Ship Stability

September 2013

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Ship Stability

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- ☑ Ch. 2 Review of Fluid Mechanics
- ☑ Ch. 3 Transverse Stability
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- ☑ Ch. 5 Free Surface Effect
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- ☑ Ch. 13 Probabilistic Damage Stability (Subdivision and Damage Stability, SDS)

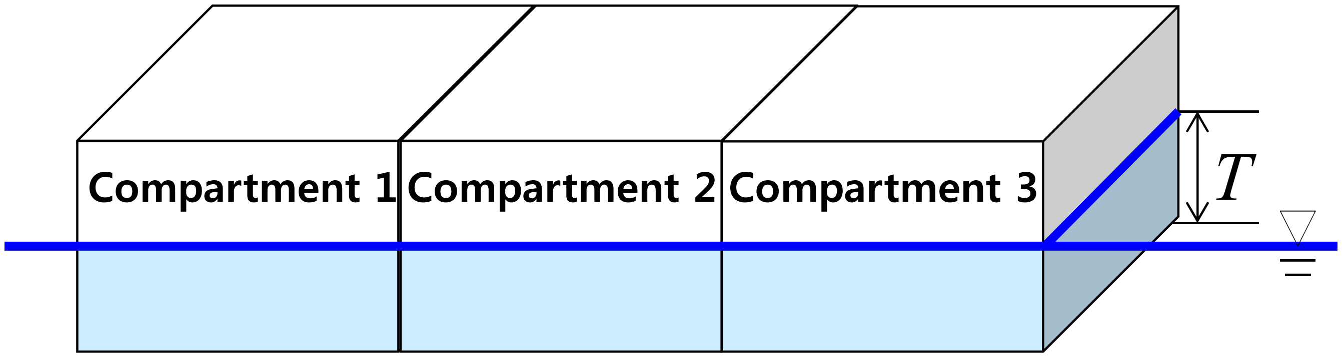
Ch. 11 Introduction to Damage Stability

Change in Position Due to Flooding
Lost Buoyancy Method
Added Weight Method

Change in Position Due to Flooding

Damage of a Box-Shaped Ship

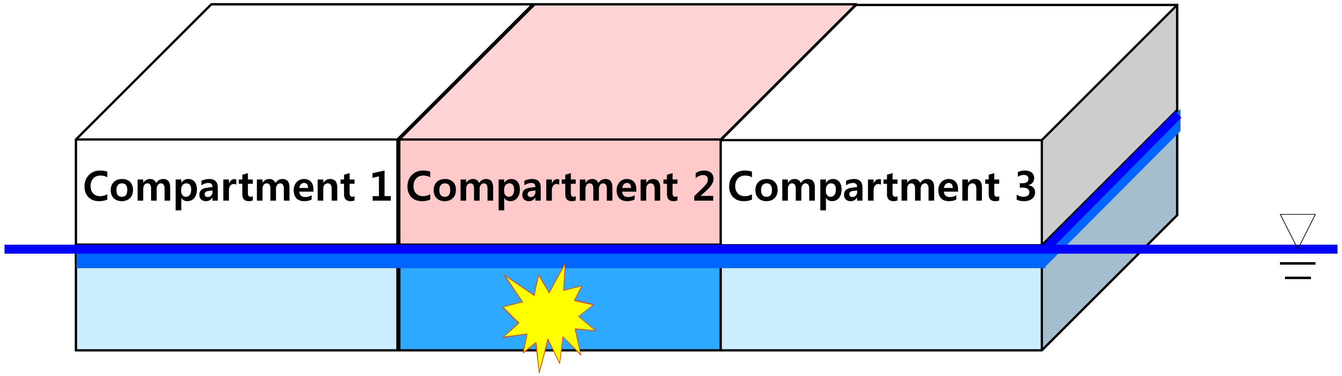
✓ A ship is composed of three compartments.



When a compartment of the ship is damaged, what is the new position of this ship?

Damage of a Box-Shaped Ship (Immersion)

When the compartment in the midship part is damaged, what is the new position of this ship?



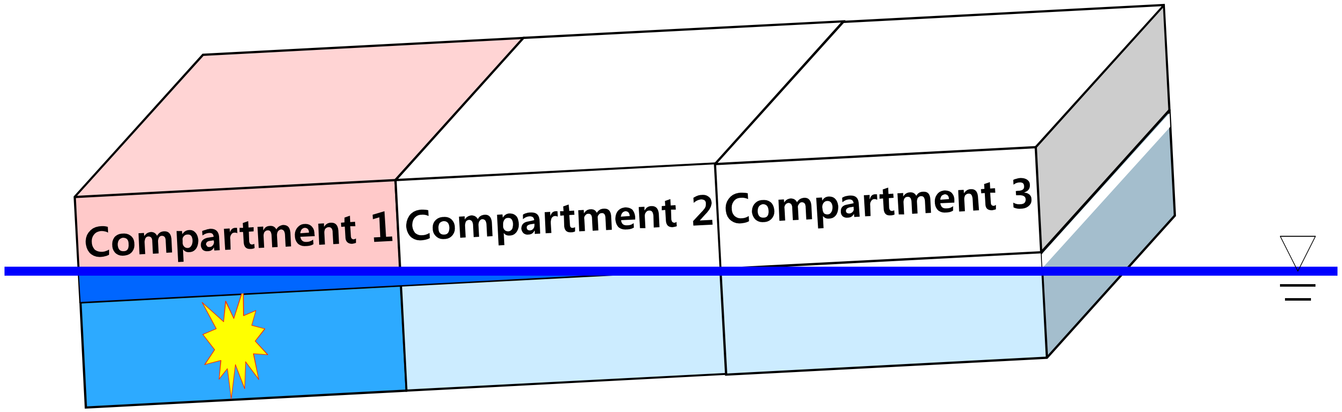
The position of the ship will be changed.

Immersion

* The new position of the ship can be calculated by **the lost buoyancy and added weight methods**.

Damage of a Box-Shaped Ship (Immersion, Trim)

When the compartment at the after part of the ship is damaged, what is the new position of this ship?



"Trim by stern" (draft at AP > draft at FP)

The position of the ship will be changed.

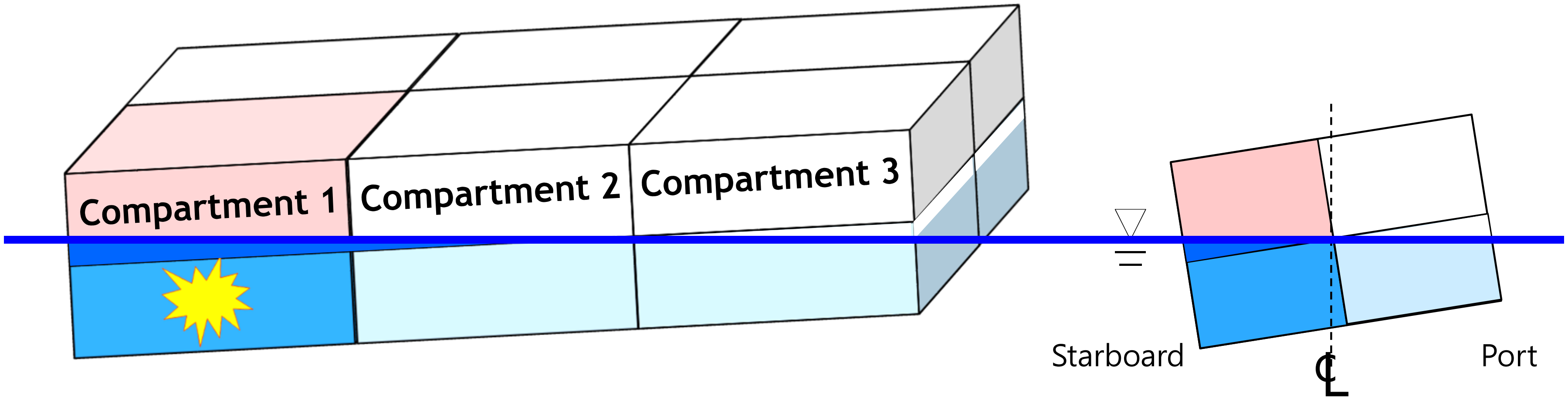
Immersion + Trim

* The new position of the ship can be calculated by **the lost buoyancy and added weight methods**.

Damage of a Box-Shaped Ship (Immersion, Trim, Heel)

✓ When the ship is composed of "six" compartments.

When the compartment at the after and right part of the ship is damaged, what is the new position of the ship?

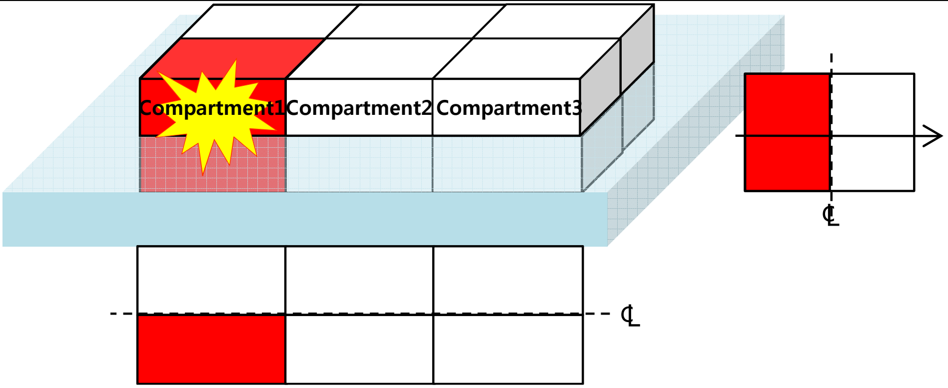


The position of the ship will be changed.

Immersion + Trim + Heel

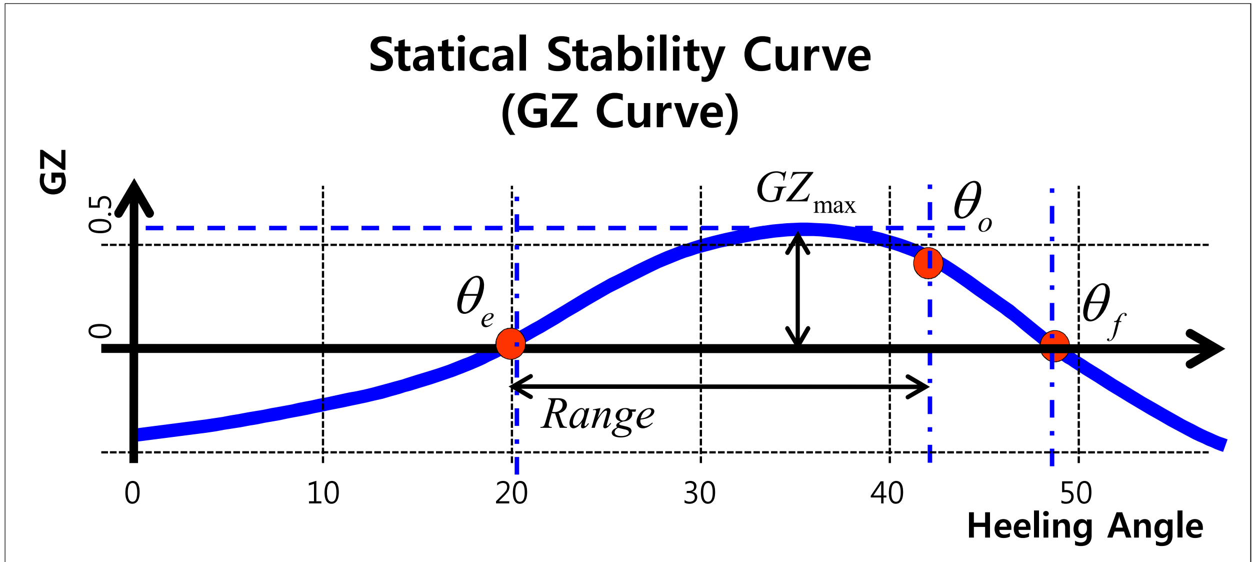
* The new position of the ship can be calculated by **the lost buoyancy and added weight methods**.

