Ship Stability

September 2013

Myung-II Roh

Department of Naval Architecture and Ocean Engineering Seoul National University

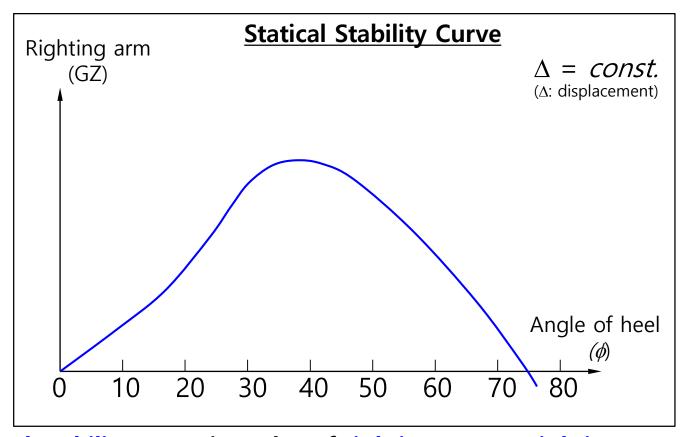
Ship Stability

- **☑** Ch. 1 Introduction to Ship Stability
- ☑ Ch. 2 Review of Fluid Mechanics
- ☑ Ch. 3 Transverse Stability
- **☑** Ch. 4 Initial Transverse Stability
- ☑ Ch. 5 Free Surface Effect
- ☑ Ch. 6 Inclining Test
- ☑ Ch. 7 Longitudinal Stability
- ☑ Ch. 8 Curves of Stability and Stability Criteria
- ☑ Ch. 9 Numerical Integration Method in Naval Architecture
- ☑ Ch. 10 Hydrostatic Values
- ☑ Ch. 11 Introduction to Damage Stability
- **☑** Ch. 12 Deterministic Damage Stability
- ☑ Ch. 13 Probabilistic Damage Stability (Subdivision and Damage Stability, SDS)

Ch. 8 Curves of Stability and Stability Criteria

Statical Stability Curve

Definition and Purpose of the Statical Stability Curve



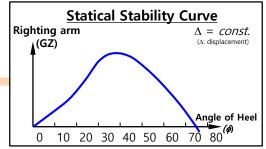
The statical stability curve is a plot of righting arm or righting moment against angle of heel for a given condition.

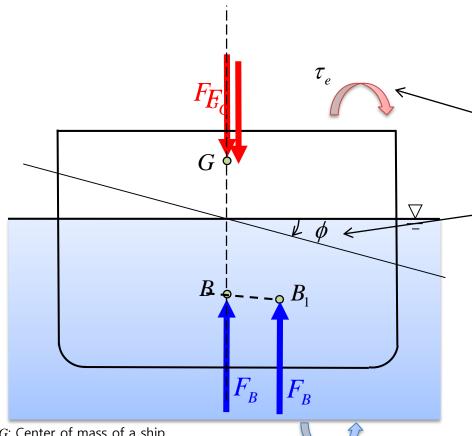
So far as the intact ship is concerned, the statical stability curve provides useful data for judging the adequacy of the ship's stability for the given condition.

Definition and Purpose of the Statical Stability Curve

- Calculation Method of "GZ" (1/2)

1. At a certain angle of heel, calculate the static equilibrium position of the ship.





The fixed angle of heel means that there is an external moment that causes the ship to heel.

G: Center of mass of a ship

 F_c : Gravitational force of a ship

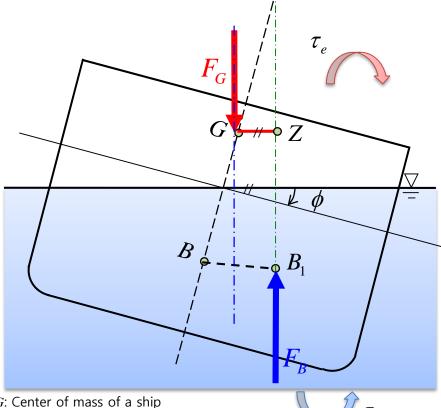
B: Center of buoyancy at initial position

 F_R : Buoyant force acting on a ship

B_i: New position of center of buoyancy after the ship has been inclined

Definition and Purpose of the Statical Stability Curve

- Calculation Method of "GZ" (2/2)
 - 2. By using the center of mass(G) and new position of the center of buoyancy(B_1), calculate the righting arm "GZ".



G: Center of mass of a ship

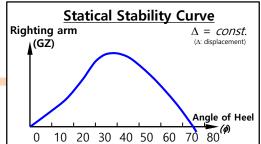
 F_c : Gravitational force of a ship

B: Center of buoyancy at initial position

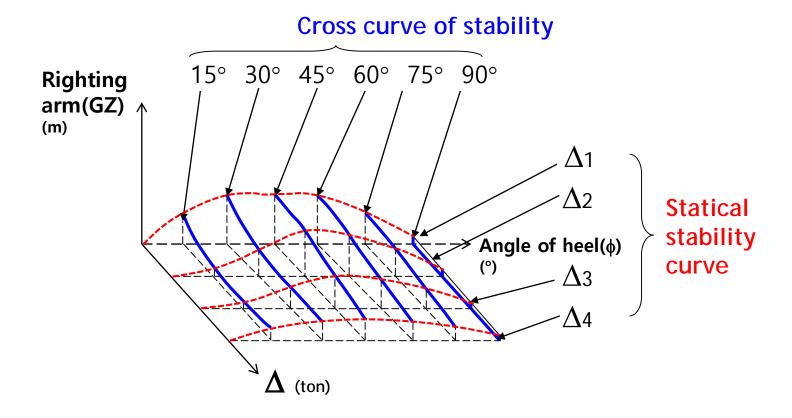
 F_B : Buoyant force acting on a ship

B₁: New position of center of buoyancy after the ship has been inclined

Z: The intersection point of a vertical line through the new position of the center of buoyancy(B₁) with the transversely parallel line to a waterline through the center of mass(G)



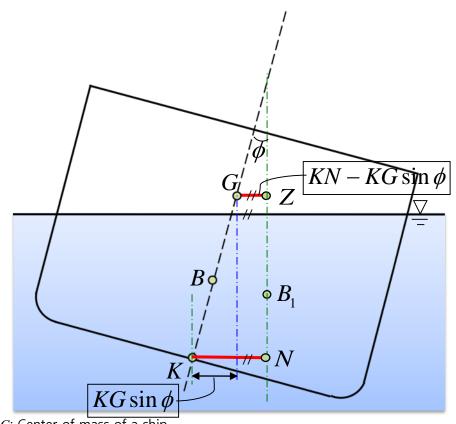
Definition and Purpose of Cross Curves of Stability



Cross curves of stability are plots of righting arm against for various values of heel and for various values of displacement.

A statical stability curve for a certain value of displacement can be obtained from the cross curves of stability.

Method for Obtaining Cross Curves of Stability



- G: Center of mass of a ship
- B: Center of buoyancy at initial position
- B₁: New position of center of buoyancy after the ship has been inclined
- Z: The intersection point of a vertical line through the new position of the center of buoyancy(B₁) with the longitudinally parallel line to a waterline through the center of mass(G)
- K: Keel
- N: The intersection point of a vertical line through the new position of the center of buoyancy(B_1) with the transversely parallel line to a waterline through the point K

Suppose that the center of mass is located at *K*.

Then, the *KN* represents the righting arm.

The *KN* depends only on the **geometry** of the submerged volume. But *KN* is not dependent on the location of the center of mass G.

$$KN = KN(\phi, \Delta)$$

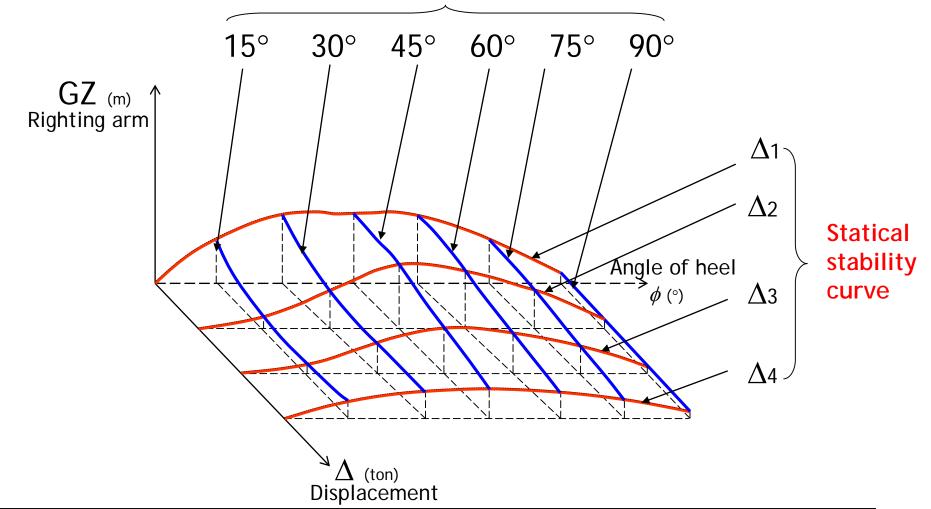
If the center of mass moves to vertical direction, so it is located at G, the values of righting arm (GZ) is

$$GZ = KN - KG \sin \phi$$

Obtaining Statical Stability Curves from Cross Curves of Stability

• Three-dimensional cross curve of stability

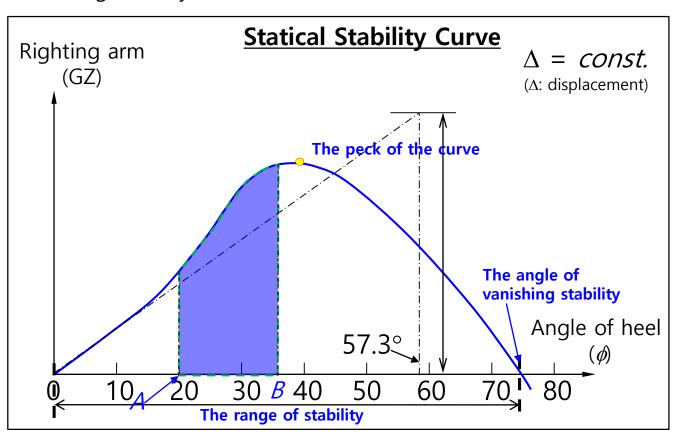
Cross curve of stability



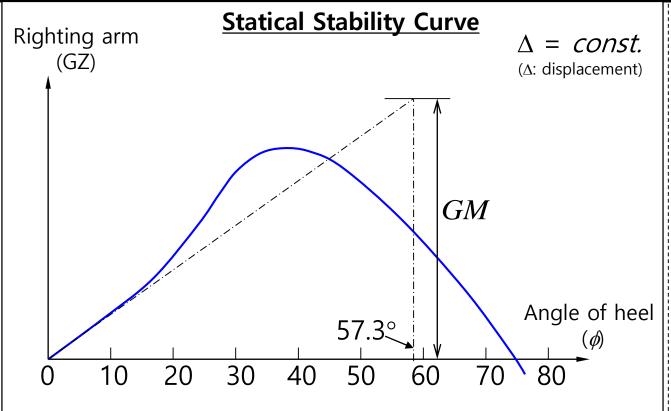
Significance of the Statical Stability Curve (1/5)

The statical stability curve has a number of features that are significant in the analysis of the ship's stability.

