Ship Stability

September 2013

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Ship Stability

Ch. 9 Numerical Integration Method in Naval Architecture

Simpson's Rule

Gaussian Quadrature

Calculation of Area by Using Green's Theorem

Calculation of Hydrostatic Values By Using Gaussian Quadrature and Green's Theorem

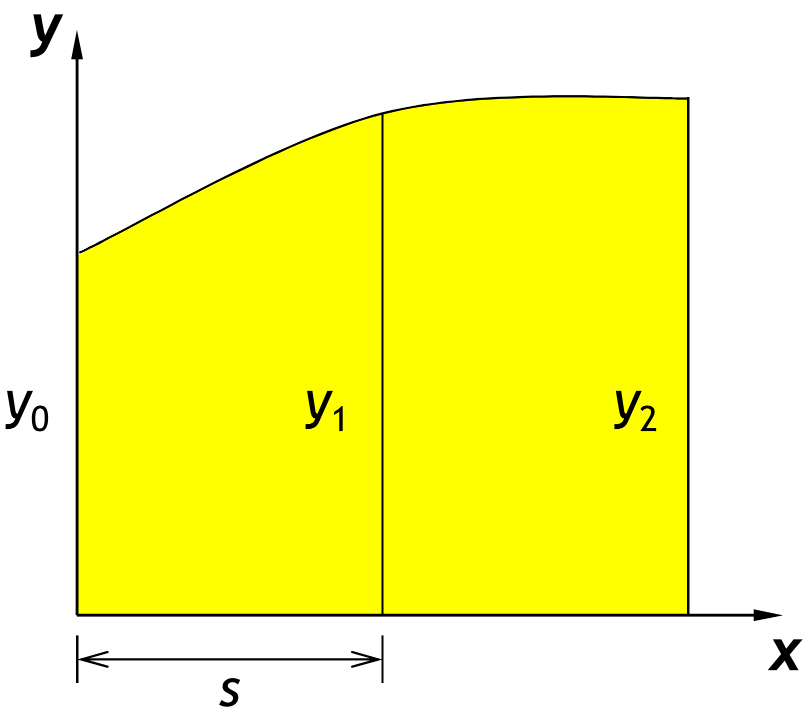
Classical Calculation Method for Ship's Surface Area

Simpson's Rule

Simpson's 1st and 2nd Rules

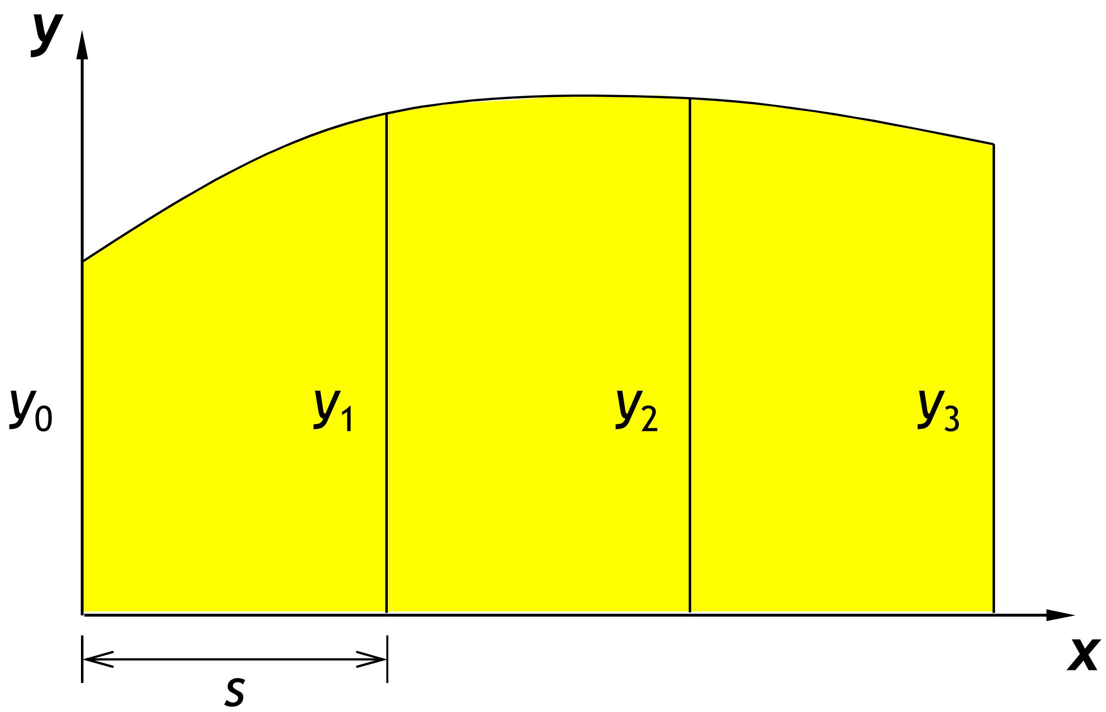
Simpson's 1st, 2nd Rules

Simpson's 1st Rule



$$Area = \frac{1}{3}s(y_0 + 4y_1 + y_2)$$

Simpson's 2nd Rule

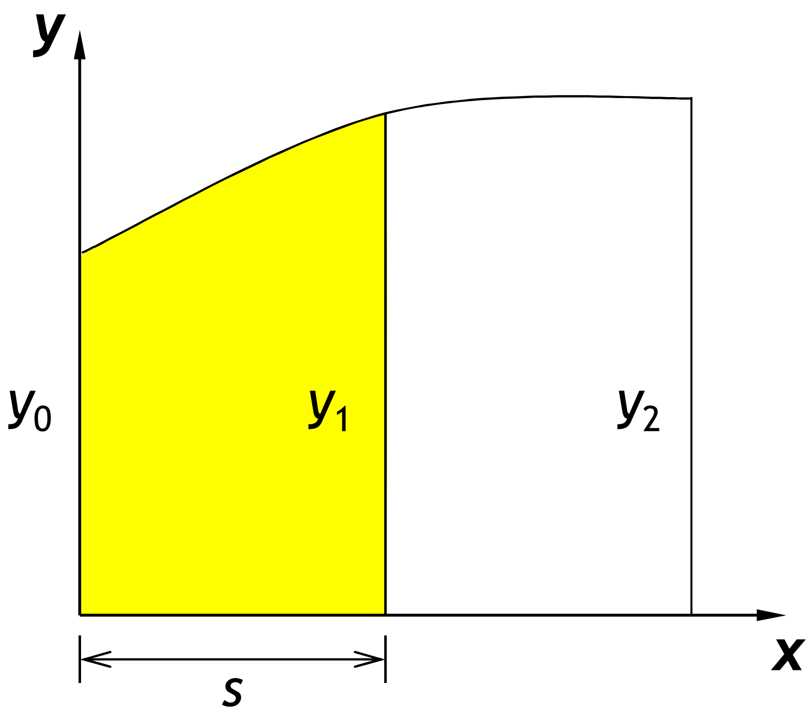


$$Area = \frac{3}{8}s(y_0 + 3y_1 + 3y_2 + y_3)$$

5·8·-1, 3·10·-1, and 7·36·-3 Rules

5·8·-1, 3·10·-1, 7·36·-3 Rules

5·8·-1 Rule



$$Area = \frac{1}{12} s(5y_0 + 8y_1 - 1y_2)$$

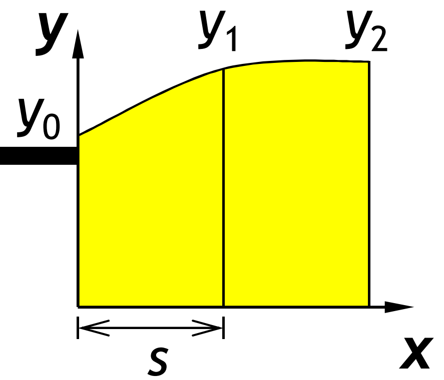
3·10·-1 Rule

$$M_y = \frac{1}{24} s^2 (3y_0 + 10y_1 - 1y_2)$$

7·36·-3 Rule

$$I_y = \frac{1}{120} s^3 (7y_0 + 36y_1 - 3y_2)$$

Derivation of Simpson's 1st Rule (1/4)



Simpson's 1st Rule:
Approximate the function y by a parabola (quadratic polynomial curve) whose equation has the form

Parabola : $y = a_0 + a_1 x + a_2 x^2$

The parabola is represented by three points defining this curve.
The three points (y_0, y_1, y_2) are obtained by dividing the given interval into equal subintervals "s".

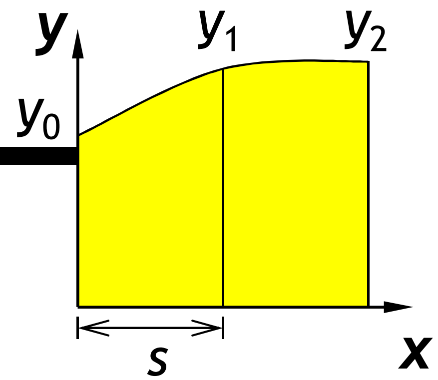
The relation between the coefficients a_0, a_1, a_2 and y_0, y_1, y_2 is

$$x = 0: \quad y_0 = a_0$$

$$x = s: \quad y_1 = a_0 + a_1 s + a_2 s^2$$

$$x = 2s: \quad y_2 = a_0 + 2a_1 s + 4a_2 s^2$$

Derivation of Simpson's 1st Rule (2/4)



$$y = a_0 + a_1 x + a_2 x^2$$

$$y_0 = a_0 \quad ①$$

$$\left.\begin{aligned} y_1 &= a_0 + a_1 s + a_2 s^2 \\ y_2 &= a_0 + 2a_1 s + 4a_2 s^2 \end{aligned}\right\} \quad \begin{aligned} &a_1 s + a_2 s^2 + y_0 - y_1 = 0 \quad ② \\ &2a_1 s + 4a_2 s^2 + y_0 - y_2 = 0 \quad ③ \end{aligned}$$

4 x ② - ③:

$$2a_1 s + 3y_0 - 4y_1 + y_2 = 0$$

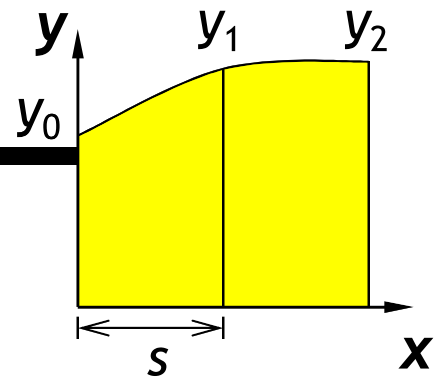
$$\therefore a_1 = \frac{1}{2s}(-3y_0 + 4y_1 - y_2)$$

③ - 2 x ②:

$$2a_2 s^2 - y_0 + 2y_1 - y_2 = 0$$

$$\therefore a_2 = \frac{1}{2s^2}(y_0 - 2y_1 + y_2)$$

Derivation of Simpson's 1st Rule (3/4)



$$y = a_0 + a_1 x + a_2 x^2$$

$$a_0 = y_0, \quad a_1 = \frac{1}{2s}(-3y_0 + 4y_1 - y_2), \qquad a_2 = \frac{1}{2s^2}(y_0 - 2y_1 + y_2)$$

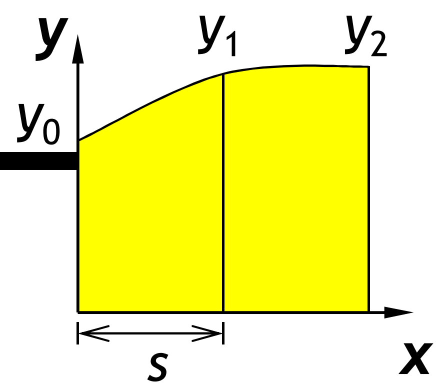
$$y = y_0 + \frac{1}{2s}(-3y_0 + 4y_1 - y_2)x + \frac{1}{2s^2}(y_0 - 2y_1 + y_2)x^2$$

Integrate the area A from 0 to 2s.

$$A = \int_0^{2s} y\,dx$$

$$= \int_0^{2s} y_0 + \frac{1}{2s}(-3y_0 + 4y_1 - y_2)x + \frac{1}{2s^2}(y_0 - 2y_1 + y_2)x^2 dx$$

Derivation of Simpson's 1st Rule (4/4)



$$A=\int_0^{2s} y_0+\frac{1}{2s}(-3y_0+4y_1-y_2)x+\frac{1}{2s^2}(y_0-2y_1+y_2)x^2dx$$

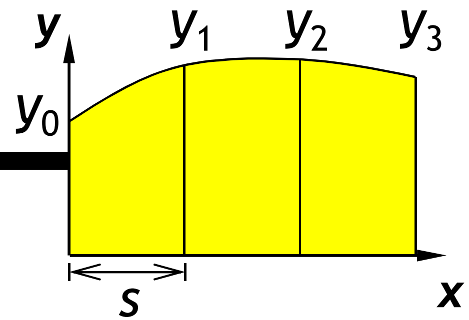
$$=y_0x+\frac{1}{4s}(-3y_0+4y_1-y_2)x^2+\frac{1}{6s^2}(y_0-2y_1+y_2)x^3\Big|_0^{2s}$$

$$=y_0(2s)+\frac{1}{4s}(-3y_0+4y_1-y_2)(2s)^2+\frac{1}{6s^2}(y_0-2y_1+y_2)(2s)^3$$

$$=2y_0s+(-3y_0+4y_1-y_2)s+\frac{4}{3}(y_0-2y_1+y_2)s$$

$$\therefore A=\frac{s}{3}(1y_0+4y_1+1y_2)$$

Derivation of Simpson's 2nd Rule (1/4)



Simpson's 2nd rule :

Approximate the function by a cubic polynomial curve whose equation has the form

Cubic polynomial curve: $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

The cubic polynomial curve is represented by four points defining this curve.

The four points (y_0, y_1, y_2, y_3) are obtained by dividing the given interval into equal subintervals "s".

The relation between the coefficients a_0, a_1, a_2, a_3 and y_0, y_1, y_2, y_3 is

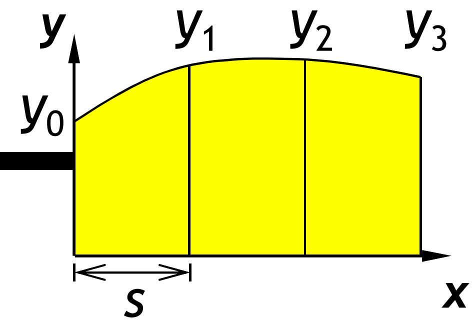
$$x = 0: \quad y_0 = a_0$$

$$x = s: \quad y_1 = a_0 + a_1 s + a_2 s^2 + a_3 s^3$$

$$x = 2s: \quad y_2 = a_0 + 2a_1 s + 4a_2 s^2 + 8s^3$$

$$x = 3s: \quad y_3 = a_0 + 3a_1 s + 9a_2 s^2 + 27s^3$$

Derivation of Simpson's 2nd Rule (2/4)



$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$y_0 = a_0, \qquad y_1 = a_0 + a_1 s + a_2 s^2 + a_3 s^3,$$

$$y_2 = a_0 + 2a_1 s + 4a_2 s^2 + 8s^3, \qquad y_3 = a_0 + 3a_1 s + 9a_2 s^2 + 27s^3$$

The unknown coefficients, a_0, a_1, a_2, a_3 lead to

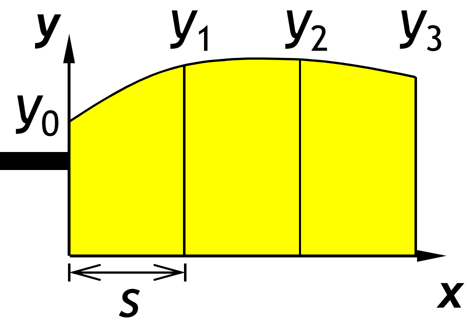
$$a_0 = y_0$$

$$a_1 = \frac{1}{6s}(-11y_0 + 18y_1 - 9y_2 + 2y_3)$$

$$a_2 = \frac{1}{2s^2}(2y_0 - 5y_1 + 4y_2 - y_3)$$

$$a_3 = \frac{1}{6s^3}(-y_0 + 3y_1 - 3y_2 + y_3)$$

Derivation of Simpson's 2nd Rule (3/4)



$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$a_0 = y_0, \qquad a_1 = \frac{1}{6s}(-11y_0 + 18y_1 - 9y_2 + 2y_3),$$

$$a_2 = \frac{1}{2s^2}(2y_0 - 5y_1 + 4y_2 - y_3), \quad a_3 = \frac{1}{6s^3}(-y_0 + 3y_1 - 3y_2 + y_3)$$

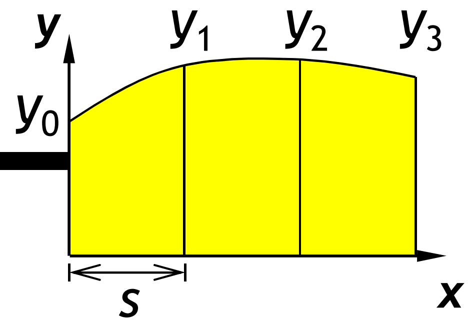
Integrate the area A from 0 to 3s.

$$A = \int_0^{3s} y\,dx = \int_0^{3s} (a_0 + a_1 x + a_2 x^2 + a_3 x^3)\,dx$$

$$= a_0 x + \frac{a_1}{2}x^2 + \frac{a_2}{3}x^3 + \frac{a_3}{4}x^4 \Bigg|_0^{3s}$$

$$= 3a_0 s + \frac{9}{2}a_1 s^2 + \frac{27}{3}a_2 s^3 + \frac{81}{4}a_3 s^4$$

Derivation of Simpson's 2nd Rule (4/4)



$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$a_0 = y_0, \qquad a_1 = \frac{1}{6s}(-11y_0 + 18y_1 - 9y_2 + 2y_3),$$

$$a_2 = \frac{1}{2s^2}(2y_0 - 5y_1 + 4y_2 - y_3), \quad a_3 = \frac{1}{6s^3}(-y_0 + 3y_1 - 3y_2 + y_3)$$

$$A = 3a_0 s + \frac{9}{2}a_1 s^2 + \frac{27}{3}a_2 s^3 + \frac{81}{4}a_3 s^4$$

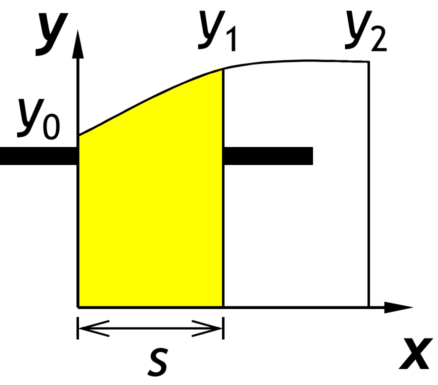
By substituting a_0, a_1, a_2 and a_3 into the equation, the Area "A" leads to

$$A = 3y_0 s + \frac{9}{2}\cdot\frac{1}{6s}(-11y_0 + 18y_1 - 9y_2 + 2y_3)s^2$$

$$+\frac{27}{3}\cdot\frac{1}{2s^2}(2y_0 - 5y_1 + 4y_2 - y_3)s^3 + \frac{81}{4}\cdot\frac{1}{6s^3}(-y_0 + 3y_1 - 3y_2 + y_3)s^4$$

$$\therefore A = \frac{3}{8}s(y_0 + 3y_1 + 3y_2 + y_3)$$

Derivation of 5·8·-1 Rule (1/4)



5·8·-1 Rule:

Approximate the function y by a parabola whose equation has the form

Parabola : $y = a_0 + a_1 x + a_2 x^2$

The parabola is represented by three points defining this curve.

The three points (y_0, y_1, y_2) are obtained by dividing the given interval into equal subintervals "s".

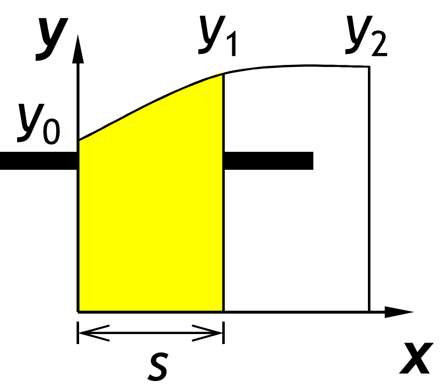
The relation between the coefficients a_0, a_1, a_2 and y_0, y_1, y_2 is

$$x = 0: \quad y_0 = a_0$$

$$x = s: \quad y_1 = a_0 + a_1 s + a_2 s^2$$

$$x = 2s: \quad y_2 = a_0 + 2a_1 s + 4a_2 s^2$$

Derivation of 5·8·-1 Rule (2/4)



$$y = a_0 + a_1 x + a_2 x^2$$

$$y_0 = a_0 \quad ①$$

$$\left.\begin{aligned} y_1 &= a_0 + a_1 s + a_2 s^2 \\ y_2 &= a_0 + 2a_1 s + 4a_2 s^2 \end{aligned}\right\} \quad \begin{aligned} a_1 s + a_2 s^2 + y_0 - y_1 = 0 \quad ② \\ 2a_1 s + 4a_2 s^2 + y_0 - y_2 = 0 \quad ③ \end{aligned}$$

4 x ② - ③:

$$2a_1 s + 3y_0 - 4y_1 + y_2 = 0$$

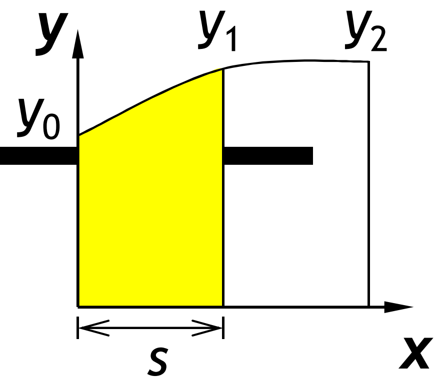
$$\therefore a_1 = \frac{1}{2s}(-3y_0 + 4y_1 - y_2)$$

③ - 2 x ②:

$$2a_2 s^2 - y_0 + 2y_1 - y_2 = 0$$

$$\therefore a_2 = \frac{1}{2s^2}(y_0 - 2y_1 + y_2)$$

Derivation of 5·8·-1 Rule (3/4)



$$y = a_0 + a_1 x + a_2 x^2$$

$$a_0 = y_0, \quad a_1 = \frac{1}{2s}(-3y_0 + 4y_1 - y_2), \qquad a_2 = \frac{1}{2s^2}(y_0 - 2y_1 + y_2)$$

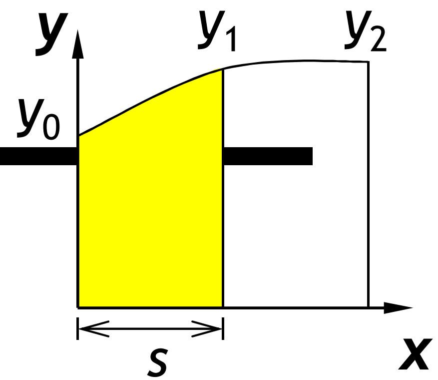
$$y = y_0 + \frac{1}{2s}(-3y_0 + 4y_1 - y_2)x + \frac{1}{2s^2}(y_0 - 2y_1 + y_2)x^2$$