Damage of a Box-Shaped Ship (GZ Curve)



Immersion + Trim + Heel

✓ To measure the damage stability, we should find the a statical stability curve(GZ curve) of this damage case by finding the new center of buoyancy(B) and center of mass(G).



θ_e: Equilibrium heel angle

θ_v: $\text{minimum}(\theta_f, \theta_o)$

(in this case, θ_v equals to θ_o)

GZ_{max}: Maximum value of GZ

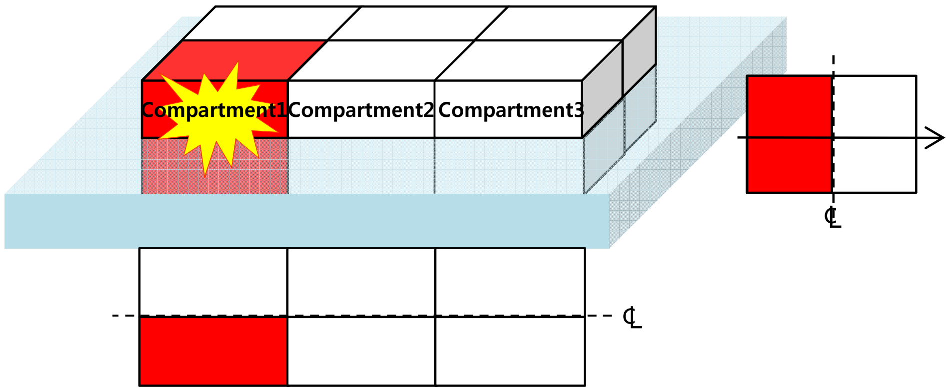
Range: Range of positive righting arm

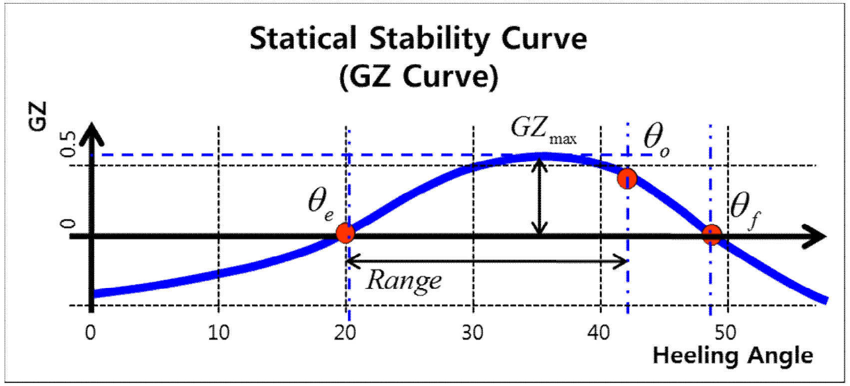
Flooding stage: Discrete step during the flooding process

θ_f: Angle of flooding (righting arm becomes negative)

θ_o: Angle at which an **"opening"** incapable of being closed weathertight becomes submerged

Two Methods to Measure the Ship's Damage Stability





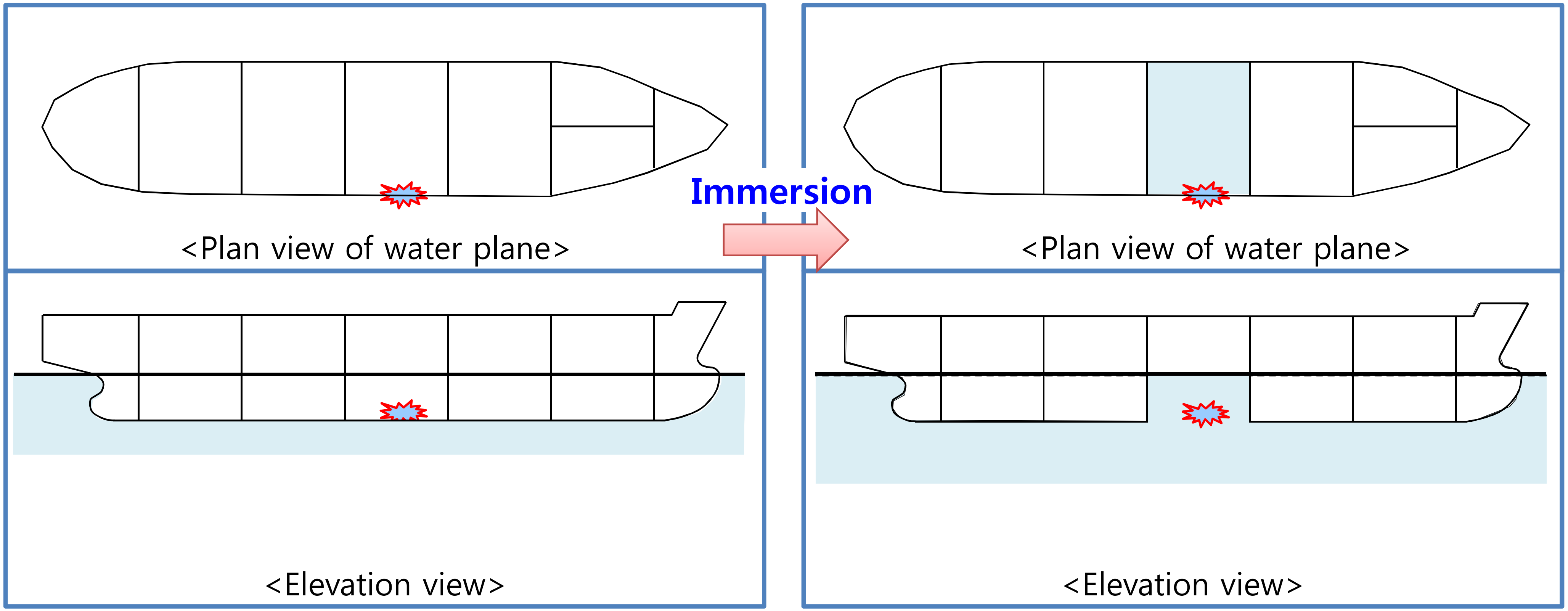
How to measure the ship's stability in a damaged condition?

Deterministic Method : Calculation of survivability of a ship based on **the position, stability, and inclination in damaged conditions**

Probabilistic Method : Calculation of survivability of a ship based on **the probability of damage**

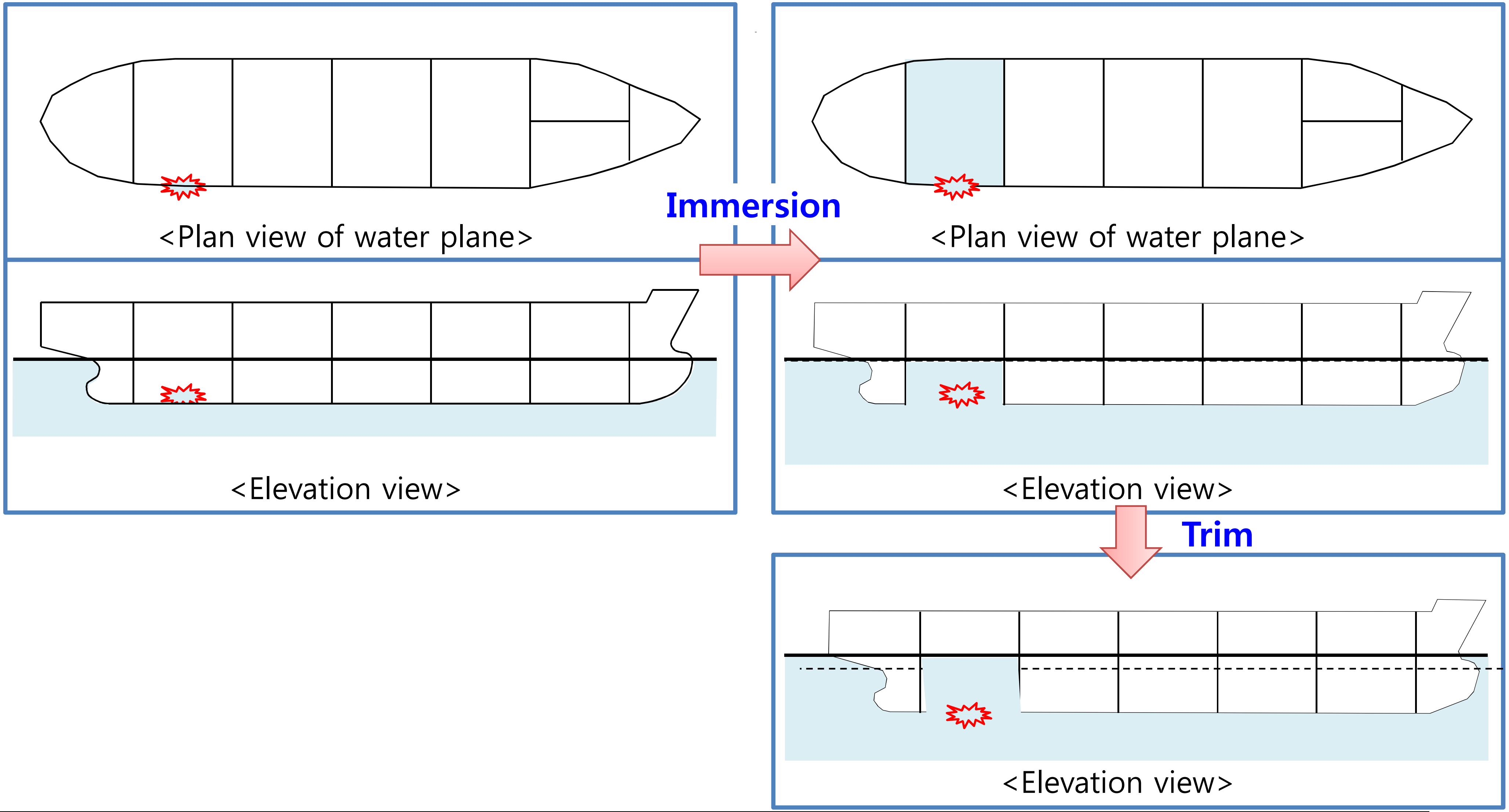
Change in Position due to Flooding (Immersion)

What happens if the compartment located in the midship part of a ship is damaged?



