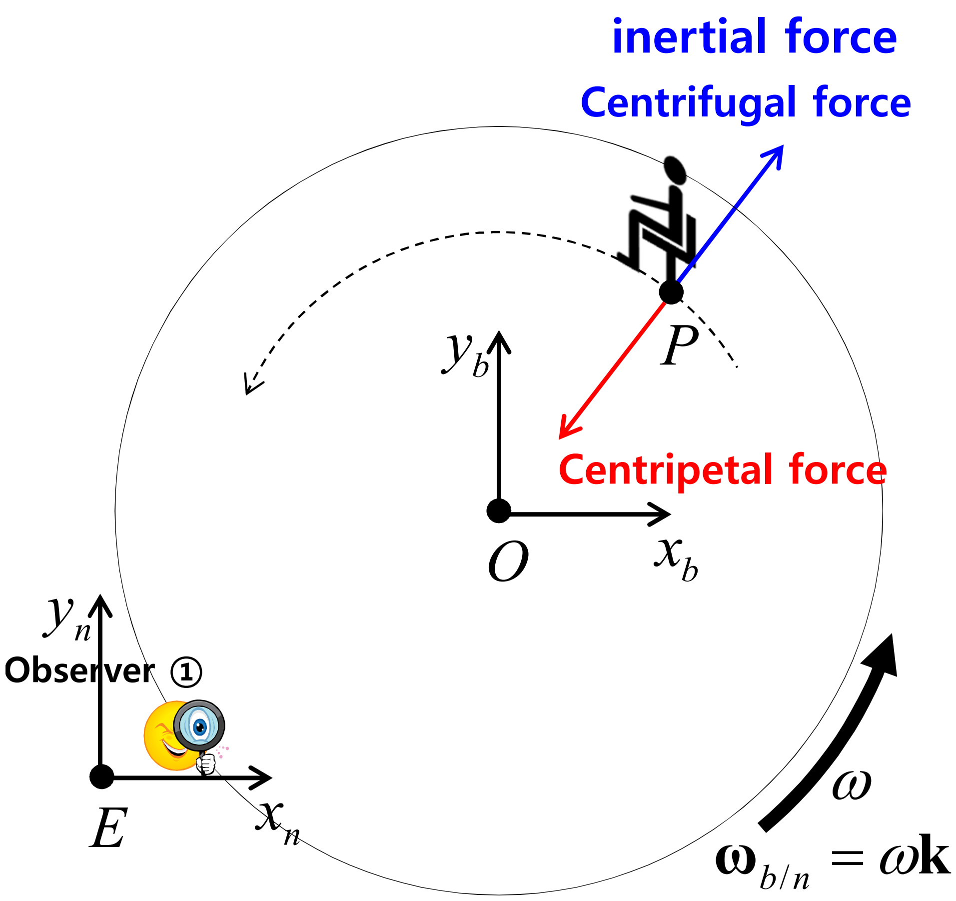


The centrifugal acceleration is proportional to the distance from the center of the space station. The farther the person goes away from the center, the perceived gravity would be greater.
Thus, the person will fly out and fall into the floor.

Relative Motion
- Examples of Rotating Reference Frame (1/8)

Case #1

- A chair is fixed on a circular disk which is rotating with an angular velocity ω.
- What kind of forces does a person sitting on the chair feel?



Description from the observer ①

The person sitting on the chair revolves around the center of the disk.

It shows that the centripetal force is exerted on the person

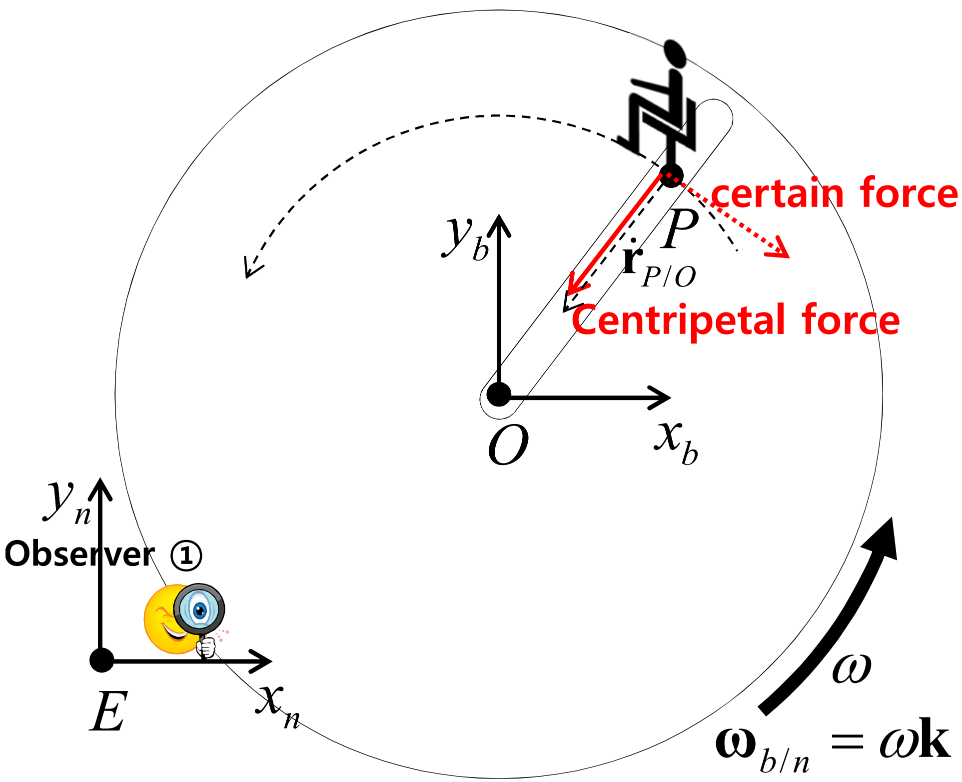
Description from the person sitting on the chair.

The person sitting on the chair feels centrifugal force. ← inertial force

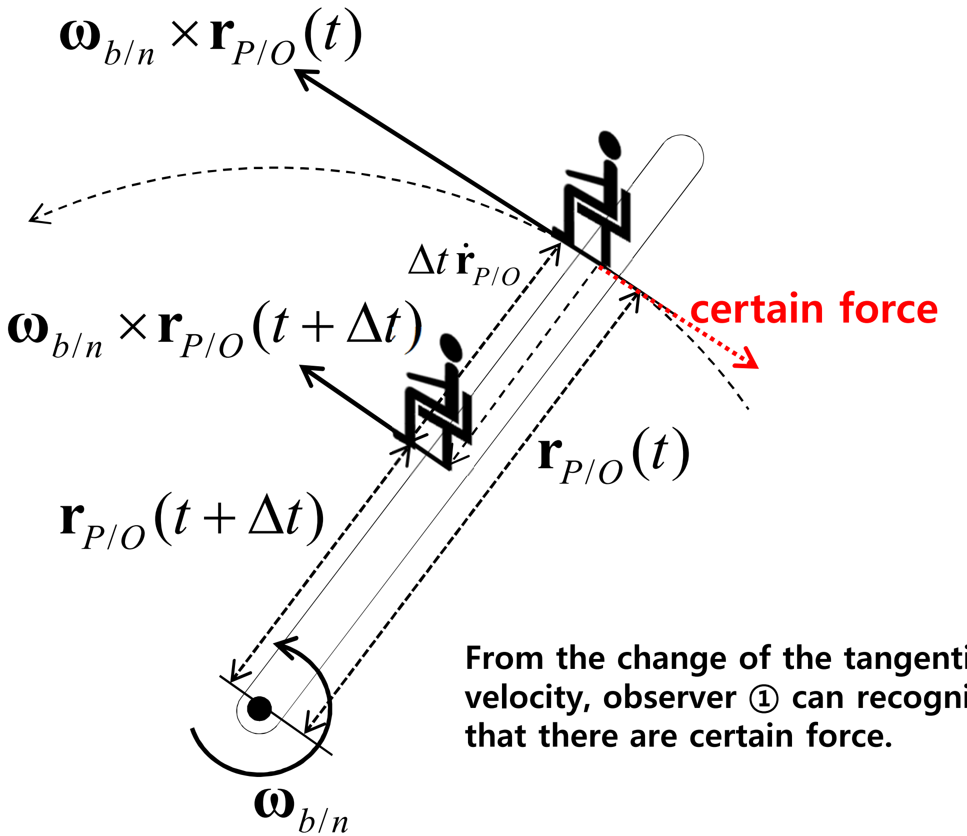
Relative Motion
- Examples of Rotating Reference Frame (2/8)

Case #2

- A chair moves with velocity v along the line on a circular disk which is rotating with an angular velocity ω.
- What kind of forces does a person sitting on the chair feel?



Description from the observer ①

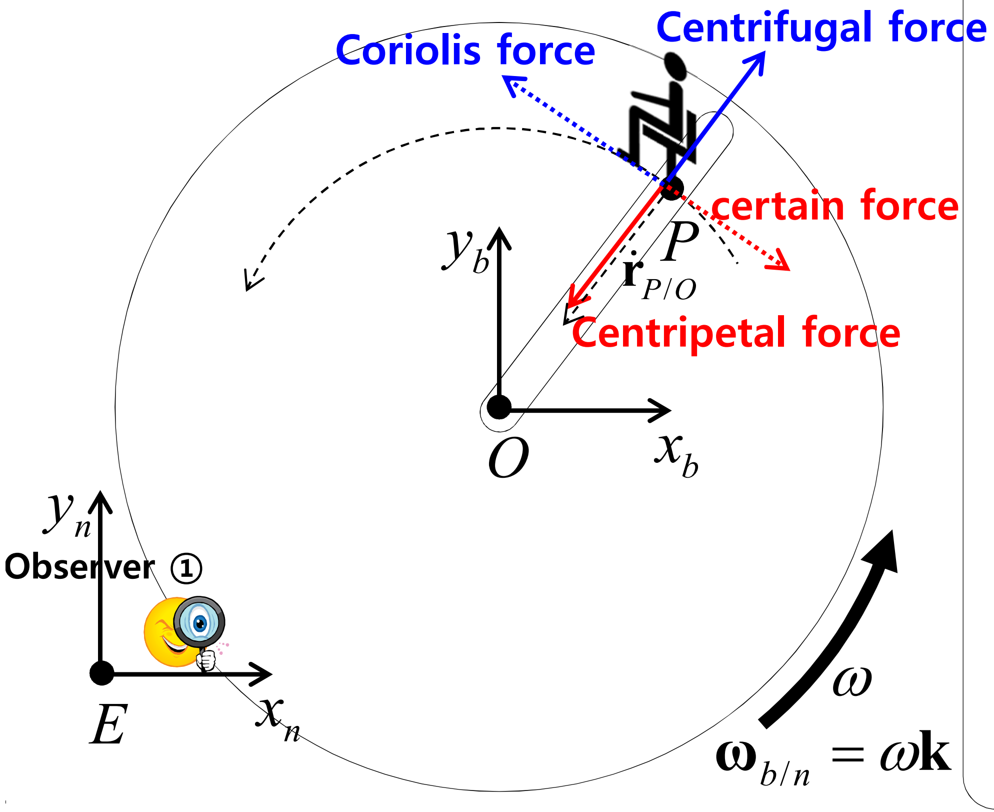


From the change of the tangential velocity, observer ① can recognize that there are certain force.