- The slope of the curve at zero degree, the peak of the curve, the range of stability, the angle of vanishing stability, and the area under the curve



Significance of the Statical Stability Curve (2/5) (1) The Slope of the Curve at Zero Degrees



The slope of the curve at zero degree is the metacentric height(GM).

This is a convenient check for major error in the initial portion of the statical stability curve.

✓ Derivation

At small angles of heel $GZ \approx GM \sin |\phi|$

Slop of the curve at
$$\phi = 0$$

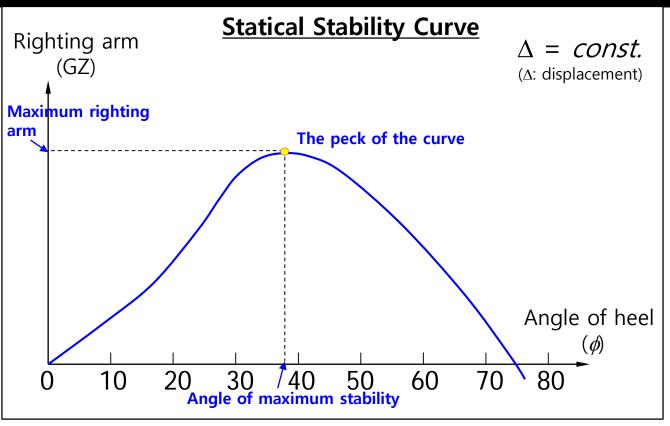
$$\lim_{\phi \to 0} \frac{GZ}{\phi} = \lim_{\phi \to 0} \frac{GM \sin \phi}{\phi}$$

$$= \frac{GM}{1(rad)} , \left(\lim_{\phi \to 0} \frac{\sin \phi}{\phi} = 1 \right)$$

$$= \frac{GM}{1(rad)} , \left(\lim_{\phi \to 0} \frac{\sin \phi}{\phi} = 1 \right)$$

If the righting arm continues to increase at the same rate as at the origin, it would be equal to GM at an inclination of 1 radian or 57.3°.

Significance of the Statical Stability Curve (3/5) (2) The Peak of the Curve



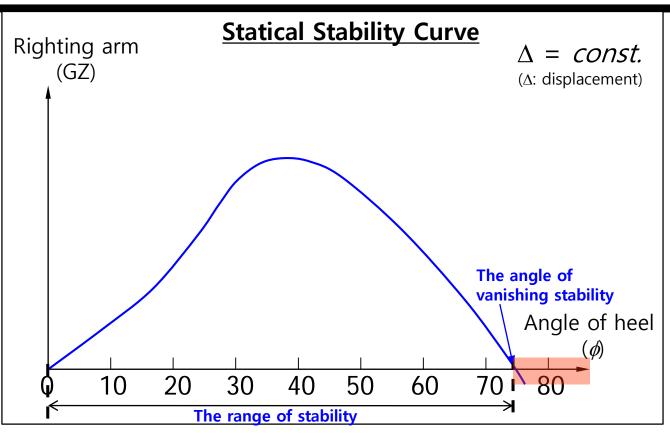
The peak of a statical stability curve identifies two quantities that are the maximum righting arm and the angle of maximum stability.

The product of the displacement and the maximum righting arm is the maximum heeling moment that the ship can experience without capsizing.

In other words, if the ship is forced over to the angle of maximum stability by an externally applied constant heeling moment, the ship will capsize.

Significance of the Statical Stability Curve (4/5)

(3) The Range of Stability and the Angle of Vanishing Stability

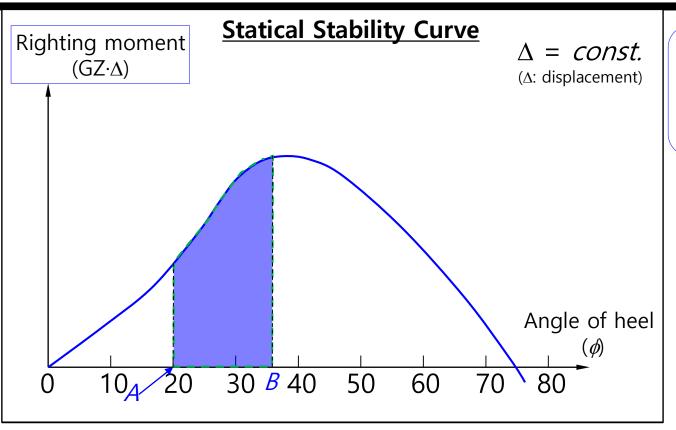


The range of stability is the range over which the ship has positive righting arms.

The angle of vanishing stability is the angle of heel at which the righting arm returns to zero.

If the ship heels beyond this angle, the moment caused by gravitation force and buoyant force will act to capsize, rather than to right, the ship.

Significance of the Statical Stability Curve (5/5) (4) The Area under the Curve



The statical stability curve can be the plot of righting moment against angle of heel for a given condition by the product of the displacement and the righting arm.

The area under the curve, such as between angle A and angle B, represents the work required to heel the ship from angle A to angle B.

✓ Derivation

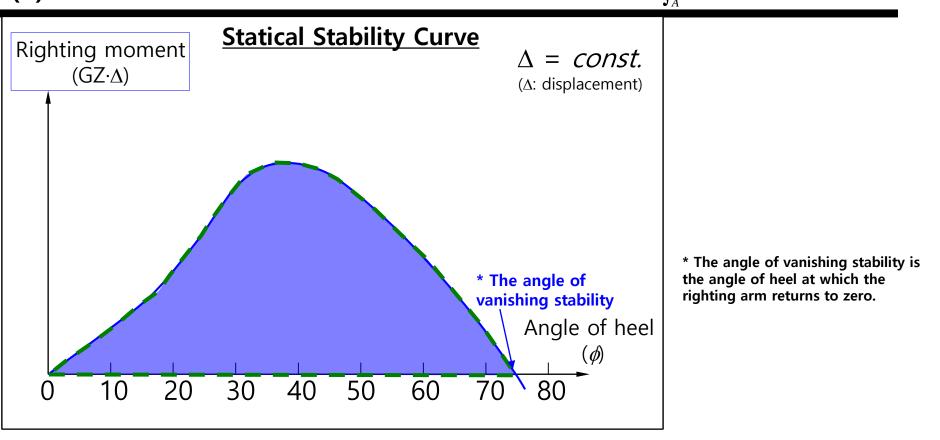
When M is the moment at any angle of heel(ϕ),

the work required to rotate the ship against this moment(M) through an angle($d\phi$)

 $= M d\phi$

The work(W) required to rotate from angle A to angle B: $W = \int_{A}^{B} M \, d\phi$, (ϕ in radinas)

Significance of the Statical Stability Curve (5/5) The work(W) required to rotate from A to B: (4) The Area under the Curve – Total Area $W = \int_{A}^{B} M \, d\phi$, (\$\phi\$ in radinas)

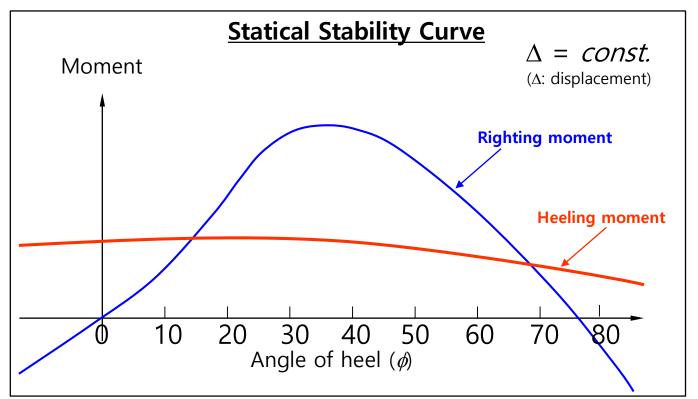


The total area between the statical stability curve (at zero degree to the angle of vanishing stability) and the horizontal axis represents the total work to capsize the ship from the upright position.

This is often referred to as dynamic stability.

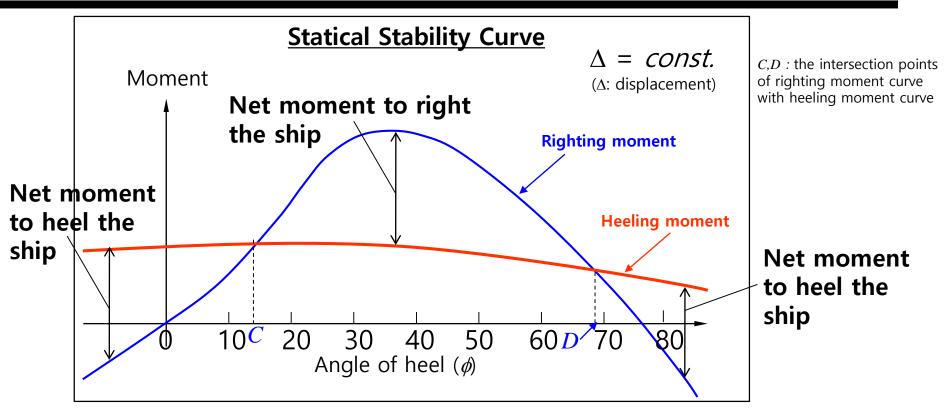
There are the nature of certain heeling force acting on the ship.

These forces may be caused by action of the beam winds, action of the waves in rolling the ship, lifting of heavy weights over the side, high-speed turn, etc.



By superimposing various heeling moment caused by these forces on a curve of righting moment, statical stability of the ship in any condition can be evaluated.

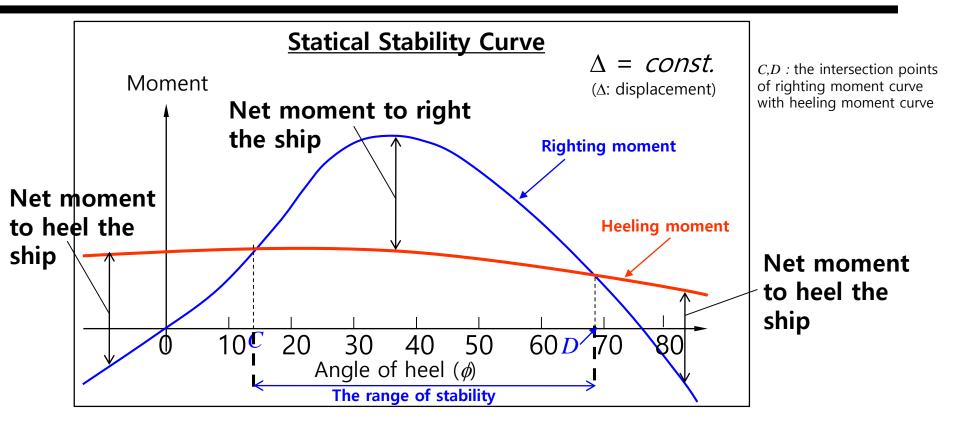
- Significance between the Statical Stability Curve and Heeling Moments Curve (1/3)



At angle C and D, the heeling moment is equal to the righting moment and the forces are in equilibrium.