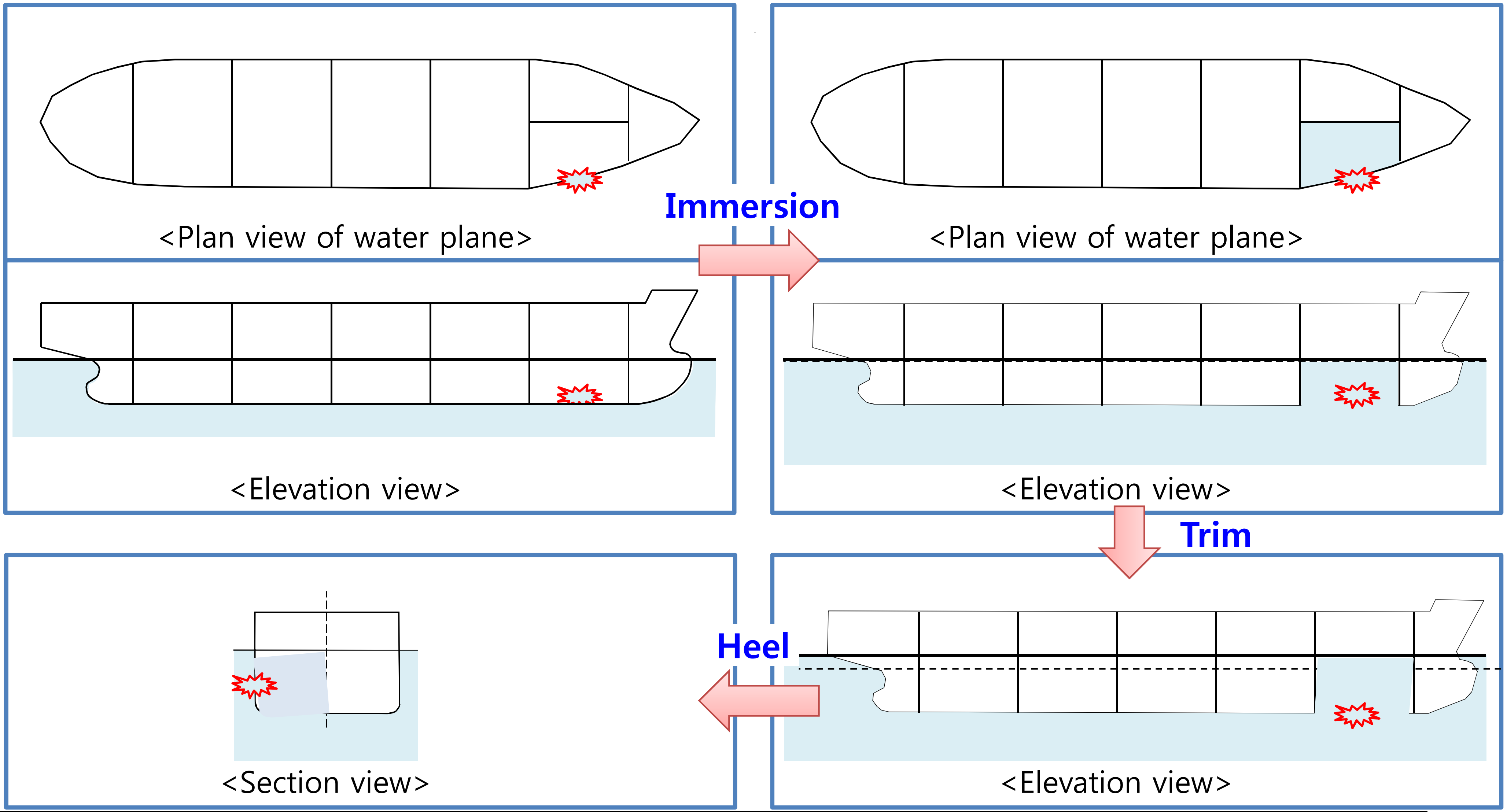
Change in Position due to Flooding (Immersion, Trim)

What happens if the compartment located in the after part of a ship is damaged?



Change in Position due to Flooding (Immersion, Trim, Heel)

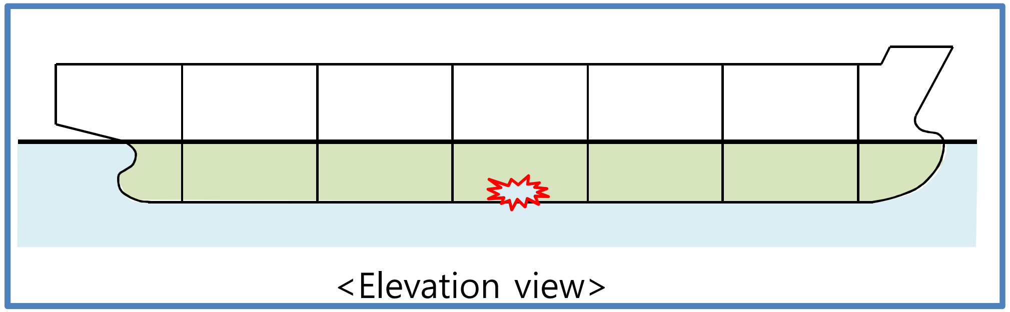
What happens if the compartment located in the fore and right part of a ship is damaged?



Lost Buoyancy Method

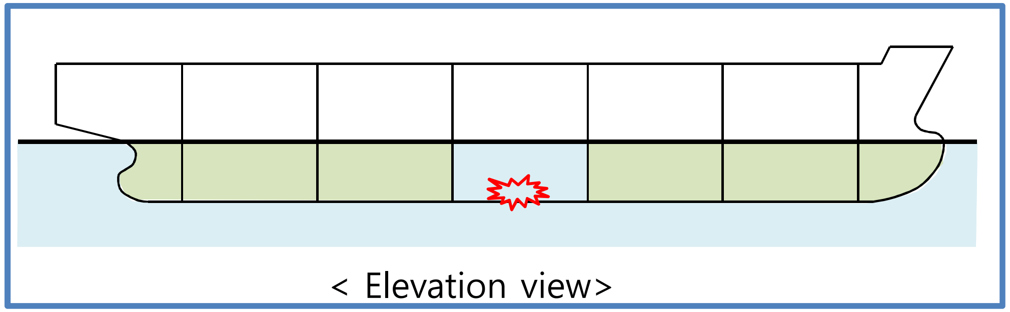
Concept of Lost Buoyancy Method (1/2)

* Hydrostatic Equilibrium
Displacement(Δ) = Buoyant Force = Weight(W)

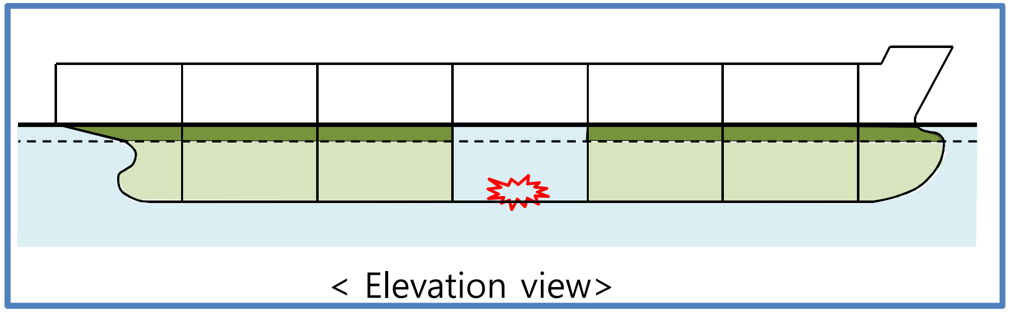


A damage occurs.

: Volume which contributes to buoyancy



The buoyancy of the flooded space is lost.



The lost buoyancy must be regained by an increase of draft.

: Additional volume which contributes to buoyancy (regained buoyancy)

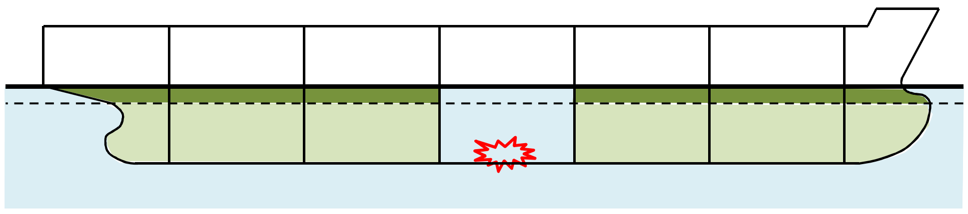
Lost buoyancy method

"The water that enters the ship is considered still part of the sea, and the buoyancy of the flooded space is lost."

Concept of Lost Buoyancy Method (2/2)

* Hydrostatic Equilibrium
Displacement(Δ) = Buoyant Force = Weight(W)

Lost buoyancy method

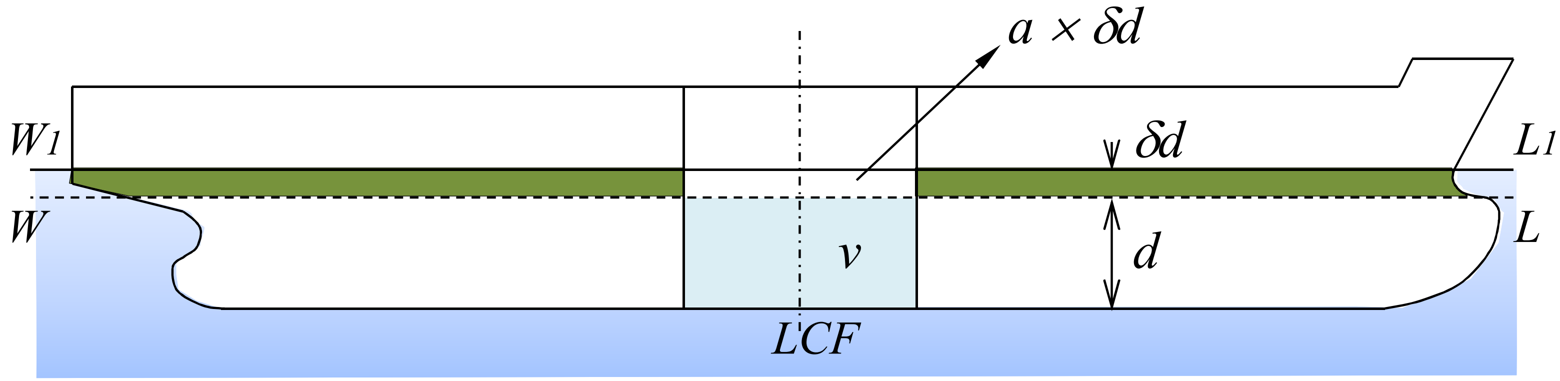


<Elevation view>

- In this method, it is assumed that the flooded compartment has free communication with the sea.
- The flooded compartment can be considered as a sieve (or filter), and that offers no buoyancy to the ship. Only the intact portions of the ship on either side of the flooded compartment contribute to the buoyancy.
- Since buoyancy has been lost, it must be regained via an increase in the draft.
- The ship will sink until the volume (or displacement) of the newly immersed portions equals the volume (or displacement) of the flooded compartment.

Lost Buoyancy Method

The water that enters the damaged compartment is considered as an still part of the sea, and the buoyancy of the flooded space is lost.
And the loss of buoyancy is regained by an increase of draft.