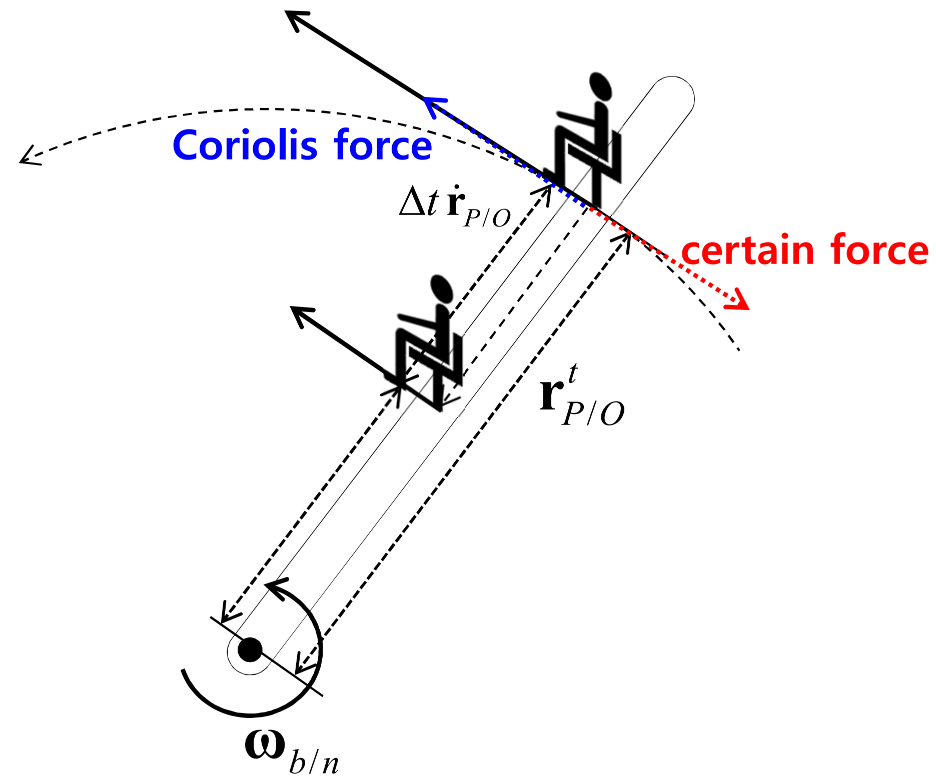
Relative Motion
- Examples of Rotating Reference Frame (3/8)

Case #2

- A chair moves with velocity v along the line on a circular disk which is rotating with an angular velocity ω.
- What kind of forces does a person sitting on the chair feel?



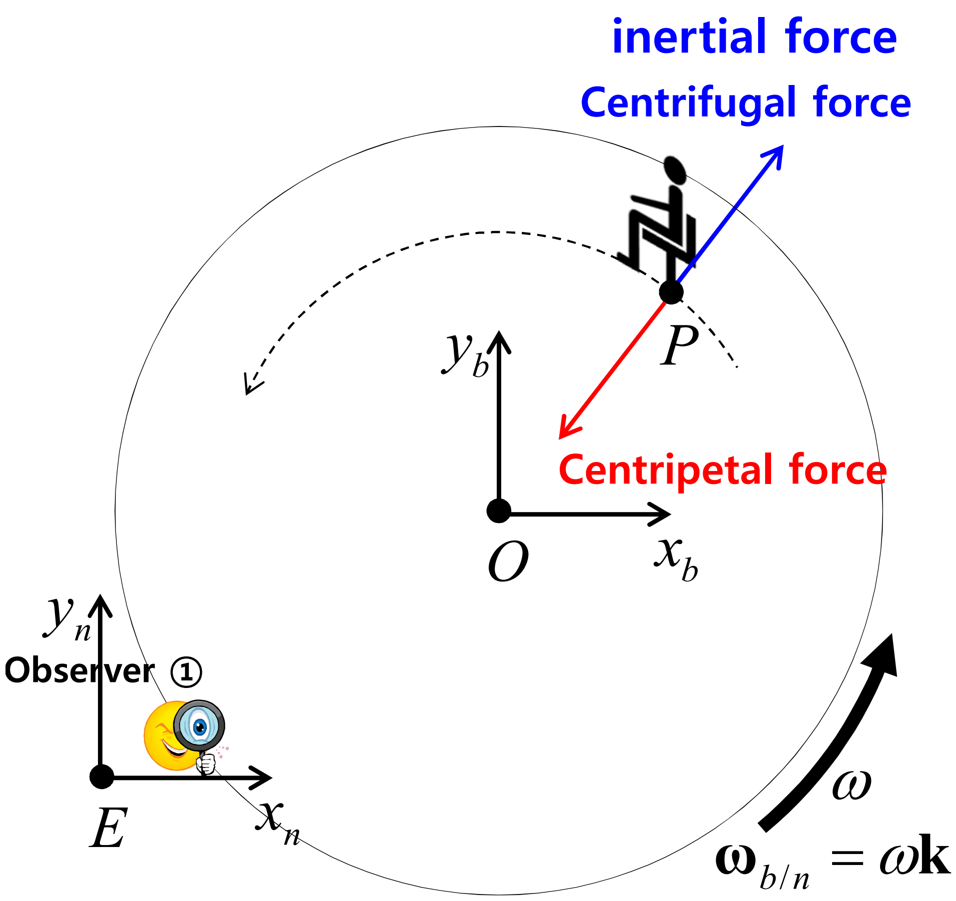
The person sitting on the chair feels **Coriolis force**.



Relative Motion
- Examples of Rotating Reference Frame (4/8)

Case #1

- A chair is fixed on a circular disk which is rotating with an angular velocity ω.
- What kind of forces does a person sitting on the chair feel?



Description from the observer ①

The person sitting on the chair revolves around the center of the disk.

It shows that the centripetal force is exerted on the person

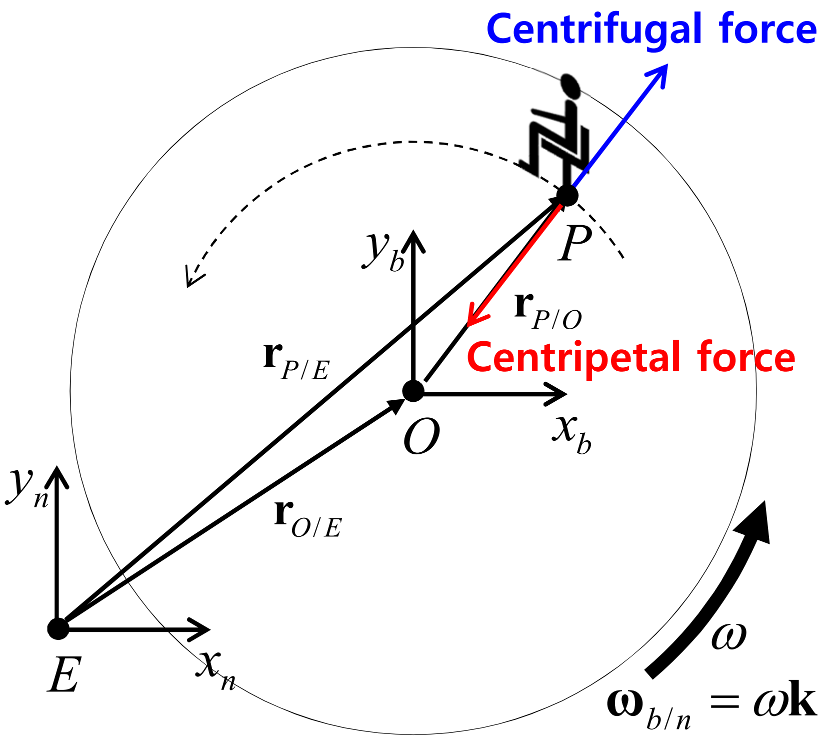
Description from the person sitting on the chair.

The person sitting on the chair feels centrifugal force. ← inertial force

Relative Motion
- Examples of Rotating Reference Frame (5/8)

Case #1

- A chair is fixed on a circular disk which is rotating with an angular velocity ω.
- What kind of forces does a person sitting on the chair feel?



- We apply Newton's 2nd law to the person on the chair

$$m_P\, {}^n\ddot{\mathbf{r}}_{P/E} = \mathbf{F}_P$$

$$m_P\, \cancel{{}^n\ddot{\mathbf{r}}_{O/E}} + m_P\, \cancel{{}^n\ddot{\mathbf{r}}_{P/O}} + m_P(\cancel{{}^n\dot{\boldsymbol{\omega}}_{b/n}} \times {}^n\mathbf{r}_{P/O})$$
$$+2m_P({}^n\boldsymbol{\omega}_{b/n} \times \cancel{{}^n\dot{\mathbf{r}}_{P/O}}) + \boxed{m_P({}^n\boldsymbol{\omega}_{b/n} \times ({}^n\boldsymbol{\omega}_{b/n} \times {}^n\mathbf{r}_{P/O}))} = \mathbf{F}_P$$

centripetal force is exerted on the person

$$\boxed{m_P\, {}^n\ddot{\mathbf{r}}_{P/O}} = \mathbf{F}_P - m_P\, \cancel{{}^n\ddot{\mathbf{r}}_{O/E}} - m_P(\cancel{{}^n\dot{\boldsymbol{\omega}}_{b/n}} \times {}^n\mathbf{r}_{P/O})$$
$$-2m_P({}^n\boldsymbol{\omega}_{b/n} \times \cancel{{}^n\dot{\mathbf{r}}_{P/O}}) \boxed{- m_P({}^n\boldsymbol{\omega}_{b/n} \times ({}^n\boldsymbol{\omega}_{b/n} \times {}^n\mathbf{r}_{P/O}))}$$

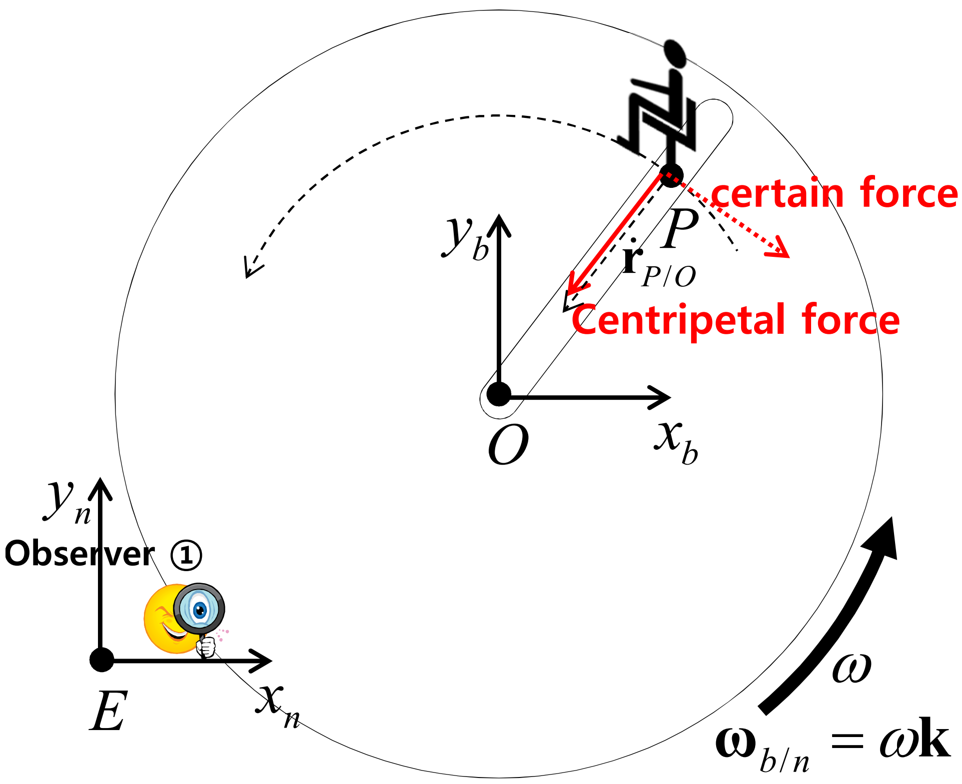
inertial force

The person feels centrifugal force

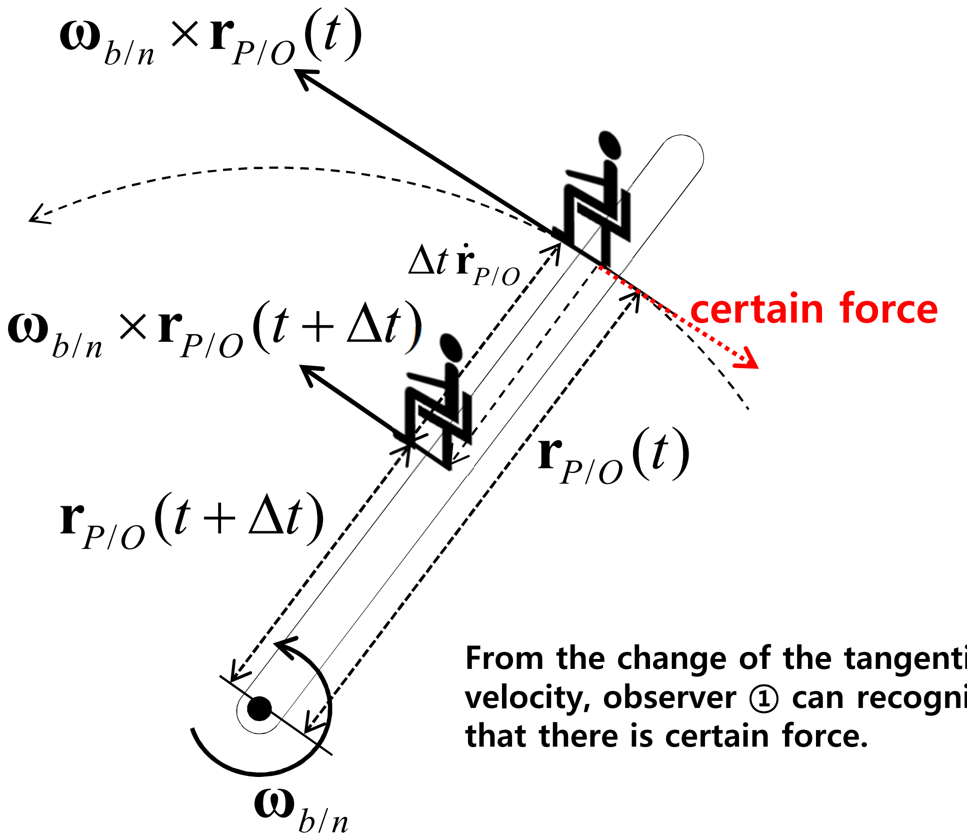
Relative Motion
- Examples of Rotating Reference Frame (6/8)