Integrate the area A from 0 to s.

$$A = \int_0^s y dx$$

$$= \int_0^s y_0 + \frac{1}{2s}(-3y_0 + 4y_1 - y_2)x + \frac{1}{2s^2}(y_0 - 2y_1 + y_2)x^2 dx$$

Derivation of 5·8·-1 Rule (4/4)



$$A = \int_0^s y_0 + \frac{1}{2s}(-3y_0 + 4y_1 - y_2)x + \frac{1}{2s^2}(y_0 - 2y_1 + y_2)x^2 dx$$

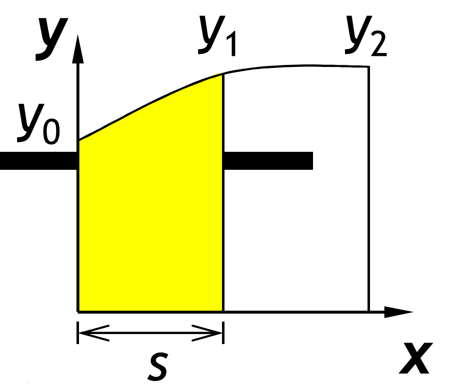
$$= y_0 x + \frac{1}{4s}(-3y_0 + 4y_1 - y_2)x^2 + \frac{1}{6s^2}(y_0 - 2y_1 + y_2)x^3 \Big|_0^s$$

$$= y_0(s) + \frac{1}{4s}(-3y_0 + 4y_1 - y_2)(s)^2 + \frac{1}{6s^2}(y_0 - 2y_1 + y_2)(s)^3$$

$$= y_0 s + \frac{1}{4}(-3y_0 + 4y_1 - y_2)s + \frac{1}{6}(y_0 - 2y_1 + y_2)s$$

$$\therefore A = \frac{s}{12}(5y_0 + 8y_1 - 1y_2)$$

Derivation of 3·10·-1 and 7·36·-3 Rules



3·10·-1 Rule: The first moment of area about y axis $M_y = \int_0^s x dA$

7·36·-3 Rule: The second moment of area about y axis $I_y = \int_0^s x^2 dA$

$$M_y = \int_0^s x dA = \int_0^s xy dx = \int_0^s a_0 x + a_1 x^2 + a_2 x^3 dx$$

$$= \boxed{\frac{1}{24} s^2 (3y_0 + 10y_1 - y_2)}$$

⬅ $a_0 = y_0,\ a_1 = \frac{1}{2s}(-3y_0 + 4y_1 - y_2),\ a_2 = \frac{1}{2s^2}(y_0 - 2y_1 + y_2)$

$$I_y = \int_0^s x^2 dA = \int_0^s x^2 y dx = \int_0^s a_0 x^3 + a_1 x^4 + a_2 x^5 dx$$

$$= \boxed{\frac{1}{120} s^3 (7y_0 + 36y_1 - 3y_2)}$$

⬅ $a_0 = y_0,\ a_1 = \frac{1}{2s}(-3y_0 + 4y_1 - y_2),\ a_2 = \frac{1}{2s^2}(y_0 - 2y_1 + y_2)$

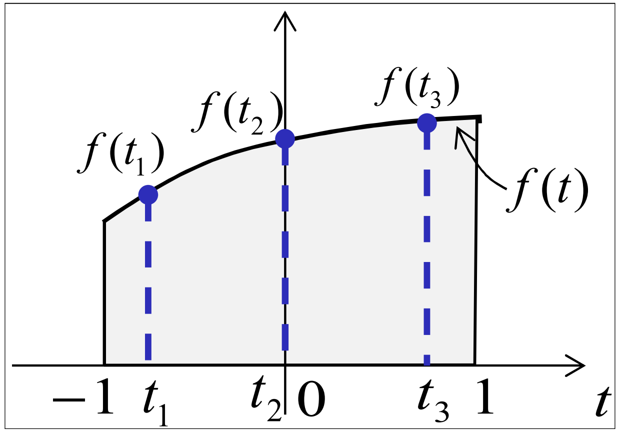
Gaussian Quadrature

Gaussian Quadrature

Gaussian quadrature: $\int_{-1}^{1} f(t)dt \approx \sum_{j=1}^{n} A_j \cdot f(t_j)$

Given: Function $f(t)$

Find: Integration of $f(t)$ at given interval [-1, 1] $\int_{-1}^{1} f(t)dt$



In the case of Cubic Gaussian quadrature,

$$\int_{-1}^{1} f(t)dt \approx A_1 \cdot f(t_1) + A_2 \cdot f(t_2) + A_3 \cdot f(t_3)$$

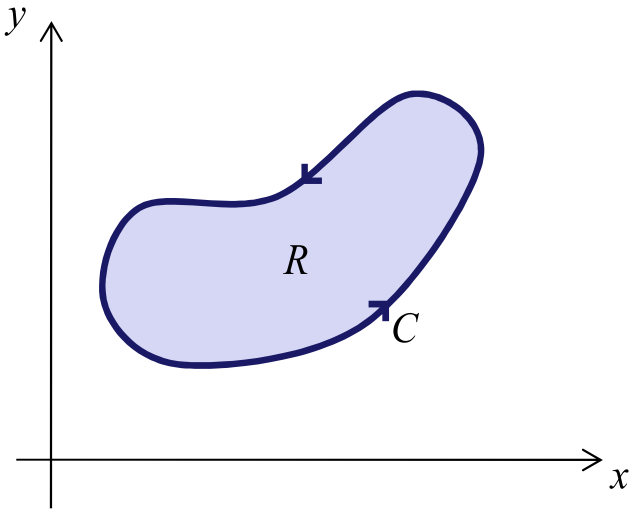
n	Coefficients A_j	Node t_j
3	A_1 = 0.5555555556	t_1 = -0.7745966692
	A_2 = 0.8888888889	t_2 = 0
	A_3 = 0.5555555556	t_3 = 0.7745966692

n	Coefficients A_j	Node t_j
4 (Quartic)	A_1 = 0.3478548451	t_1 = -0.8611363115
	A_2 = 0.6521451548	t_2 = -0.3399810435
	A_3 = 0.6521451548	t_3 = 0.3399810435
	A_4 = 0.3478548451	t_4 = 0.8611363115
5 (Quintic)	A_1 = 0.2369268850	t_1 = -0.9061798459
	A_2 = 0.4786286704	t_2 = -0.5384693101
	A_3 = 0.6521451548	t_3 = 0.0
	A_4 = 0.4786286704	t_4 = 0.5384693101
	A_5 = 0.2369268850	t_5 = 0.9061798459

Calculation of Area by Using Green's Theorem*

* Erwin Kreyszig, Advanced Engineering Mathematics, 9th Edition, pp.439-445, 2006

Calculation of Area by Using Green's Theorem



$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \oint_C (Mdx + Ndy)$$

Surface Integral **Line Integral**

M, N: The functions of x and y. And M, N, dM/dy, and dN/dx are continuous on R.

✓Calculation of area $\left(A = \int dA = \iint dxdy\right)$

If $M = -y, \ N = x$

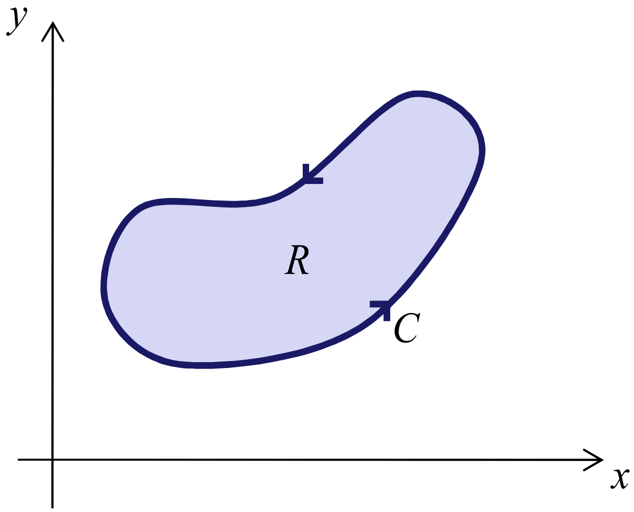
L.H.S = $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \iint_R \left(\frac{\partial}{\partial x}(x) - \frac{\partial}{\partial y}(-y)\right) dxdy = \iint_R 2dxdy = 2A$ (A: Area)

R.H.S = $\oint_C (Mdx + Ndy) = \oint_C (-ydx + xdy) = \oint_C (xdy - ydx)$

$$\therefore 2A = \oint_C (xdy - ydx)$$

$$A = \frac{1}{2}\oint_C (xdy - ydx)$$

Calculation of First Moment of Area by Using Green's Theorem (1/2)



$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dxdy = \oint_C (Mdx + Ndy)$$

Surface Integral | **Line Integral**

M, N: The functions of x and y. And M, N, dM/dy, and dN/dx are continuous on R.

✓First moment of area about the y-axis in x direction $\left(M_{A,y} = \int xdA = \iint xdxdy \right)$

If $M = -xy,\ N = \dfrac{x^2}{2}$

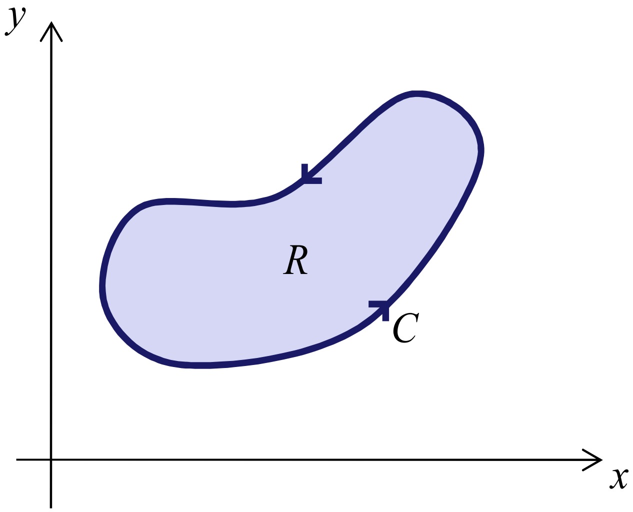
L.H.S = $\displaystyle\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dxdy = \iint_R \left(\frac{\partial}{\partial x}\left(\frac{x^2}{2}\right) - \frac{\partial}{\partial y}(-xy) \right) dxdy = \iint_R 2xdxdy = 2M_{A,y}$

R.H.S = $\displaystyle\oint_C (Mdx + Ndy) = \oint_C \left(-xydx + \frac{x^2}{2}dy \right) = \oint_C \left(\frac{x^2}{2}dy - xydx \right)$

$$\therefore 2M_{A,y} = \oint_C \left(\frac{x^2}{2}dy - xydx \right)$$

$$M_{A,y} = \frac{1}{2}\oint_C \left(\frac{x^2}{2}dy - xydx \right)$$

Calculation of First Moment of Area by Using Green's Theorem (2/2)



$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dxdy = \oint_C (Mdx + Ndy)$$

Surface Integral **Line Integral**

M, N: The functions of x and y. And M, N, dM/dy, and dN/dx are continuous on R.

✓First moment of area about the x-axis in y direction $\left(M_{A,x} = \int ydA = \iint ydxdy \right)$

If $M = -\frac{y^2}{2}$, $N = xy$

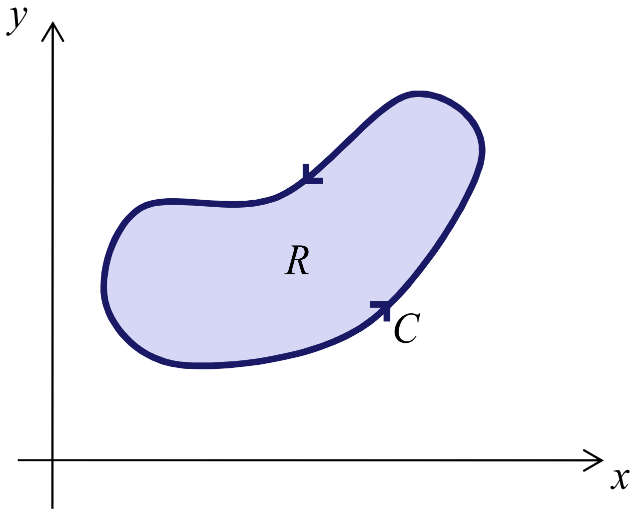
L.H.S = $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dxdy = \iint_R \left(\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial y}(-\frac{y^2}{2}) \right) dxdy = \iint_R 2ydxdy = 2M_{A,x}$

R.H.S = $\oint_C (Mdx + Ndy) = \oint_C \left(-\frac{y^2}{2} dx + xydy \right) = \oint_C \left(xydy - \frac{y^2}{2} dx \right)$

$\therefore 2M_{A,x} = \oint_C \left(xydy - \frac{y^2}{2} dx \right)$

$$M_{A,x} = \frac{1}{2} \oint_C \left(xydy - \frac{y^2}{2} dx \right)$$

Calculation of Second Moment of Area by Using Green's Theorem (1/2)



$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dxdy = \oint_C (Mdx + Ndy)$$

Surface Integral **Line Integral**

M, N: The functions of x and y. And M, N, dM/dy, and dN/dx are continuous on R.

✓Second moment of area about the y-axis in x direction $\left(I_{A,y} = \int x^2 dA = \iint x^2 dxdy \right)$

If $M = -x^2 y,\ N = \frac{x^3}{3}$

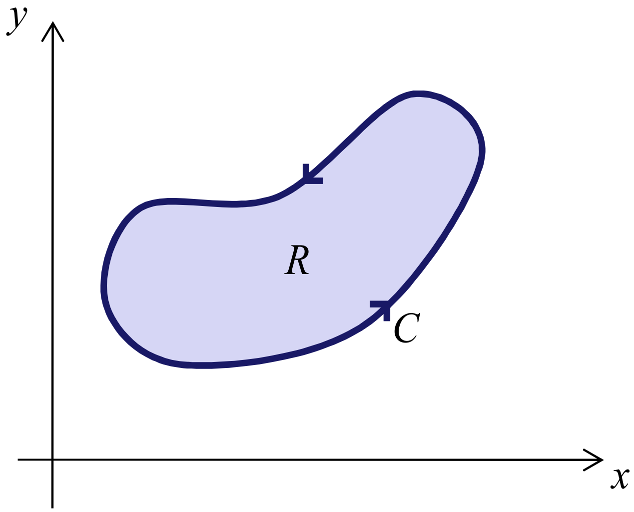
L.H.S = $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dxdy = \iint_R \left(\frac{\partial}{\partial x}\left(\frac{x^3}{3}\right) - \frac{\partial}{\partial y}(-x^2 y) \right) dxdy = \iint_R 2x^2 dxdy = 2I_{A,y}$

R.H.S = $\oint_C (Mdx + Ndy) = \oint_C \left(-x^2 ydx + \frac{x^3}{3} dy \right) = \oint_C \left(\frac{x^3}{3} dy - x^2 ydx \right)$

$$\therefore 2I_{A,y} = \oint_C \left(\frac{x^3}{3} dy - x^2 ydx \right)$$

$$I_{A,y} = \frac{1}{2} \oint_C \left(\frac{x^3}{3} dy - x^2 ydx \right)$$

Calculation of Second Moment of Area by Using Green's Theorem (2/2)



$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dxdy = \oint_C (Mdx + Ndy)$$

Surface Integral **Line Integral**

M, N: The functions of x and y. And M, N, dM/dy, and dN/dx are continuous on R.

✓Second moment of area about the x-axis in y direction $\left(I_{A,x} = \int y^2 dA = \iint y^2 dxdy \right)$

If $M = -\frac{y^3}{3}$, $N = xy^2$

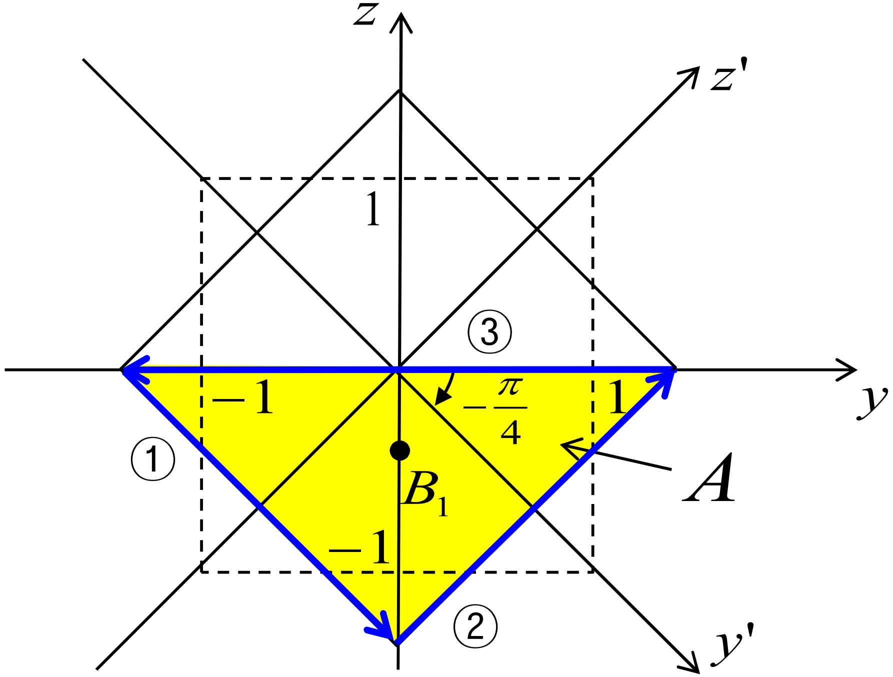
L.H.S = $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dxdy = \iint_R \left(\frac{\partial}{\partial x}(xy^2) - \frac{\partial}{\partial y}(-\frac{y^3}{3}) \right) dxdy = \iint_R 2y^2 dxdy = 2I_{A,x}$

R.H.S = $\oint_C (Mdx + Ndy) = \oint_C \left(-\frac{y^3}{3} dx + xy^2 dy \right) = \oint_C \left(xy^2 dy - \frac{y^3}{3} dx \right)$

$\therefore 2I_{A,x} = \oint_C \left(xy^2 dy - \frac{y^3}{3} dx \right)$

$$I_{A,x} = \frac{1}{2} \oint_C \left(xy^2 dy - \frac{y^3}{3} dx \right)$$

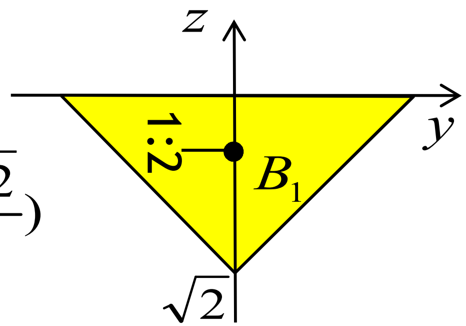
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (1/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓Area A

$$A = \int dA = \iint dydz$$

⇩ Green's theorem

$$= \frac{1}{2}\oint_C ydz - zdy$$

Segment ①: $y(t) = t, \quad z(t) = -t - \sqrt{2}, \quad -\sqrt{2} \le t \le 0$

Using the chain rule, convert the line integral for y and z into the integral for only one parameter t.

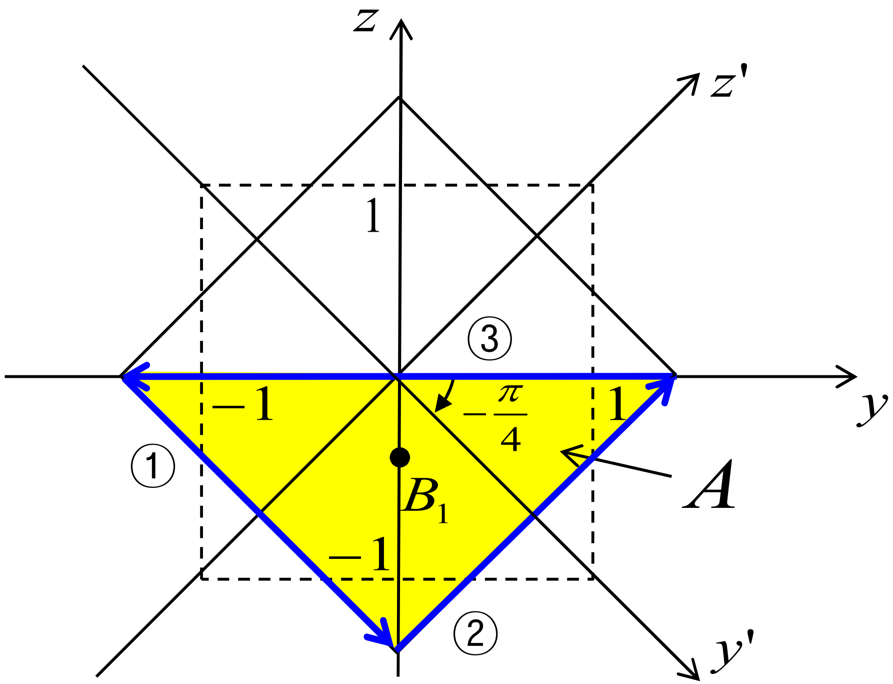
$$\frac{1}{2}\int_{①} ydz - zdy = \frac{1}{2}\int_{-\sqrt{2}}^{0}\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-\sqrt{2}}^{0}\left(t(-1) - (-t-\sqrt{2})\cdot 1\right)dt$$

$$= \frac{1}{2}\int_{-\sqrt{2}}^{0}\sqrt{2}dt = \frac{1}{2}\sqrt{2}t\Big|_{-\sqrt{2}}^{0}$$

$$= \frac{1}{2}\sqrt{2}\sqrt{2} = 1$$

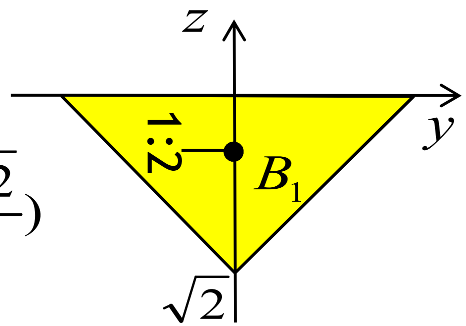
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (2/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2} \cdot 2 \cdot 2 = 2$$