Loss of buoyancy: Sea water flooded into the damaged compartment is considered as part of the sea

Loss of buoyancy = Regained buoyancy by the increase of draft

$$\rho \cdot g \cdot v = \rho \cdot g \cdot (A_{WP} - a) \cdot \delta d$$

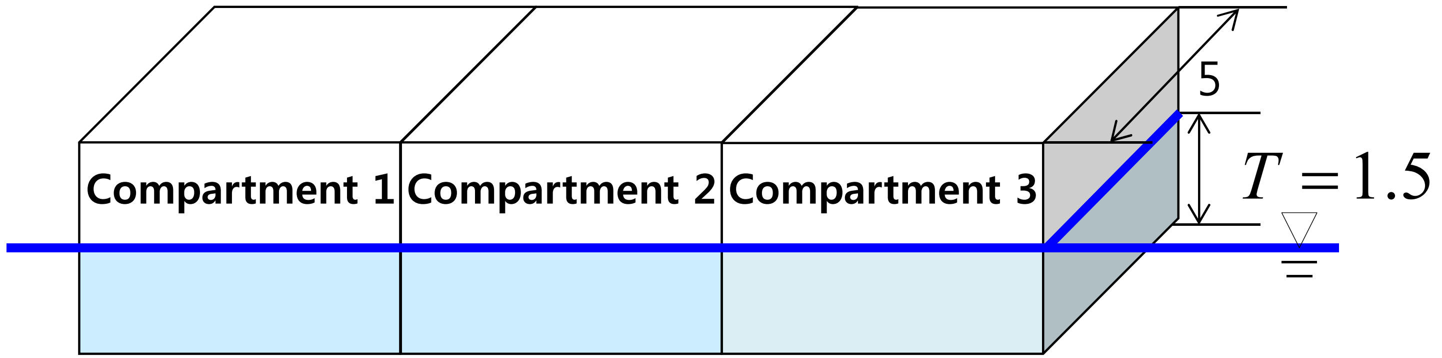
Changed draft due to lost buoyancy:

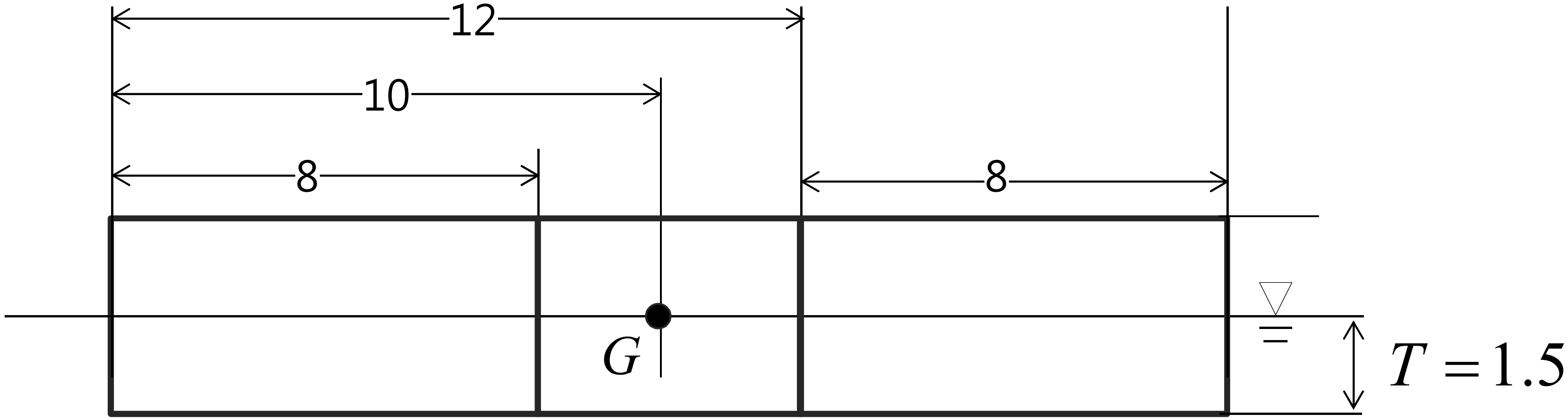
$$\delta d = \frac{v}{A_{WP} - a}$$

A_{WP}: water plane area of the ship
(Including water plane area of the damaged compartment)
a: water plane area of the damaged compartment
d: Draft before the compartment is not damaged
δd: Draft change due to damaged compartment
v: Volume of damaged compartment below initial water plane

[Example] Damage of a Box-Shaped Ship (Immersion) (1/6)

✓ A ship is composed of three compartments.



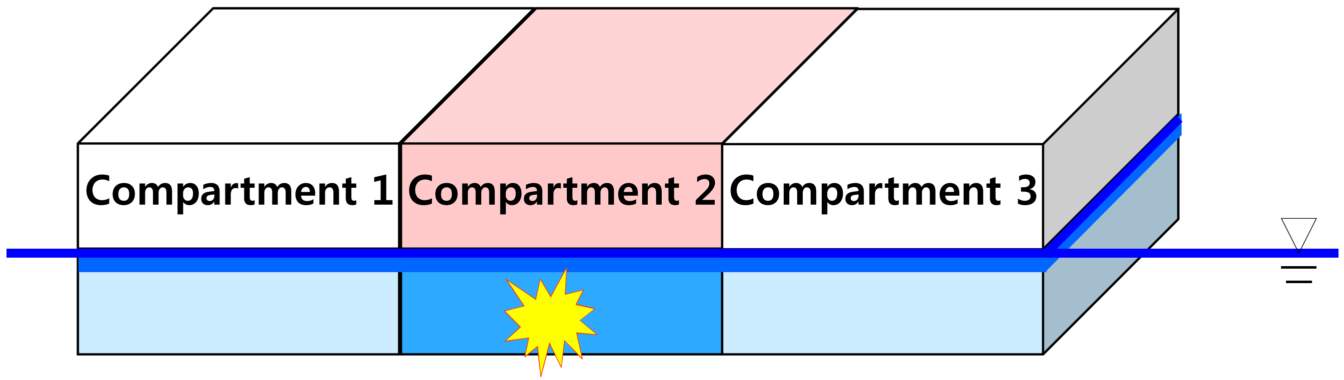


Initial displacement volume: $\nabla_I = LBT = 20\times 5\times 1.5 = 150m^3$

When a compartment of the ship is damaged, what is the new position of this ship?

[Example] Damage of a Box-Shaped Ship (Immersion) (2/6)

When the compartment in the midship part is damaged, what is the new position of this ship?

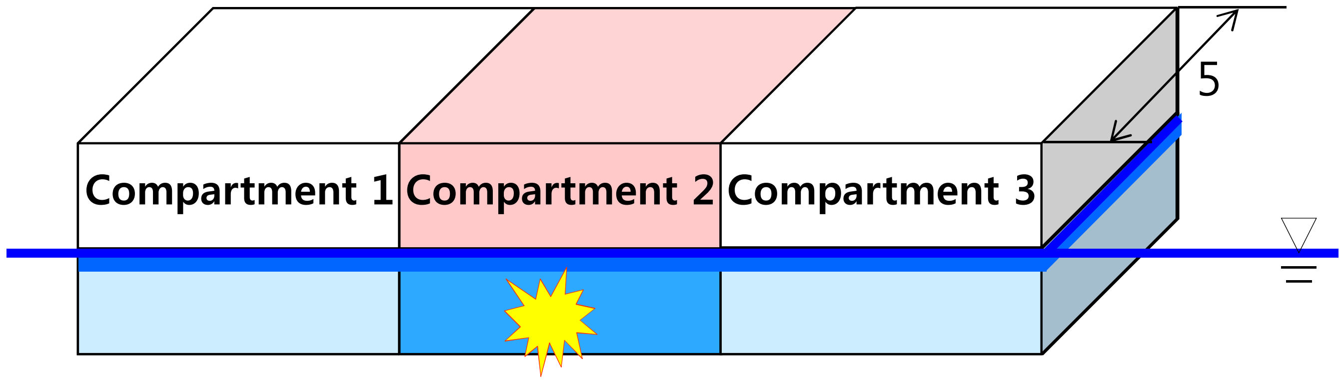


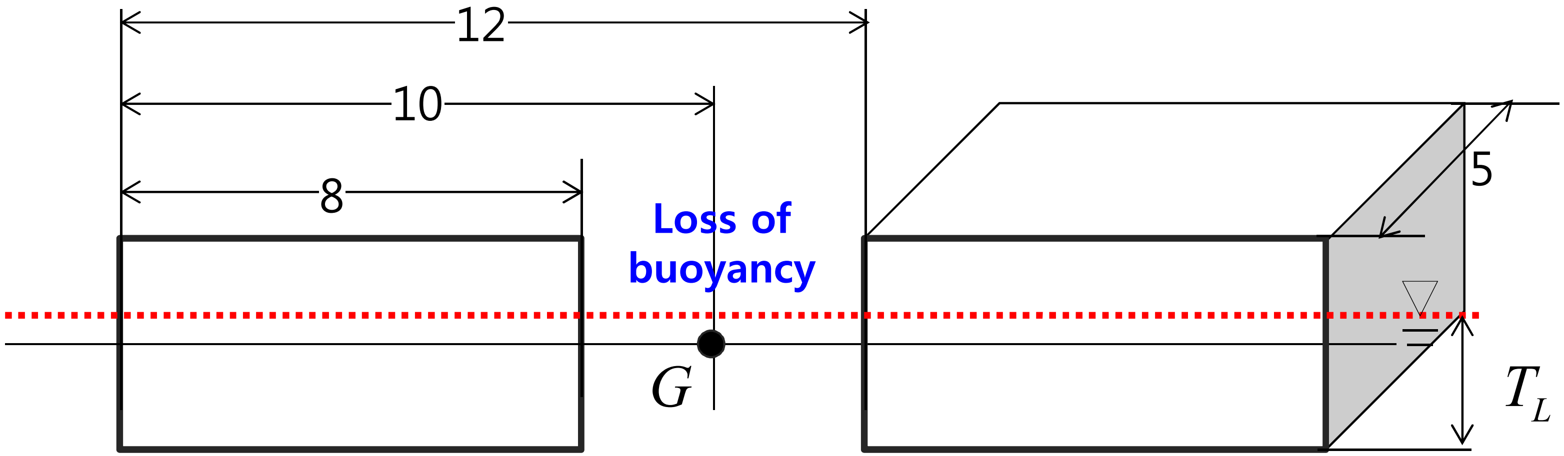
The position of the ship will be changed.