Case #2

- A chair moves with velocity v along the line on a circular disk which is rotating with an angular velocity ω.
- What kind of forces does a person sitting on the chair feel?



Description from the observer ①

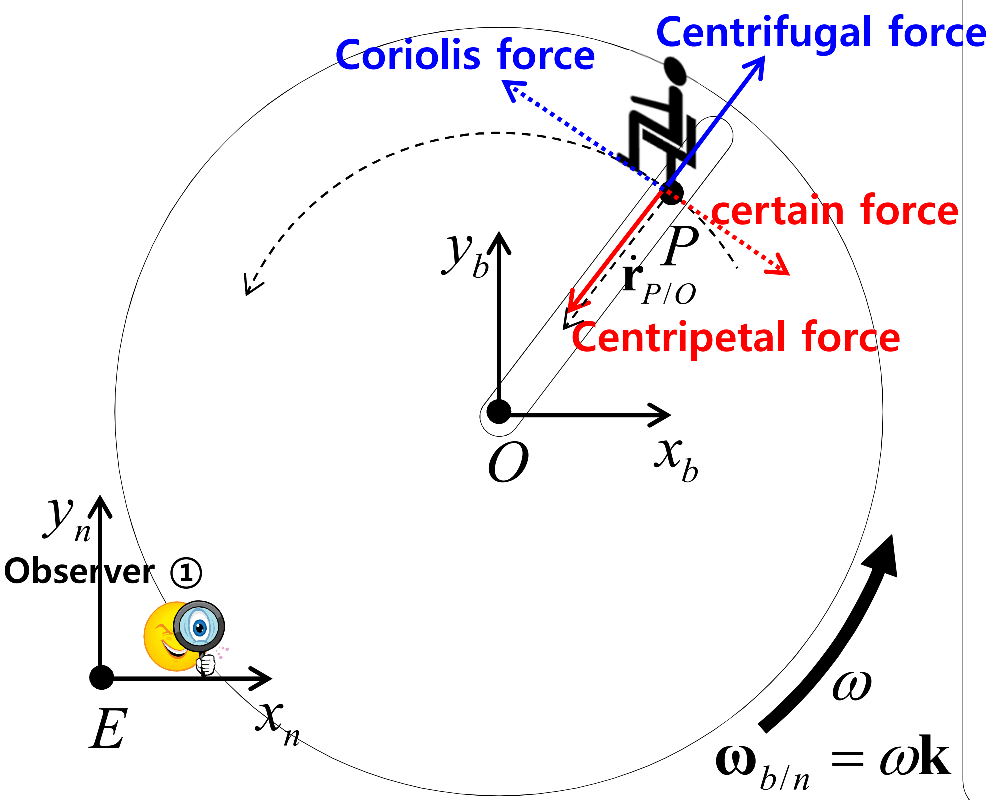


From the change of the tangential velocity, observer ① can recognize that there is certain force.

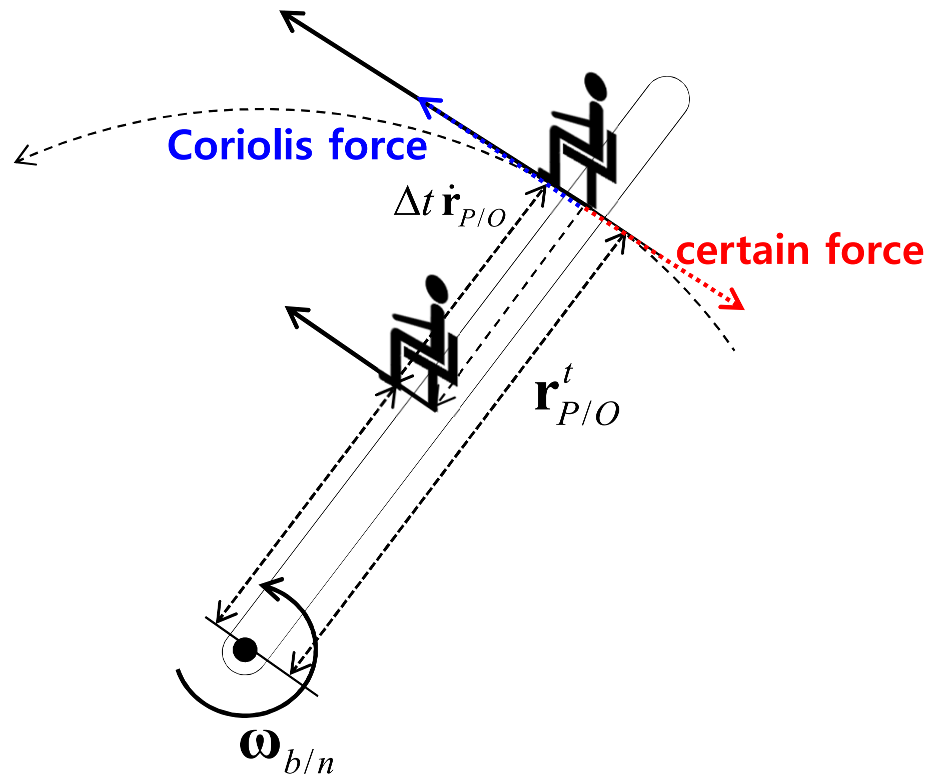
Relative Motion
- Examples of Rotating Reference Frame (7/8)

Case #2

- A chair moves with velocity v along the line on a circular disk which is rotating with an angular velocity ω.
- What kind of forces does a person sitting on the chair feel?



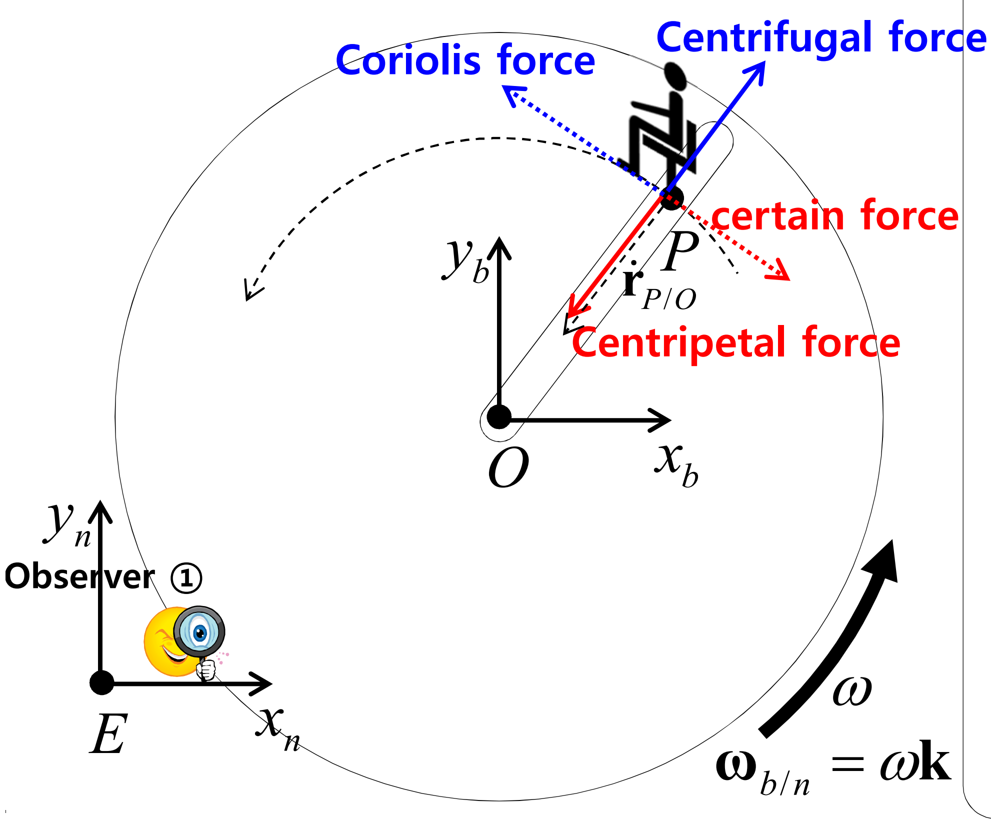
The person sitting on the chair feels **Coriolis force**.



Relative Motion
- Examples of Rotating Reference Frame (8/8)

Case #2

- A chair moves with velocity v along the line on a circular disk which is rotating with an angular velocity ω.
- What kind of forces does a person sitting on the chair feel?



- We apply Newton's 2nd law to the person on the chair

$$m_P\,{}^n\ddot{\mathbf{r}}_{P/E} = \mathbf{F}_P$$

$$m_P\,{}^n\cancel{\ddot{\mathbf{r}}_{O/E}} + m_P\,{}^n\cancel{\ddot{\mathbf{r}}_{P/O}} + m_P({}^n\cancel{\dot{\boldsymbol{\omega}}_{b/n}} \times {}^n\mathbf{r}_{P/O})$$
$$+\boxed{2m_P({}^n\boldsymbol{\omega}_{b/n} \times {}^n\dot{\mathbf{r}}_{P/O})} + \boxed{m_P({}^n\boldsymbol{\omega}_{b/n} \times ({}^n\boldsymbol{\omega}_{b/n} \times {}^n\mathbf{r}_{P/O}))} = \mathbf{F}_P$$

certain and centripetal force is exerted on the person

$$\boxed{m_P\,{}^n\ddot{\mathbf{r}}_{P/O}} = \mathbf{F}_P - m_P\,{}^n\cancel{\ddot{\mathbf{r}}_{O/E}} - m_P({}^n\cancel{\dot{\boldsymbol{\omega}}_{b/n}} \times {}^n\mathbf{r}_{P/O})$$

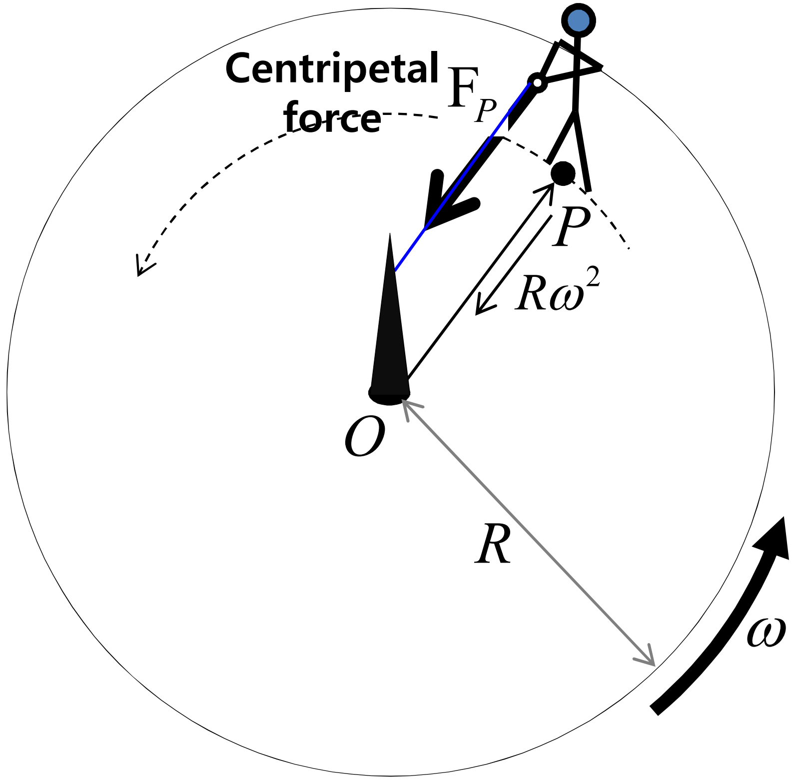
inertial force

$$\boxed{-2m_P({}^n\boldsymbol{\omega}_{b/n} \times {}^n\dot{\mathbf{r}}_{P/O})} \boxed{-m_P({}^n\boldsymbol{\omega}_{b/n} \times ({}^n\boldsymbol{\omega}_{b/n} \times {}^n\mathbf{r}_{P/O}))}$$

The person feels Coriolis and centrifugal force

Examples of a Person on the Rotating Disk (1/2)

The person "P" is sitting on a chair which is fixed on a large disk rotating with constant angular velocity ω.