$$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓Area A

$$A = \frac{1}{2} \oint_C ydz - zdy$$

Segment ①: $\frac{1}{2} \int_{①} ydz - zdy = 1$

Segment ②: $y(t) = t, \quad z(t) = t - \sqrt{2}, \quad 0 \le t \le \sqrt{2}$

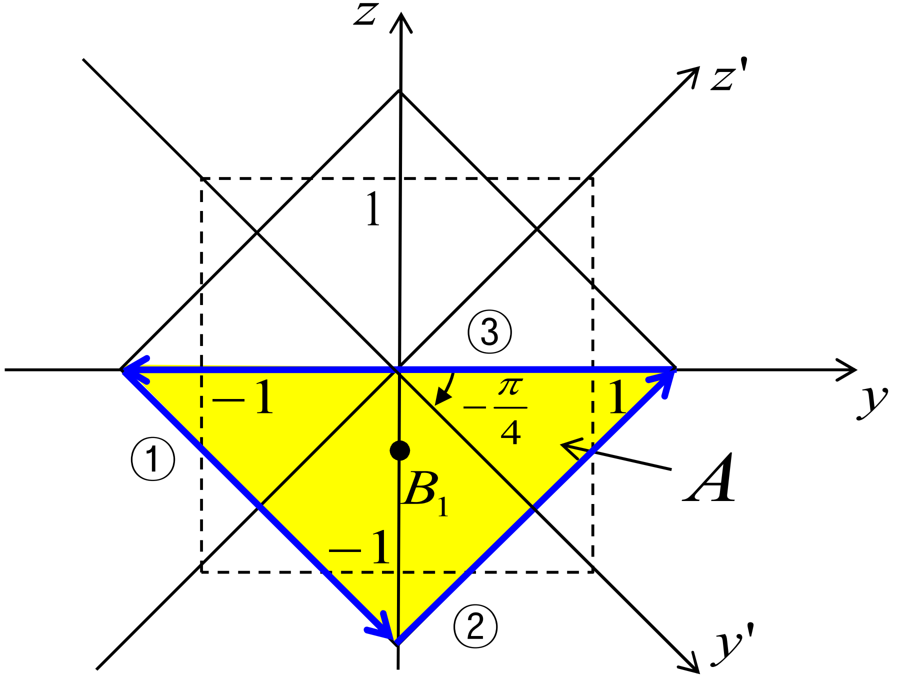
$$\frac{1}{2} \int_{②} ydz - zdy = \frac{1}{2} \int_0^{\sqrt{2}} \left(y \frac{dz}{dt} - z \frac{dy}{dt} \right) dt$$

$$= \frac{1}{2} \int_0^{\sqrt{2}} \left(t \cdot 1 - (t - \sqrt{2}) \cdot 1 \right) dt$$

$$= \frac{1}{2} \int_0^{\sqrt{2}} \sqrt{2} dt = \frac{1}{2} \sqrt{2} t \Big|_0^{\sqrt{2}}$$

$$= \frac{1}{2} \sqrt{2} \sqrt{2} = 1$$

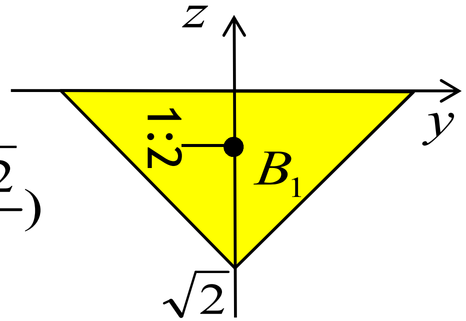
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (3/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓Area A

$$A = \frac{1}{2}\oint_C ydz - zdy$$

Segment ①: $\frac{1}{2}\int_{①} ydz - zdy = 1$

Segment ②: $\frac{1}{2}\int_{②} ydz - zdy = 1$

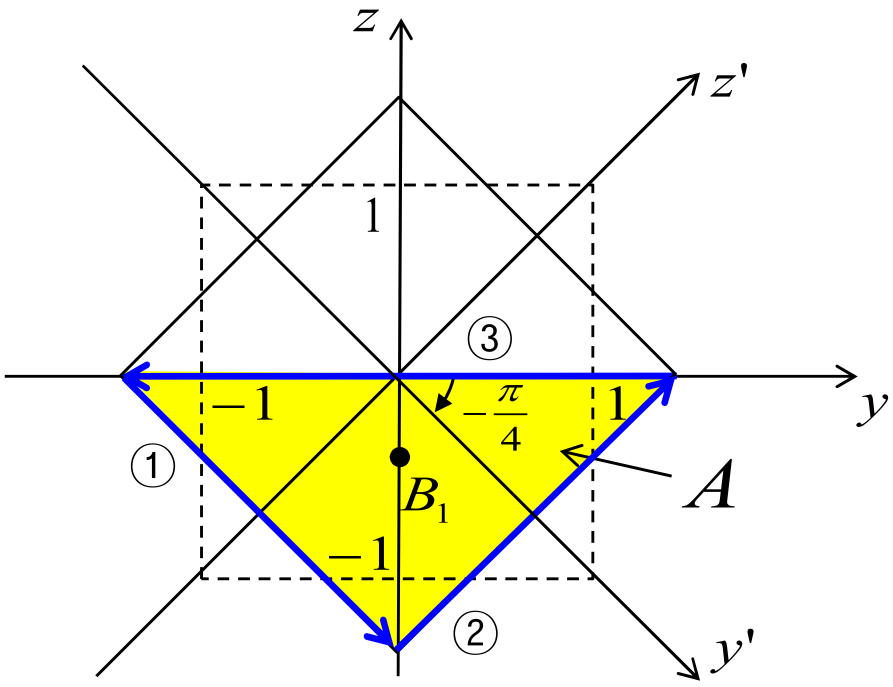
Segment ③: $y(t) = t, \quad z = 0, \quad -\sqrt{2} \le t \le \sqrt{2}$

$$\frac{1}{2}\int_{③} ydz - zdy = \frac{1}{2}\int_{\sqrt{2}}^{-\sqrt{2}}\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{\sqrt{2}}^{-\sqrt{2}}(t\cdot 0 - 0\cdot 1)dt = 0$$

$$\therefore A = \frac{1}{2}\oint_C ydz - zdy = \underset{①}{1} + \underset{②}{1} + \underset{③}{0} = 2$$

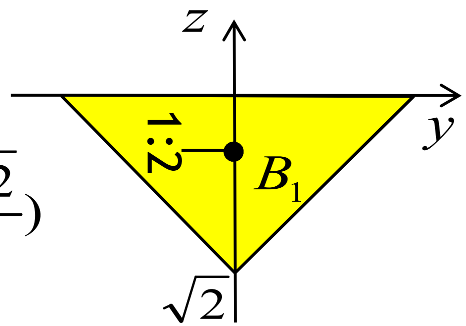
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (4/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



$oy'z'$: Body fixed coordinate

oyz: Water plane fixed coordinate

✓First moment of area about the z-axis in y direction $M_{A,z}$

$$M_{A,z} = \int y dA = \iint y dy dz$$

⇩ Green's theorem

$$= \frac{1}{2}\oint_C \frac{y^2}{2} dz - yz dy$$

Segment ①: $y(t) = t, \quad z(t) = -t - \sqrt{2}, \quad -\sqrt{2} \le t \le 0$

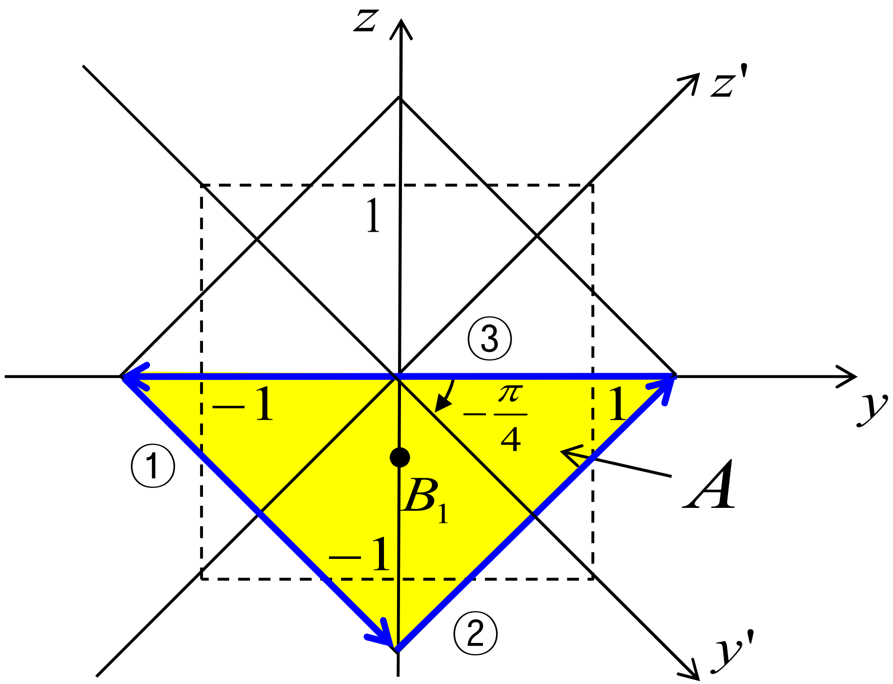
$$\frac{1}{2}\int_{①} \frac{y^2}{2} dz - yz dy = \frac{1}{2}\int_{-\sqrt{2}}^{0}\left(\frac{y^2}{2}\frac{dz}{dt} - yz\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-\sqrt{2}}^{0}\left(\frac{t^2}{2}(-1) - t(-t-\sqrt{2})\cdot 1\right)dt$$

$$= \frac{1}{2}\int_{-\sqrt{2}}^{0}\left(\frac{t^2}{2} + \sqrt{2}t\right)dt = \frac{1}{2}\left[\frac{t^3}{6} + \frac{\sqrt{2}}{2}t^2\right]_{-\sqrt{2}}^{0}$$

$$= -\frac{\sqrt{2}}{3}$$

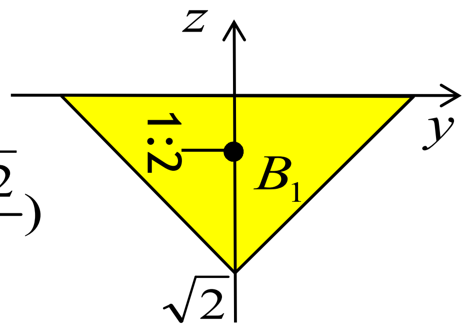
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (5/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓First moment of area about the z-axis in y direction $\boldsymbol{M_{A,z}}$

$$M_{A,z} = \frac{1}{2}\oint_C \frac{y^2}{2}dz - yzdy$$

Segment ①: $\frac{1}{2}\int_{①} \frac{y^2}{2}dz - yzdy = -\frac{\sqrt{2}}{3}$

Segment ②: $y(t) = t,\quad z(t) = t - \sqrt{2},\quad 0 \le t \le \sqrt{2}$

$$\frac{1}{2}\int_{②} \frac{y^2}{2}dz - yzdy = \frac{1}{2}\int_0^{\sqrt{2}} \left(\frac{y^2}{2}\frac{dz}{dt} - yz\frac{dy}{dt}\right)dt$$

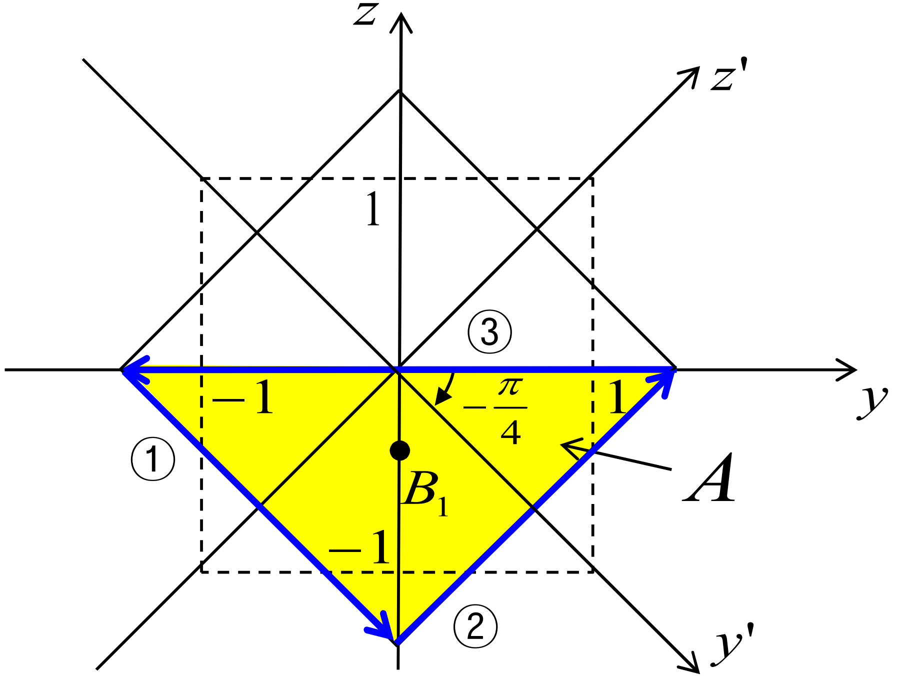
$$= \frac{1}{2}\int_0^{\sqrt{2}} \left(\frac{t^2}{2}\cdot 1 - t(t-\sqrt{2})\cdot 1\right)dt$$

$$= \frac{1}{2}\int_0^{\sqrt{2}} \left(-\frac{t^2}{2} + \sqrt{2}t\right)dt$$

$$= \frac{1}{2}\left[-\frac{t^3}{6} + \frac{\sqrt{2}}{2}t^2\right]_0^{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{3}$$

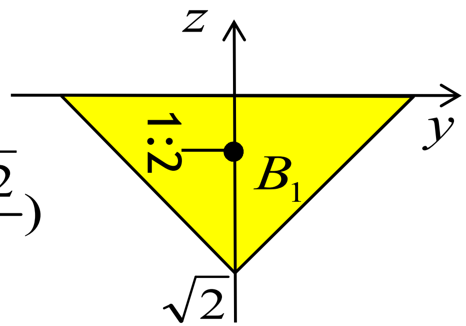
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (6/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓First moment of area about the z-axis in y direction $\boldsymbol{M_{A,z}}$

$$M_{A,z} = \frac{1}{2}\oint_C \frac{y^2}{2}dz - yzdy$$

Segment ①: $\frac{1}{2}\int_{①} \frac{y^2}{2}dz - yzdy = -\frac{\sqrt{2}}{3}$

Segment ②: $\frac{1}{2}\int_{②} \frac{y^2}{2}dz - yzdy = \frac{\sqrt{2}}{3}$

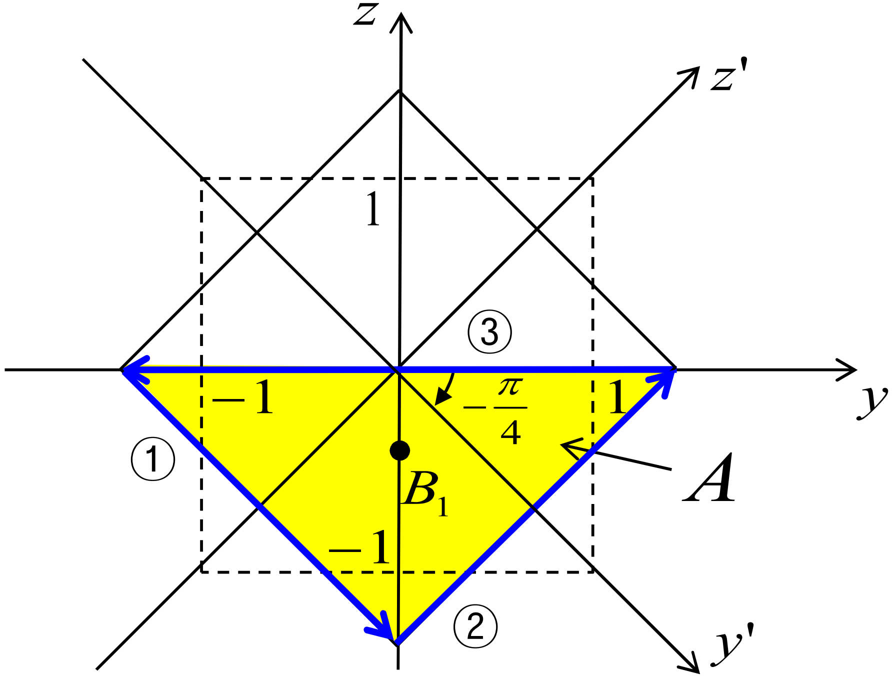
Segment ③: $y(t) = t, \quad z = 0, \quad -\sqrt{2} \le t \le \sqrt{2}$

$$\frac{1}{2}\int_{③} \frac{y^2}{2}dz - yzdy = \frac{1}{2}\int_{\sqrt{2}}^{-\sqrt{2}} \left(\frac{y^2}{2}\frac{dz}{dt} - yz\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{\sqrt{2}}^{-\sqrt{2}} \left(\frac{t^2}{2}\cdot 0 - t\cdot 0\cdot 1\right)dt = 0$$

$$\therefore M_{A,z} = \frac{1}{2}\oint_C \frac{y^2}{2}dz - yzdy = \underset{①}{-\frac{\sqrt{2}}{3}} + \underset{②}{\frac{\sqrt{2}}{3}} + \underset{③}{0} = 0$$

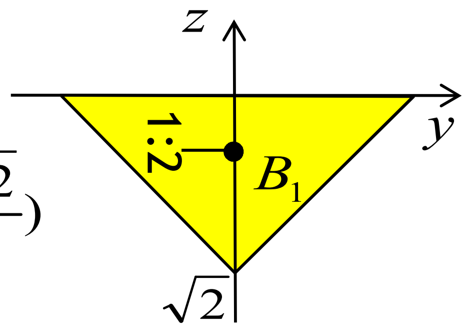
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (7/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A=\frac{1}{2}\cdot 2\cdot 2=2$$

$$(y_{B_1}, z_{B_1})=(0,-\frac{\sqrt{2}}{3})$$



oy'z': Body fixed coordinate

oyz: Water plane fixed coordinate

✓First moment of area about the y-axis in z direction $\boldsymbol{M_{A,y}}$

$$M_{A,y}=\frac{1}{2}\oint_C yzdz-\frac{z^2}{2}dy$$

⇩ Green's theorem

$$=\frac{1}{2}\oint_C yzdz-\frac{z^2}{2}dy$$

Segment ①: $y(t)=t,\quad z(t)=-t-\sqrt{2},\quad -\sqrt{2}\le t\le 0$

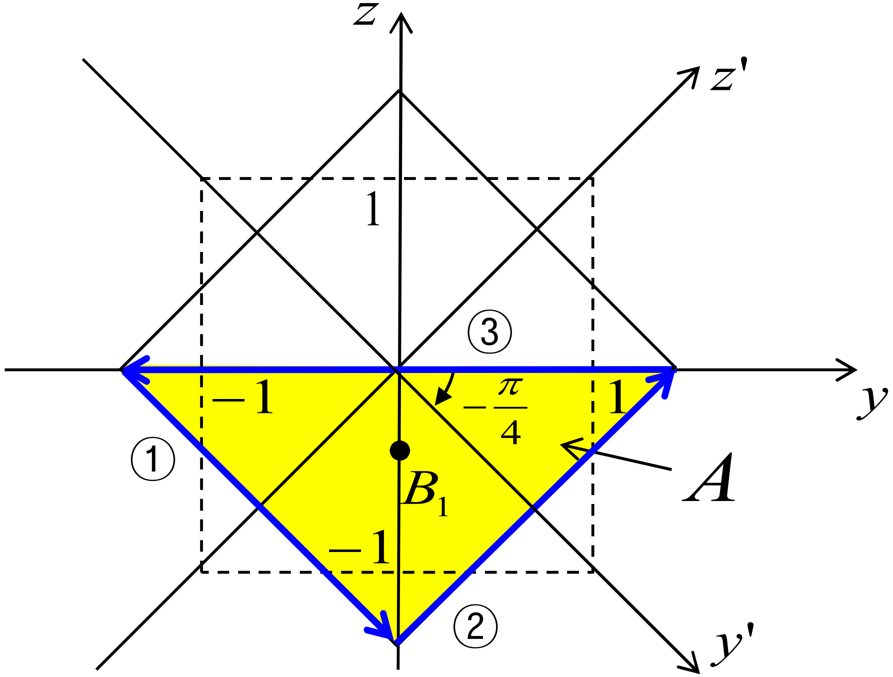
$$\frac{1}{2}\int_{①} yzdz-\frac{z^2}{2}dy=\frac{1}{2}\int_{-\sqrt{2}}^{0}\left(yz\frac{dz}{dt}-\frac{z^2}{2}\frac{dy}{dt}\right)dt$$

$$=\frac{1}{2}\int_{-\sqrt{2}}^{0}\left(t(-t-\sqrt{2})(-1)-\frac{(-t-\sqrt{2})^2}{2}\cdot 1\right)dt$$

$$=\frac{1}{2}\int_{-\sqrt{2}}^{0}\left(t^2+\sqrt{2}t-\frac{t^2+2\sqrt{2}t+2}{2}\right)dt$$

$$=\frac{1}{2}\int_{-\sqrt{2}}^{0}\left(\frac{t^2}{2}-1\right)dt=\frac{1}{2}\left[\frac{t^3}{6}-t\right]_{-\sqrt{2}}^{0}=-\frac{\sqrt{2}}{3}$$

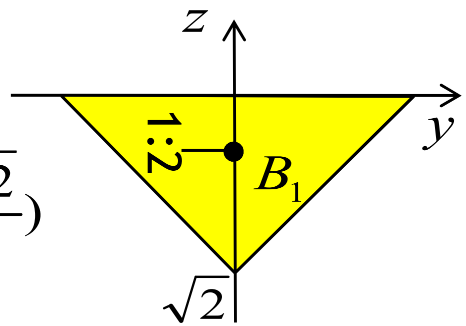
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (8/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓First moment of area about the y-axis in z direction $M_{A,y}$

$$M_{A,y} = \frac{1}{2}\oint_C yzdz - \frac{z^2}{2}dy$$

Segment ①: $\frac{1}{2}\int_{①} yzdz - \frac{z^2}{2}dy = -\frac{\sqrt{2}}{3}$

Segment ②: $y(t) = t, \quad z(t) = t - \sqrt{2}, \quad 0 \le t \le \sqrt{2}$

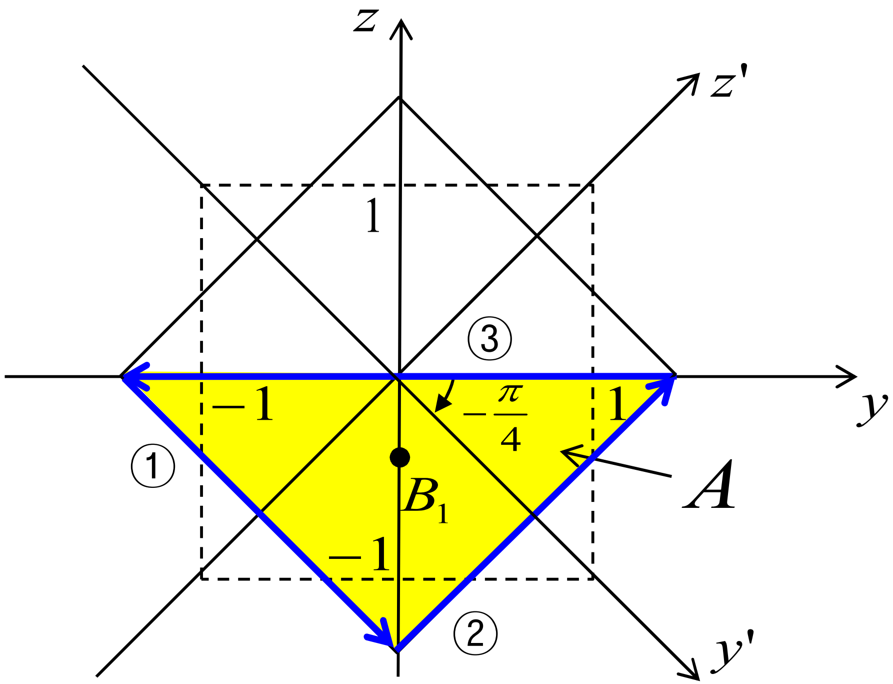
$$\frac{1}{2}\int_{②} yzdz - \frac{z^2}{2}dy = \frac{1}{2}\int_0^{\sqrt{2}}\left(yz\frac{dz}{dt} - \frac{z^2}{2}\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_0^{\sqrt{2}}\left(t(t-\sqrt{2})\cdot 1 - \frac{(t-\sqrt{2})^2}{2}\cdot 1\right)dt$$

$$= \frac{1}{2}\int_0^{\sqrt{2}}\left(t^2 - \sqrt{2}t - \frac{t^2 - 2\sqrt{2}t + 2}{2}\right)dt$$

$$= \frac{1}{2}\int_0^{\sqrt{2}}\left(\frac{t^2}{2} - 1\right)dt = \frac{1}{2}\left[\frac{t^3}{6} - t\right]_0^{\sqrt{2}} = -\frac{\sqrt{2}}{3}$$