Immersion

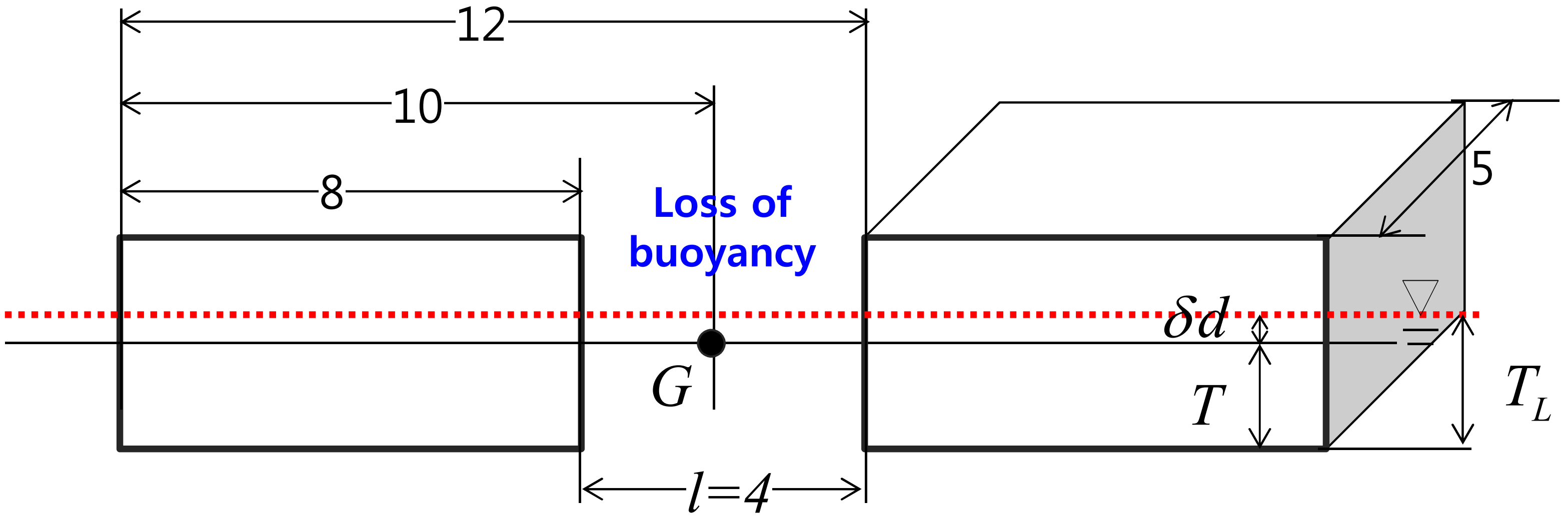
[Example] Damage of a Box-Shaped Ship (Immersion) (3/6)

When the compartment in the midship part is damaged, what is the new position of this ship?





[Example] Damage of a Box-Shaped Ship (Immersion) (4/6)



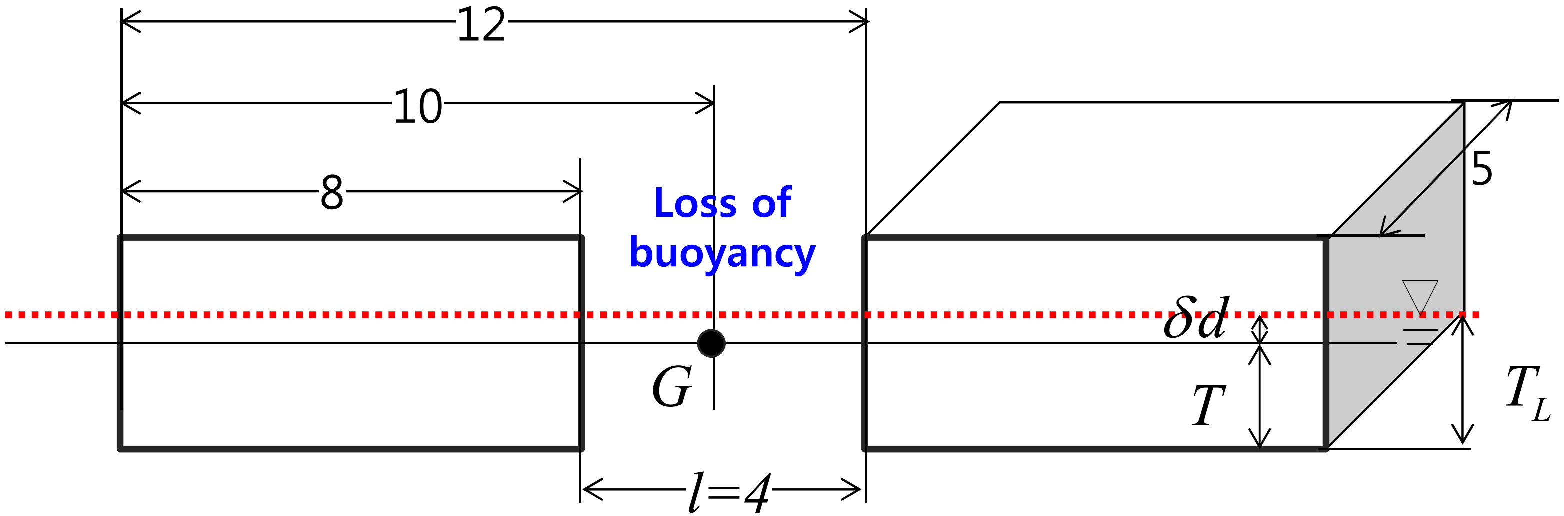
Simplest method using the formula

Draft after immersion: $$\delta d = \frac{v}{A_{WP} - a} = \frac{(4\times5\times1.5)}{(20\times5)-(4\times5)} = 0.375m$$

$$T_L = T + \delta d = 1.5 + 0.375 = 1.875m$$

A_{WP}: water plane area of the ship
(Including water plane area of the damaged compartment)
a: water plane area of the damaged compartment
δd: Draft change due to damaged compartment
v: Volume of damaged compartment below initial water plane

[Example] Damage of a Box-Shaped Ship (Immersion) (5/6)



Another method

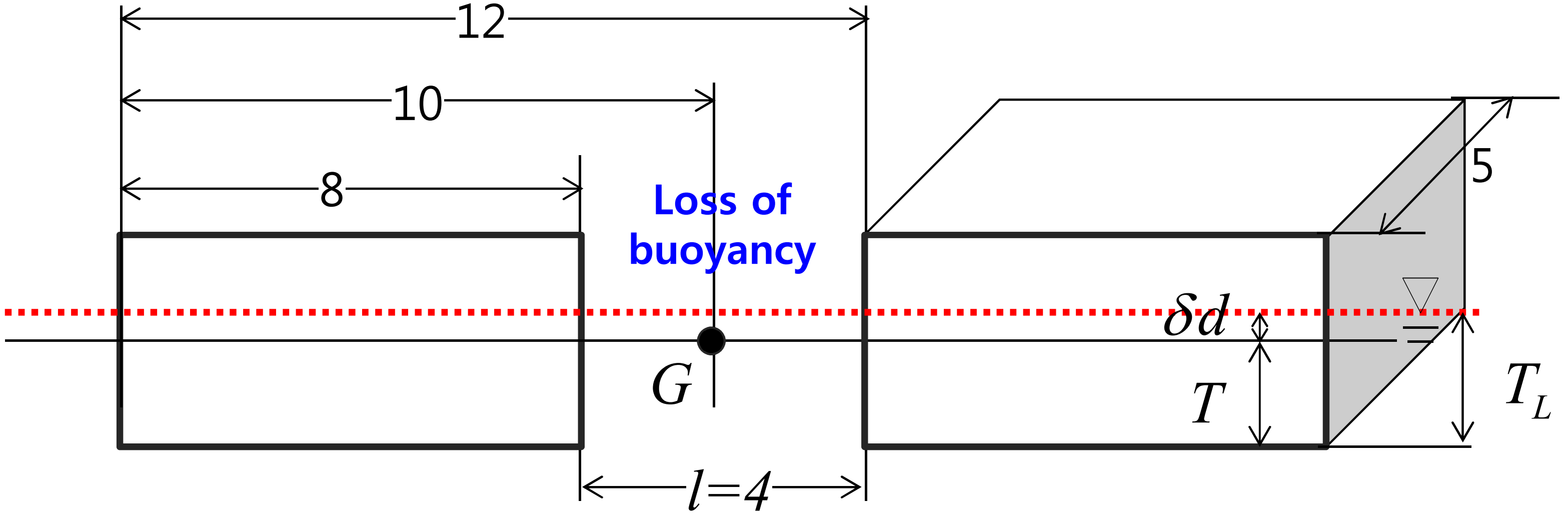
Water plane area: $A_L = (L-l)B = (20-4)\times 5 = 80m^2$

Draft after immersion: $T_L = \dfrac{\nabla_I}{A_L} = \dfrac{150}{80} = 1.875m$, where $\nabla_I = 150m^3$

$$KB_L = \frac{T_L}{2} = \frac{1.875}{2} = 0.938m$$

Moment of inertia of water plane area about transverse axis through point G: $I_L = \dfrac{B^3 \cdot (L-l)}{12} = \dfrac{5^3(20-4)}{12}$
$= 166.6667m^4$

[Example] Damage of a Box-Shaped Ship (Immersion) (6/6)



Metacentric radius: $BM_L = \dfrac{I_L}{\nabla_I} = \dfrac{166.6667}{150} = 1.111m$

Metacentric Height: $GM_L = KB_L + BM_L - KG = 0.938 + 1.111 - 1.5 = 0.549m$

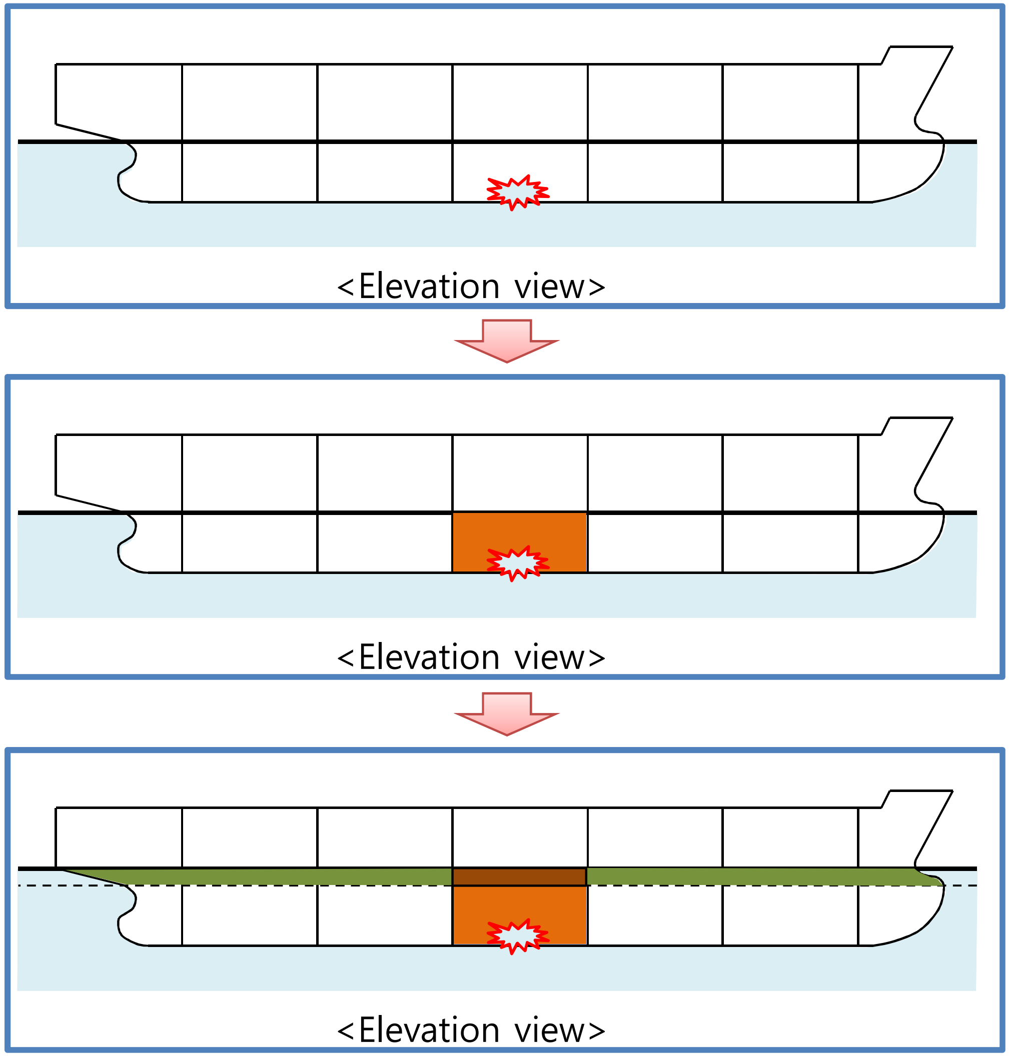
The righting moment for small angle of heel by lost buoyancy method:

$$M_{RL} = \Delta_I \cdot GM_L \cdot \sin\phi = 150 \times 1.025 \times 0.549 \sin\phi = 84.349 \sin\phi (ton \cdot m)$$

Added Weight Method

Concept of Added Weight Method (1/2)

* Hydrostatic Equilibrium
Displacement(Δ) = Buoyant Force = Weight(W)



A damage occurs.