1. In inertial frame

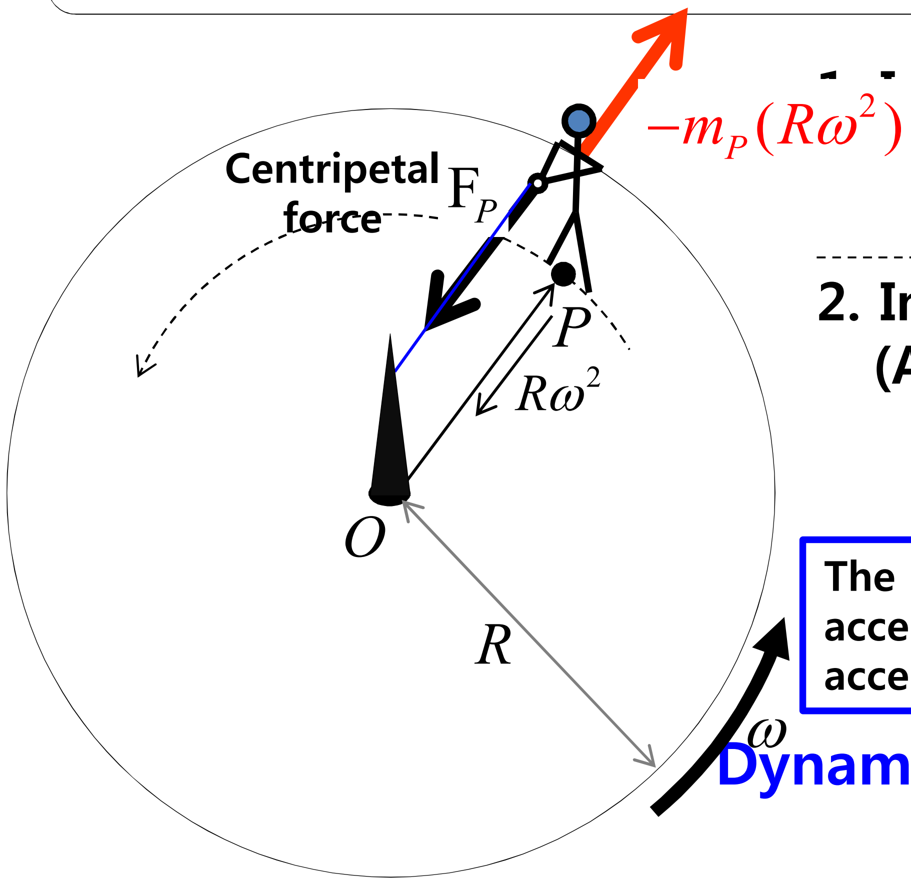
$$m_P(\underline{R\omega^2}) = \underline{F_P} \quad \cdots \text{(1)}$$

The person "P" is accelerated in inward direction with an acceleration "$R\omega^2$"

The external force exerted on the person "P", this is the "Centripetal force".

Examples of a Person on the Rotating Disk (2/2)

The person "P" is sitting on a chair which is fixed on a large disk rotating with constant angular velocity ω.



1. In inertial frame

$$m_P(R\omega^2) = F_P \quad \cdots \text{ (1)}$$

2. In non-inertial frame
(According to D'Alembert Principle)

$$\underline{0} = \underline{F_P - m_P(R\omega^2)} \quad \cdots \text{ (2)}$$

The person "P" is not accelerated. (the acceleration is zero)

Dynamic Equilibrium

→ The force "$-m_P(R\omega^2)$" is a centrifugal force which is also inertial force.

The person "P" perceives the centripetal force and an additional force " $m_P(R\omega^2)$ " in outward direction. i.e. The person perceives the tension on his arm caused by the centripetal force and the centrifugal force "$-m_P(R\omega^2)$"

3. What is the magnitude of the inertial force " $-m_P(R\omega^2)$ "?

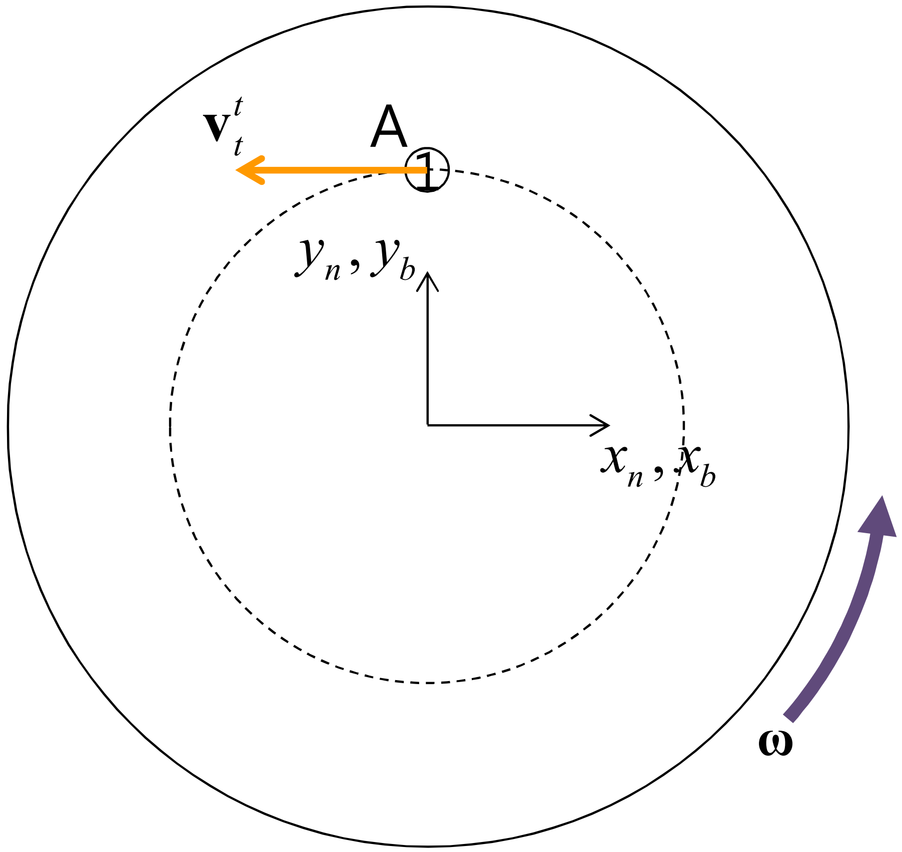
➡ From the equation (1), $-m_P(R\omega^2) = -F_P$

Centrifugal and Coriolis Accereation

Example) Rotating Disk - Centripetal, Centrifugal (1/5)

A point "A" is fixed on a rotating disk rotating with a constant angular velocity.

Velocity of the point A observed in n-frame.

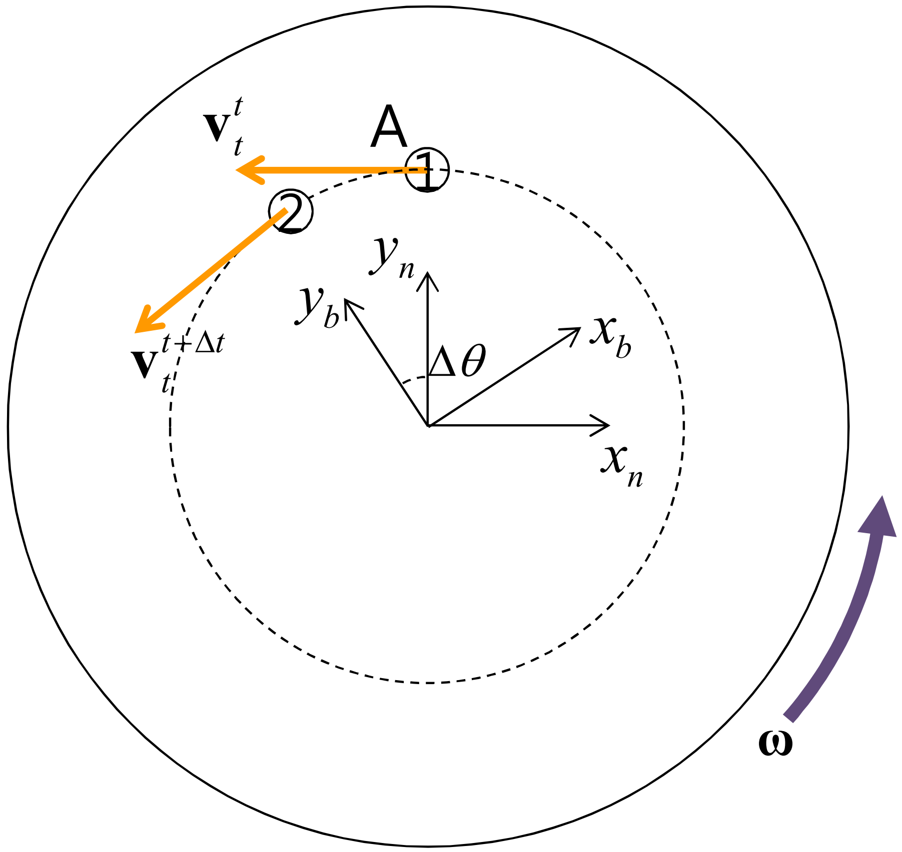


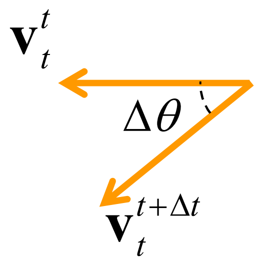
n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.

Example) Rotating Disk - Centripetal, Centrifugal (2/5)

A point "A" is fixed on a rotating disk rotating with a constant angular velocity.

Velocity of the point A observed in n-frame.