The vertical distance between the heeling moment and righting moment curves at any angles represents the net moment acting at that angle either to heel or right the ship, depending on the relative magnitude of the righting and heeling moments.

- Significance between the Statical Stability Curve and Heeling Moments Curve (2/3)

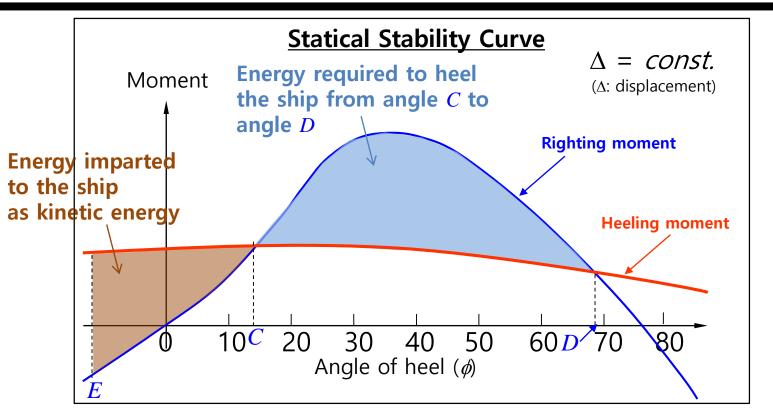


If the ship is heeled to angle C, an inclination in either direction will generate a moment tending to restore the ship to angle C.

If the ship is heeled to beyond angle D, the ship will capsize.

The range of stability is decreased by the effect of the heeling moment.

- Significance between the Statical Stability Curve and Heeling Moments Curve (3/3)



Assumption) The ship was heeled to the left to angle E, has come to rest, and is about to be hilled in the opposite direction.

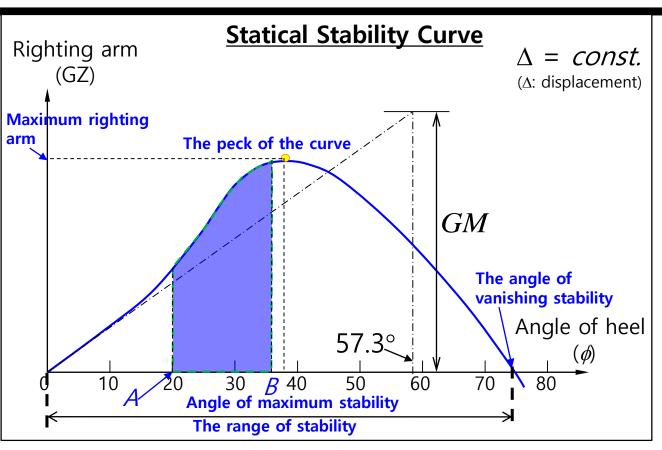
If the energy imparted to the ship as kinetic energy is larger than the energy required to heel the ship from angle C to angle D, the ship will capsize.

To reduce the danger of capsizing under these conditions, the area between the heeling and righting moment curves and between angle C and angle D should be greater, by some margin, than that between angle E and angle C.

20

Evaluation of Stability

[Review] Statical Stability Curve



✓ Slope of the curve at zero degree: metacentric height(GM)

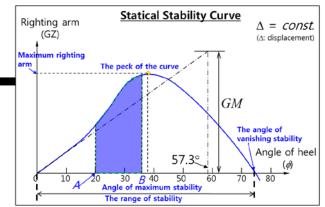
$$GZ \approx GM \sin |\phi|$$

- √The peak of a statical stability curve: maximum righting arm, angle of maximum stability
- √The product of the displacement and the maximum righting arm: the maximum heeling moment that the ship can experience without capsizing
- √The range of stability: the range over which the ship has positive righting arms
- ✓ The area under the curve, such as between angle *A* and angle *B*: the work required to heel the ship from angle *A* to angle *B*.



What criteria is considered to evaluate the ship's stability? What is satisfactory stability? How much stable a ship must be?

Stability Criteria in General



☑ Various researchers and regulatory bodies prescribed criteria for deciding if the stability is satisfactory.

In this chapter, we present <u>examples of such criteria</u> based on <u>consideration of actual shape and characteristics of the curves</u> of righting and heeling moment (or arm) for an <u>undamaged ship</u> (intact ship) through <u>large angles of heel</u>.

☑ Features of the curves that warrant consideration from a purely static viewpoint are:

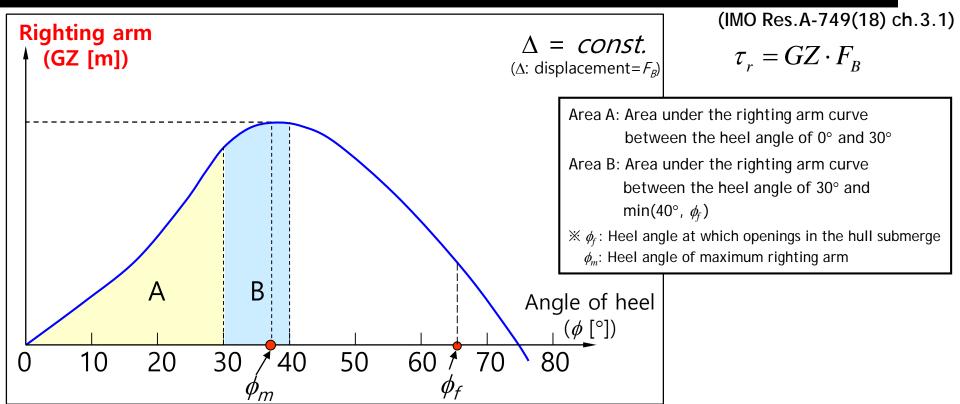
Static considerations

- The angle of steady heel
- The range of positive stability
- The relative magnitudes of the heeling arm and the maximum righting arm.

The work and energy considerations (dynamic stability)

Stability Criteria

- IMO Regulations for Intact Stability



IMO Regulations for Intact Stability

- (a) Area A ≥ 0.055 (m·rad)
- (b) Area A + B ≥ 0.09 (m·rad)
- (c) Area B \geq 0.030 (m·rad)
- (d) $GZ \ge 0.20$ (m) at an angle of heel equal to or greater than 30° .
- (e) GZ_{max} should occur at an angle of heel equal to or greater than 25°.
- (f) The initial metacentric height GM_o should not be less than 0.15 (m).

 After receiving the approval for the intact and damage stability of IMO regulation from owner and classification society, ship construction can be proceed.

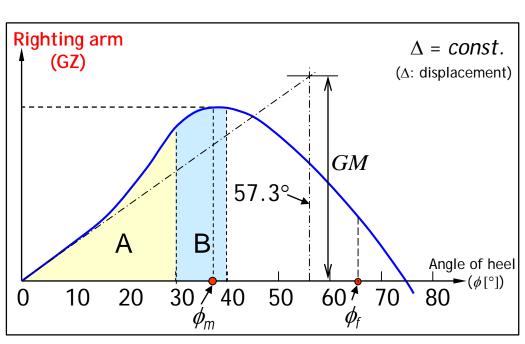


Merchant Ship Stability Criteria

- IMO Regulations for Intact Stability

(IMO Res.A-749(18) ch.3.1)

☑ IMO recommendation on intact stability for passenger and cargo ships



Area A: Area under the righting arm curve between the heel angle of 0° and 30° Area B: Area under the righting arm curve between the heel angle of 30° and $\min(40^{\circ}, \phi_f)$ $\times \phi_f$: Heel angle at which openings in the hull submerge ϕ_m : Heel angle of maximum righting arm



※ After receiving approval of calculation of IMO regulation from Owner and Classification Society, ship construction can proceed.

IMO Regulations for Intact Stability

- (a) Area A ≥ 0.055 (m·rad)
- (b) Area A + B \geq 0.09 (m·rad)
- (c) Area B \geq 0.030 (m·rad)
- (d) GZ ≥ 0.20 (m) at an angle of heel equal to or greater than 30°
- (e) GZ_{max} should occur at an angle of heel preferably exceeding 30° but not less than 25° .
- (f) The initial metacentric height GM_o should not be less than 0.15 (m).

The work and energy considerations (dynamic stability)

Static considerations

Merchant Ship Stability Criteria

- IMO Regulations for Intact Stability

☑ Special Criteria for Certain Types of Ships

- Containerships greater than 100 m

These requirements apply to containerships greater than 100 m. They may also be applied to other cargo ships with considerable flare or large water plane areas. The administration may apply the following criteria instead of those in paragraphs of previous slide.

IMO Regulations for containerships greater than 100 m

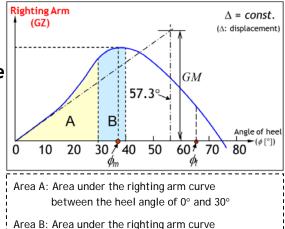
- (a) Area A $\geq 0.009/C$ (m-rad)
- (b) Area A + B \geq 0.016/C (m-rad)
- (c) Area B $\geq 0.006/C$ (m-rad)
- (d) GZ ≥ 0.033/C (m) at an angle of heel equal to or greater than 30°
- (e) $GZ_{max} \ge 0.042/C$ (m)
- (f) The total area under the righting arm curve (GZ curve) up to the angle of flooding ϕ_F should not be less than 0.029/C (m-rad)

In the above criteria the form factor C should be calculated using the formula and figure on the right-hand side.

$$C = \frac{dD'}{B_m^2} \sqrt{\frac{d}{KG}} \left(\frac{C_B}{C_W}\right) \sqrt[2]{\frac{100}{L}} \quad \text{where, d: Mean draught (m)} \\ D' = D + h \frac{2b - B_D}{B_D} \left(\frac{2\sum l_H}{L}\right) \quad \text{as defined in figure on the right-hand side.}$$

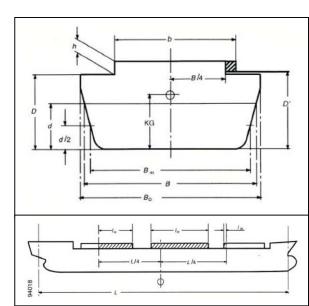
- D: Moulded depth of the ship (m)
- B: Moulded breadth of the ship (m)
- KG: Height of the centre of gravity in (m) above the keel not to be taken as less than d
- C_R: Block coefficient
- C_W: Water plane coefficient

(IMO Res.A-749(18) ch.4.9)



Area B: Area under the righting arm curve between the heel angle of 30° and min(40°, ϕ_j)

 $(*) \times \phi_{r}$: Heel angle at which openings in the hull submerge ϕ_{m} : Heel angle of maximum righting arm



Design Criteria Applicable to All Ships

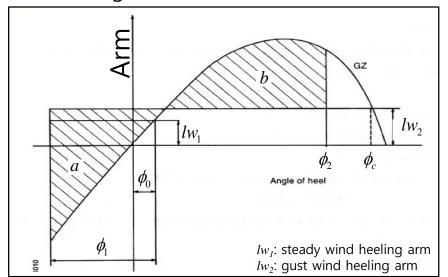
- IMO Regulations for Severe Wind and Rolling Criteria (Weather Criteria)

☑ Scope

(IMO Res.A-749(18) ch.3.2)

The weather criteria should govern the minimum requirements for passenger or cargo ships

of 24 m in length and over.