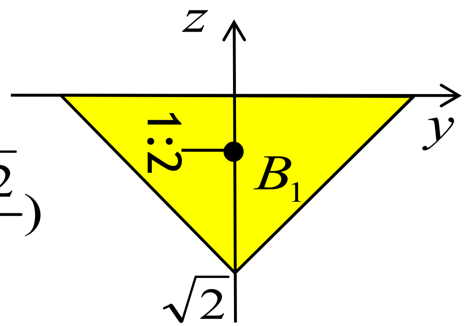
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (9/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



oy'z': Body fixed coordinate

oyz : Water plane fixed coordinate

✓First moment of area about the y-axis in z direction $M_{A,y}$

$$M_{A,y} = \frac{1}{2}\oint_C yzdz - \frac{z^2}{2}dy$$

Segment ①: $\frac{1}{2}\int_{①} yzdz - \frac{z^2}{2}dy = -\frac{\sqrt{2}}{3}$

Segment ②: $\frac{1}{2}\int_{②} yzdz - \frac{z^2}{2}dy = -\frac{\sqrt{2}}{3}$

Segment ③: $y(t) = t, \quad z = 0, \quad -\sqrt{2} \le t \le \sqrt{2}$

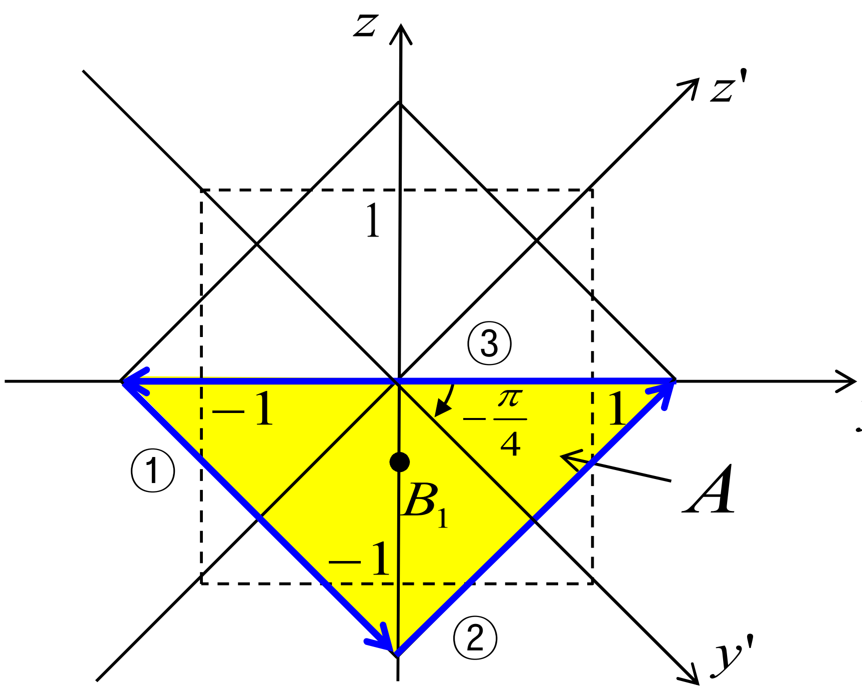
$$\frac{1}{2}\int_{③} yzdz - \frac{z^2}{2}dy = \frac{1}{2}\int_{\sqrt{2}}^{-\sqrt{2}}\left(yz\frac{dz}{dt} - \frac{z^2}{2}\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{\sqrt{2}}^{-\sqrt{2}}\left(t\cdot 0\cdot 1 - \frac{0^2}{2}\cdot 1\right)dt = 0$$

$$\therefore M_{A,y} = \frac{1}{2}\oint_C yzdz - \frac{z^2}{2}dy = -\frac{\sqrt{2}}{3} - \frac{\sqrt{2}}{3} + 0 = -\frac{2\sqrt{2}}{3}$$

① ② ③

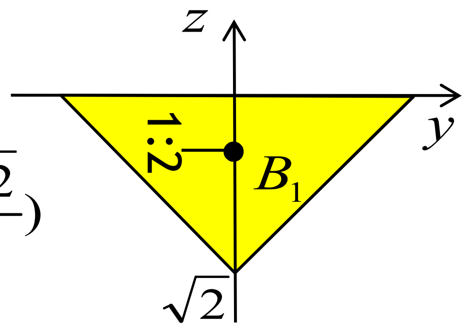
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (10/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2} \cdot 2 \cdot 2 = 2$$

$$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓Area ***A***

$$A = \frac{1}{2}\oint_C ydz - zdy = 2$$

✓First moment of area about the z-axis in y direction $\boldsymbol{M_{A,z}}$

$$M_{A,z} = \frac{1}{2}\oint_C \frac{y^2}{2}dz - yzdy = 0$$

✓First moment of area about the y-axis in z direction $\boldsymbol{M_{A,y}}$

$$M_{A,y} = \frac{1}{2}\oint_C yzdz - \frac{z^2}{2}dy = -\frac{2\sqrt{2}}{3}$$

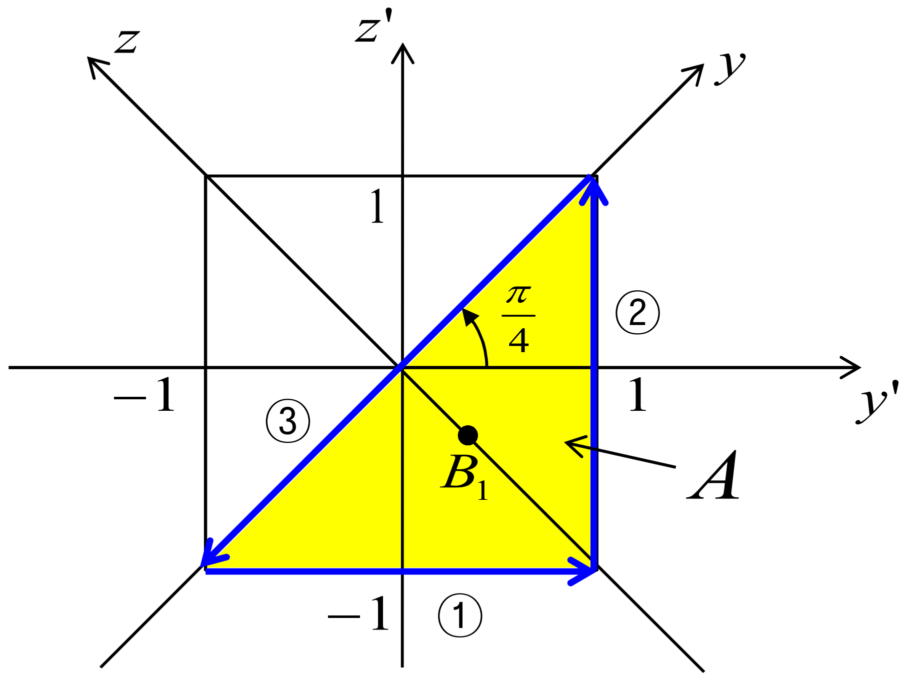
✓Centroid

$$(y_{B_1}, z_{B_1}) = \left(\frac{M_{A,z}}{A}, \frac{M_{A,y}}{A}\right)$$

$$= \left(\frac{0}{2}, \frac{1}{2}\cdot\left(-\frac{2\sqrt{2}}{3}\right)\right)$$

$$= \left(0, -\frac{\sqrt{2}}{3}\right)$$

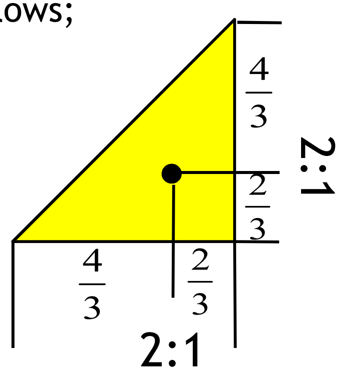
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (1/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓Area A

$$A = \int dA = \iint dy'dz'$$

Green's theorem

$$= \frac{1}{2}\oint_C y'dz' - z'dy'$$

Segment ①: $y'(t) = t, \quad z'(t) = -1, \quad -1 \le t \le 1$

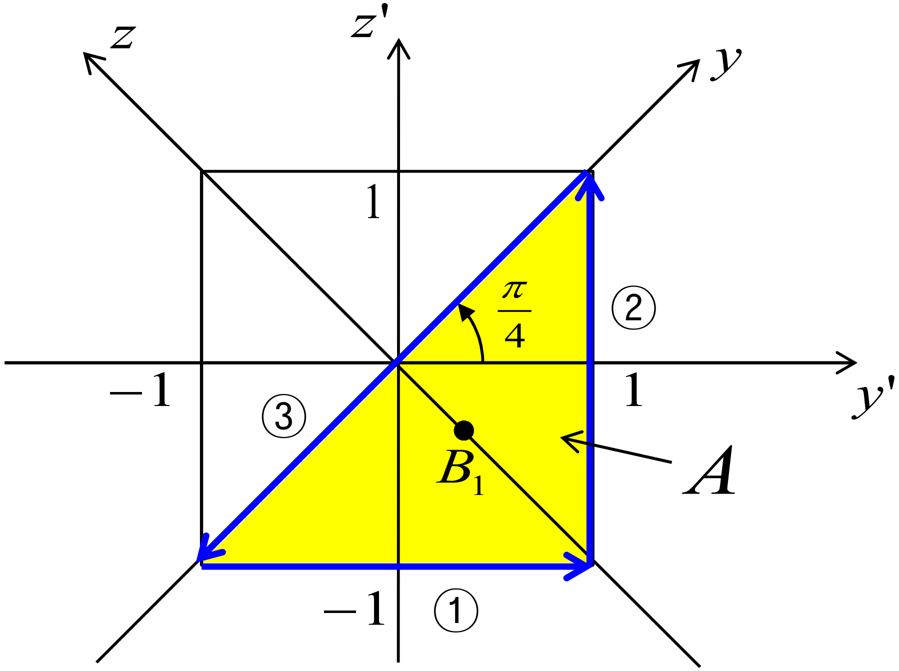
Using the chain rule, convert the line integral for y' and z' into the integral for only one parameter t.

$$\frac{1}{2}\int_{①} y'dz' - z'dy' = \frac{1}{2}\int_{-1}^{1}\left(y'\frac{dz}{dt} - z'\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}(t\cdot 0 - (-1)\cdot 1)dt$$

$$= \frac{1}{2}\int_{-1}^{1} 1dt = \frac{1}{2}t\Big|_{-1}^{1} = 1$$

[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (2/10)



✓Area A

$$A = \frac{1}{2}\oint_C y'dz' - z'dy'$$

Segment ①: $\frac{1}{2}\int_{①} y'dz' - z'dy' = 1$

Segment ②: $y'(t) = 1, \quad z'(t) = t, \quad -1 \le t \le 1$

$$\frac{1}{2}\int_{②} y'dz' - z'dy' = \frac{1}{2}\int_{-1}^{1}\left(y'\frac{dz}{dt} - z'\frac{dy}{dt}\right)dt$$

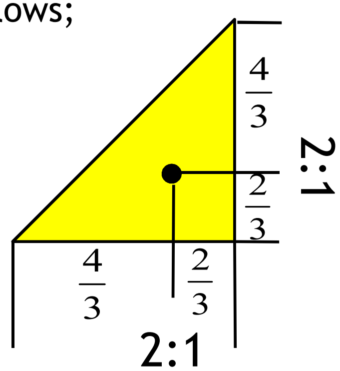
$$= \frac{1}{2}\int_{-1}^{1}(1\cdot 1 - t\cdot 0)dt$$

$$= \frac{1}{2}t\Big|_{-1}^{1} = 1$$

Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

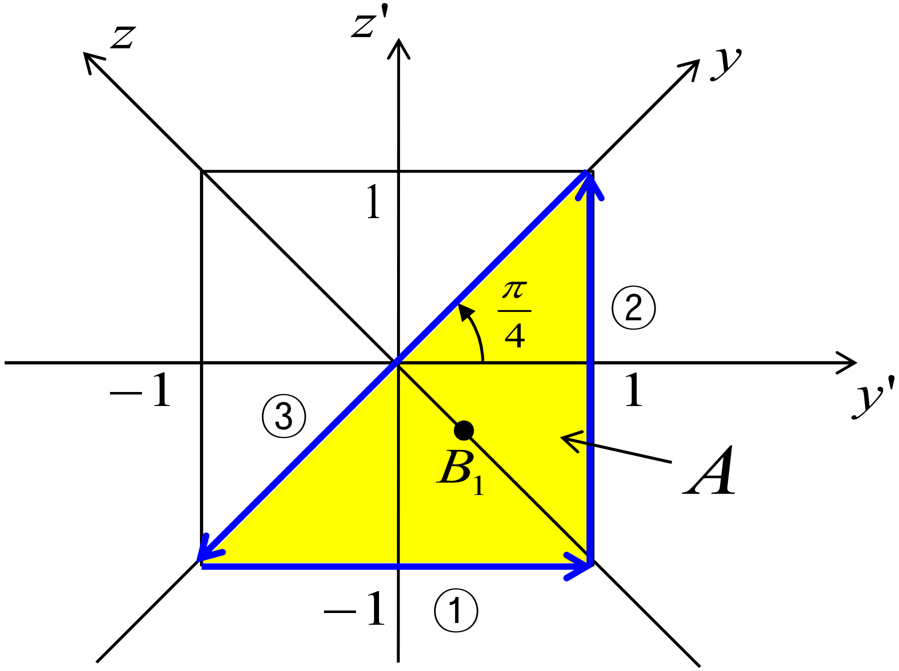
$$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

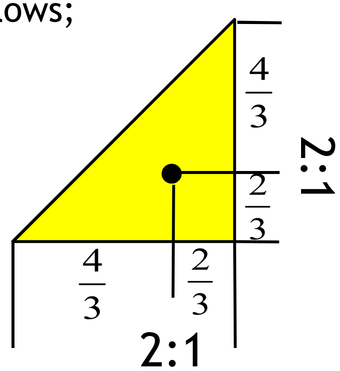
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (3/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A=\frac{1}{2}\cdot 2\cdot 2=2$$

$$(y'_{B_1},z'_{B_1})=(\frac{1}{3},-\frac{1}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓Area A

$$A=\frac{1}{2}\oint_C y'dz'-z'dy'$$

Segment ①: $\frac{1}{2}\int_{①} y'dz'-z'dy'=1$

Segment ②: $\frac{1}{2}\int_{②} y'dz'-z'dy'=1$

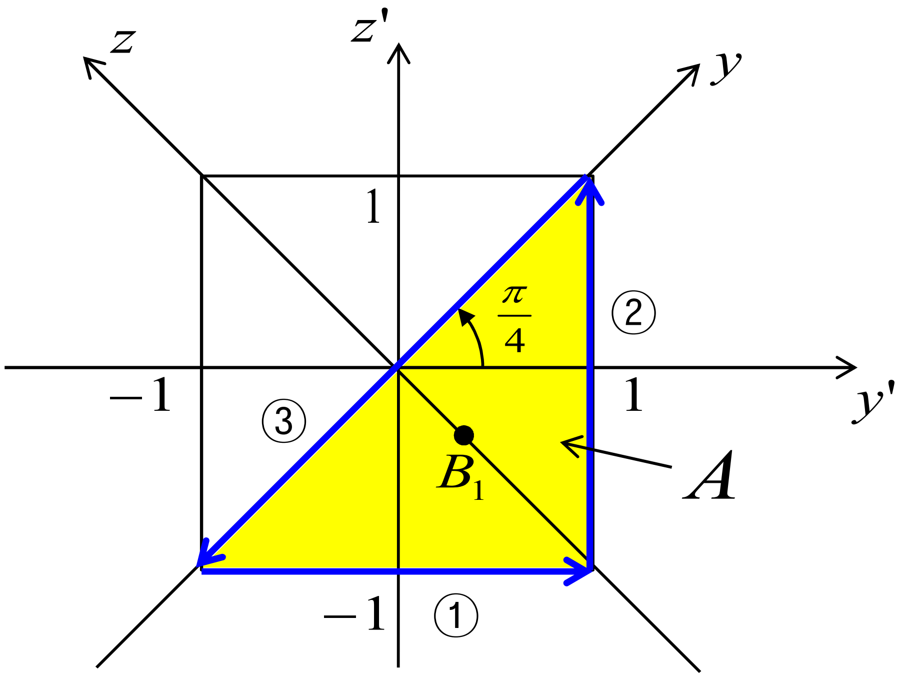
Segment ③: $y'(t)=t,\quad z'(t)=t,\quad -1\le t\le 1$

$$\frac{1}{2}\int_{③} y'dz'-z'dy'=\int_1^{-1}\left(y'\frac{dz}{dt}-z'\frac{dy}{dt}\right)dt$$

$$=\int_1^{-1}(1\cdot 1-1\cdot 1)dt=0$$

$$\therefore\ A=\frac{1}{2}\oint_C y'dz'-z'dy'=\underset{①}{1}+\underset{②}{1}+\underset{③}{0}=2$$

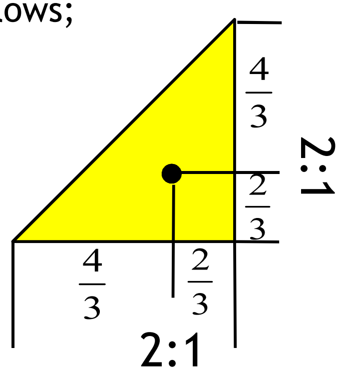
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (4/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y'_{B_1}, z'_{B_1}) = \left(\frac{1}{3}, -\frac{1}{3}\right)$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓First moment of area about the z′-axis in y′ direction $M_{A,z'}$

$$M'_{A,z'} = \int y'\,dA = \iint y'\,dy'\,dz'$$

⇩ Green's theorem

$$= \frac{1}{2}\oint_C \frac{y'^2}{2}dz' - y'z'dy'$$

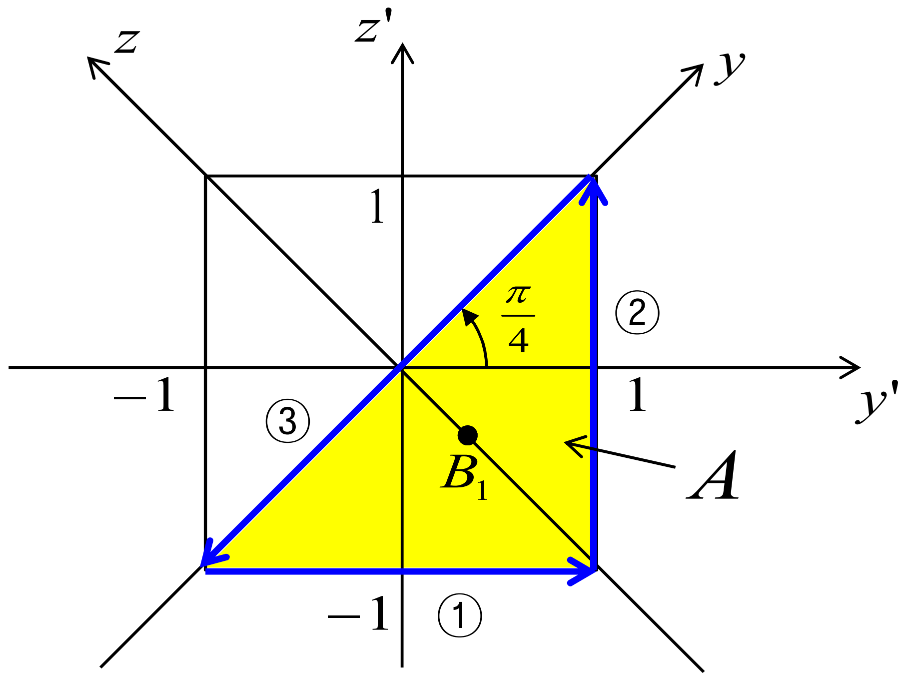
Segment ①: $y'(t) = t,\quad z'(t) = -1,\quad -1 \le t \le 1$

$$\frac{1}{2}\int_{①} \frac{y'^2}{2}dz' - y'z'dy' = \frac{1}{2}\int_{-1}^{1}\left(\frac{y'^2}{2}\frac{dz'}{dt} - y'z'\frac{dy'}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}\left(\frac{t^2}{2}\cdot 0 - t(-1)\cdot 1\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1} t\,dt = \frac{1}{4}t^2\Big|_{-1}^{1} = 0$$

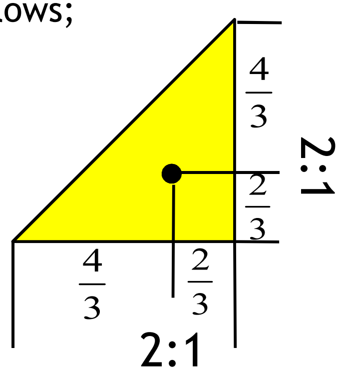
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (5/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓First moment of area about the z′-axis in y′ direction $M_{A,z'}$

$$M'_{A,z'} = \frac{1}{2}\oint_C \frac{y'^2}{2}dz' - y'z'dy'$$

Segment ①: $\frac{1}{2}\int_{①} \frac{y'^2}{2}dz' - y'z'dy' = 0$

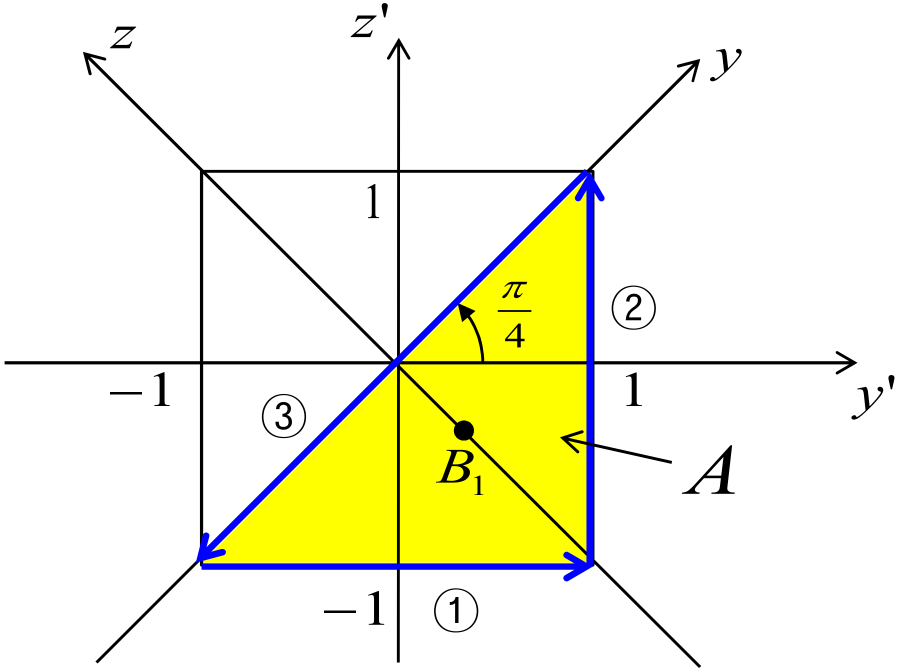
Segment ②: $y'(t) = 1, \quad z'(t) = t, \quad -1 \le t \le 1$

$$\frac{1}{2}\int_{②} \frac{y'^2}{2}dz' - y'z'dy' = \frac{1}{2}\int_{-1}^{1}\left(\frac{y'^2}{2}\frac{dz'}{dt} - y'z'\frac{dy'}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}\left(\frac{1^2}{2}\cdot 1 - 1\cdot t\cdot 0\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}\frac{1}{2}dt = \frac{1}{4}t\Big|_{-1}^{1} = \frac{1}{2}$$