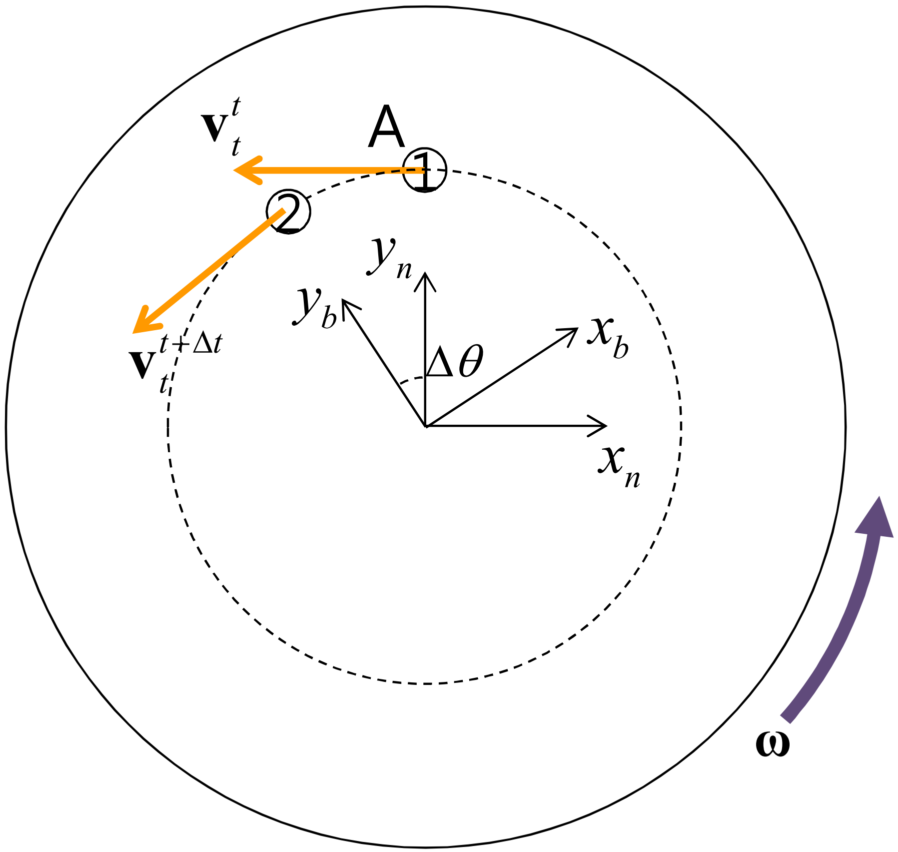


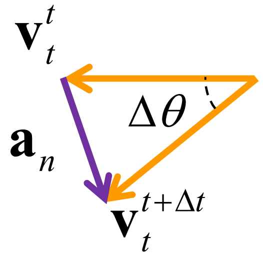
n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.

Example) Rotating Disk - Centripetal, Centrifugal (3/5)

A point "A" is fixed on a rotating disk rotating with a constant angular velocity.

Velocity of the point A observed in n-frame.



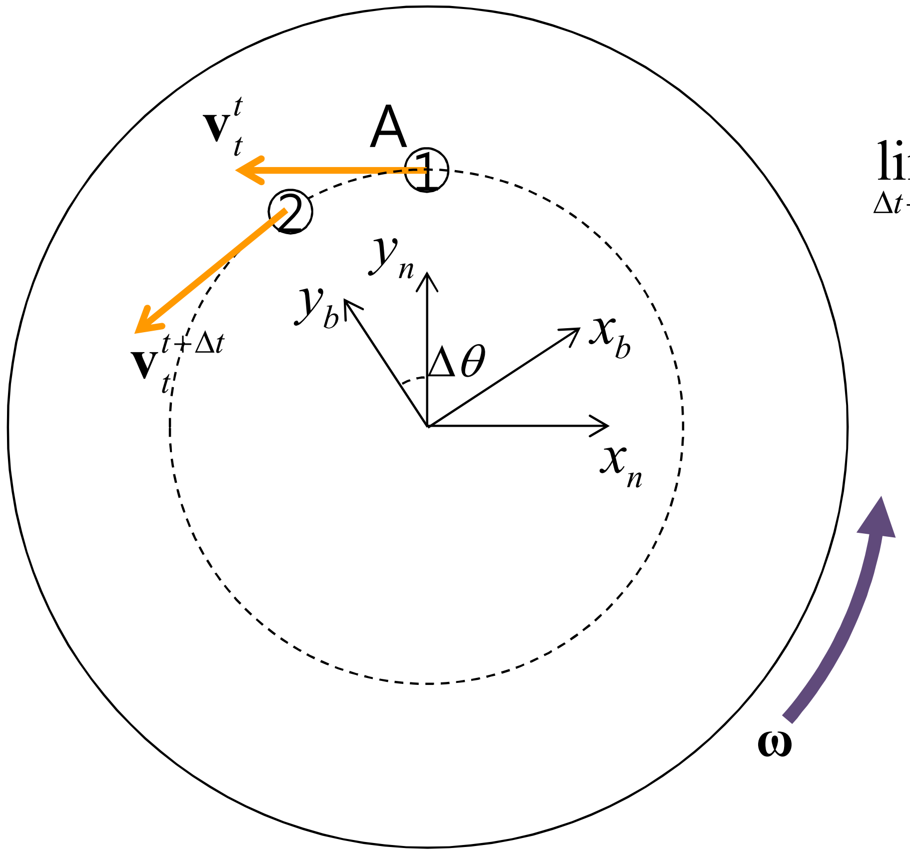


n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.

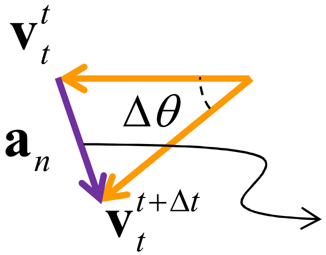
Example) Rotating Disk - Centripetal, Centrifugal (4/5)

A point "A" is fixed on a rotating disk rotating with a constant angular velocity.

Velocity of the point A observed in n-frame.



$$\lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{a}_n$$



Centripetal Acceleration

$$\boldsymbol{\omega} \times \left(\boldsymbol{\omega} \times \mathbf{r}_{A/B} \right)$$

Magnitude of $\mathbf{a}_n$

$$\mathbf{v}_t^t = \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$\left| \mathbf{v}_t^t \right| = \omega r$$

$$\lim_{\Delta \theta \to 0} \left| \Delta \mathbf{v} \right| \approx \left| \mathbf{v}_t^t \right| \cdot \Delta\theta = \omega r \cdot \Delta\theta$$

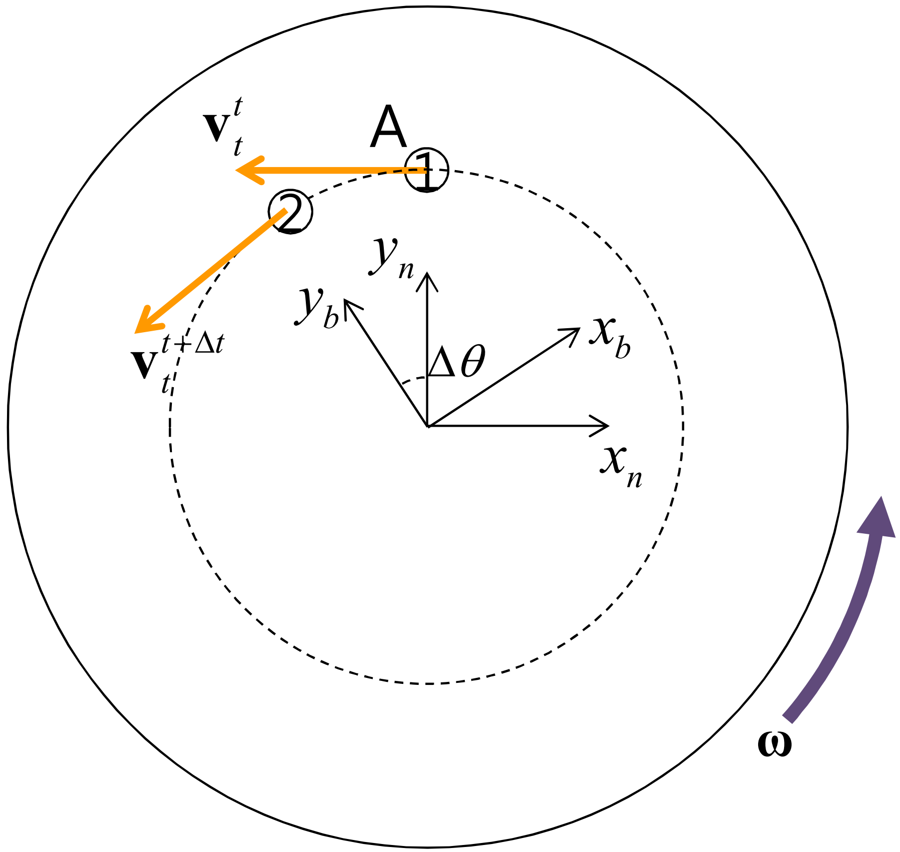
$$\left| \mathbf{a}_n \right| = \lim_{\Delta t \to 0} \left| \frac{\Delta \mathbf{v}}{\Delta t} \right| \approx \left| \mathbf{v}_t^t \right| \cdot \frac{\Delta\theta}{\Delta t} = \omega r \cdot \omega = \omega^2 r$$

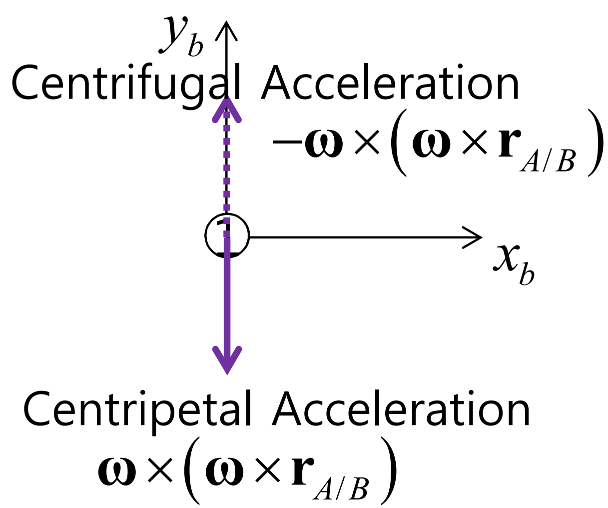
n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.

Example) Rotating Disk - Centripetal, Centrifugal (5/5)

A point "A" is fixed on a rotating disk rotating with a constant angular velocity.

Velocity of the point A observed in b-frame.





Since, the point A observed in b-frame is not accelerated, there should be an additional force exerted on the point A except the centripetal force.

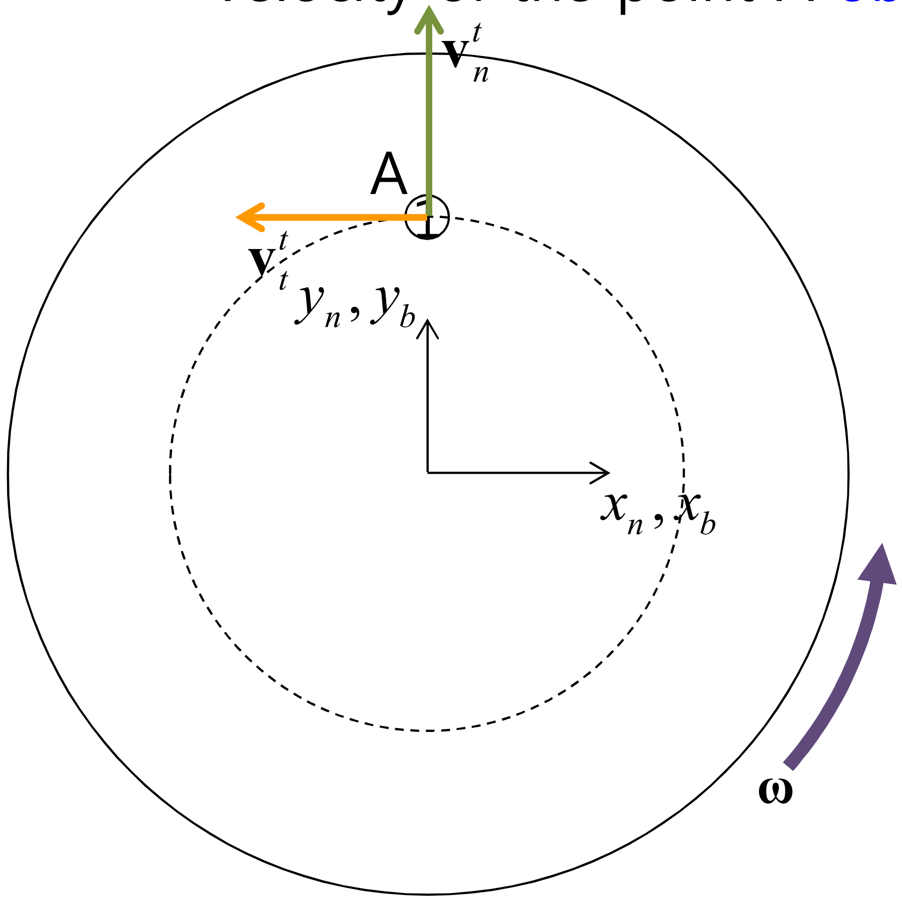
The additional force is a centrifugal force.