 ϕ_0 : Angle of heel under action of steady wind

 ϕ_l : Angle of roll to windward due to wave action

 ϕ_2 : Angle of down flooding(ϕ_f) or 50°, whichever is less where,

 ϕ_f : Angle of heel at which openings in the hull submerge

 ϕ_c : Angle of the second intersection between wind heeling arm and GZ(righting arm) curves

Area a: The shaded area between angle ϕ_1 and the first intersection of righting arm curve with heeling arm curve

Area b: The shaded area between the first intersection of righting arm curve with heeling arm curve and angle ϕ_2

IMO Regulations for Severe Wind and Rolling Criteria (Weather Criteria)

(a) ϕ_0 should be limited to 16° or 80% of the angle of deck edge immersion(ϕ_0), whichever is less.

The ship is subjects to a steady wind pressure acting perpendicular to the ship's center line which results in a steady wind heeling arm (lw_l) .

 $lw_1 = \frac{P \cdot A \cdot Z}{1000 \cdot g \cdot \nabla} (m)$ $(P = 504 \ N / m^2, \ g = 9.81 m / s^2)$

A: Lateral projected area above water line.

Z: Vertical distance from the center of wind pressure to the center of water pressure

(b) Under these circumstances, *area b* should be equal to or greater than *area a*. \Box The ship is subjected to a gust wind pressure which results in a gust wind heeling arm (lw_2) .

The work and energy considerations (dynamic stability)

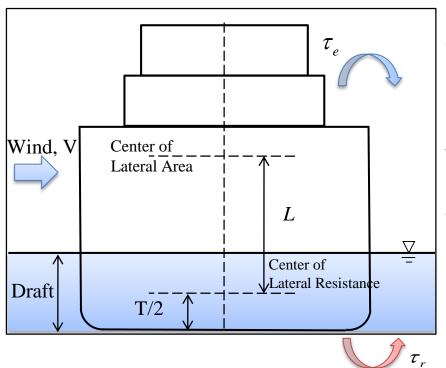
$$lw_2 = 1.5 \cdot lw_1 \ (m)$$

War Ship Stability Criteria

- U.S. Navy Criteria (1/2)

☑ General U.S. Navy criteria are intended to ensure the adequacy of stability of all types and sizes of naval ships, as evidenced by sufficient righting energy to withstand various types of upsetting of heeling moments.

(Example) Beam Winds Combined with Rolling



When winds of high velocity exist, beam winds and rolling are considered simultaneously.

If the water were still, the ship would require only sufficient righting moment to overcome the heeling moment produced by the action of the wind on the ship's "sail area".

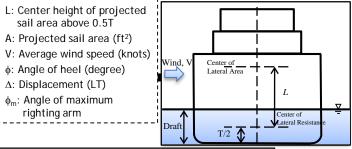
However, when the probability of wave action is taken into account, an additional allowance of dynamic stability is required to absorb the energy imparted to the ship by the rolling motion.

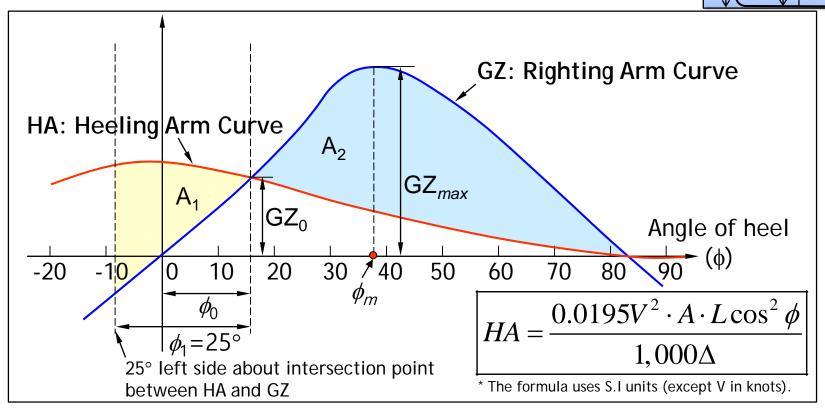
- L: Center height of projected sail area above 0.5T
- A: Projected sail area (ft²), V: average wind speed (knots)
- ϕ : Angle of heel (degree), Δ : Displacement (LT)
- $\varphi_m :$ Angle of maximum righting arm (degree)

War Ship Stability Criteria

- U.S. Navy Criteria (2/2)

☑ Stability is considered satisfactory if:





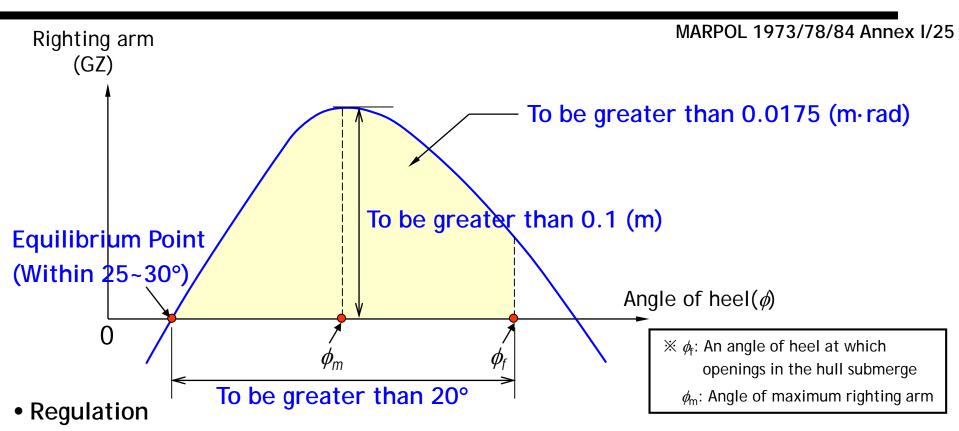
Regulation

(a) $GZ_0 \le 0.6 \cdot GZ_{max}$: Static considerations

(b) $A_2 \ge 1.4 \cdot A_1$: The work and energy considerations (dynamic stability)

Stability Criteria of Damage Stability

MARPOL Regulation for Damage Stability

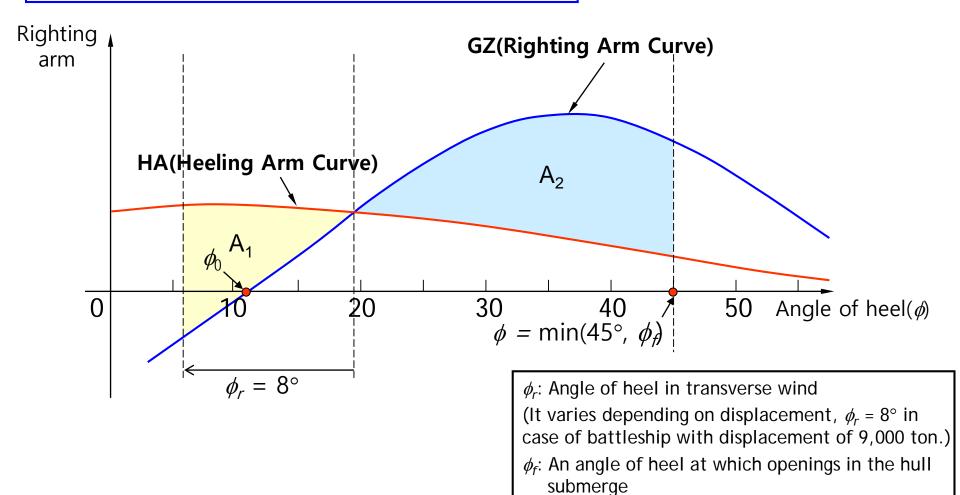


- (a) The final waterline shall be below the lower edge of any opening through which progressive flooding may take place.
- (b) The angle of heel due to unsymmetrical flooding shall not exceed 25 degrees, provided that this angle may be increased up to 30 degrees if no deck edge immersion occurs.
- (c) The statical stability curve has at least a range of 20 degrees beyond the position of equilibrium in association with a maximum residual righting arm of at least 0.1 meter within the 20 degrees range
- (d) The area under the curve within this range shall not be less than 0.0175 meter-radians.

Damage Stability Criteria in Battleship*

Regulation

 ϕ_0 (Initial Angle of Heel) $\leq 15^\circ$, $A_2 \geq 1.4 \cdot A_1$



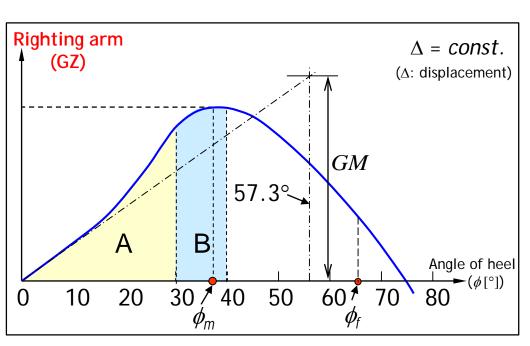
[Example] 7,000 TEU Container Carrier at Homo. Scantling Arrival Condition (14mt)

Merchant Ship Stability Criteria

- IMO Regulations for Intact Stability

(IMO Res.A-749(18) ch.3.1)

☑ IMO recommendation on intact stability for passenger and cargo ships



Area A: Area under the righting arm curve between the heel angle of 0° and 30° Area B: Area under the righting arm curve between the heel angle of 30° and $\min(40^{\circ}, \phi_f)$ $\not = \phi_f$: Heel angle at which openings in the hull submerge ϕ_m : Heel angle of maximum righting arm



※ After receiving approval of calculation of IMO regulation from owner and classification society, ship construction can proceed.