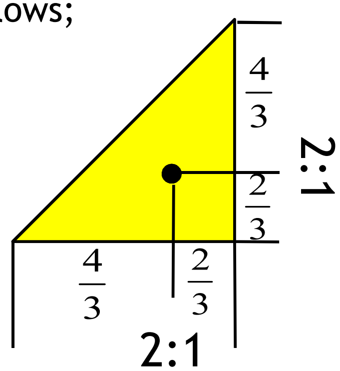
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (6/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓First moment of area about the z′-axis in y′ direction $M_{A,z'}$

$$M'_{A,z'} = \frac{1}{2}\oint_C \frac{y'^2}{2}dz' - y'z'dy'$$

Segment ①: $\frac{1}{2}\int_{①} \frac{y'^2}{2}dz' - y'z'dy' = 0$

Segment ②: $\frac{1}{2}\int_{②} \frac{y'^2}{2}dz' - y'z'dy' = \frac{1}{2}$

Segment ③: $y'(t) = t, \quad z'(t) = t, \quad -1 \le t \le 1$

$$\frac{1}{2}\int_{②} \frac{y'^2}{2}dz' - y'z'dy' = \frac{1}{2}\int_{-1}^{1}\left(\frac{y'^2}{2}\frac{dz'}{dt} - y'z'\frac{dy'}{dt}\right)dt$$

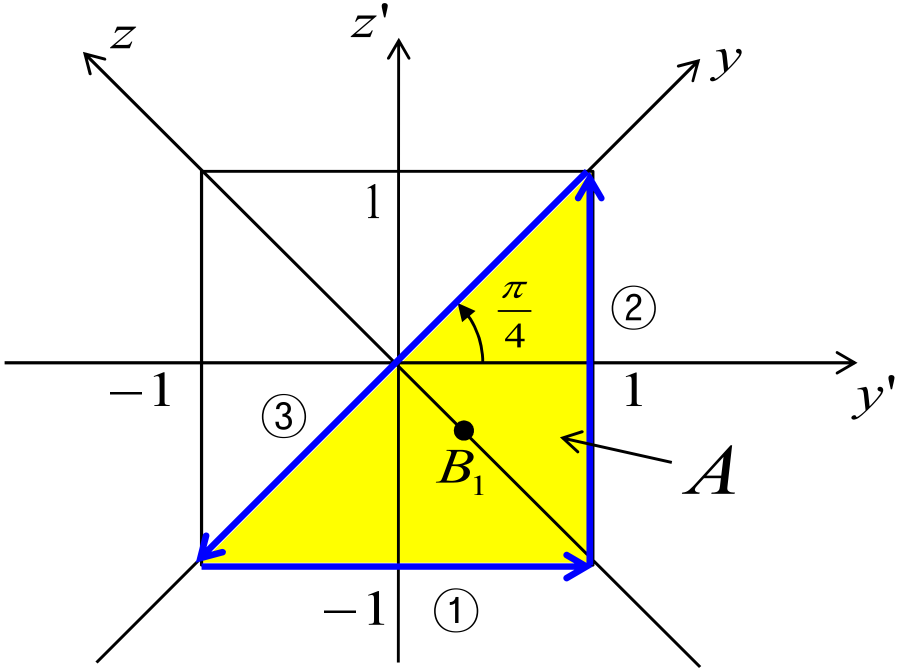
$$= \frac{1}{2}\int_{-1}^{1}\left(\frac{t^2}{2}\cdot 1 - t\cdot t\cdot 1\right)dt = \frac{1}{2}\int_{-1}^{1}\left(-\frac{t^2}{2}\right)dt = -\frac{t^3}{12}\Big|_{-1}^{1} = -\frac{1}{6}$$

$$\therefore M'_{A,z'} = \frac{1}{2}\oint_C \frac{y'^2}{2}dz' - y'z'dy'$$

$$= 0 + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}$$

① ② ③

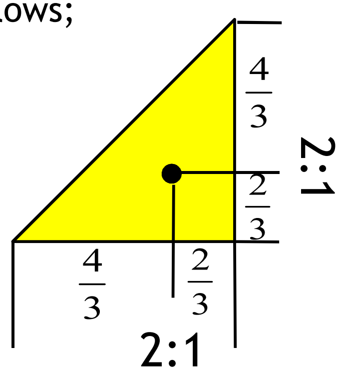
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (7/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y'_{B_1}, z'_{B_1}) = \left(\frac{1}{3}, -\frac{1}{3}\right)$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓First moment of area about the y′-axis in z′ direction $M_{A,y'}$

$$M'_{A,y'} = \int z'\,dA = \iint z'\,dy'\,dz'$$

Green's theorem

$$= \frac{1}{2}\oint_C y'z'\,dz' - \frac{z'^2}{2}dy'$$

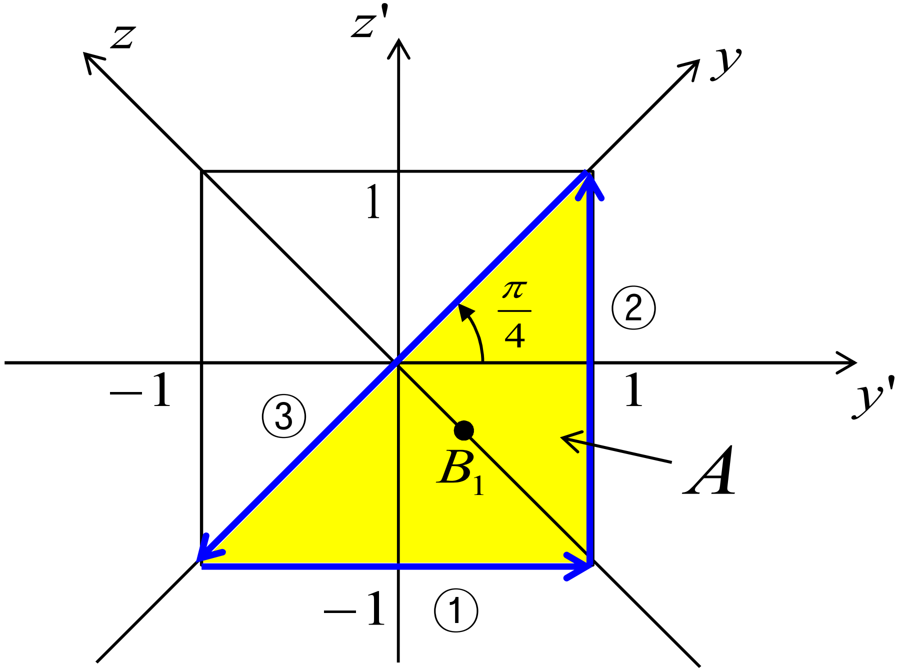
Segment ①: $y'(t) = t, \quad z'(t) = -1, \quad -1 \le t \le 1$

$$\frac{1}{2}\int_{①} y'z'\,dz' - \frac{z'^2}{2}dy' = \frac{1}{2}\int_{-1}^{1}\left(y'z'\frac{dz'}{dt} - \frac{z'^2}{2}\frac{dy'}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}\left(t(-1)\cdot 0 - \frac{(-1)^2}{2}\cdot 1\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}\left(-\frac{1}{2}\right)dt = -\frac{1}{4}t\Big|_{-1}^{1} = -\frac{1}{2}$$

[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (8/10)



✓First moment of area about the y′-axis in z′ direction $M_{A,y'}$

$$M'_{A,y'} = \frac{1}{2}\oint_C y'z'dz' - \frac{z'^2}{2}dy'$$

Segment ①: $\frac{1}{2}\int_{①} y'z'dz' - \frac{z'^2}{2}dy' = -\frac{1}{2}$

Segment ②: $y'(t) = 1, \quad z'(t) = t, \quad -1 \le t \le 1$

$$\frac{1}{2}\int_{②} y'z'dz' - \frac{z'^2}{2}dy' = \frac{1}{2}\int_{-1}^{1}\left(y'z'\frac{dz'}{dt} - \frac{z'^2}{2}\frac{dy'}{dt}\right)dt$$

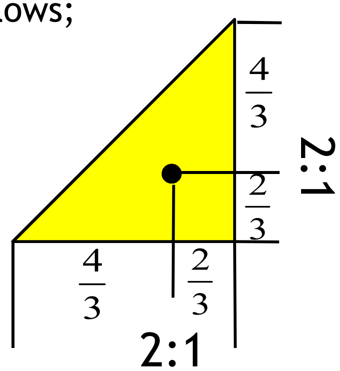
$$= \frac{1}{2}\int_{-1}^{1}\left(1\cdot t\cdot 1 - \frac{t^2}{2}\cdot 0\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1} t\,dt = \frac{1}{4}t^2\Big|_{-1}^{1} = 0$$

Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

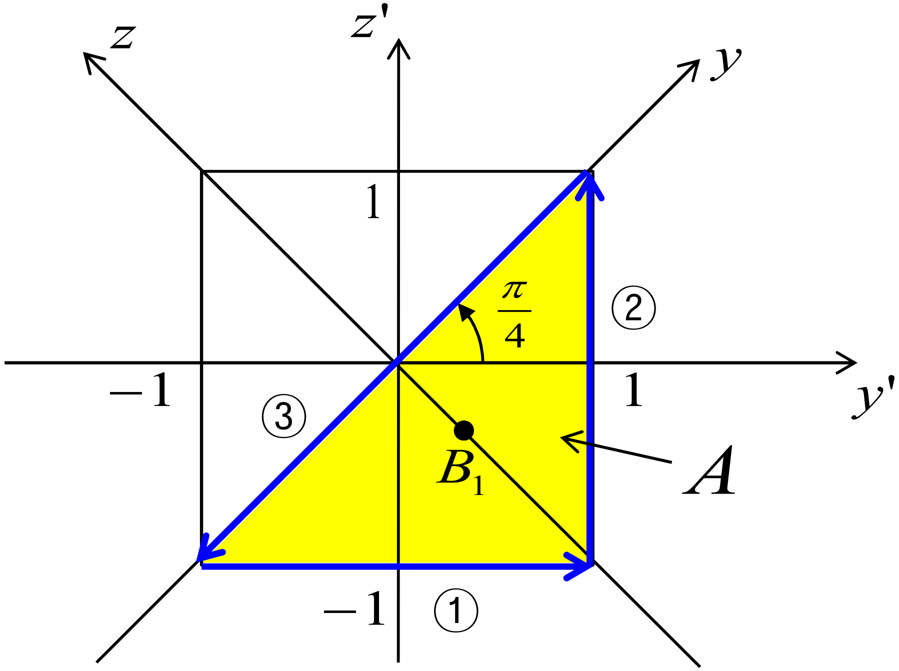
$$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

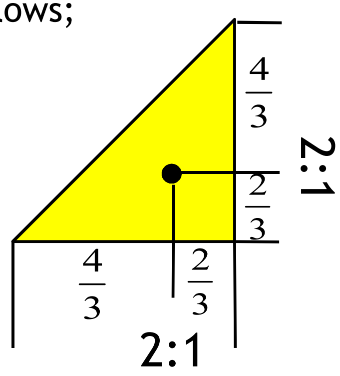
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (9/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A=\frac{1}{2}\cdot 2\cdot 2=2$$

$$(y'_{B_1},z'_{B_1})=(\frac{1}{3},-\frac{1}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓First moment of area about the y′-axis in z′ direction $M_{A,y'}$

$$M'_{A,y'}=\frac{1}{2}\oint_C y'z'dz'-\frac{z'^2}{2}dy'$$

Segment ①: $\frac{1}{2}\int_{①} y'z'dz'-\frac{z'^2}{2}dy'=-\frac{1}{2}$

Segment ②: $\frac{1}{2}\int_{②} y'z'dz'-\frac{z'^2}{2}dy'=0$

Segment ③: $y'(t)=t,\quad z'(t)=t,\quad -1\le t\le 1$

$$\frac{1}{2}\int_{③} y'z'dz'-\frac{z'^2}{2}dy'=\frac{1}{2}\int_{-1}^{1}\left(y'z'\frac{dz'}{dt}-\frac{z'^2}{2}\frac{dy'}{dt}\right)dt$$

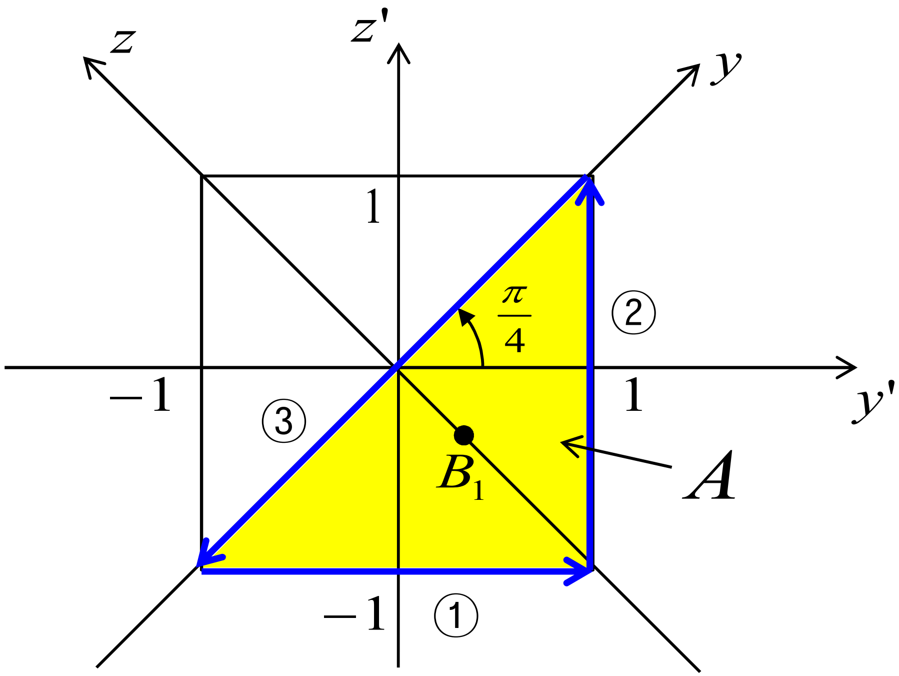
$$=\frac{1}{2}\int_{-1}^{1}\left(t\cdot t\cdot 1-\frac{t^2}{2}\cdot 1\right)dt=\frac{1}{2}\int_{-1}^{1}\frac{t^2}{2}dt=\left.\frac{t^3}{12}\right|_{-1}^{1}=\frac{1}{6}$$

$$\therefore M'_{A,y}=\frac{1}{2}\oint_C y'z'dz'-\frac{z'^2}{2}dy'$$

$$=0-\frac{1}{2}+\frac{1}{6}=-\frac{2}{3}$$

① ② ③

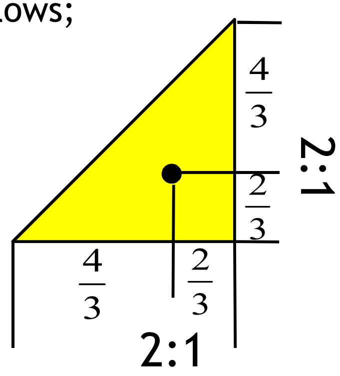
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (10/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓Area $\boldsymbol{A}$

$$A = \frac{1}{2}\oint_C y'dz' - z'dy' = 2$$

✓First moment of area about the z′-axis in y′ direction $\boldsymbol{M_{A,z'}}$

$$M'_{A,z'} = \frac{1}{2}\oint_C \frac{y'^2}{2}dz' - y'z'dy' = \frac{2}{3}$$

✓First moment of area about the y′-axis in z′ direction $\boldsymbol{M_{A,y'}}$

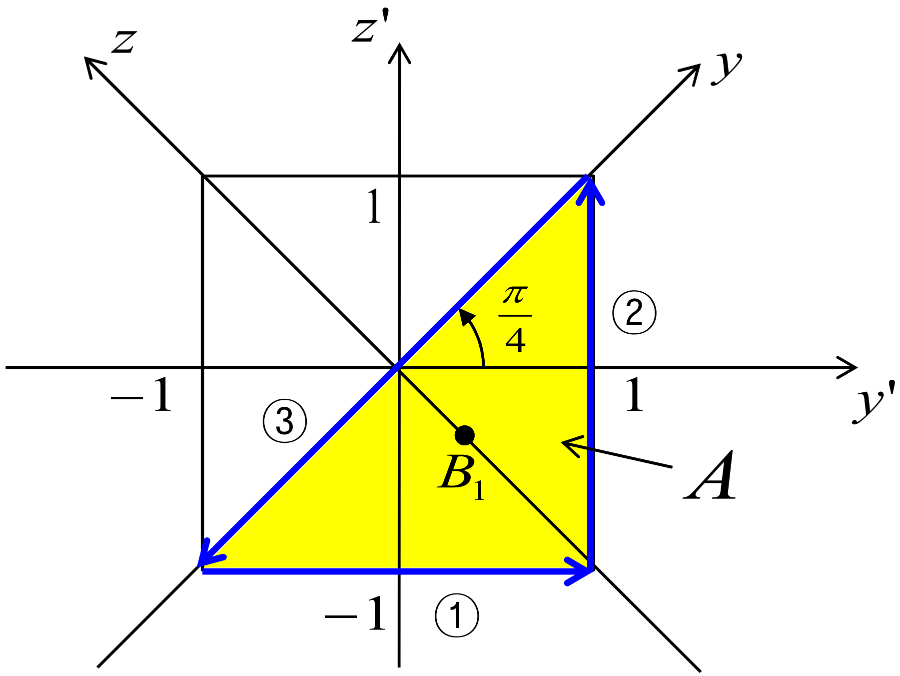
$$M'_{A,y'} = \frac{1}{2}\oint_C y'z'dz' - \frac{z'^2}{2}dy' = -\frac{2}{3}$$

✓Centroid

$$(y'_{B_1}, z'_{B_1}) = \left(\frac{M'_{A,z'}}{A}, \frac{M'_{A,y'}}{A}\right)$$

$$= \left(\frac{1}{2}\cdot\frac{2}{3}, \frac{1}{2}\cdot\left(-\frac{2}{3}\right)\right) = \left(\frac{1}{3}, -\frac{1}{3}\right)$$

[Example] Calculation of Area, First Moment of Area, and Centroid - Transform the Position Vectors with Respect to the Inertial Frame



$$A = 2$$

$$M'_{A,z'} = \frac{2}{3} \qquad M'_{A,y'} = -\frac{2}{3}$$

$$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$$

oy'z': Body fixed coordinate

oyz : Water plane fixed coordinate

✓Calculation of centroid(Center of buoyancy B_1) in the body fixed frame and inertial frame

Body fixed frame	Inertial frame
$\left(\frac{1}{3}, -\frac{1}{3}\right)$	$\left(0, -\frac{\sqrt{2}}{3}\right)$

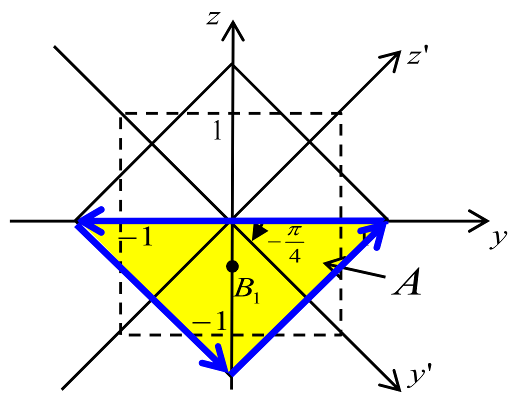
✓Transform the center of buoyancy in *oy'z'* frame into *oyz* frame by rotating the point about the negative x'-axis with an angle of $\frac{\pi}{4}$. Then the result is the same as the calculation result of centroid in the inertial frame.

$$\mathbf{r}_{B_1} = \begin{bmatrix} y_{B_1} \\ z_{B_1} \end{bmatrix} = \begin{bmatrix} \cos\left(-\frac{\pi}{4}\right) & -\sin\left(-\frac{\pi}{4}\right) \\ \sin\left(-\frac{\pi}{4}\right) & \cos\left(-\frac{\pi}{4}\right) \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$$

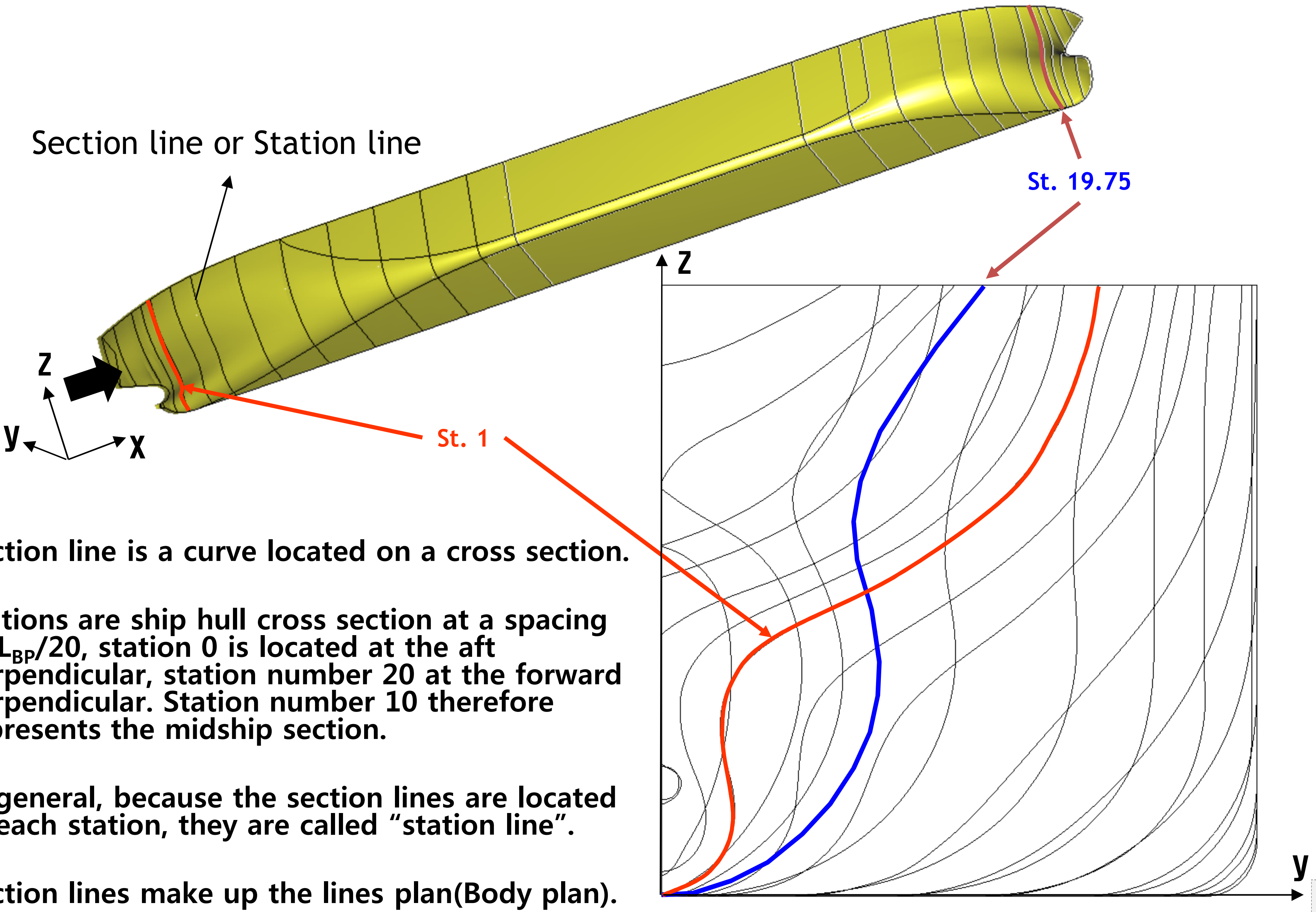
$$= \begin{bmatrix} 0 \\ -\frac{\sqrt{2}}{3} \end{bmatrix}$$

$$\therefore (y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



Calculation of Hydrostatic Values By Using Gaussian Quadrature and Green's Theorem

Section Line & Body Plan



☑Section line is a curve located on a cross section.