Flooded water is considered as the added weight.

: Added weight

Added weight will be equilibrium with the buoyancy regained by an increase of draft.

: Additional added weight

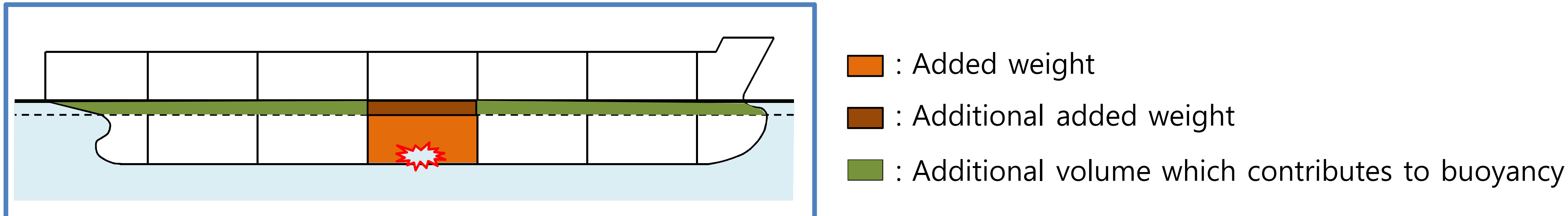
: Additional volume which contributes to buoyancy

Added weight method

"The water that enters the damaged compartment is considered as an **added weight** with no loss of buoyancy."

Concept of Added Weight Method (2/2)

Added weight method

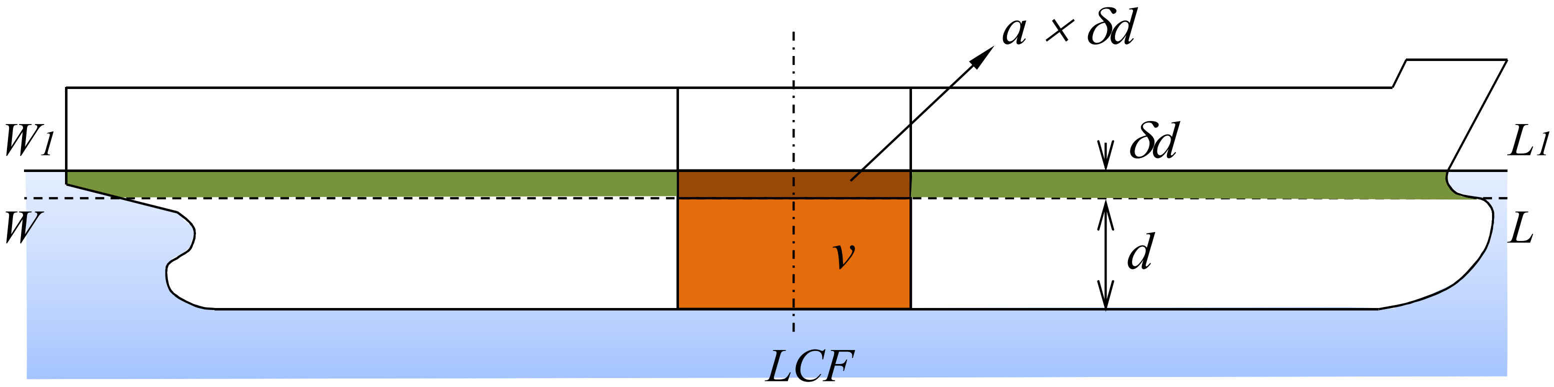


<Elevation view>

- The water that enters the damaged compartment is considered as an **added weight** with no loss of buoyancy.
- This is a **misnomer**, since water in space open to the sea and free to run in or out does not actually add to a ship's weight.
- For calculation purposes, it is **convenient** to regard such flooding water as adding to the displacement.
- However, it must be remembered that the resulting (virtual) displacement not only differ from the initial displacement, but varies with change in trim or heel.
- Since the added weight method involves a direct integration of volumes up to water plane at the damaged condition, it is just as well adapted to dealing with complex flooding conditions as with simple ones.

Added Weight Method

The water that enters the damaged compartment is considered as an added weight with no loss of buoyancy.



Weight of sea water due to the damaged compartment: $w = \rho \cdot g \cdot (v + a \cdot \delta d)$

Increased buoyancy due to the change in draft: $b = \rho \cdot g \cdot \left(A_{WP} \cdot \delta d \right)$

$$w = b \Rightarrow \rho \cdot g \cdot (v + a \cdot \delta d) = \rho \cdot g \cdot \left(A_{WP} \cdot \delta d \right)$$

Changed draft due to compensated weight of damaged compartment:

$$\delta d = \frac{v}{A_{WP} - a}$$

A_{WP}: water plane area of the ship
(Including water plane area of the damaged compartment)
a: water plane area of the damaged compartment
d: Draft before the compartment is not damaged
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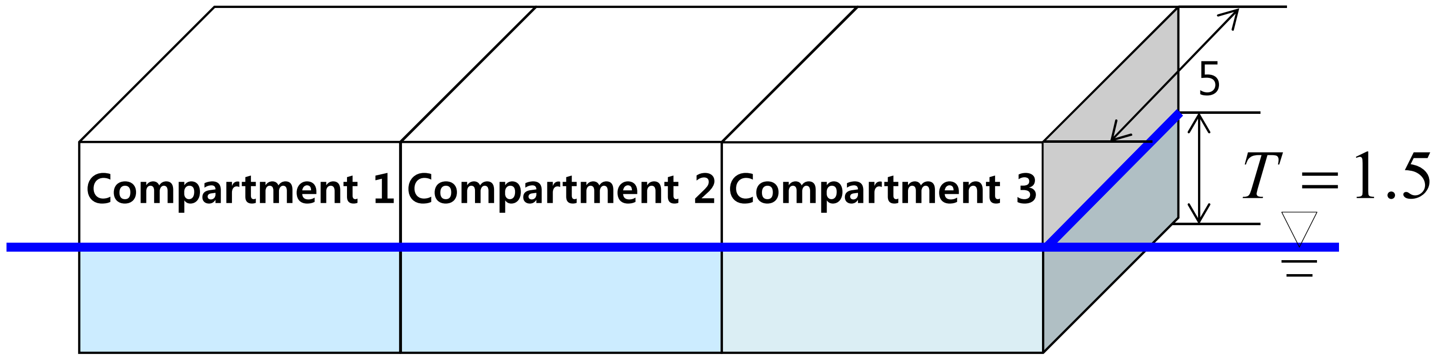
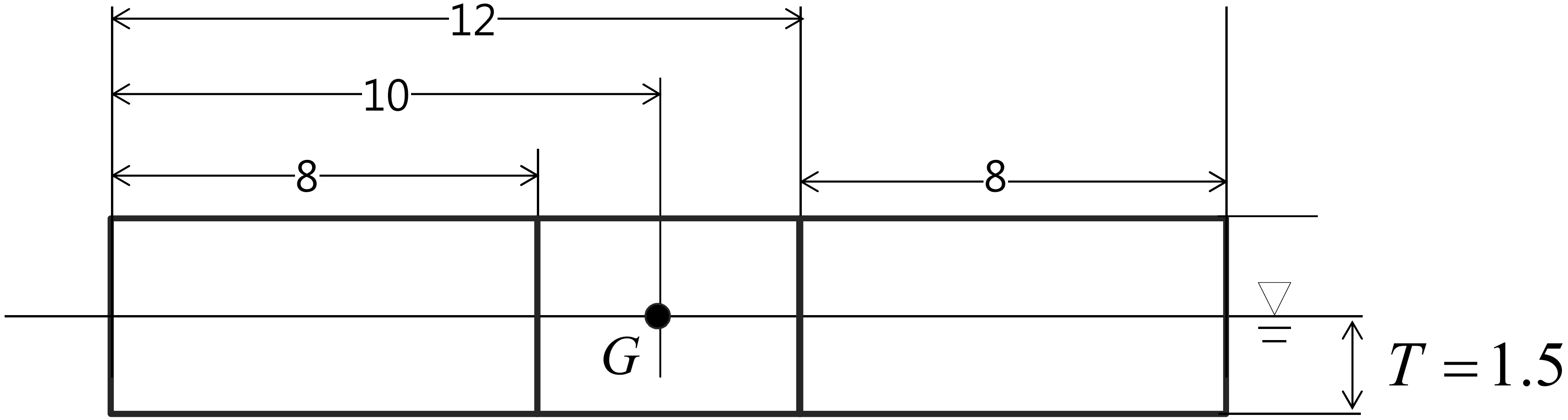
v: Volume of damaged compartment below initial water plane

✓ **A ship is composed of three compartments.**



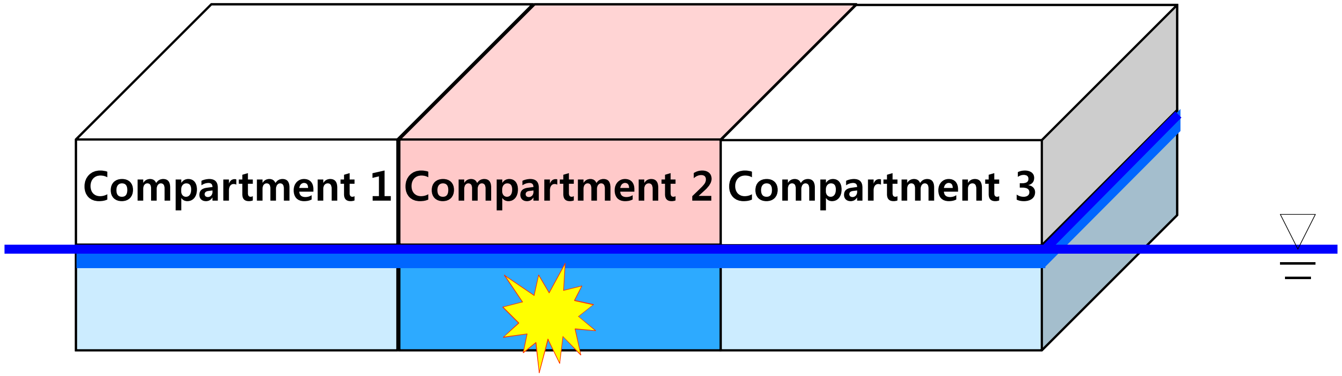


Initial displacement volume: $\nabla_I = LBT = 20 \times 5 \times 1.5 = 150m^3$

When a compartment of the ship is damaged, what is the new position of this ship?