IMO Regulations for Intact Stability

- (a) Area A ≥ 0.055 (m·rad)
- (b) Area A + B \geq 0.09 (m·rad)
- (c) Area B \geq 0.030 (m·rad)
- (d) GZ ≥ 0.20 (m) at an angle of heel equal to or greater than 30°
- (e) GZ_{max} should occur at an angle of heel preferably exceeding 30° but not less than 25° .
- (f) The initial metacentric height GM_o should not be less than 0.15 (m).

The work and energy considerations (dynamic stability)

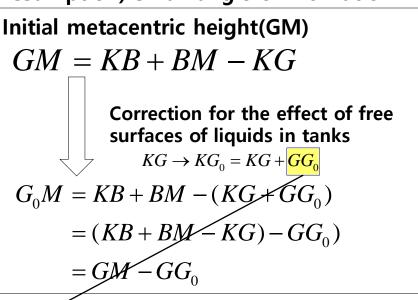
Static considerations

Effect of Free Surfaces of Liquids in Tanks

(IMO Res.A-749(18) ch.3.3)

For all conditions, the initial metacentric height(GM) and the stability curves should be corrected for the effect of free surfaces of liquids in tanks.

(Assumption) Small angle of inclination



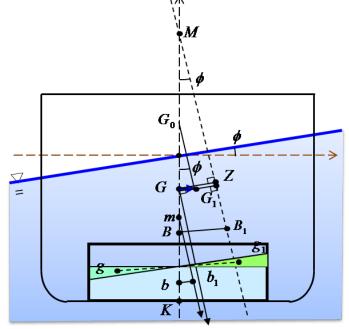
$$GG_0 = \frac{\sum \rho_F \cdot i_T}{\rho_{SW} \nabla}$$
: Free surface moment

 i_{τ} : Moment of inertia of liquid plane area in tank about longitudinal axis

 $\rho_{\rm F}$: Density of liquid in tank

 ρ_{SW} : Density of sea water

 ∇ : Displacement volume of the ship



- G: Center of total mass
- G_0 : Virtual risen center of gravity
- G_1 : New position of center of total mass
- G_1 ': The intersection of the line GZ with G_0G_1
- B: Center of buoyancy
- B_1 : New position of center of buoyancy after the ship has been inclined
- q: Center of the emerged volume
- q_1 : Center of the submerged volume
- b: Center of liquid in tank
- b_1 : New position of center of liquid in tank
- m: Metacenter of cargo hold

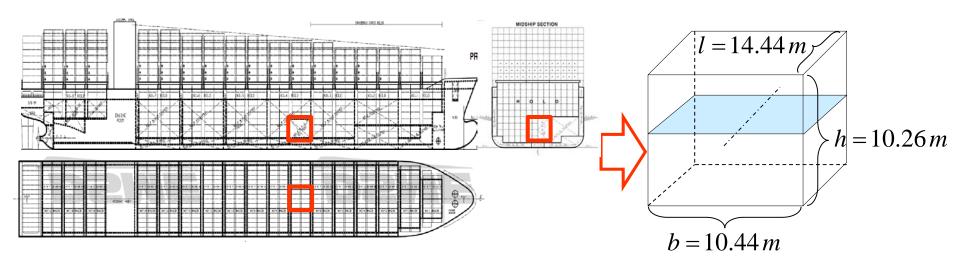
How can you get the value of free surface moment?

Effect of Free Surfaces of Liquids in Tanks

- 7,000 TEU Container Carrier at Homo. Scantling Arrival Condition

$$GG_0 = \frac{\sum_{\rho_F} \rho_F \cdot i_T}{\rho_{SW} \nabla}$$
: Free surface moment

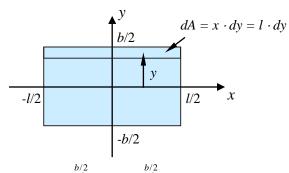
(Example) Free surface moment of No. 4 heavy fuel oil tank (center) in No. 4 hold



$$i_T = \frac{l \cdot b^3}{12} = \frac{(14.44) \cdot (10.44)^3}{12} = 1,369(m^4)$$

Free surface moment_{NO.4HFOT(C)} = $\rho_F \cdot i_T$ $=0.98 \cdot 1,369$ $=1,342(m \cdot ton)$

 i_{τ} : Moment of inertia of liquid plane area in tank about longitudinal axis $\rho_{\rm E}$: Density of liquid in tank (heavy fuel oil) = 0.98 (ton/m³)



$$i_{T} = i_{x} = \int_{-b/2}^{b/2} y^{2} dA = \int_{-b/2}^{b/2} y^{2} (l \cdot dy)$$

$$= I \int_{-b/2}^{b/2} y^{2} dy = \frac{l \cdot b^{3}}{b^{3}}$$

$$= l \int_{-1}^{b/2} y^2 dy = \frac{l \cdot b^3}{12}$$

Effect of Free Surfaces of Liquids in Tanks

- 7,000 TEU Container Carrier at Homo. Scantling Arrival Condition

$$GG_0 = \frac{\sum \rho_F \cdot i_T}{\rho_{SW} \nabla}$$
 Free surface moment

Free surface moment_{NO.4HFOT(C)} = 1,342 $(m \cdot ton)$ –

Free surface moments of all heavy fuel oil tanks

WEIGHT ITEMS	FILL.	S.G	WEIGHT	L.C.G	V.C.G	F.S.N
NO4 HFOT(C)	0.00	0.9800	0.0	179.625	7.184	1341.
NO4 HFOT(P)	0.00	0.9800	0.0	179.625	7.870	1822.
NO4 HFOT(S)	0.00	0.9800	0.0	179.625	7.870	1822.
NO5 DEEP HFOT (P)	0.00	0.9800	0.0	129.975	11.297	299.
NO5 DEEP HFOT(S)	0.00	0.9800	0.0	129.975	11.297	299.
NO6 DEEP HFOT (P)	0.00	0.9800	0.0	101.075	11.297	299.
NO6 DEEP HFOT(S)	0.00	0.9800	0.0	101.075	11.297	299.
NO7 DEEP HFOT (P)	54.06	0.9800	222.1	72.100	7.931	296.
NO7 DEEP HFOT(S)	54.06	0.9800	222.1	72.100	7.931	296.
NO1 HFO SETT.TK	98.00	0.9800	89.0	69.900	17.500	83.
NO2 HFO SETT.TK	98.00	0.9800	89.0	69.900	17.500	83.
NO1 HFO SERV.TK	98.00	0.9800	89.0	69.900	17.500	83.
NO2 HFO SERV.TK	98.00	0.9800	89.0	69.900	17.500	83.

Free surface moment $_{Total\ HFOT} = 7,109.2 (m \cdot ton)$

Effect of Free Surfaces of Liquids in Tanks

- 7,000 TEU Container Carrier at Homo. Scantling Arrival Condition

$$GG_0 = \frac{\sum \rho_F \cdot i_T}{\rho_{SW} \nabla}$$
 Free surface moment

Calculating free surface moment of other tank at homo. scantling arrival condition(14mt)

DTTT					
			L.C.G		F.S.N
(%)		(MT)	(M)	(M)	
100.00	1.0250	560.1	228.280	2.640	0.0
100.00	1.0250	560.1	228.280	2.640	0.0
100.00	1.0250	940.7	200.357	2.015	0.0
100.00	1.0250	940.7	200.357	2.015	0.0
100.00	1.0250	1070.1	201.907	11.873	0.0
100.00	1.0250	1070.1	201.907	11.873	0.0
100.00	1.0250	1266.8	173.078	1.923	0.0
100.00	1.0250	1266.8	173.078	1.923	0.0
100.00	1.0250	1145.4	143.534	1.690	0.0
100.00	1.0250	1145.4	143.534	1.690	0.0
100.00	1.0250	977.8	143.500	12.369	24.3
100.00	1.0250	977.8	143.500	12.369	24.3
100.00	1.0250	1143.6	114.585	1.690	0.0
100.00	1.0250	1143.6	114.585	1.690	0.0
100.00	1.0250	1031.2	85.978	1.778	0.0
100.00	1.0250	1031.2	85.978	1.778	0.0
		16271.3	156.848	4.463	48.
		41.6	45.600	12.757	20.
		800.0	71.121	12.188	7109.3
		40.0	66.300	11.175	60.
		47.4	66.318	7.861	14.
					0.
					7253.
		27710	122.656		
					7253.3
	100.00 100.00 100.00 100.00 100.00 100.00 100.00 100.00 100.00 100.00	100.00 1.0250 100.00 1.0250	100.00 1.0250 560.1 100.00 1.0250 940.7 100.00 1.0250 940.7 100.00 1.0250 1070.1 100.00 1.0250 1266.8 100.00 1.0250 1266.8 100.00 1.0250 1145.4 100.00 1.0250 977.8 100.00 1.0250 977.8 100.00 1.0250 143.6 100.00 1.0250 1143.6 100.00 1.0250 1031.2 100.00 1.0250 1031.2 100.00 1.0250 1031.2	100.00 1.0250 560.1 228.280 100.00 1.0250 560.1 228.280 100.00 1.0250 940.7 200.357 100.00 1.0250 940.7 200.357 100.00 1.0250 1070.1 201.907 100.00 1.0250 1070.1 201.907 100.00 1.0250 1266.8 173.078 100.00 1.0250 1266.8 173.078 100.00 1.0250 1145.4 143.534 100.00 1.0250 1145.4 143.534 100.00 1.0250 977.8 143.500 100.00 1.0250 977.8 143.500 100.00 1.0250 1143.6 114.585 100.00 1.0250 1143.6 114.585 100.00 1.0250 1031.2 85.978 100.00 1.0250 1031.2 85.978 100.00 1.0250 1031.2 85.978 100.00 1.0250 1031.2 85.978 100.00 1.0250 1031.2 85.978 16271.3 156.848	100.00 1.0250 560.1 228.280 2.640 100.00 1.0250 940.7 200.357 2.015 100.00 1.0250 940.7 200.357 2.015 100.00 1.0250 1070.1 201.907 11.873 100.00 1.0250 1070.1 201.907 11.873 100.00 1.0250 1266.8 173.078 1.923 100.00 1.0250 1266.8 173.078 1.923 100.00 1.0250 1145.4 143.534 1.690 100.00 1.0250 1145.4 143.534 1.690 100.00 1.0250 977.8 143.500 12.369 100.00 1.0250 977.8 143.500 12.369 100.00 1.0250 1143.6 114.585 1.690 100.00 1.0250 1143.6 114.585 1.690 100.00 1.0250 1031.2 85.978 1.778 100.00 1.0250 1031.2 85.978 1.778 16271.3 156.848 4.463 41.6 45.600 12.757 800.0 71.121 12.188 40.0 66.300 11.175 47.4 66.318 7.861