☑Stations are ship hull cross section at a spacing of $L_{BP}/20$, station 0 is located at the aft perpendicular, station number 20 at the forward perpendicular. Station number 10 therefore represents the midship section.

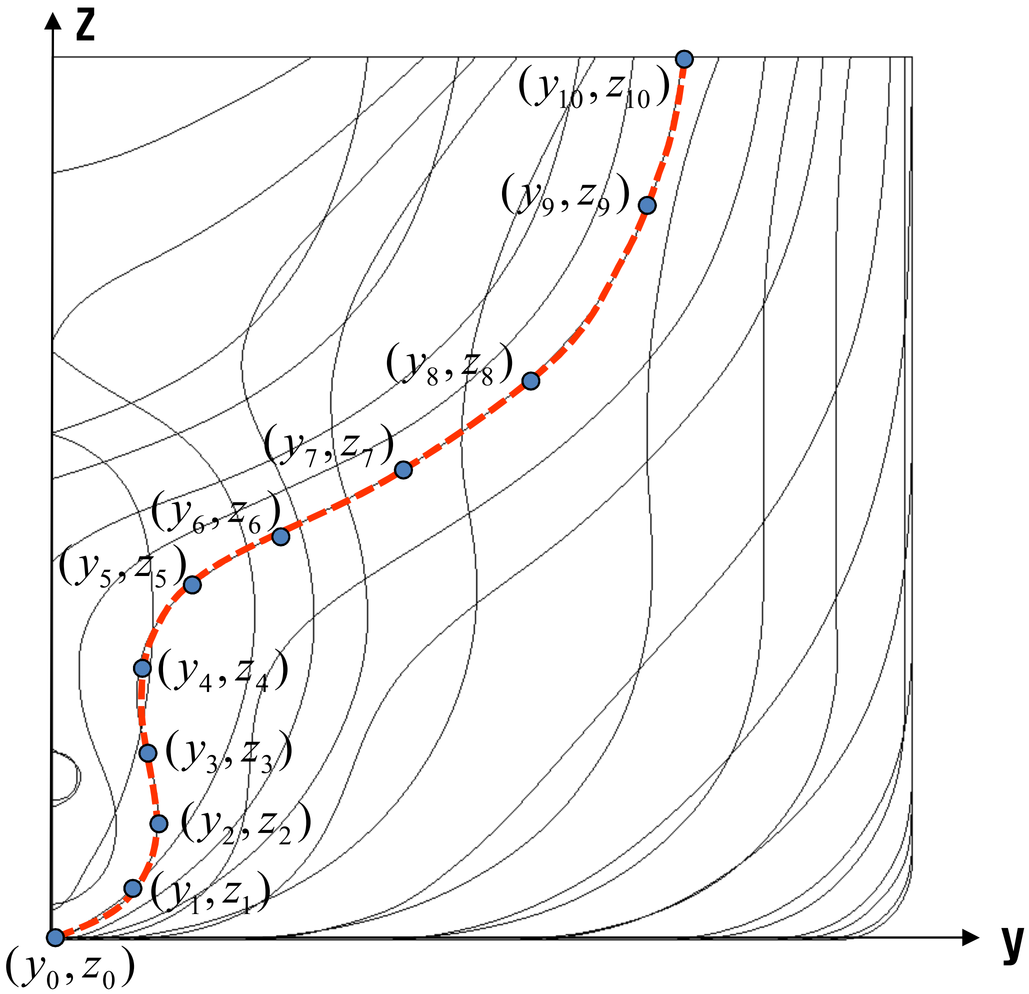
☑In general, because the section lines are located at each station, they are called "station line".

☑Section lines make up the lines plan(Body plan).

Description of Section Lines (1/2)

1. Make text file for describing the body plan of a ship.

Given: Body plan of a Ship
Find: Text file describing the body plan of a ship



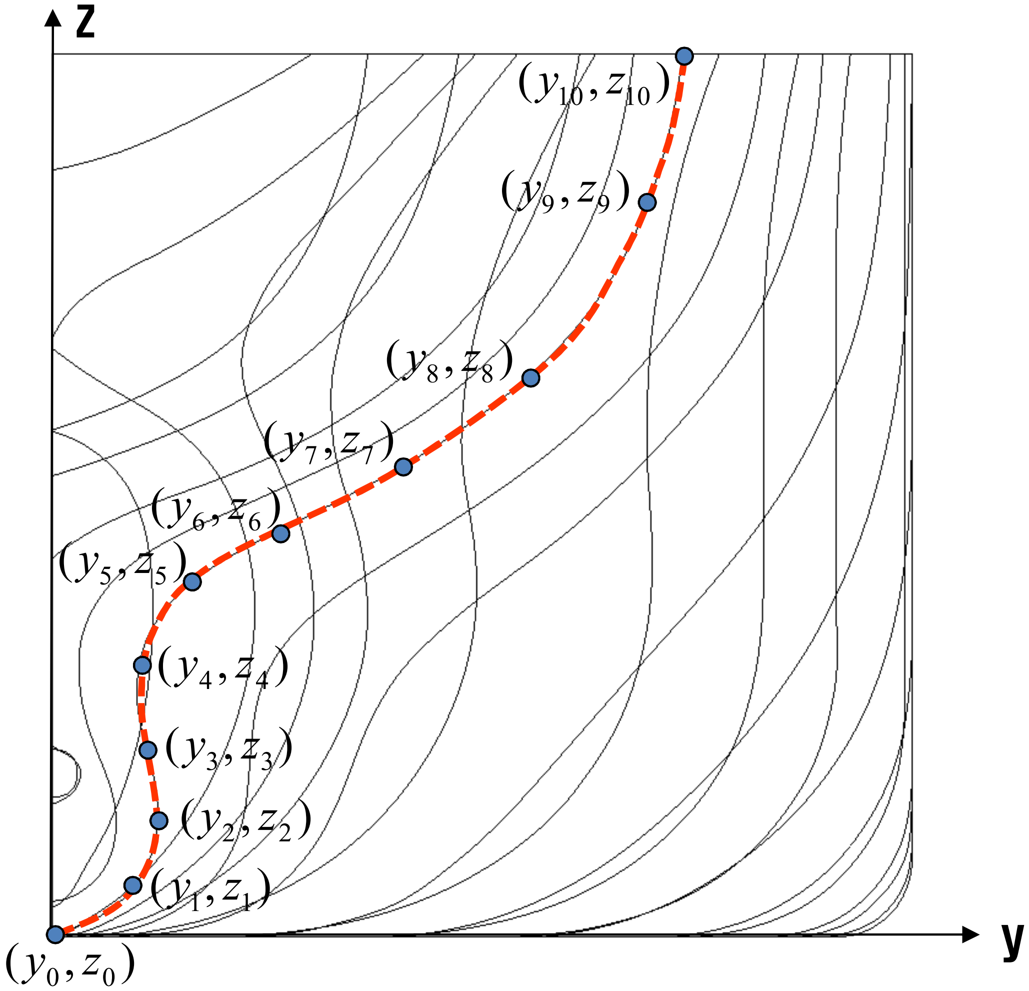
Example of text file for describing the body plan of a ship

300.0 50.0 27.0 18.0	**// *LBP*, *B*$_{mld}$, *D*$_{mld}$, *T***
27	**// Section Line Num.**
...	
1.0 11	**// Station, Point Num.**
y_0 z_0	**// Y coord., Z coord.**
y_1 z_1	
y_2 z_2	
...	
y_{10} z_{10}	
1.5 10	
...	

Description of Section Lines (2/2)

2. Find cubic B-spline curve passing the points on the section lines.

Given: Data of the points on the section line that describe the body plan of a ship
Find: Cubic B-Spline curve which passes through the points on the section line



Make cubic B-spline curve which passes through the given points

➡ Refer to the Part "Curve and Surface"

(Computer Aided Ship Design for 3rd Year Undergraduate Course)

$$\mathbf{r}(u) = \mathbf{d}_0 N_0^3(u) + \mathbf{d}_1 N_1^3(u) + \mathbf{d}_2 N_2^3(u) + \cdots + \mathbf{d}_{D-1} N_{D-1}^3(u)$$

$\mathbf{d}_i$: de Boor points (control points), $i = 0, 1, \ldots, D-1$

$N_i^n(u)$: B-splines basis function of degree $n(=3)$

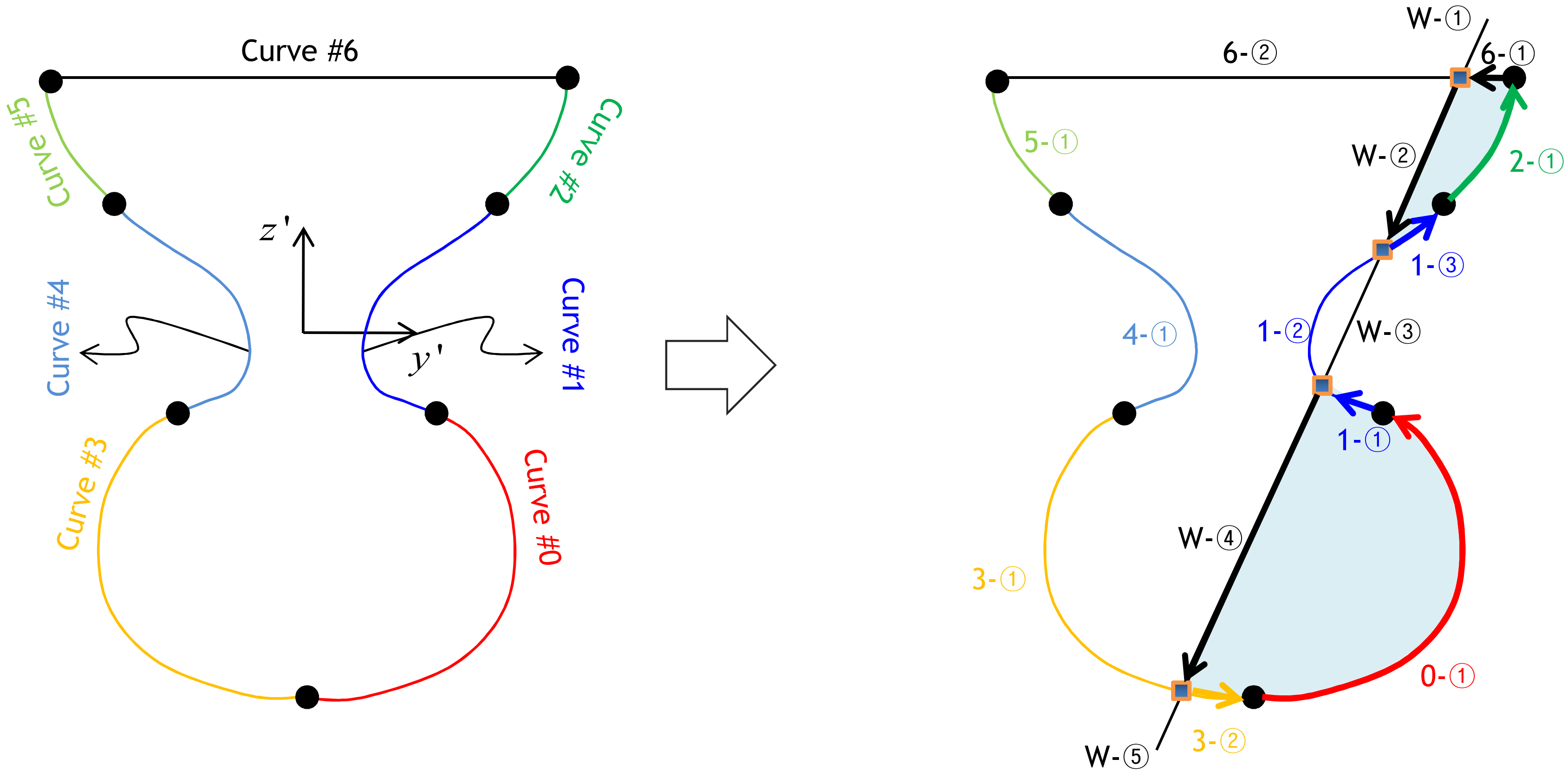
u_j : Knots, $j = 0, 1, \ldots, K-1$, where $K = D + n + 1$

$$N_i^n(u) = \frac{u - u_{i-1}}{u_{i+n-1} - u_{i-1}} N_i^{n-1}(u) + \frac{u_{i+n} - u}{u_{i+n} - u_i} N_{i+1}^{n-1}(u)$$

$$N_i^0(u) = \begin{cases} 1 & \textbf{if } \ u_{i-1} \le u < u_i \\ 0 & \textbf{else} \end{cases}, \quad \sum_{i=0}^{D-1} N_i^n(u) = 1$$

Calculation of Area and 1st Moment of Sectional Area Under the Water Plane (1/4)

Given: B-spline curve, the intersection points between the B-spline curves and water plane, and B-spline parameter "u" at each end point of the line segments
Find: Area and 1st moment of section



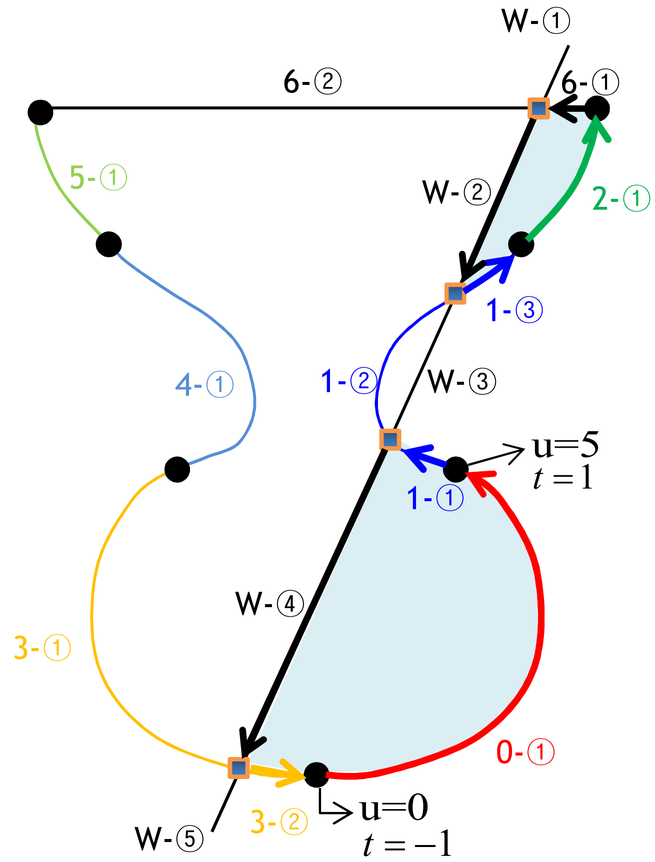
The section is represented by Curve #0 ~ Curve #6

The area and 1st moment of the section under the waterline is calculated by integration of the following line segments.

3-② ➡ 0-① ➡ 1-① ➡ W-④,
1-③ ➡ 2-① ➡ 6-① ➡ W-②

Calculation of Area and 1st Moment of Sectional Area Under the Water Plane (2/4)

Given: B-spline curve, the intersection points between the B-spline curve and water plane, and B-spline parameter "u" at each end point of the line segments
Find: Area and 1st moment of section



✓ Relation between the Parameter u and t

$$u = \frac{(t+1)(u_{\max} - u_{\min})}{2} + u_{\min}$$

$$u = \frac{(t+1)(5-0)}{2} + 0$$

<Surface integral> Green's Theorem <Line integral >

$$A = \iint_R dy'dz' \;\Rightarrow\; = \frac{1}{2}\oint_C (y'dz' - z'dy')$$

For example, integrate the line segment 0-①

For the line integral of the segment in the y'z' coordinate, the interval for integration has to be determined.

> Since the parameter u increases monotone, the interval can be found easily.

> Using the chain rule, convert the line integral for y' and z' into the integral for only one parameter 'u'.

$$\frac{1}{2}\int_0^5 \left(y'(u)\frac{dz'}{du}du - z'(u)\frac{dy'}{du}du \right)$$

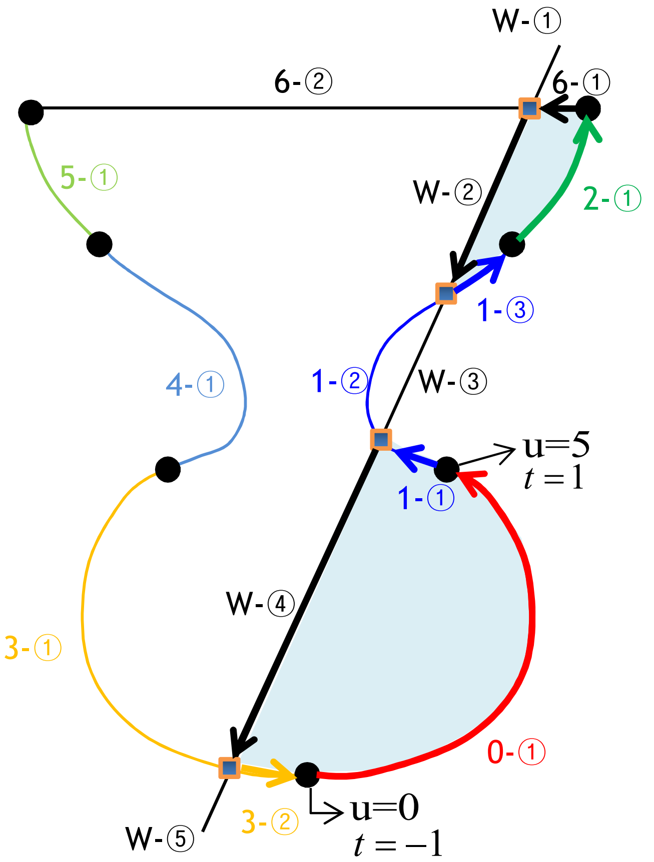
$$= \frac{1}{2}\int_0^5 \left(y'(u)\frac{dz'}{du} - z'(u)\frac{dy'}{du} \right) du = \frac{1}{2}\int_0^5 g(u)du$$

➡To use Gaussian quadrature, convert the integration parameter 'u' and the interval [0, 5] into 't' and [-1,1]

$$\frac{1}{2}\int_{-1}^1 \left(y'(u(t))\frac{dz'}{du} - z'(u(t))\frac{dy'}{du} \right)\frac{du}{dt}dt = \frac{1}{2}\int_{-1}^1 f(t)dt$$

✓ In the same way, integrate the remained line segments using Gaussian quadrature.

Calculation of Area and 1st Moment of Sectional Area Under the Water Plane (3/4)

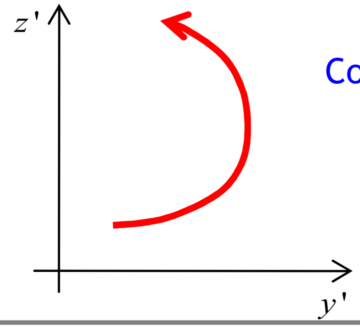


✓ Relation between the Parameter u and t

$$u = \frac{(t+1)(u_{max} - u_{min})}{2} + u_{min}$$

$$u = \frac{(t+1)(5-0)}{2} + 0$$

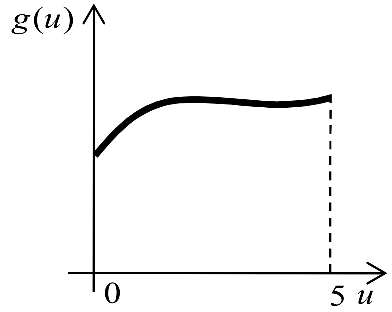
※ Procedure for calculation of the area and 1st moment of sectional area under the water plane



Convert surface integral into line integral

$$= \frac{1}{2}\oint_C y'dz' - z'dy'$$

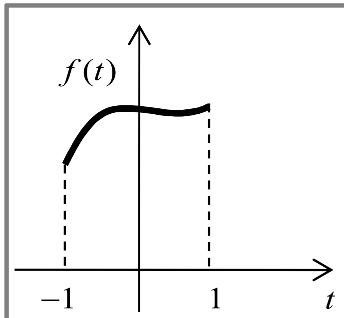
Using the chain rule, convert the line integral for y' and z' into the integral for only one parameter 'u'.



$$\frac{1}{2}\int_0^5 y'(u)\frac{dz'}{du}du - z'(u)\frac{dy'}{du}du$$

$$= \frac{1}{2}\int_0^5 \left(y'(u)\frac{dz'}{du} - z'(u)\frac{dy'}{du} \right) du$$

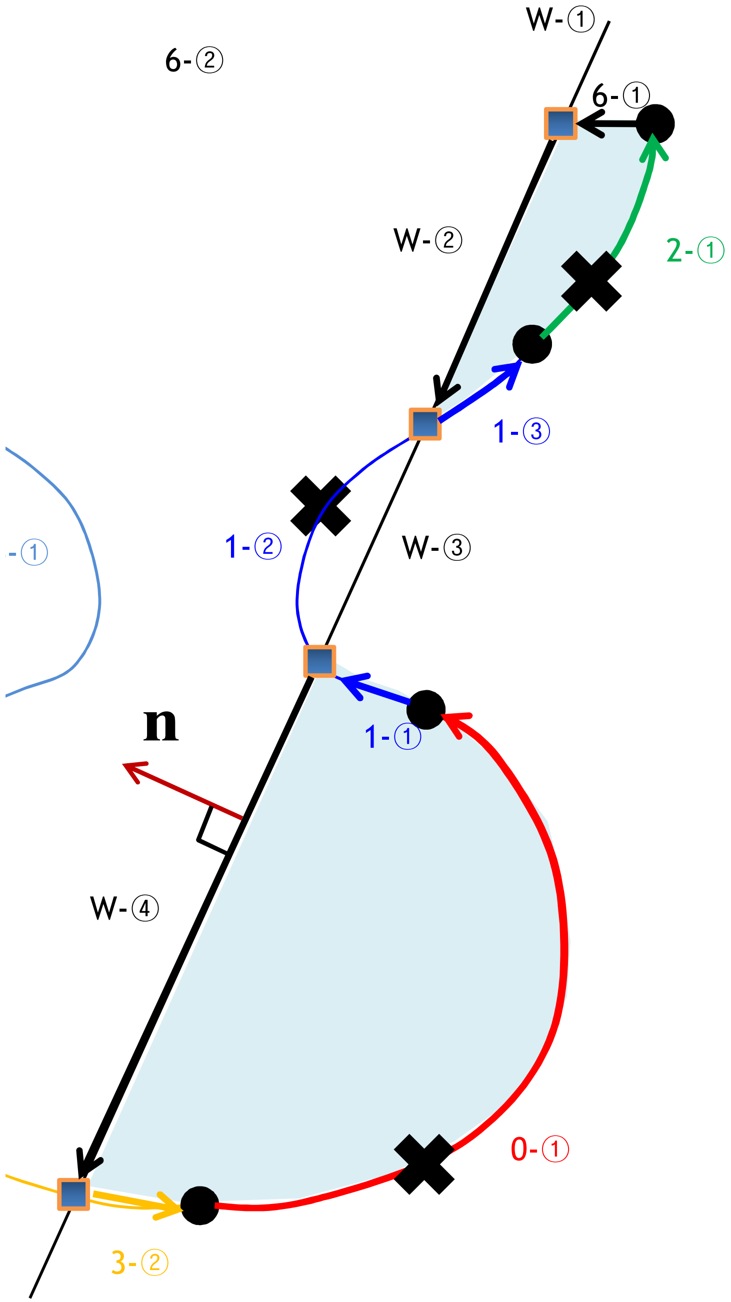
$$= \frac{1}{2}\int_0^5 g(u)du$$



To use Gaussian quadrature, convert the parameter and the interval into 't' and [-1,1].