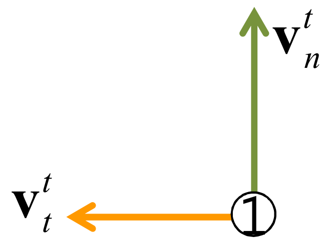
n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.

Example) Rotating Disk - Coriolis Acceleration(1/7)

A point "A" is moving along a slot with a constant velocity, and the slot is on a disk rotating with a constant angular velocity.

Velocity of the point A observed in n-frame.



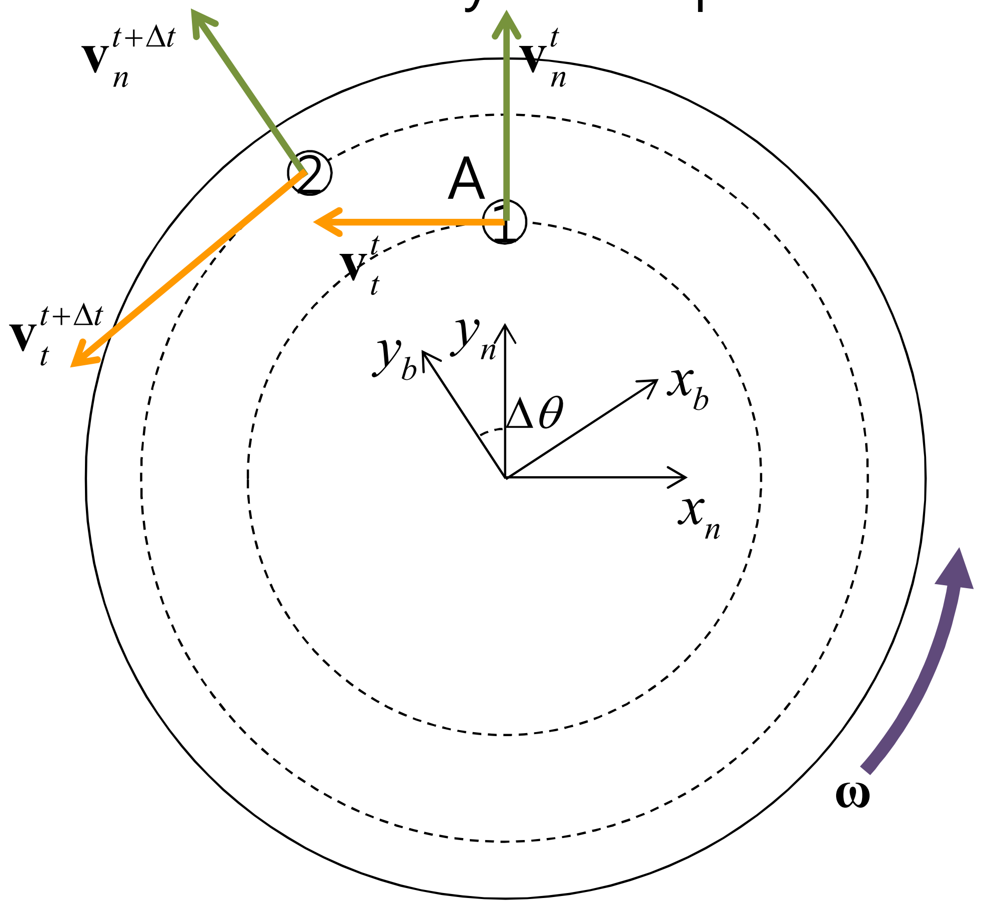


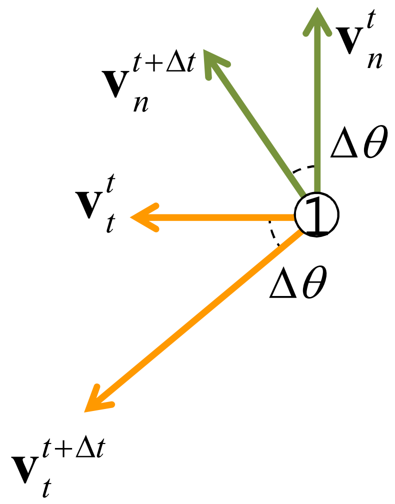
n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.

Example) Rotating Disk - Coriolis Acceleration(2/7)

A point “A” is moving along a slot with a constant velocity, and the slot is on a disk rotating with a constant angular velocity.

Velocity of the point A observed in n-frame.



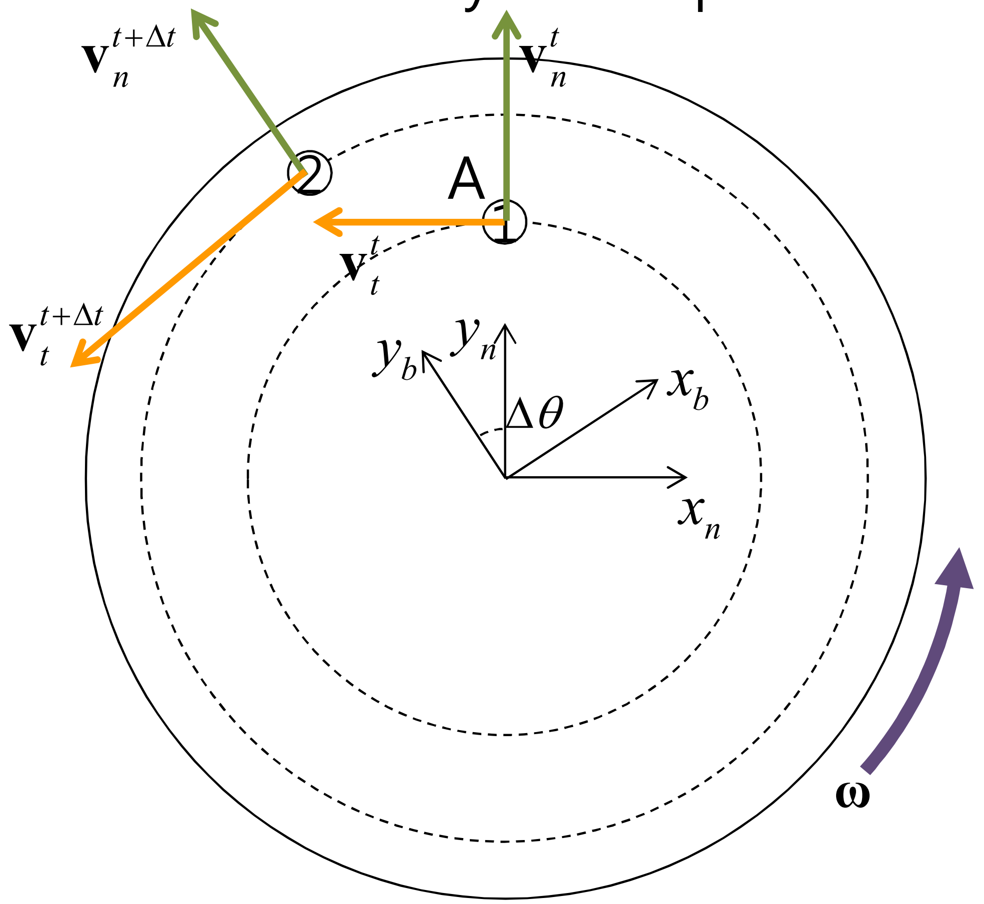


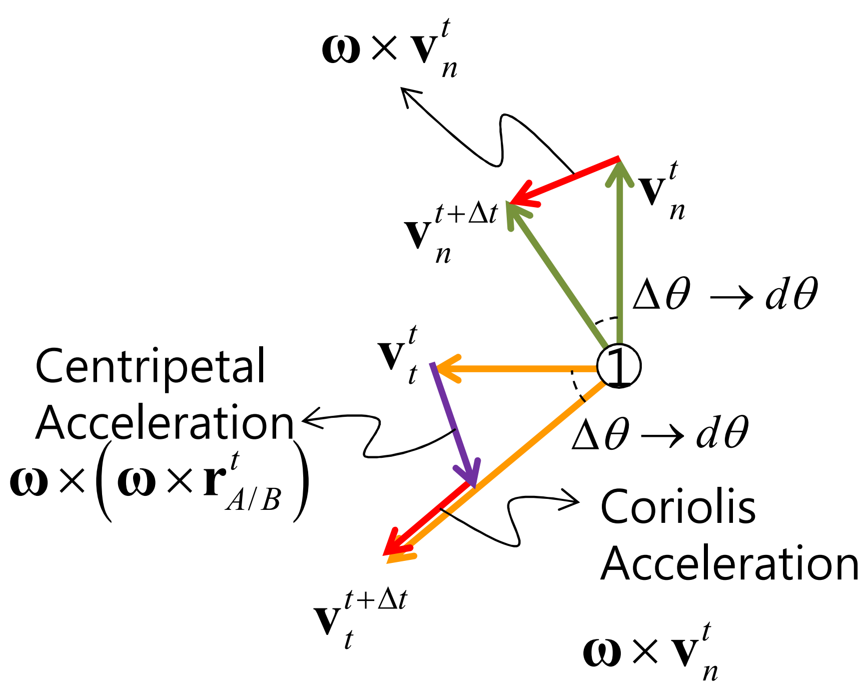
n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.

Example) Rotating Disk - Coriolis Acceleration(3/7)

A point "A" is moving along a slot with a constant velocity, and the slot is on a disk rotating with a constant angular velocity.

Velocity of the point A observed in n-frame.



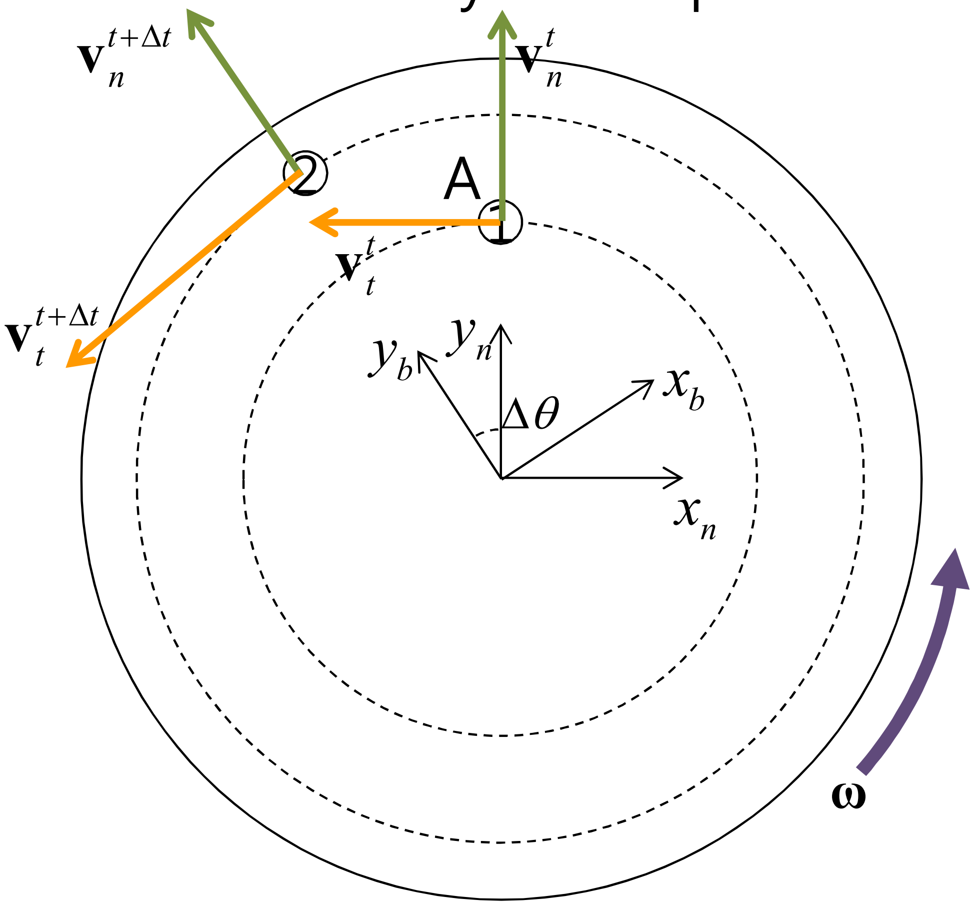


n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.