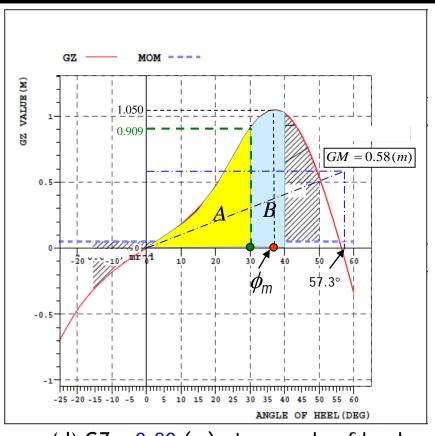
$$GG_0 = \frac{\sum \rho_F \cdot i_T}{\rho_{SW} \nabla} = \frac{7,253.3}{120,038} = 0.06(m)$$

Correction for effect of free surface of liquid in tanks is as follows:

$$G_0M = GM - GG_0$$
 Initial metacentric height(GM) at this loading condition = 0.64(m)
$$= 0.64 - 0.06 = 0.58(m)$$

IMO Regulations for General Intact Stability Criteria

- 7,000 TEU Container Carrier at Homo. Scantling Arrival Condition



Area A: Area under the righting arm curve between the heel angle of 0° and 30°

Area B: Area under the righting arm curve between the heel angle of 30° and min(40°, ϕ_f)

 $\times \phi_f$: Heel angle at which openings in the hull

$$\phi_f = 50^{\circ}$$

Area B: Area under the righting arm curve between the heel angle of 30° and 40°

 $\times \phi_m$: Heel angle of maximum righting arm = 36.8°

- (a) Area A \geq 0.055 (m·rad) $Area \ A = 0.148(m \cdot rad) \geq 0.055(m \cdot rad)$
- (b) Area A + B \geq 0.09 (m·rad) $Area \ A + B = 0.301(m \cdot rad) \geq 0.090(m \cdot rad)$
- (c) Area B ≥ 0.030 (m·rad) $Area \ B = 0.153(m \cdot rad) \geq 0.030(m \cdot rad)$
- (d) GZ \geq 0.20 (m) at an angle of heel equal to or greater than 30° $GZ_{atangle\ of\ heel=30^{\circ}}=0.909(m)\geq0.20(m)$
- (e) GZ_{max} should occur at an angle of heel preferably exceeding 30° but not less than 25°. $\phi_m = 36.8^{\circ} \ge 25^{\circ}$
- (f) The initial metacentric height G_0M should not be less than 0.15 (m).

$$GM = 0.58 m \ge 0.15(m)$$

All regulations are satisfied.

Merchant Ship Stability Criteria

- IMO Regulations for Intact Stability

☑ Special Criteria for Certain Types of Ships

- Containerships greater than 100 m

These requirements apply to containerships greater than 100 m. They may also be applied to other cargo ships with considerable flare or large water plane areas. The administration may apply the following criteria instead of those in paragraphs of previous slide.

IMO Regulations for containerships greater than 100 m

- (a) Area A $\geq 0.009/C$ (m-rad)
- (b) Area A + B \geq 0.016/C (m-rad)
- (c) Area B $\geq 0.006/C$ (m-rad)
- (d) $GZ \ge 0.033/C$ (m) at an angle of heel equal to or greater than 30°
- (e) $GZ_{max} \ge 0.042/C$ (m)
- (f) The total area under the righting lever curve (GZ curve) up to the angle of flooding ϕ_F should not be less than 0.029/C (m-rad)

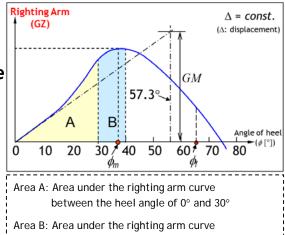
In the above criteria the form factor C should be calculated using the formula and figure on the right-hand side.

$$C = \frac{dD'}{B_m^2} \sqrt{\frac{d}{KG}} \left(\frac{C_B}{C_W}\right) \sqrt[2]{\frac{100}{L}} \quad \text{where, d: Mean draught (m)}$$

$$D' = D + h \frac{2b - B_D}{B_D} \left(\frac{2\sum l_H}{L}\right) \quad \text{as defined in figure on the right-hand side.}$$

- D: Moulded depth of the ship (m)
- B: Moulded breadth of the ship (m)
- KG: Height of the centre of gravity in (m) above the keel not to be taken as less than d
- C_R: Block coefficient
- C_W: Water plane coefficient

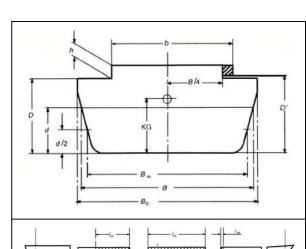
(IMO Res.A-749(18) ch.4.9)



between the heel angle of 30° and min(40°, ϕ_r)

 $\times \phi_{f}$: Heel angle at which openings in the hull submerge

φ...: Heel angle of maximum righting arm



IMO Regulations for Intact Stability: Containerships greater than 100 m - 7,000 TEU Container Carrier at Homo. Scantling Arrival Condition

In the above criteria the form factor C should be calculated using the formula and figure on the right-hand side.

$$C = \frac{dD'}{B_m^2} \sqrt{\frac{d}{KG}} \left(\frac{C_B}{C_W}\right) \sqrt[2]{\frac{100}{L}} \quad \text{where, d: Mean draught (m)}$$

$$D' = D + h \frac{2b - B_D}{B_D} \left(\frac{2\sum l_H}{L}\right)$$

KG: Height of the centre of gravity in m above the keel D: Moulded depth of the ship (m)

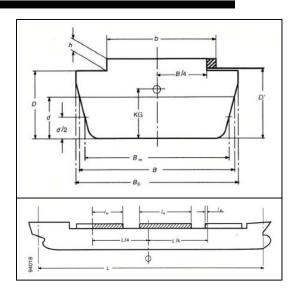
(m) not to be taken as less than d

B: Moulded breadth of the ship (m)

C_R: Block coefficient

C_w: Water plane coefficient

••	•							
d		D'		D	В	KG	CB	CW
	14.15		25.452125	24.2	40	17.852	0.71693	0.89044
						KG0		
						17.913		
h		b		BD	IH	L		
	1.8		35.9	40	126	288		
С						72		A 11
0.076	54965							> All I



egulations are satisfied.

IMO regulations for containerships greater than 100 m

(a) Area
$$A \ge 0.009/C = 0.117$$
 (m·rad)

Area
$$A = 0.148(m \cdot rad)$$

(b) Area A + B
$$\geq$$
 0.016/C = 0.209 (m·rad)

Area
$$A + B = 0.301(m \cdot rad)$$

(c) Area B
$$\geq$$
 0.006/C = 0.078 (m·rad)

Area
$$B = 0.153(m \cdot rad)$$

(d)
$$G7 > 0.033/C = 0.431$$
 (m) at an angle of heel

(d)
$$GZ \ge 0.033/C = 0.431$$
 (m) at an angle of heel equal to or greater than 30° $GZ_{at angle \ of \ heel = 30^{\circ}} = 0.909(m)$

(e)
$$GZ_{max} \ge 0.042/C = 0.549$$
 (m)

$$GZ_{max} = 1.050(m)$$

(f) The total area under the righting arm curve (GZ curve) up to $Area\ GZ\ Curve = 0.4644(m \cdot rad)$ the angle of flooding ϕ_f should not be less than 0.029/C = 0.379 (m·rad)

Design Criteria Applicable to All Ships

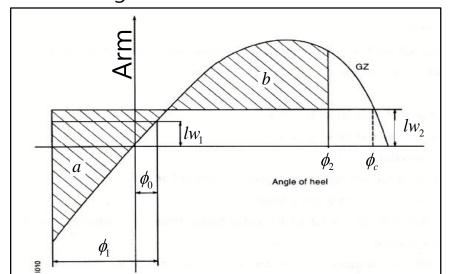
- IMO Regulations for Severe Wind and Rolling Criteria (Weather Criteria)

☑ Scope

(IMO Res.A-749(18) ch.3.2)

The weather criteria should govern the minimum requirements for passenger or cargo ships

of 24 m in length and over.



 ϕ_0 : Angle of heel under action of steady wind

 ϕ_i : Angle of roll to windward due to wave action

 ϕ_2 : Angle of down flooding(ϕ_f) or 50°, whichever is less where,

 ϕ_f : Angle of heel at which openings in the hull submerge

 ϕ_c : Angle of the second intersection between wind heeling arm and GZ(righting arm) curves

Area a: The shaded area between angle ϕ_1 and the first intersection of righting arm curve with heeling arm curve

Area b: The shaded area between the first intersection of righting arm curve with heeling arm curve and angle ϕ_2

IMO Regulations for Severe Wind and Rolling Criteria (Weather Criteria)

(a) ϕ_0 should be limited to 16° or 80% of the angle of deck edge immersion(ϕ_i), whichever is less.

The ship is subjects to a steady wind pressure acting perpendicular to the ship's center line which results in a steady wind heeling arm (lw_l) . $lw_l = \frac{P \cdot A \cdot Z}{1000 \cdot g \cdot \nabla} \quad (m) \qquad (P = 504 \ N / m^2, \ g = 9.81 m / s^2)$

A: Lateral projected area above water line.

Z: Vertical distance from the center of wind pressure to the center of water pressure

(b) Under these circumstances, *area b* should be equal to or greater than *area a*. \Box The ship is subjected to a gust wind pressure which results in a gust wind heeling arm (lw_2) .

The work and energy considerations (dynamic stability)

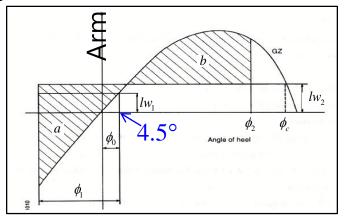
$$lw_2 = 1.5 \cdot lw_1 \ (m)$$

IMO Regulations for Severe Wind and Rolling Criteria (Weather Criteria) - 7,000 TEU Container Carrier at Homo. Scantling Arrival Condition

(a) ϕ_0 is suggested to 16° or 80% of the angle of deck edge immersion(ϕ_f), whichever is less.

First, we have to know the value of a steady wind heeling arm (lw_1) .

$$lw_1 = \frac{P \cdot A \cdot Z}{1000 \cdot g \cdot \nabla} \quad (m) \qquad P = 504(N/m^2), \ g = 9.81(m/s^2)$$



- A: Lateral projected area above water line = $9.871(m^2)$
- Z: Vertical distance from center of wind pressure to center of water pressure
- = (Vertical distance from base line to center of wind pressure) (Vertical distance from base line to center of water pressure)
- = 29.749 7.714 = 22.04(m)

$$lw_1 = \frac{P \cdot A \cdot Z}{1000 \cdot g \cdot \nabla} = \frac{504 \cdot 9,871 \cdot 22.04}{1000 \cdot 9.81 \cdot 117,110} = 0.1(m)$$

Second, the angle of heel under action of steady wind(ϕ_0) is angle of intersection between GZ curve and lw₁. The ϕ_0 is 4.5°. The regulation (a) is satisfied.