[Example] Damage of a Box-Shaped Ship (Immersion) (2/9)

When the compartment in the midship part is damaged, what is the new position of this ship?



The position of the ship will be changed.

Immersion

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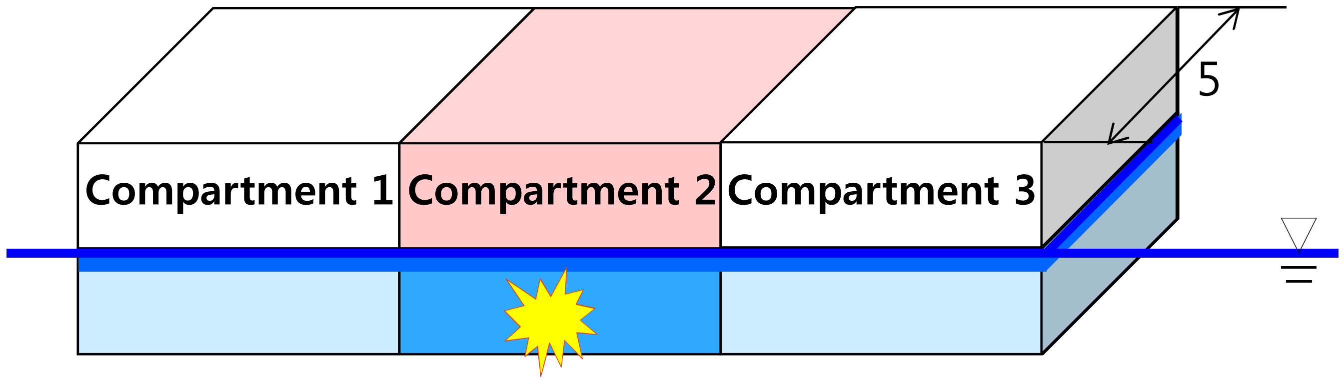
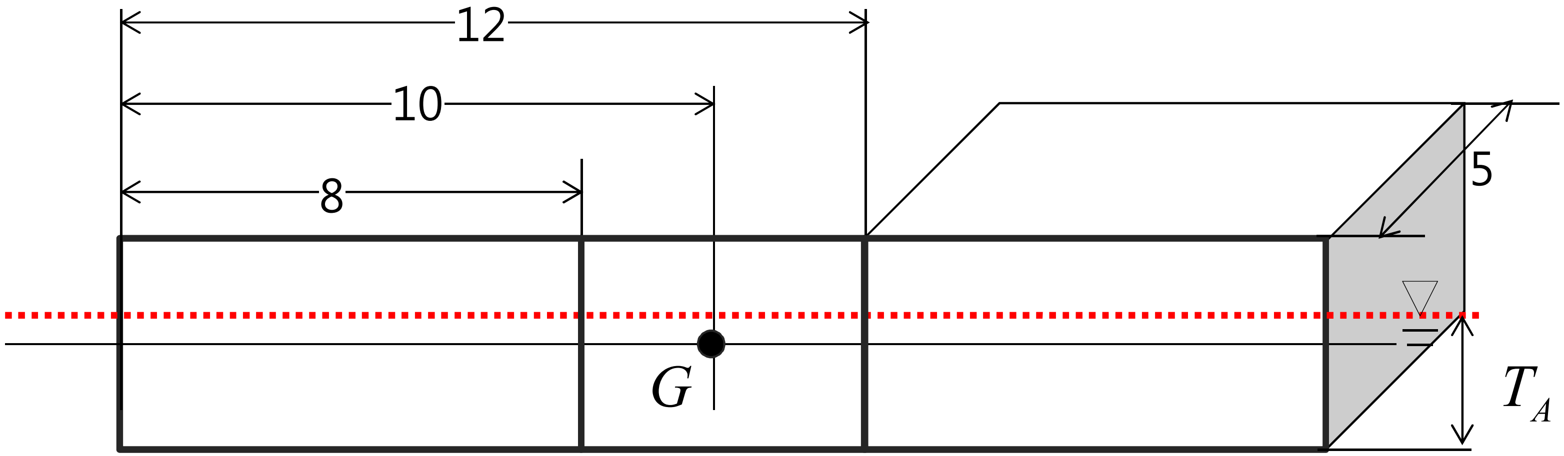
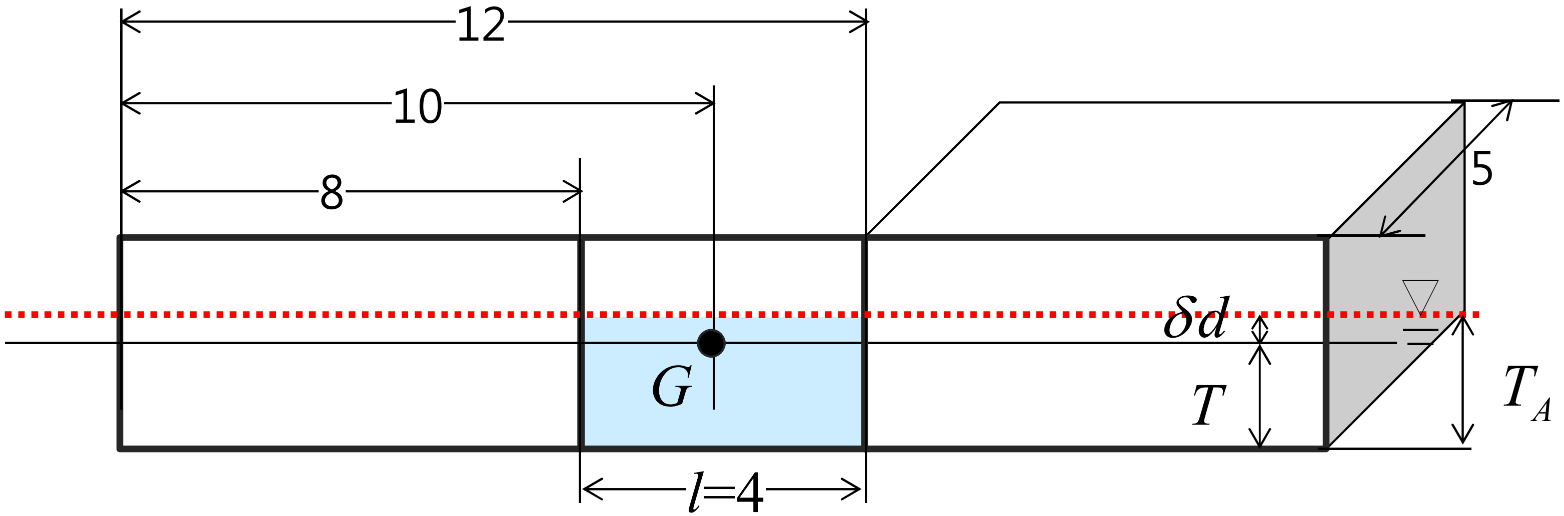




[Example] Damage of a Box-Shaped Ship (Immersion) (4/9)



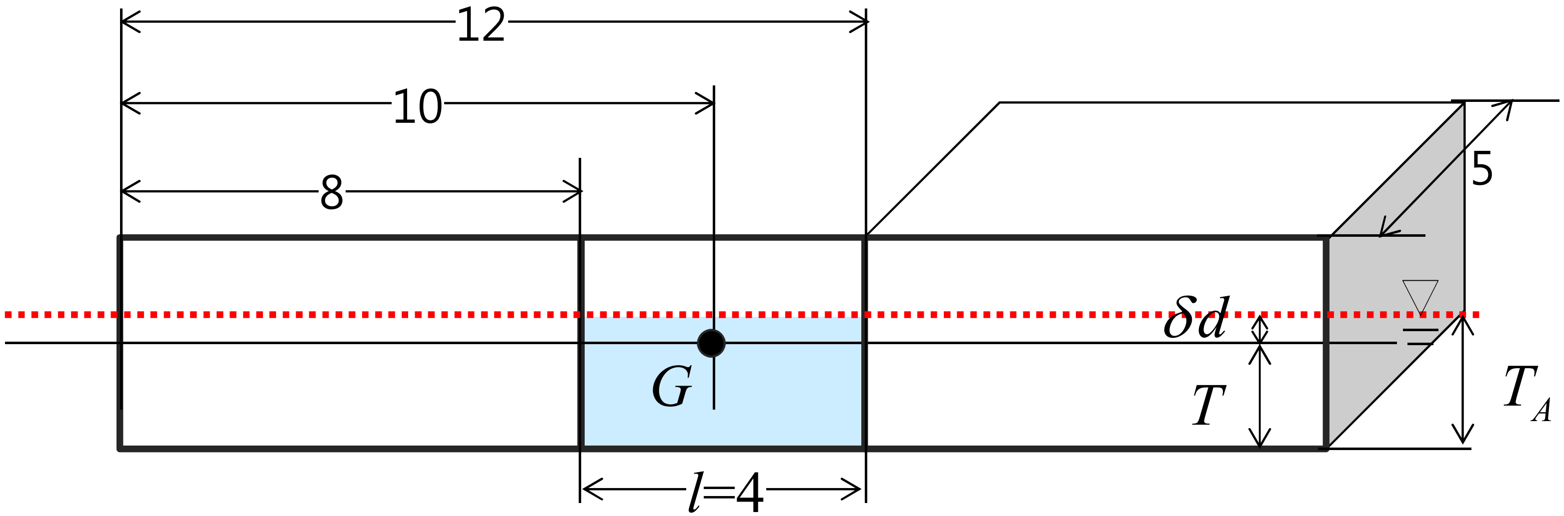
Simplest method using the formula

Draft after immersion: $\delta d = \dfrac{v}{A_{WP} - a} = \dfrac{(4 \times 5 \times 1.5)}{(20 \times 5) - (4 \times 5)} = 0.375m$

$$T_A = T + \delta d = 1.5 + 0.375 = 1.875m$$

A_{WP}: water plane area of the ship
(Including water plane area of the damaged compartment)
a: water plane area of the damaged compartment
δd: Draft change due to damaged compartment
v: Volume of damaged compartment below initial water plane

[Example] Damage of a Box-Shaped Ship (Immersion) (5/9)



Another method

The volume of flooding water: $v + a \times \delta d = l \cdot B \cdot T_A = l \cdot B \cdot (T + \delta d)$