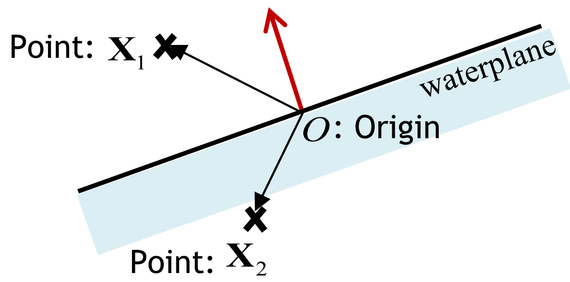
$$\frac{1}{2}\int_{-1}^1 \left(y'(u(t))\frac{dz'}{du} - z'(u(t))\frac{dy'}{du} \right) \frac{du}{dt}dt = \frac{1}{2}\int_{-1}^1 f(t)dt$$

Calculation of Area and 1st Moment of Sectional Area Under the Water Plane (4/4)



※ Method to check to check if the line segments are located under the water plane or not

- To calculate the sectional area under the water plane, it is required to check if the points on the line segments are located under the water plane or not. $\mathbf{n}$: Normal vector



✓Check the location of the point by using the sign of dot product of normal vector of the water plane and position vector of the point

$\mathbf{n} \bullet (\mathbf{X} - \mathbf{O}) > 0$: The point is above the water plane.

$\mathbf{n} \bullet (\mathbf{X} - \mathbf{O}) \leq 0$: The point is on or below the water plane.

✓Perform only line integration for the segments which are on or below the water plane.

In this example, the line integration is performed as follows:

The line segment 0-① : $\mathbf{n} \bullet (\mathbf{X} - \mathbf{O}) \leq 0$ ➡ Perform integration

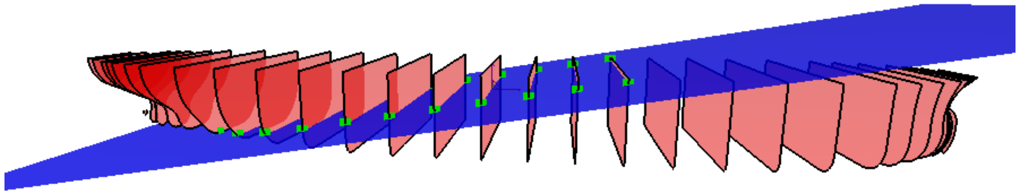
The line segment 1-② : $\mathbf{n} \bullet (\mathbf{X} - \mathbf{O}) > 0$ ➡ No integration

The line segment 2-① : $\mathbf{n} \bullet (\mathbf{X} - \mathbf{O}) \leq 0$ ➡ Perform integration

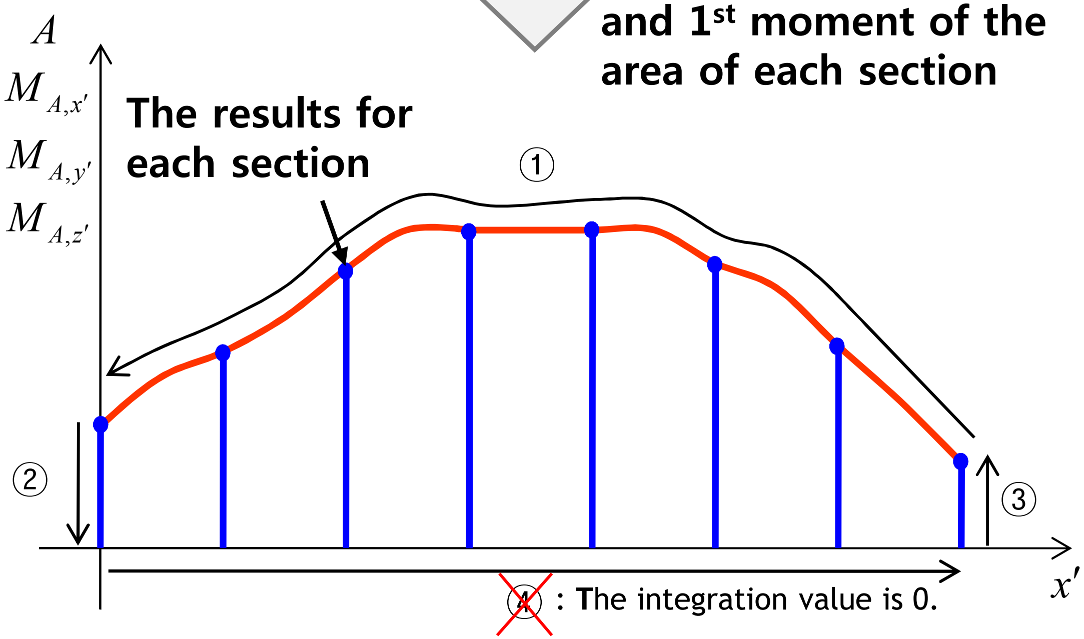
($\mathbf{X}$: the middle point of the each line segment)

Calculation of Ship's Displacement Volume, 1st Moment of Displaced Volume, LCB, TCB, and KB

Given: Sectional areas and 1st moment of the sectional area under water
Find: Displacement volume, 1st moment of displacement volume, LCB, TCB, and KB



Calculate sectional area and 1st moment of the area of each section



$$V = \nabla = \int A(x')dx'$$

↑
Displacement Volume

$$M_{\nabla,y'z'} = \int M_{A,y'z'}(x')dx'$$

$$M_{\nabla,x'z'} = \int M_{A,x'z'}(x')dx'$$

$$M_{\nabla,x'y'} = \int M_{A,x'y'}(x')dx'$$

Calculation procedure

✓ Calculate the displacement volume and 1st moment of the volume by integrating the sectional area and 1st moment of the sectional area over ship's length.

1) Make the ordinate set along ship's length by using the results for each section.
2) Generate B-spline curve which interpolates the ordinates.
3) Perform the line integration counter-clockwise using Green's theorem and Gaussian quadrature.

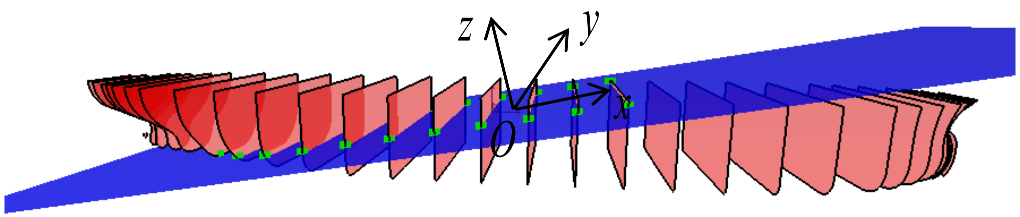
Displacement: $\Delta = \rho_{sw} \cdot \nabla$

$$LCB = \frac{M_{\nabla,y'z'}}{\nabla}, TCB = \frac{M_{\nabla,x'z'}}{\nabla}, VCB = \frac{M_{\nabla,x'y'}}{\nabla}$$

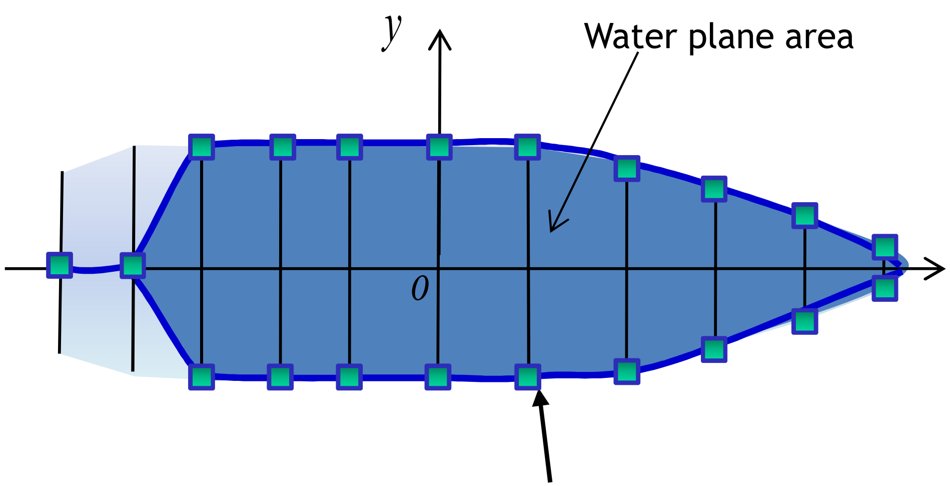
$$KB = VCB + T_d$$

Calculation of Water Plane Area, 1st and 2nd Moment of Water Plane Area

Given: Intersection points between the water plane and the section lines
Find: Water plane area, 1st moment and 2nd moment of the water plane area



Calculate sectional area or 1st moment of area of each section



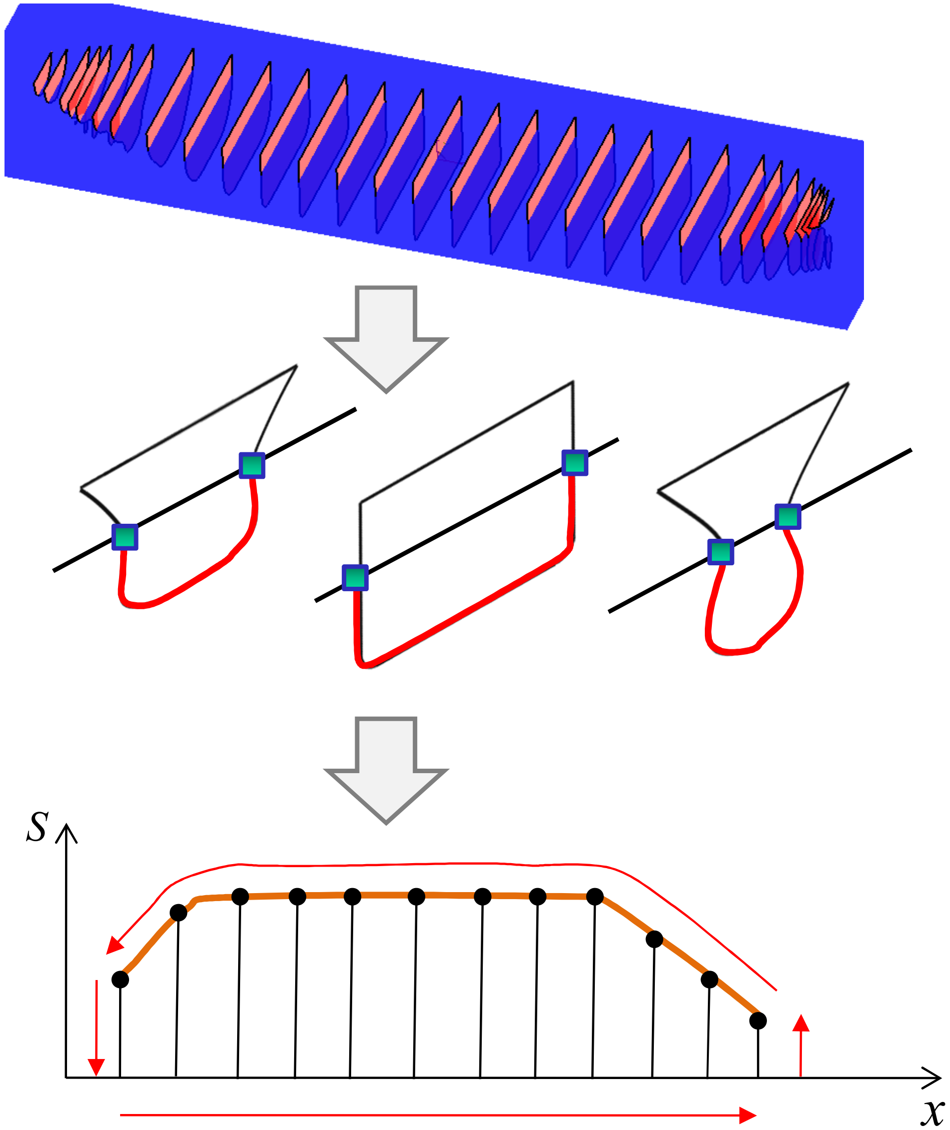
Intersection point between the water plane area and the section lines

Calculation procedure

- ✓ Transform the intersection points decomposed in body fixed frame into the points decomposed in water plane fixed frame(inertial frame).
- ✓ Generate the curve which interpolates the intersection points. If a section 'x' has no intersection point, input the point as (x, 0, 0).
- ✓ Calculate the area, 1st moment and 2nd moment of area using Green's theorem or Gaussian quadrature.

Calculation of Wetted Surface Area

Given: Intersection points between the water plane and the section lines
Find: Wetted surface area



Calculation procedure

1) Calculate the girth length of the section lines under water.

$$s = \int_{t_0}^{t_1} ds = \int_{t_0}^{t_1} \left\| \dot{\mathbf{r}}(t) \right\| dt$$

2) Calculate the sectional area surrounded by the girth length and water plane
3) Make the ordinate set of the sectional area
4) Generate B-spline curve which interpolates the ordinates
5) Integrate the area along ship's length using Green's theorem or Gaussian quadrature

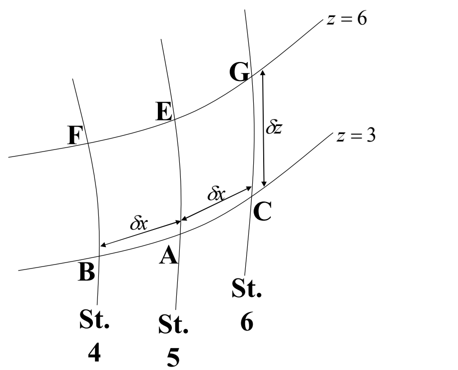
➡ Wetted surface area is calculated

Classical Calculation Method for Ship's Surface Area

Example of Calculation for Ship's Surface Area (1/7)

Using the "Calculation for ship's surface area", calculate the wetted surface area of the ship between 3m and 6m of waterline.

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)^1/2	(10) S.M	(11) Prod.
5	19.66	18.41	0.42 (1)	0.17	6	20.12	19.84	4	17.56	15.56	-0.12 (2)	0.01	1.18	1.09	1	1.09
4	17.56	15.47	0.70	0.49	5	19.66	18.41	3	13.38	11.16	-0.24	0.06	1.55	1.24	3	3.72
3	13.38	11.16	0.74	0.55	4	17.56	15.47	2	8.14	6.64	-0.33	0.11	1.66	1.29	3	3.87
2	8.14	6.64	0.50	0.25	3	13.38	11.16	1	2.62	2.16	-0.35	0.13	1.38	1.17	1.444	1.69
1 1/2	5.43	4.39	0.35	0.12	2	8.14	6.64	1 (3)	2.62	2.16	-0.36	0.13	1.25	1.12	1.778	1.99
1	2.62	2.16	0.15	0.02	1 1/2	5.43	4.39	1/2	-0.22*	-0.28*	-0.37	0.14	1.16	1.08	0.444	0.48
															Σ =	12.84



HB: Half-breadth for waterline

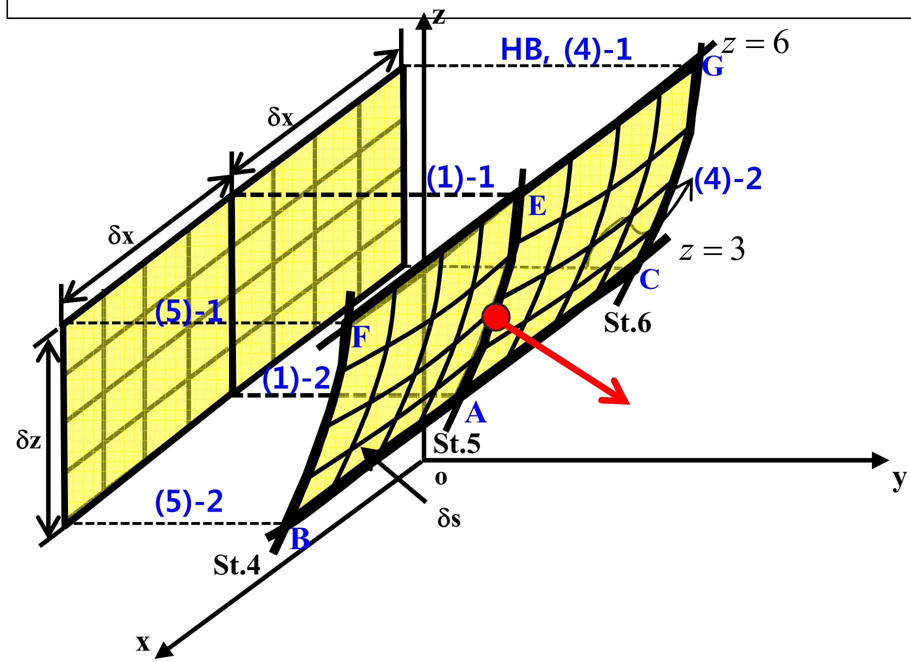
HB_A: Half-breadth afterward

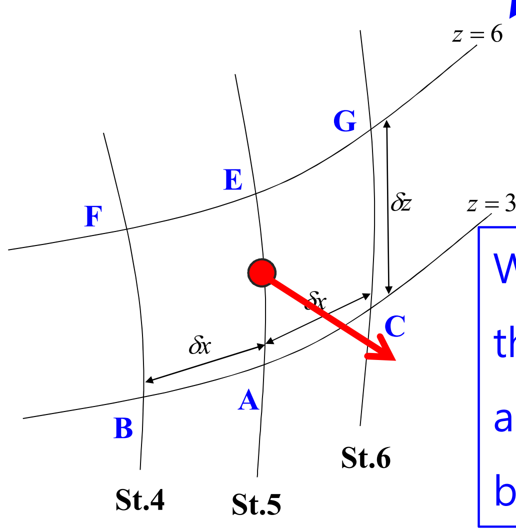
HB_f: Half-breadth forward

S: Wetted surface area of the ship

$$Sum = 1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2$$

$$\delta x = \text{Station interval} = 13.94\ m$$



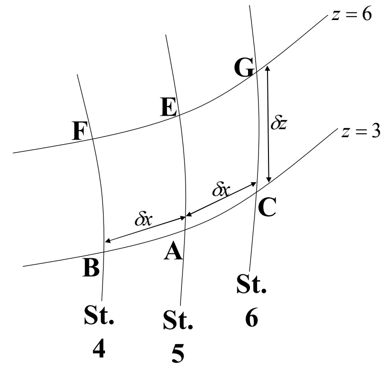


We can find the vertical station shape slope $\frac{dy}{dz}$ and longitudinal water line slope $\frac{dy}{dx}$ by using the central difference.

Example of Calculation for Ship's Surface Area (2/7)

Using the "Calculation for ship's surface area", calculate the wetted surface area of the ship between 3m and 6m of waterline.

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)^1/2	(10) S.M	(11) Prod.
5	19.66	18.41	0.42 (1)	0.17	6	20.12	19.84	4	17.56	15.56	-0.12 (2)	0.01	1.18	1.09	1	1.09
4	17.56	15.47	0.70	0.49	5	19.66	18.41	3	13.38	11.16	-0.24	0.06	1.55	1.24	3	3.72
3	13.38	11.16	0.74	0.55	4	17.56	15.47	2	8.14	6.64	-0.33	0.11	1.66	1.29	3	3.87
2	8.14	6.64	0.50	0.25	3	13.38	11.16	1	2.62	2.16	-0.35	0.13	1.38	1.17	1.444	1.69
1 1/2	5.43	4.39	0.35	0.12	2	8.14	6.64	1 (3)	2.62	2.16	-0.36	0.13	1.25	1.12	1.778	1.99
1	2.62	2.16	0.15	0.02	1 1/2	5.43	4.39	1/2	-0.22*	-0.28*	-0.37	0.14	1.16	1.08	0.444	0.48
															Σ	= 12.84



HB: Half-breadth for waterline

HB_A: Half-breadth afterward

HB_f: Half-breadth forward

S: Wetted surface area of the ship

$$Sum = 1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2$$

$$\delta x = \text{Station interval} = 13.94\ m$$

1. Approximated formula for ship's surface area: $S = \delta z \int_{Sta.1}^{Sta.5} \sqrt{1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dz}\right)^2}\, dx$

1) $\frac{dy}{dz} \approx \frac{\delta y}{\delta z}$

$$\delta z = (6-3) = 3\,m$$

In the table,

$$\delta y = HB_{W.L.=6m} - HB_{W.L.=3m}$$ ---------------- [(1.2) – (1.1)]

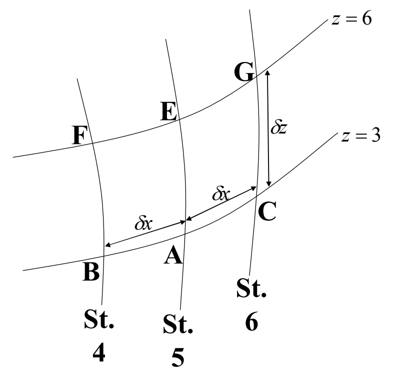
$$\frac{dy}{dz} \approx \frac{HB_{W.L.=6m} - HB_{W.L.=3m}}{\delta z}$$ --------------------- (2)

$$\left(\frac{dy}{dz}\right)^2 \approx \left(\frac{HB_{W.L.=6m} - HB_{W.L.=3m}}{\delta z}\right)^2$$ ------------------ (3)

Example of Calculation for Ship's Surface Area (3/7)

Using the "Calculation for ship's surface area", calculate the wetted surface area of the ship between 3m and 6m of waterline.