IMO Regulations for Severe Wind and Rolling Criteria (Weather Criteria) - 7,000 TEU Container Carrier at Homo. Scantling Arrival Condition

(b) Under these circumstances, area b should be equal to or greater than area a.

First, we have to know the value of a gust wind heeling arm (lw_2) .

$$lw_2 = 1.5 \cdot lw_1$$

= 1.5 \cdot 0.1 = 0.15 (m)

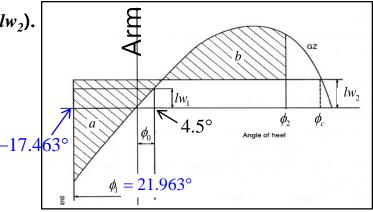
Second, we have to know the angle of roll (ϕ_i) .

$$\phi_{1} = 109 \cdot k \cdot x_{1} \cdot x_{2} \cdot \sqrt{r \cdot s}$$

$$= 109 \cdot 0.99 \cdot 0.93 \cdot 1 \cdot \sqrt{0.887 \cdot 0.054}$$

$$= 21.963^{\circ}$$





Area a: The shaded area between angle ϕ_1 and the first intersection of righting arm curve with heeling arm curve Area b: The shaded area between the first intersection of righting arm curve with heeling arm curve and angle ϕ_2

Third, ϕ_2 is angle of down flooding(ϕ_f) or 50°, whichever is less. We assume that ϕ_2 for this container carrier is 50°.

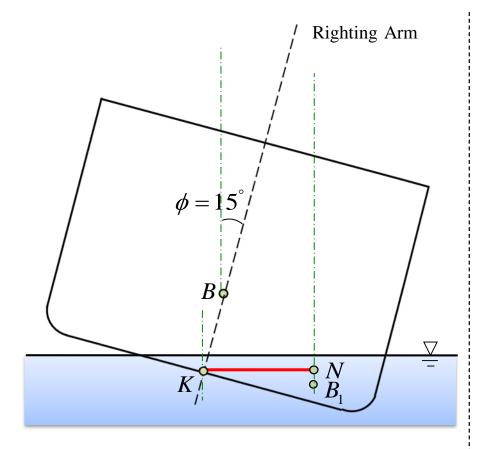
Forth, the area a is 0.0384 (m·rad) and the area b is 0.3661 (m·rad).



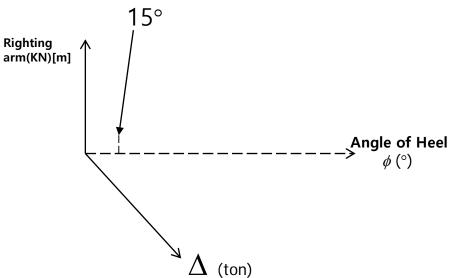
Reference Slides

Statical Stability Curve

- Calculating for a Number of Waterline at Various Drafts and Angles of Heel (1/4)



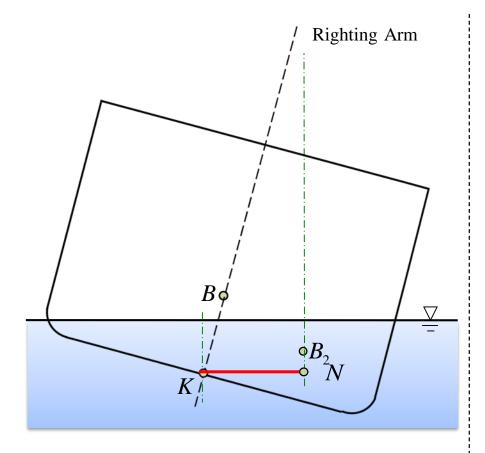
- *K*: Keel
- B: Center of buoyancy at initial position
- B_i: New position of center of buoyancy corresponding displacement
- N: The intersection point of a vertical line through the new position of the center of buoyancy with the longitudinally parallel line to a waterline through the point K



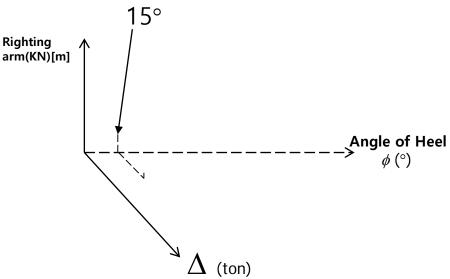
Righting arm(KN) is calculated for a number of waterlines at various drafts and angles of heel.

✓ Assumption: There is a complete watertight envelope consisting of bottom, side shell and weather deck*.

- Calculating for a Number of Waterline at Various Drafts and Angles of Heel (2/4)



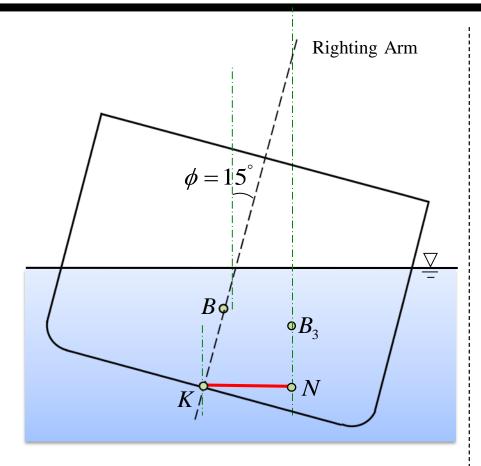
- *K*: Keel
- B: Center of buoyancy at initial position
- B₂: New position of center of buoyancy corresponding displacement
- N: The intersection point of a vertical line through the new position of the center of buoyancy with the longitudinally parallel line to a waterline through the point K



Righting arm(KN) is calculated for a number of waterlines at <u>various drafts</u> and angles of heel.

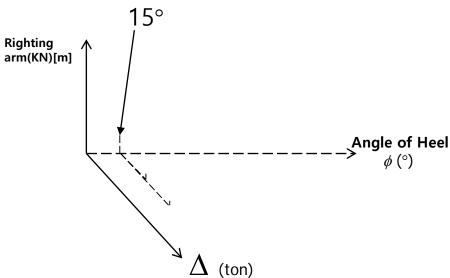
√Assumption: There is a complete watertight envelope consisting of bottom, side shell and weather deck*.

- Calculating for a Number of Waterline at Various Drafts and Angles of Heel (3/4)





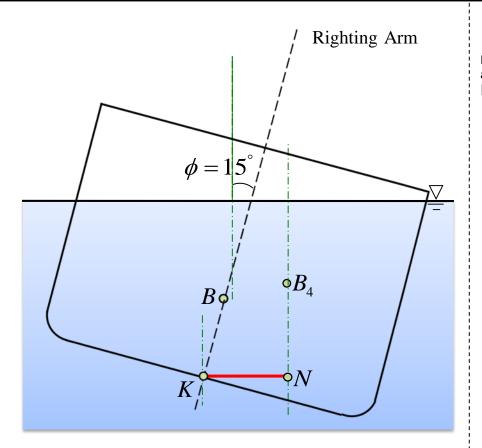
- B: Center of buoyancy at initial position
- B₃: New position of center of buoyancy corresponding displacement
- N: The intersection point of a vertical line through the new position of the center of buoyancy with the longitudinally parallel line to a waterline through the point K



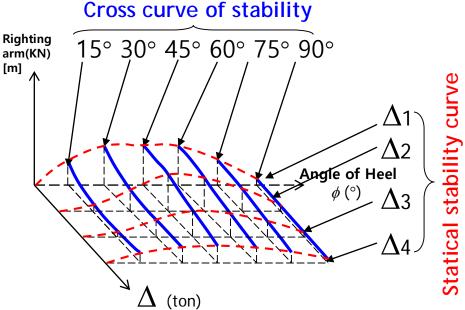
Righting arm(KN) is calculated for a number of waterlines at <u>various drafts</u> and angles of heel.

✓ Assumption: There is a complete watertight envelope consisting of bottom, side shell and weather deck*.

- Calculating for a Number of Waterline at Various Drafts and Angles of Heel (4/4)



- K: Keel
- B: Center of buoyancy at initial position
- B_{a} : New position of center of buoyancy corresponding displacement
- N: The intersection point of a vertical line through the new position of the center of buoyancy with the longitudinally parallel line to a waterline through the point K



Righting arm(KN) is calculated for a number of waterlines at <u>various drafts</u> and angles of heel.

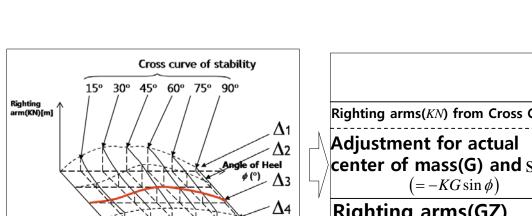
✓ Assumption: There is a complete watertight envelope consisting of bottom, side shell and weather deck*.

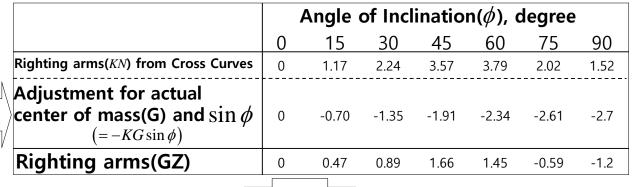
Obtaining the Statical Stability Curves from the Cross Curves of Stability

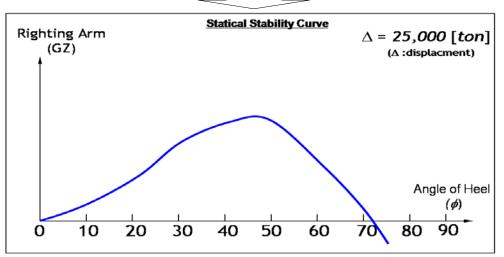
Example) Given condition

 Δ (ton)

- Displacement = 25,000 [ton]
- -KG = 2.7 [m]