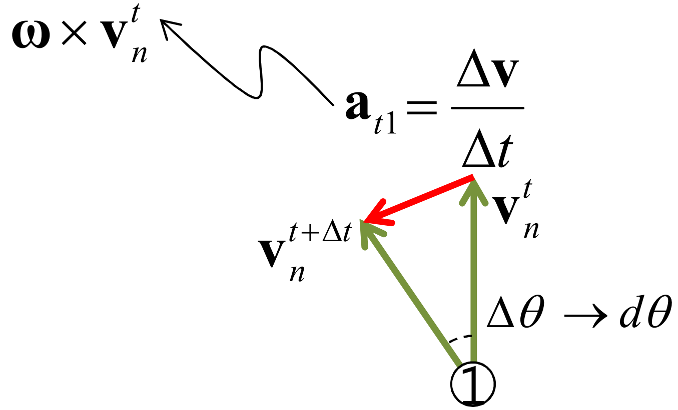
Example) Rotating Disk - Coriolis Acceleration(3-1/7)

A point "A" is moving along a slot with a constant velocity, and the slot is on a disk rotating with a constant angular velocity.

Velocity of the point A observed in n-frame.



Coriolis Acceleration



Magnitude of $\mathbf{a}_t$

$$\lim_{\Delta\theta \to 0} |\Delta \mathbf{v}| \approx |\mathbf{v}_n^t| \cdot \Delta\theta = \mathbf{v}_n^t \cdot \Delta\theta$$

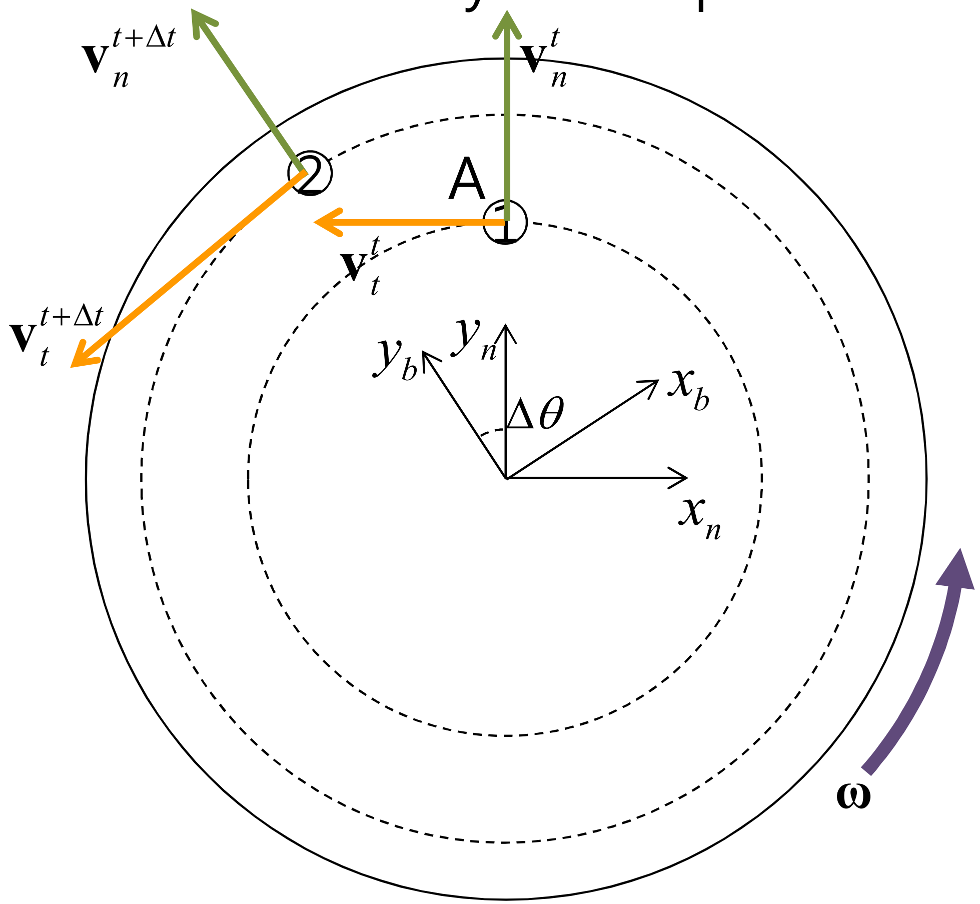
$$|\mathbf{a}_{t1}| = \lim_{\Delta t \to 0} \left| \frac{\Delta \mathbf{v}}{\Delta t} \right| \approx |\mathbf{v}_n^t| \cdot \frac{\Delta\theta}{\Delta t} = |\mathbf{v}_n^t| \omega$$

n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.

Example) Rotating Disk - Coriolis Acceleration(3-2/7)

A point "A" is moving along a slot with a constant velocity, and the slot is on a disk rotating with a constant angular velocity.

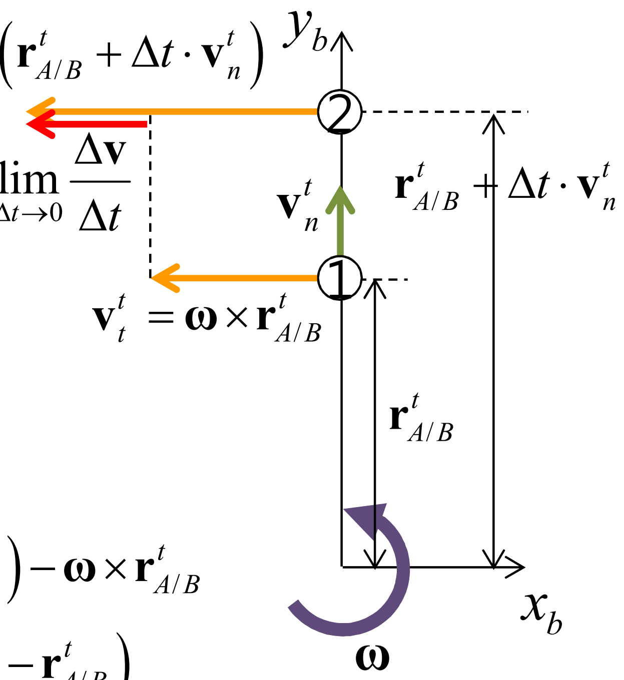
Velocity of the point A observed in n-frame.



$$\mathbf{v}_t^{t+\Delta t} = \boldsymbol{\omega} \times \left(\mathbf{r}_{A/B}^t + \Delta t \cdot \mathbf{v}_n^t \right)$$

$$\mathbf{a}_{t2} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t}$$

$$\mathbf{v}_t^t = \boldsymbol{\omega} \times \mathbf{r}_{A/B}^t$$



$$\begin{aligned}
\Delta \mathbf{v} &= \mathbf{v}_t^{t+\Delta t} - \mathbf{v}_t^t \\
&= \boldsymbol{\omega} \times \left(\mathbf{r}_{A/B}^t + \Delta t \cdot \mathbf{v}_n^t \right) - \boldsymbol{\omega} \times \mathbf{r}_{A/B}^t \\
&= \boldsymbol{\omega} \times \left(\mathbf{r}_{A/B}^t + \Delta t \cdot \mathbf{v}_n^t - \mathbf{r}_{A/B}^t \right) \\
&= \boldsymbol{\omega} \times \Delta t \cdot \mathbf{v}_n^t
\end{aligned}$$

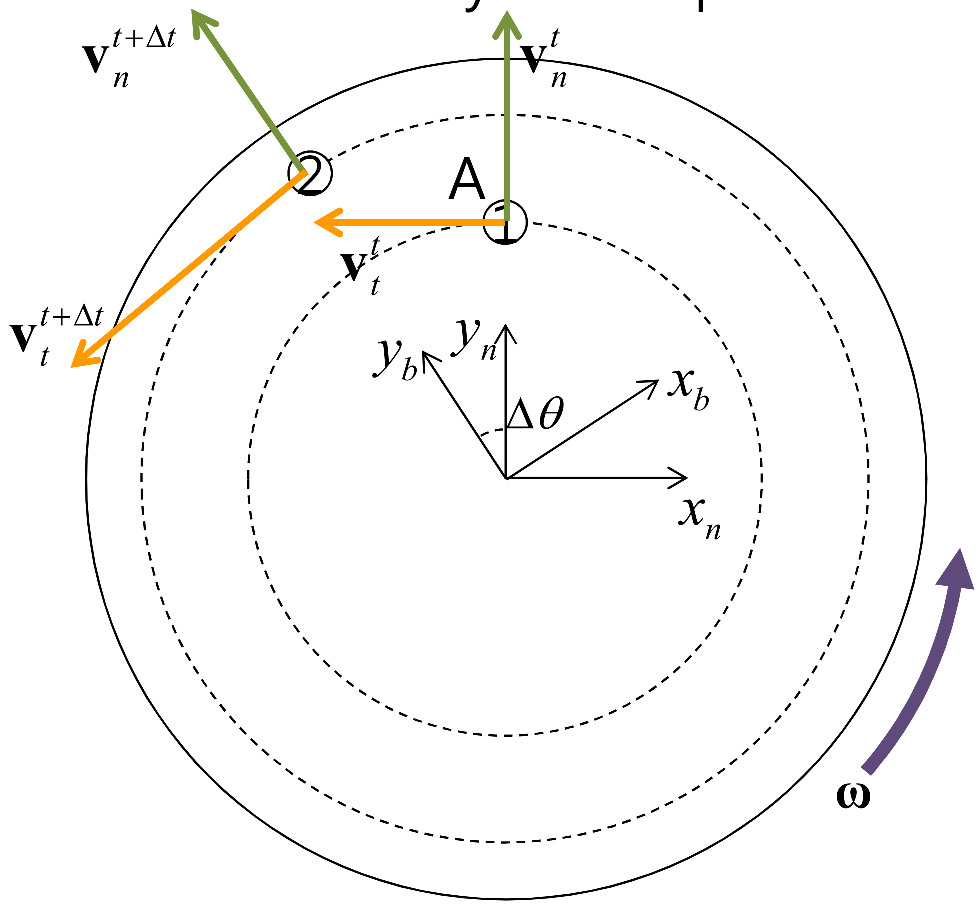
$$\mathbf{a}_{t2} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\boldsymbol{\omega} \times \Delta t \cdot \mathbf{v}_n^t}{\Delta t} = \boldsymbol{\omega} \times \mathbf{v}_n^t$$

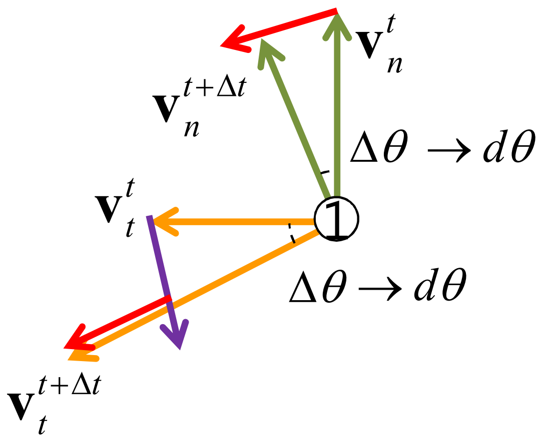
n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.

Example) Rotating Disk - Coriolis Acceleration(4/7)

A point "A" is moving along a slot with a constant velocity, and the slot is on a disk rotating with a constant angular velocity.

Velocity of the point A observed in n-frame.