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)^1/2	(10) S.M	(11) Prod.
5	19.66	18.41	0.42 (1)	0.17	6	20.12	19.84	4	17.56	15.56	-0.12 (2)	0.01	1.18	1.09	1	1.09
4	17.56	15.47	0.70	0.49	5	19.66	18.41	3	13.38	11.16	-0.24	0.06	1.55	1.24	3	3.72
3	13.38	11.16	0.74	0.55	4	17.56	15.47	2	8.14	6.64	-0.33	0.11	1.66	1.29	3	3.87
2	8.14	6.64	0.50	0.25	3	13.38	11.16	1	2.62	2.16	-0.35	0.13	1.38	1.17	1.444	1.69
1 1/2	5.43	4.39	0.35	0.12	2	8.14	6.64	1 (3)	2.62	2.16	-0.36	0.13	1.25	1.12	1.778	1.99
1	2.62	2.16	0.15	0.02	1 1/2	5.43	4.39	1/2	-0.22*	-0.28*	-0.37	0.14	1.16	1.08	0.444	0.48
															Σ	= 12.84



HB: Half-breadth for waterline

HB_A: Half-breadth afterward

HB_f: Half-breadth forward

S: Wetted surface area of the ship

$$Sum = 1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2$$

$$\delta x = \text{Station interval} = 13.94\ m$$

1. Approximated formula for ship's surface area: $S = \delta z \int_{Sta.1}^{Sta.5} \sqrt{1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dz}\right)^2}\, dx$

2) $$\frac{dy}{dx} = \frac{1}{2}\left(\left.\frac{dy}{dx}\right|_{W.L.=6m} + \left.\frac{dy}{dx}\right|_{W.L.=3m}\right)$$

In the table,

$$\left.\frac{dy}{dx}\right|_{W.L.=6m} \approx \left.\frac{\delta y}{\delta x}\right|_{W.L.=6m} = \frac{HB_{A,W.L.=6m} - HB_{F,W.L.=6m}}{2\cdot\delta x}$$ -------------------------------- [(5.1) – (4.1)]/2δx

(W.L.: Waterline)

$$\left.\frac{dy}{dx}\right|_{W.L.=3m} \approx \left.\frac{\delta y}{\delta x}\right|_{W.L.=3m} = \frac{HB_{A,W.L.=3m} - HB_{F,W.L.=3m}}{2\cdot\delta x}$$ -------------------------------- [(5.2) – (4.2)]/2δx

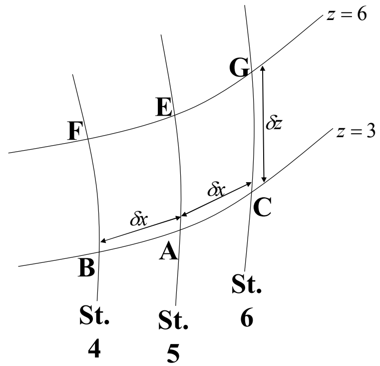
$$\frac{dy}{dx} \approx \frac{1}{2}\left(\frac{HB_{A,W.L.=6m} - HB_{F,W.L.=6m}}{2\cdot\delta x} + \frac{HB_{A,W.L.=3m} - HB_{F,W.L.=3m}}{2\cdot\delta x}\right)$$ ------------------------ (6)

$$\left(\frac{dy}{dx}\right)^2 \approx \left[\frac{1}{2}\left(\frac{HB_{A,W.L.=6m} - HB_{F,W.L.=6m}}{2\cdot\delta x} + \frac{HB_{A,W.L.=3m} - HB_{F,W.L.=3m}}{2\cdot\delta x}\right)\right]^2$$ ------------------------ (7)

Example of Calculation for Ship's Surface Area (4/7)

Using the "Calculation for ship's surface area", calculate the wetted surface area of the ship between 3m and 6m of waterline.

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)^2	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)^2	(8) Sum	(9) (Sum)$^{1/2}$	(10) S.M	(11) Prod.
5	19.66	18.41	0.42 (1)	0.17	6	20.12	19.84	4	17.56	15.56	-0.12 (2)	0.01	1.18	1.09	1	1.09
4	17.56	15.47	0.70	0.49	5	19.66	18.41	3	13.38	11.16	-0.24	0.06	1.55	1.24	3	3.72
3	13.38	11.16	0.74	0.55	4	17.56	15.47	2	8.14	6.64	-0.33	0.11	1.66	1.29	3	3.87
2	8.14	6.64	0.50	0.25	3	13.38	11.16	1	2.62	2.16	-0.35	0.13	1.38	1.17	1.444	1.69
1 1/2	5.43	4.39	0.35	0.12	2	8.14	6.64	1 (3)	2.62	2.16	-0.36	0.13	1.25	1.12	1.778	1.99
1	2.62	2.16	0.15	0.02	1 1/2	5.43	4.39	1/2	-0.22*	-0.28*	-0.37	0.14	1.16	1.08	0.444	0.48
															Σ	= 12.84



HB: Half-breadth for waterline

HB_A: Half-breadth afterward

HB_f: Half-breadth forward

S: Wetted surface area of the ship

$$Sum = 1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2$$

$$\delta x = \text{Station interval} = 13.94\ m$$

1) $\dfrac{dy}{dz} \approx \dfrac{HB_{W.L.=6m} - HB_{W.L.=3m}}{\delta z}$

2) $\dfrac{dy}{dx} \approx \dfrac{1}{2}\left(\dfrac{HB_{A,W.L.=6m} - HB_{F,W.L.=6m}}{2\cdot\delta x} + \dfrac{HB_{A,W.L.=3m} - HB_{F,W.L.=3m}}{2\cdot\delta x}\right)$

1. Approximated formula for ship's surface area: $S = \delta z \int_{Sta.1}^{Sta.5} \sqrt{1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dz}\right)^2}\, dx$

$(8) = 1 + (7) + (3)$

2. Substituting 1) and 2) into the formula.

$$S \approx \delta z \int_{Sta.1}^{Sta.5} \sqrt{1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2}\, dx$$

$$= \delta z \int_{Sta.1}^{Sta.5} \sqrt{1 + \left(\frac{1}{2}\left(\frac{HB_{A,W.L.=6m} - HB_{F,W.L.=6m}}{2\cdot\delta x} + \frac{HB_{A,W.L.=3m} - HB_{F,W.L.=3m}}{2\cdot\delta x}\right)\right)^2 + \left(\frac{HB_{W.L.=6m} - HB_{W.L.=3m}}{\delta z}\right)^2}\, dx$$

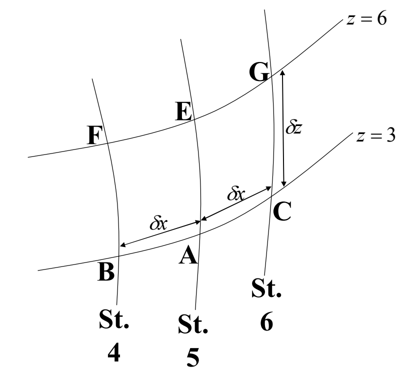
$(9) = \sqrt{(8)}$

3. By using the Simpson's 1st and 2nd rules, calculate the ship's surface area(wetted surface area).

Example of Calculation for Ship’s Surface Area (5/7)

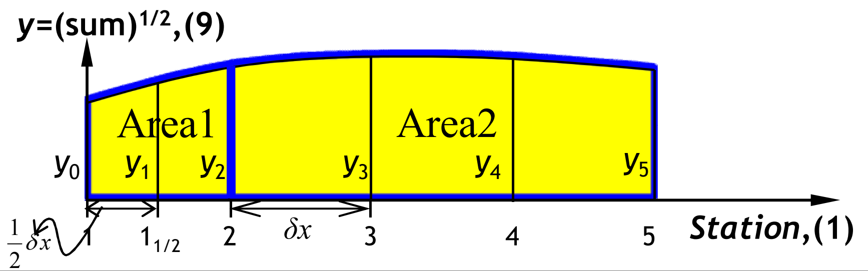
Using the “Calculation for ship’s surface area”, calculate the wetted surface area of the ship between 3m and 6m of waterline.

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)^1/2	(10) S.M	(11) Prod.
5	19,66	18,41	0,42 (1)	0,17	6	20,12	19,84	4	17,56	15,56	-0,12 (2)	0,01	1,18	1,09	1	1,09
4	17,56	15,47	0,70	0,49	5	19,66	18,41	3	13,38	11,16	-0,24	0,06	1,55	1,24	3	3,72
3	13,38	11,16	0,74	0,55	4	17,56	15,47	2	8,14	6,64	-0,33	0,11	1,66	1,29	3	3,87
2	8,14	6,64	0,50	0,25	3	13,38	11,16	1	2,62	2,16	-0,35	0,13	1,38	1,17	1,444	1,69
1 1/2	5,43	4,39	0,35	0,12	2	8,14	6,64	1 (3)	2,62	2,16	-0,36	0,13	1,25	1,12	1,778	1,99
1	2,62	2,16	0,15	0,02	1 1/2	5,43	4,39	1/2	-0,22*	-0,28*	-0,37	0,14	1,16	1,08	0,444	0,48



3. By using the Simpson’s 1st and 2nd rules, calculate the ship’s surface area.

1) Simpson’s multiplier (10)



Simpson’s 1st Rule: $Area1 = \frac{1}{3}\cdot\frac{1}{2}\delta x\cdot\left(y_0 + 4y_1 + y_2\right)$

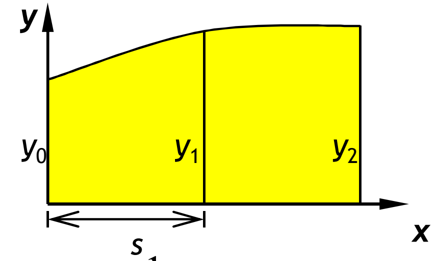
Simpson’s 2nd Rule: $Area2 = \frac{3}{8}\cdot\delta x\cdot\left(y_2 + 3y_3 + 3y_4 + y_5\right)$

Total Area: $Area1 + Area2 = \frac{3}{8}\cdot\delta x\cdot\left(\frac{8}{3}\cdot\frac{1}{3}\cdot\frac{1}{2}y_0 + \frac{8}{3}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot 4y_1 + \frac{8}{3}\cdot\frac{1}{3}\cdot\frac{1}{2}y_2 + y_2 + 3y_3 + 3y_4 + y_5\right)$

$= \frac{3}{8}\cdot\delta x\cdot\left(\boxed{0.444}y_0 + \boxed{1.778}y_1 + \boxed{1.444}y_2 + \boxed{3}y_3 + \boxed{3}y_4 + \boxed{1}y_5\right)$

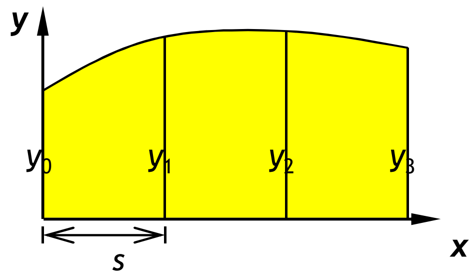
□: S.M, (10)

Simpson’s 1st Rule



$$Area = \frac{1}{3}s\left(y_0 + 4y_1 + y_2\right)$$

Simpson’s 2nd Rule

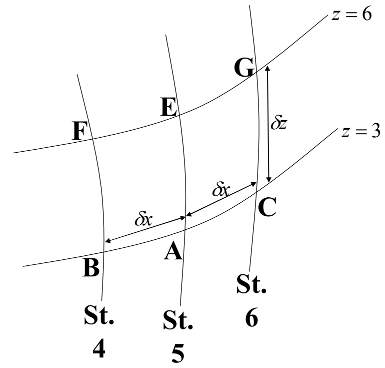


$$Area = \frac{3}{8}s\left(y_0 + 3y_1 + 3y_2 + y_3\right)$$

Example of Calculation for Ship's Surface Area (6/7)

Using the "Calculation for ship's surface area", calculate the wetted surface area of the ship between 3m and 6m of waterline.

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)$^{1/2}$	(10) S.M	(11) Prod.
5	19.66	18.41	0.42 (1)	0.17	6	20.12	19.84	4	17.56	15.56	-0.12 (2)	0.01	1.18	1.09	1	1.09
4	17.56	15.47	0.70	0.49	5	19.66	18.41	3	13.38	11.16	-0.24	0.06	1.55	1.24	3	3.72
3	13.38	11.16	0.74	0.55	4	17.56	15.47	2	8.14	6.64	-0.33	0.11	1.66	1.29	3	3.87
2	8.14	6.64	0.50	0.25	3	13.38	11.16	1	2.62	2.16	-0.35	0.13	1.38	1.17	1.444	1.69
1 1/2	5.43	4.39	0.35	0.12	2	8.14	6.64	1 (3)	2.62	2.16	-0.36	0.13	1.25	1.12	1.778	1.99
1	2.62	2.16	0.15	0.02	1 1/2	5.43	4.39	1/2	-0.22*	-0.28*	-0.37	0.14	1.16	1.08	0.444	0.48
															Σ	= 12.84



HB: Half-breadth for waterline

HB_A: Half-breadth afterward

HB_f: Half-breadth forward

S: Wetted surface area of the ship

$$Sum = 1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2$$

$$\delta x = 13.94\ m, \delta z = 3\ m$$

3. By using the Simpson's 1st and 2nd rules, calculate the ship's surface area.

$$S \approx \delta z \int_{Sta.1}^{Sta.5} \sqrt{1 + \left(\frac{1}{2}\left(\frac{HB_{A,W.L.=6m} - HB_{F,W.L.=6m}}{2\cdot\delta x} + \frac{HB_{A,W.L.=3m} - HB_{F,W.L.=3m}}{2\cdot\delta x}\right)\right)^2 + \left(\frac{HB_{W.L.=6m} - HB_{W.L.=3m}}{\delta z}\right)^2}\, dx$$

$$= \delta z \cdot \frac{3}{8}\cdot\delta x \cdot \sum \left[\underbrace{S.M}_{(10)} \cdot \underbrace{\sqrt{1 + \left(\frac{1}{2}\left(\frac{HB_{A,W.L.=6m} - HB_{F,W.L.=6m}}{2\cdot\delta x} + \frac{HB_{A,W.L.=3m} - HB_{F,W.L.=3m}}{2\cdot\delta x}\right)\right)^2 + \left(\frac{HB_{W.L.=6m} - HB_{W.L.=3m}}{\delta z}\right)^2}}_{(9)} \right]$$

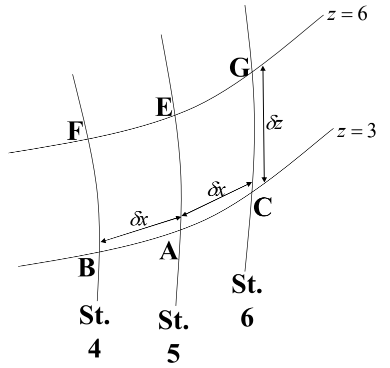
$$= \delta z \cdot \frac{3}{8}\cdot\delta x \cdot \sum \underbrace{\text{Prod.}}_{(11)}$$

$$= 3\cdot\frac{3}{8}\cdot 13.94 \cdot 12.84 = 201.36\ \ (m^2)$$

Example of Calculation for Ship's Surface Area (7/7)

Using the "Calculation for ship's surface area", calculate the wetted surface area of the ship between 3m and 6m of waterline.

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)$^{1/2}$	(10) S.M	(11) Prod.
5	19.66	18.41	0.42 (1)	0.17	6	20.12	19.84	4	17.56	15.56	-0.12 (2)	0.01	1.18	1.09	1	1.09
4	17.56	15.47	0.70	0.49	5	19.66	18.41	3	13.38	11.16	-0.24	0.06	1.55	1.24	3	3.72
3	13.38	11.16	0.74	0.55	4	17.56	15.47	2	8.14	6.64	-0.33	0.11	1.66	1.29	3	3.87
2	8.14	6.64	0.50	0.25	3	13.38	11.16	1	2.62	2.16	-0.35	0.13	1.38	1.17	1.444	1.69
1½	5.43	4.39	0.35	0.12	2	8.14	6.64	1 (3)	2.62	2.16	-0.36	0.13	1.25	1.12	1.778	1.99
1	2.62	2.16	0.15	0.02	1½	5.43	4.39	½	-0.22*	-0.28*	-0.37	0.14	1.16	1.08	0.444	0.48
															Σ	= 12.84



HB: Half-breadth for waterline

HB_A: Half-breadth afterward

HB_f: Half-breadth forward

S: Wetted surface area of the ship

$$Sum = 1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2$$

3. By using the Simpson's 1st and 2nd rules, calculate the ship's surface area.

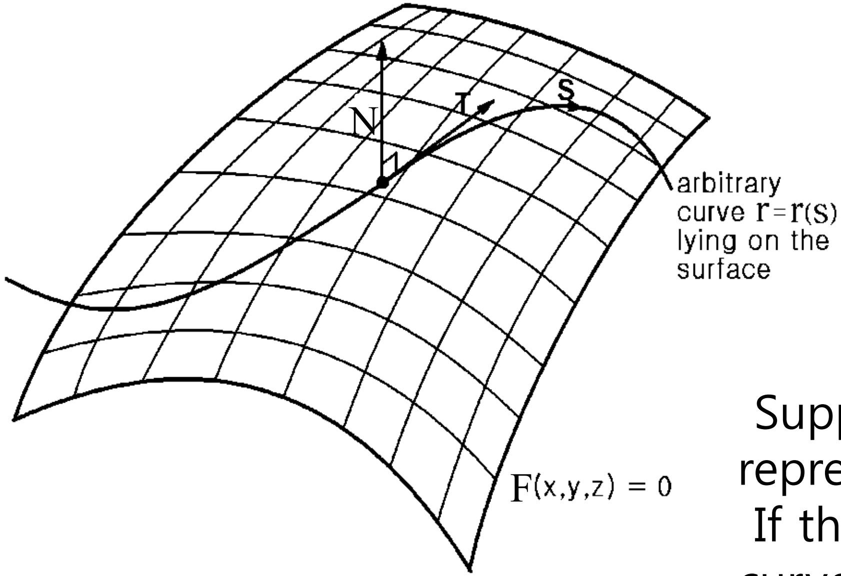
$$S \approx 201.36\ m^2$$

4. Calculate the wetted surface area of both sides of the ship

$$Wetted\ Surface,\ Both\ sides = 2 \cdot S \approx 2 \cdot 201.36 = 402.7\ \left(m^2\right)$$

Reference Slides

Derivation of *cosβ* Using Differential Geometry (1/2)



A curve $\mathbf{r}(s)$ in space is written as follows;

$$\mathbf{r}(s) = x(s)\mathbf{i} + y(s)\mathbf{j} + z(s)\mathbf{k}$$

where, s is a parameter for this curve.

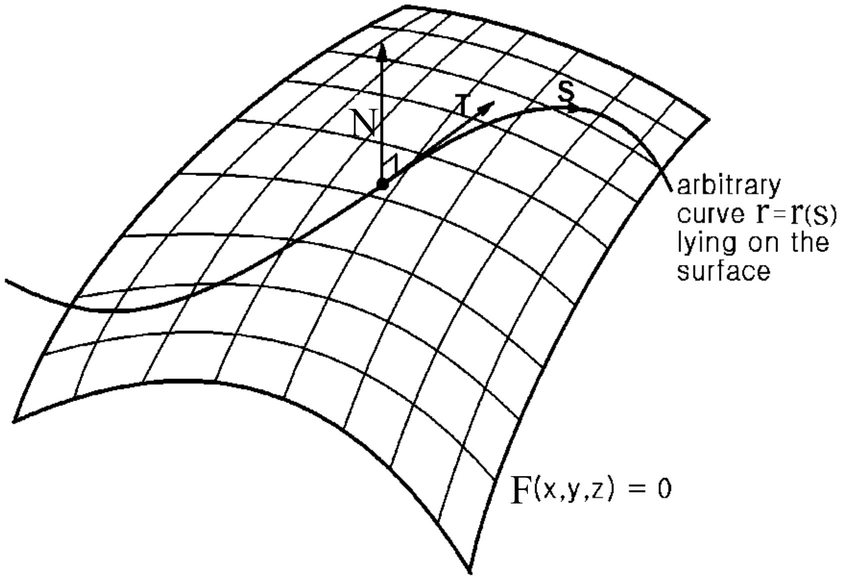
Suppose that a surface as depicted in the figure is represented as an implicit function, $F(x, y, z)=0$.
If the curve $\mathbf{r}(s)$ is on this surface, the components of this curve should satisfy the equation of the surface as follows;

$$F(x(s), y(s), z(s)) = 0$$

The normal vector $\mathbf{N}$ at a point on a the surface $F(x, y, z)=0$, is perpendicular to the curve $\mathbf{r}(s)$ crossing the point. Therefore the normal vector on the point is perpendicular to the unit tangent vector, $\mathbf{T} = d\mathbf{r}/ds$ (where, s is the arc length of the curve $\mathbf{r}(s)$.)

$$\mathbf{N} \perp \mathbf{T} \qquad \text{where } \mathbf{T}=d\mathbf{r} / ds$$

Derivation of *cosβ* Using Differential Geometry (2/2)



$$\mathbf{N} \perp \mathbf{T} \qquad \text{where } \mathbf{T}=d\mathbf{r} / ds$$

Because the value of the implicit function $F(x, y, z)=0$ is not changed along the curve $\mathbf{r}(s)$, it can be written as

$$\frac{dF}{ds} = 0$$

By using Chain rule, $dF/ds = 0$ can be expressed as

$$\frac{dF}{ds} = \frac{\partial F}{\partial x}\frac{dx}{ds} + \frac{\partial F}{\partial y}\frac{dy}{ds} + \frac{\partial F}{\partial z}\frac{dz}{ds} = 0$$

The equation leads to ∇F.

$$\left(\frac{\partial F}{\partial x}\mathbf{i} + \frac{\partial F}{\partial y}\mathbf{j} + \frac{\partial F}{\partial z}\mathbf{k}\right) \cdot \frac{d\mathbf{r}}{ds} = 0$$

Because $\left(\partial F/\partial x\,\mathbf{i} + \partial F/\partial y\,\mathbf{j} + \partial F/\partial z\,\mathbf{k}\right)$ represents the gradient vector ∇F and inner product of ∇F and $d\mathbf{r}/ds$ equals zero, that means ∇F is normal to the unit tangent vector $d\mathbf{r}/ds$ at a point of the curve on the surface.

Thus, ∇F represents the normal vector at a point on the surface. It also means ∇F is the direction of the maximum increase of a function.

Therefore, the normal vector on the surface is given as follows;

$$\mathbf{N} = \frac{\nabla F}{\left|\nabla F\right|}$$