 $\overline{KN} - KG \sin \phi$

 $\bullet B_1$

KG sin 4

Calculation of Static Equilibrium Position of a Ship

✓ Governing equation of hydrostatic equilibrium

$$\begin{bmatrix} F(\mathbf{r}) \\ M_{T}(\mathbf{r}) \\ M_{L}(\mathbf{r}) \end{bmatrix} = \mathbf{0} \quad where \begin{bmatrix} F(\mathbf{r}) \\ M_{T}(\mathbf{r}) \\ M_{L}(\mathbf{r}) \end{bmatrix} = \begin{bmatrix} F_{B}(\mathbf{r}) + F_{G}(\mathbf{r}) + F_{ext}(\mathbf{r}) \\ M_{BT}(\mathbf{r}) + M_{GT}(\mathbf{r}) + M_{extT}(\mathbf{r}) \\ M_{BL}(\mathbf{r}) + M_{GL}(\mathbf{r}) + M_{extL}(\mathbf{r}) \end{bmatrix},$$

$$\mathbf{r} = \begin{bmatrix} z & \phi & \theta \end{bmatrix}^{T}$$



Assumption

$$z^* = z^{(0)} + \delta z^{(0)}, \ \phi^* = \phi^{(0)} + \delta \phi^{(0)}, \ \theta^* = \theta^{(0)} + \delta \theta^{(0)}$$

$$\begin{bmatrix}
-F\left(z^{(0)},\phi^{(0)},\theta^{(0)}\right) \\
-M_{T}\left(z^{(0)},\phi^{(0)},\theta^{(0)}\right) \\
-M_{L}\left(z^{(0)},\phi^{(0)},\theta^{(0)}\right)
\end{bmatrix} = \begin{bmatrix}
\frac{\partial F}{\partial z} & \frac{\partial F}{\partial \phi} & \frac{\partial F}{\partial \theta} \\
\frac{\partial M_{T}}{\partial z} & \frac{\partial M_{T}}{\partial \phi} & \frac{\partial M_{T}}{\partial \theta} \\
\frac{\partial M_{L}}{\partial z} & \frac{\partial M_{L}}{\partial \phi} & \frac{\partial M_{L}}{\partial \theta}
\end{bmatrix}_{z^{(0)},\phi^{(0)},\theta^{(0)}}
\begin{bmatrix}
\delta z^{(0)} \\
\delta \phi^{(0)} \\
\delta \theta^{(0)}
\end{bmatrix}$$
Given



$$z^* = z^{(0)} + \delta z^{(0)}, \ \phi^* = \phi^{(0)} + \delta \phi^{(0)}, \ \theta^* = \theta^{(0)} + \delta \theta^{(0)}$$

$$\begin{bmatrix} \delta z^{(0)} \\ \delta \phi^{(0)} \\ \delta \theta^{(0)} \end{bmatrix}$$

$$z^* = z^{(0)} + \delta z^{(0)}, \ \phi^* = \phi^{(0)} + \delta \phi^{(0)}, \ \theta^* = \theta^{(0)} + \delta \theta^{(0)}$$



$$E(x^*)$$

$$=0.M_{\pi}(\mathbf{r}^*)$$

$$F(\mathbf{r}^*) = 0, M_T(\mathbf{r}^*) = 0, M_L(\mathbf{r}^*) = 0$$
Check
$$\theta^{(1)} = \phi^{(0)} + \delta\phi^{(0)}$$

$$\theta^{(1)} = \theta^{(0)} + \delta\theta^{(0)}$$







No
$$z^{(1)} = z^{(0)} + \delta z^{(0)}$$

 $\phi^{(1)} = \phi^{(0)} + \delta \phi^{(0)}$

$$\phi^{(1)} = \phi^{(0)} + \delta\phi^{(0)}$$
$$\theta^{(1)} = \theta^{(0)} + \delta\theta^{(0)}$$

We find the solution, z^*, ϕ^*, θ^* .

Given:
$$\mathbf{r}^{(0)} = \begin{bmatrix} z^{(0)} & \phi^{(0)} & \theta^{(0)} \end{bmatrix}^T$$
, $F(\mathbf{r}^{(0)}), M_T(\mathbf{r}^{(0)}), M_L(\mathbf{r}^{(0)})$

Find:
$$\mathbf{r}^* = \begin{bmatrix} z^* & \phi^* & \theta^* \end{bmatrix}^T$$
, where $F(\mathbf{r}^*) = 0, M_T(\mathbf{r}^*) = 0, M_L(\mathbf{r}^*) = 0$

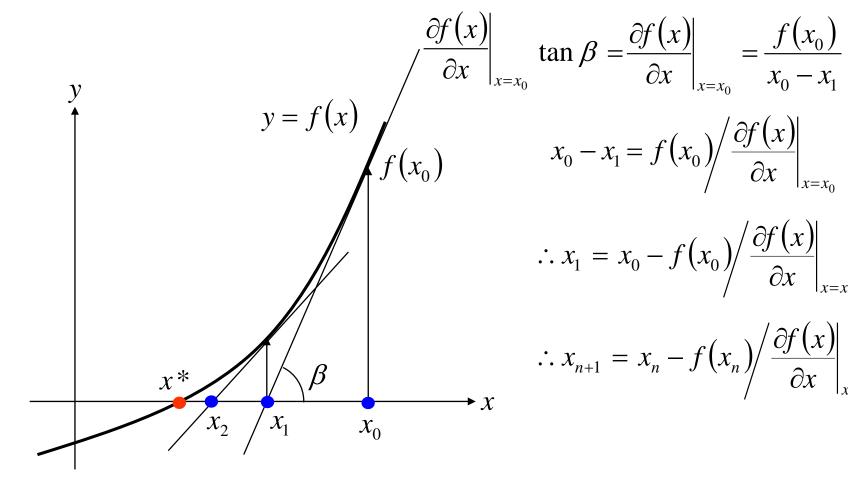
Assumption:

$$z^* = z^{(0)} + \delta z^{(0)}, \ \phi^* = \phi^{(0)} + \delta \phi^{(0)},$$

$$\boldsymbol{\theta}^* = \boldsymbol{\theta}^{(0)} + \delta \boldsymbol{\theta}^{(0)}$$

Newton-Rhapson Method

$$F(\mathbf{r}^*) = 0, M_T(\mathbf{r}^*) = 0, M_L(\mathbf{r}^*) = 0$$
 Check



$$\tan \beta = \frac{\partial f(x)}{\partial x} \bigg|_{x=x_0} = \frac{f(x_0)}{x_0 - x_1}$$

$$x_0 - x_1 = f(x_0) / \frac{\partial f(x)}{\partial x} \Big|_{x=x_0}$$

$$\therefore x_1 = x_0 - f(x_0) \left/ \frac{\partial f(x)}{\partial x} \right|_{x = x_0}$$

$$\therefore x_{n+1} = x_n - f(x_n) / \frac{\partial f(x)}{\partial x} \Big|_{x = x_n}$$

[Reference] IMO Regulations for Severe Wind and Rolling Criteria (Weather Criteria)

The Angle of Roll(ϕ_1) Rolling Period(T)

IMO Regulations for Severe Wind and Rolling Criteria (Weather Criteria) - The Angle of Roll (ϕ_1)

The Angle of Roll (ϕ_1) should be calculated as follows:

(IMO Res.A-749(18) ch.3.2.2.3)

$$\phi_1 = 109 \cdot k \cdot x_1 \cdot x_2 \cdot \sqrt{r \cdot s}$$
 (degrees)

where:

 x_l : factor as shown in Table 1

 x_2 : factor as shown in Table 2

k: factor as follows:

k = 1.0 for round-bilged ship having no bilge or bar keels

k = 0.7 for a ship having sharp bilges

k = as shown in table 3 for a ship having bilge keels, a bar keel of both

 $r = 0.73 \pm 0.6 \cdot OG / d$

with OG = distance between the center of mass and the waterline (m)

('+' if center of mass is above the waterline, '-' if it is below)

s: factor as shown in table 4

L: Waterline Length of the ship (m)

B: Moulded breadth of the ship (m)

d: Mean moulded draft of the ship (m)

 C_R : Block coefficient

 A_K : Total overall area of bilge keels, or area of the lateral projection of the bar keel, or sum of these areas (m²)

Table 1 – Values of factor x_i

B/d	X_{1}
≤2.4	1.0
2.5	0.98
2.6	0.96
2.7	0.95
2.8	0.93
2.9	0.91
3.0	0.90
3.1	0.88
3.2	0.86
3.4	0.82
≥3.5	0.80

Table 2 – Values of factor x_2

C_B	X 2
≤0.45	0.75
0.50	0.82
0.55	0.89
0.60	0.95
0.65	0.97
≥0.70	1.0

Table 3 – Values of factor K

$\frac{A_K \times 100}{L \times B}$	K
0	1.0
1.0	0.98
1.5	0.95
2.0	0.88
2.5	0.79
3.0	0.74
3.5	0.72
≥4.0	0.70

Table 4 – Values of factor S

T	S
≤6	0.100
7	0.098
8	0.093
12	0.065
14	0.053
16	0.044
18	0.038
≥20	0.035

(Intermediate values in Tables 1-4 should be obtained by linear interpolation.)

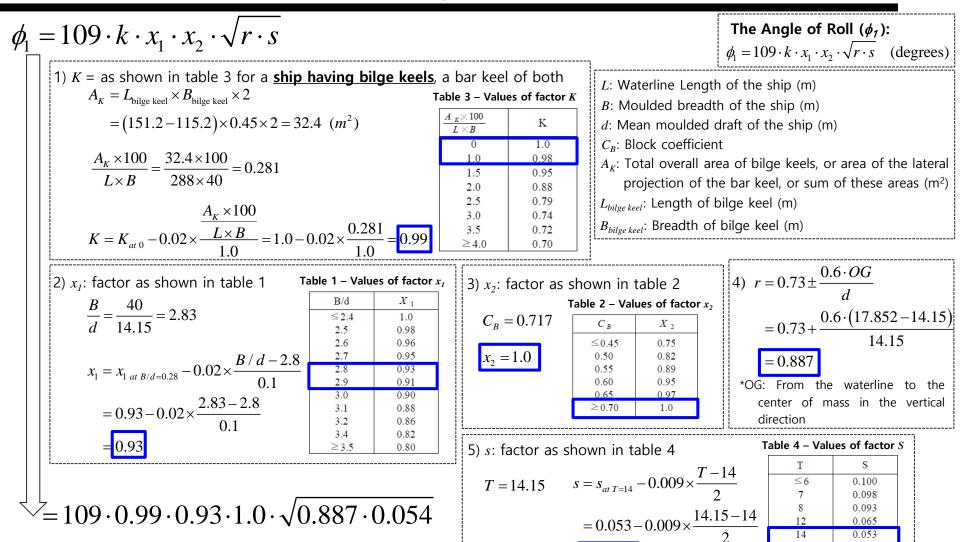
IMO Regulations for Severe Wind and Rolling Criteria (Weather Criteria)

- The Angle of Roll (φ₁)

 $=21.963^{\circ}$

- 7,000 TEU Container Carrier at Homo. Scantling Arrival Condition

(IMO Res.A-749(18) ch.3.2.2.3)



=0.054

14

18

0.065

0.053 0.044

0.038

IMO Regulations for Severe Wind and Rolling Criteria (Weather Criteria)

- Rolling Period (T)

(IMO Res.A-749(18) ch.3.2.2.3)

$$T = \frac{2 \cdot C \cdot B}{\sqrt{GM}} \quad (s)$$

where, C = 0.373 + 0.023(B/d) - 0.043(L/100)

The symbols in the above formula for the rolling period are defined as follows:

L: Waterline Length of the ship (m)

B: Moulded breadth of the ship (m)

d: Mean moulded draft of the ship (m)

 C_R : Block coefficient

 A_K : Total overall area of bilge keels, or area of the lateral projection of the bar keel, or sum of these areas (m²)

GM: Metacentric height corrected for free surface effect (m)