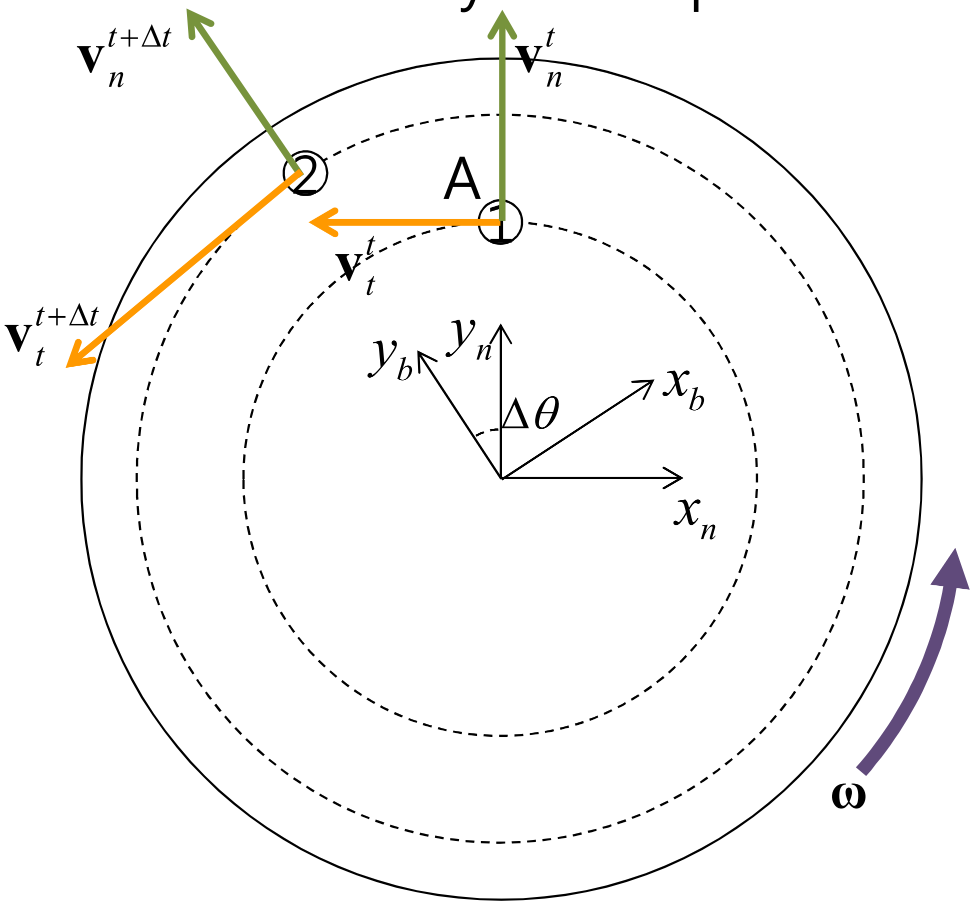


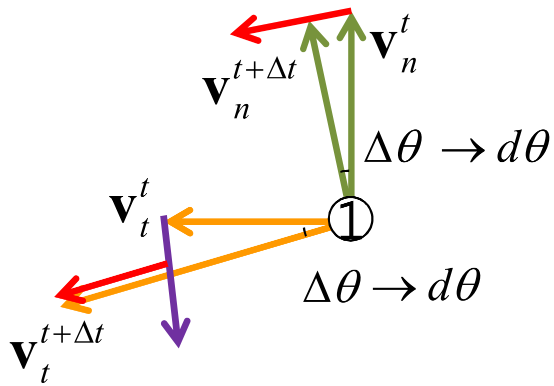
n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.

Example) Rotating Disk - Coriolis Acceleration(5/7)

A point "A" is moving along a slot with a constant velocity, and the slot is on a disk rotating with a constant angular velocity.

Velocity of the point A observed in n-frame.



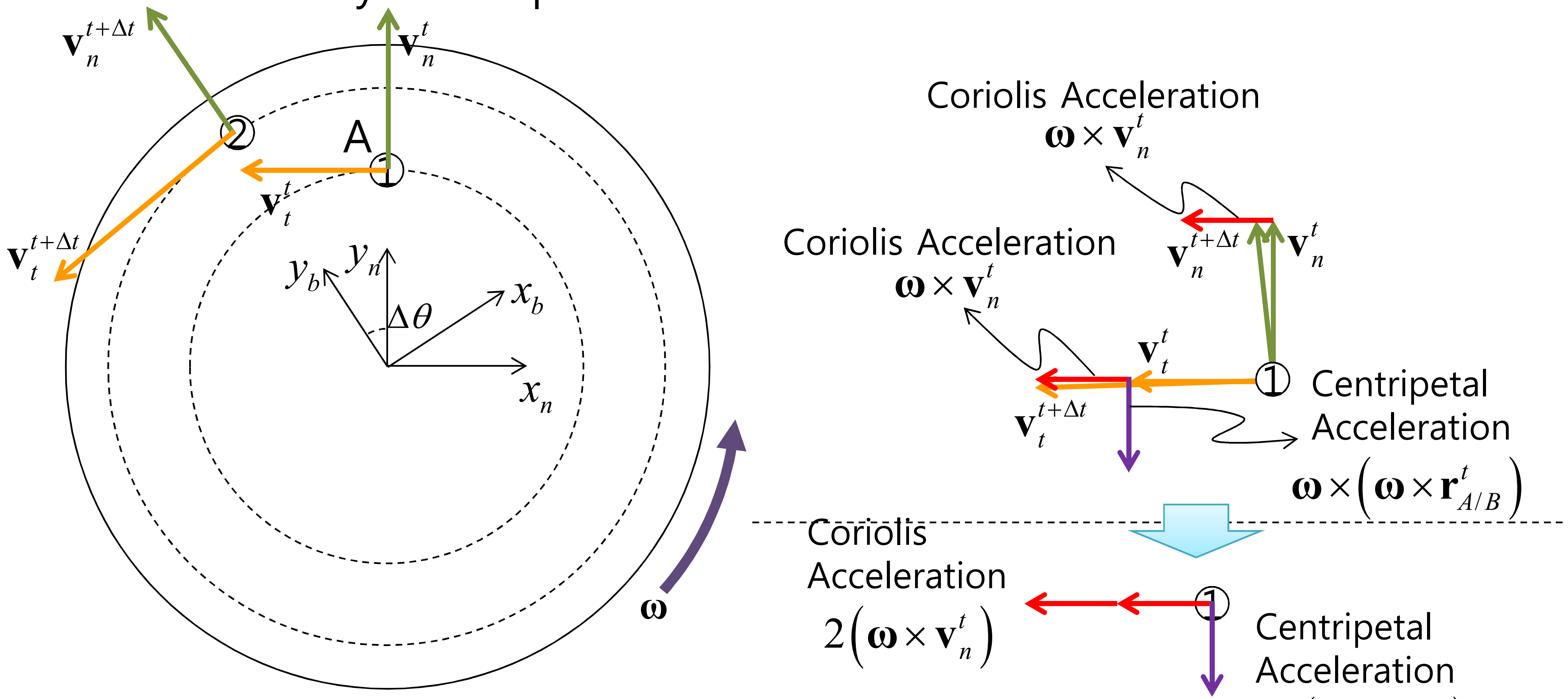


n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.

Example) Rotating Disk - Coriolis Acceleration(6/7)

A point "A" is moving along a slot with a constant velocity, and the slot is on a disk rotating with a constant angular velocity.

Velocity of the point A observed in n-frame.

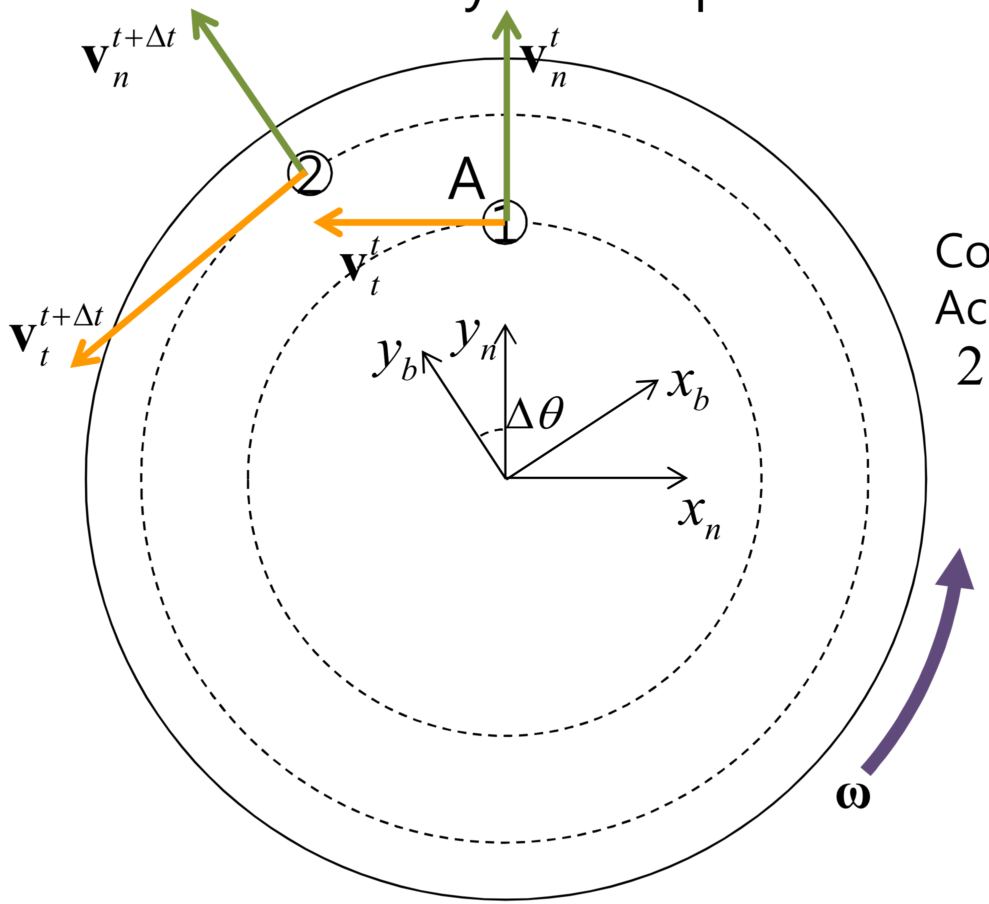


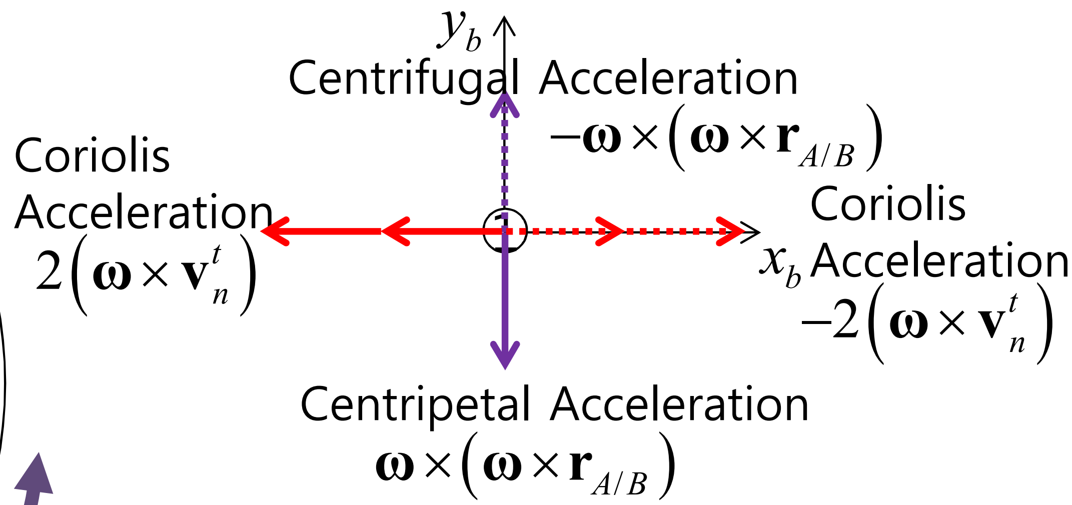
n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.

Example) Rotating Disk - Coriolis Acceleration(7/7)

A point "A" is moving along a slot with a constant velocity, and the slot is on a disk rotating with a constant angular velocity.

Velocity of the point A observed in b-frame.





Since, the point A observed in b-frame is not accelerated, there should be an additional force exerted on the point A except the centripetal force.

The additional force is a centrifugal force and Coriolis force.

n-frame: an inertial frame.
b-frame: a frame fixed on the center of the disk.