The additional buoyant volume: $\delta\nabla = L \cdot B \cdot \delta d$

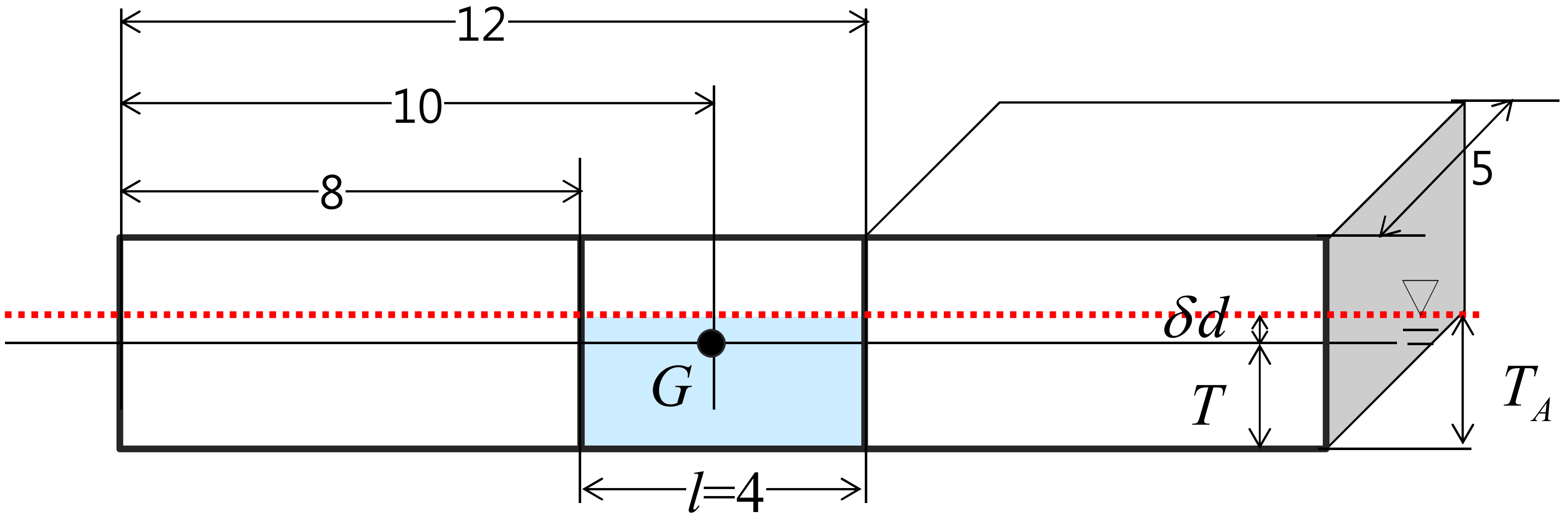
Because $v + a \times \delta d = \delta\nabla$,

$$lB(T + \delta d) = L \cdot B \cdot \delta d$$

$$l(T + \delta d) = L \cdot \delta d$$

$$l \cdot T = (L - l) \cdot \delta d \qquad \delta d = \frac{l \cdot T}{L - l} = \frac{4 \times 1.5}{20 - 4} = 0.375\,m$$

[Example] Damage of a Box-Shaped Ship (Immersion) (6/9)



The draft after flooding: $T_A = T + \delta d$
$= 1.500 + 0.375 = 1.875m$

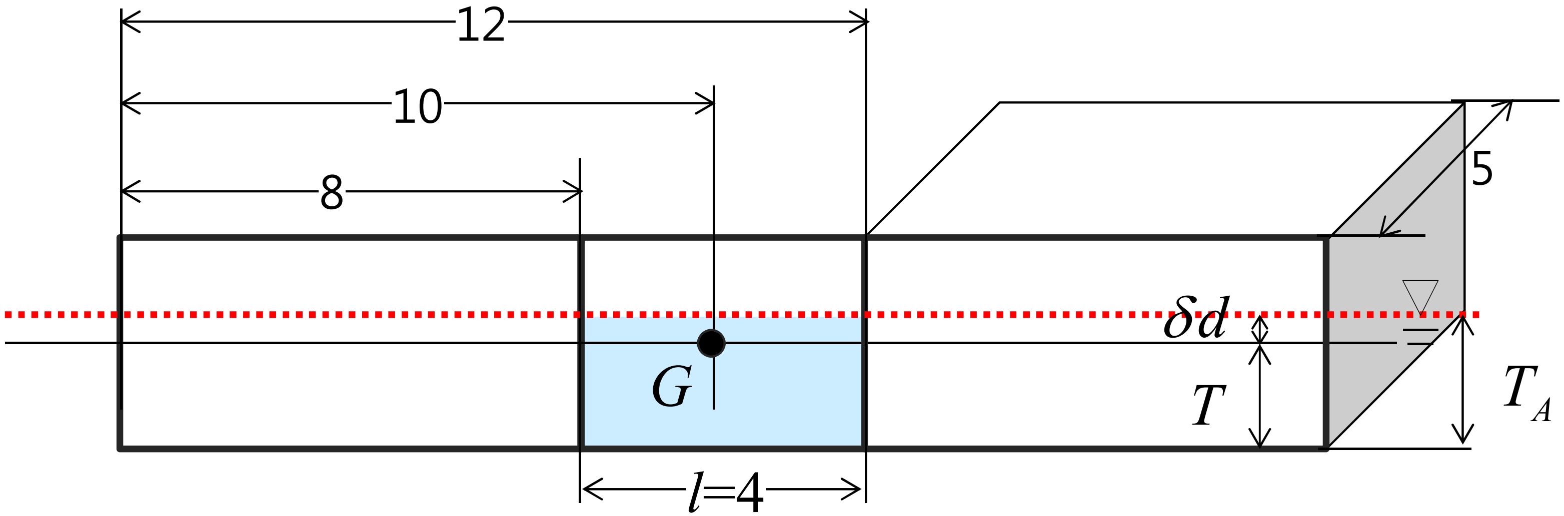
The volume of flooding water: $v + a \times \delta d = l \cdot B \cdot T_A = 4 \times 5 \times 1.875 = 37.5m^3$

The height of its center of gravity: $kg = \dfrac{T_A}{2} = \dfrac{1.875}{2} = 0.938m$

The displacement volume alter flooding:

$$\nabla_A = L \cdot B \cdot T_A = 20 \times 5 \times 1.875 = 187.5m^3$$

[Example] Damage of a Box-Shaped Ship (Immersion) (7/9)



KG by the added weight method:

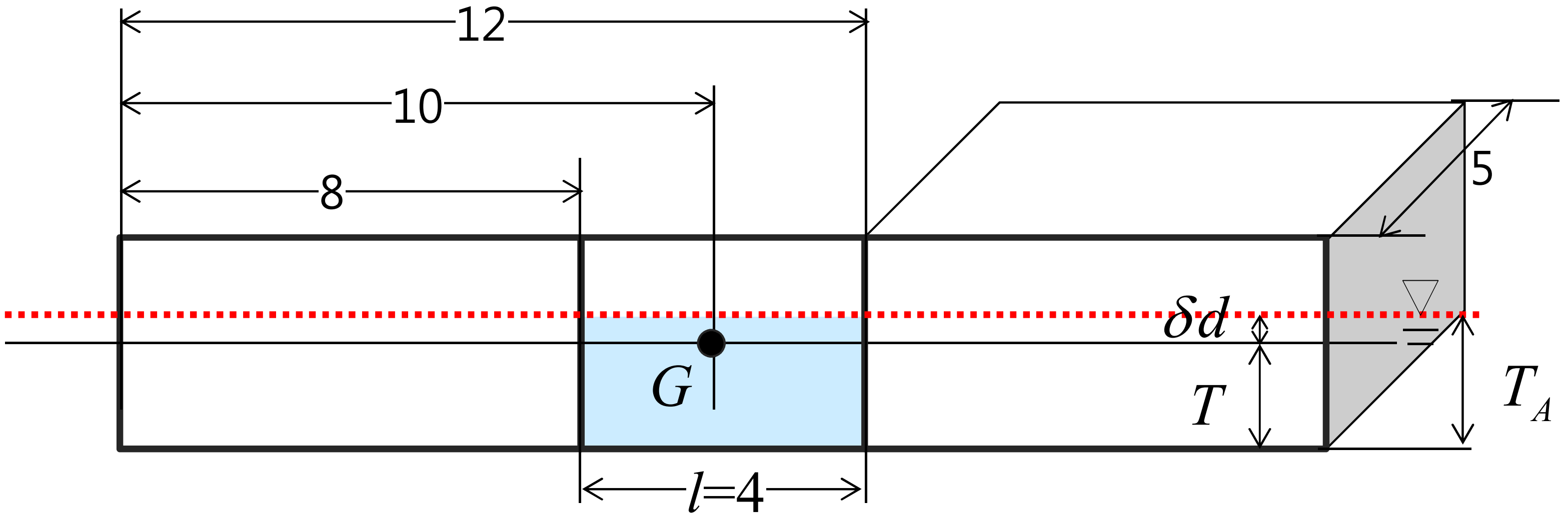
	Volume	Centre of gravity	Moment
Initial	150.0	1.500	225.000
Added	37.5	0.938	35.156
Total	187.5	1.388	260.156

$$KG_A = 1.388m$$

Moment of inertia of water plane area about transverse axis through point G: $I_A = \dfrac{B^3 \cdot L}{12} = \dfrac{5^3 \times 20}{12} = 208.333m^4$

Metacentric radius: $BM_A = \dfrac{I_A}{\nabla_A} = \dfrac{208.333}{187.5} = 1.111m$

[Example] Damage of a Box-Shaped Ship (Immersion) (8/9)



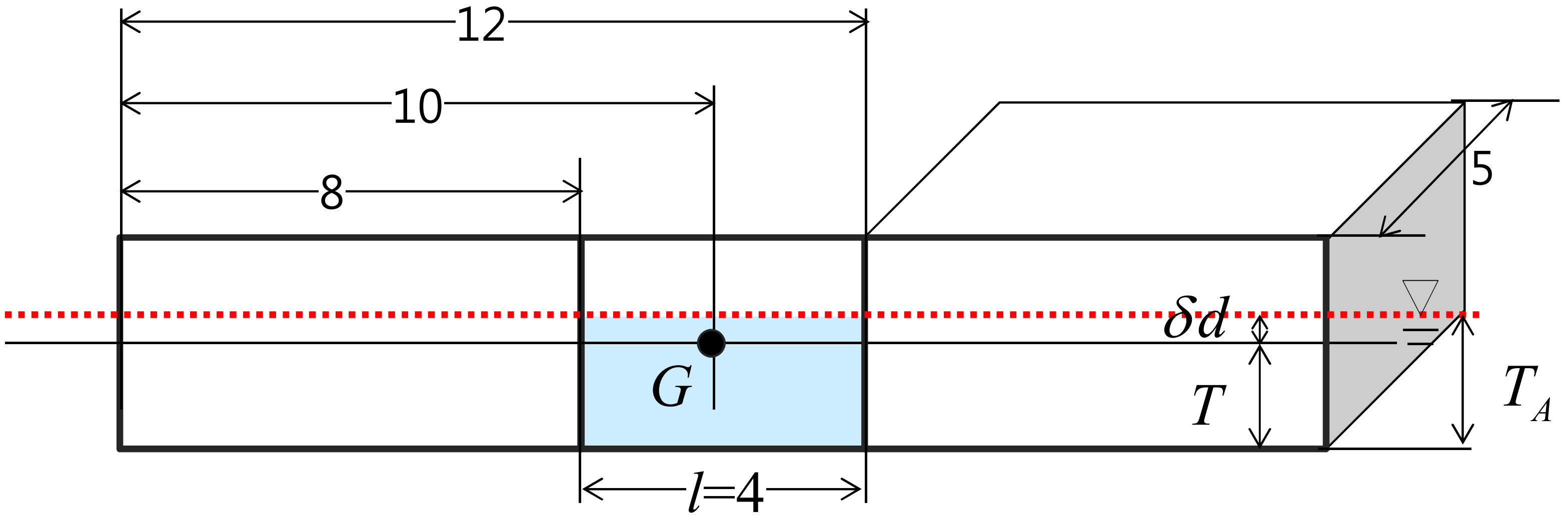
Free surface effect caused by the flooding water:

The moment of inertia of the free surface in the flooded compartment: $i = \dfrac{B^3 \cdot l}{12} = \dfrac{5^3 \times 4}{12} = 41.667 m^4$

The moment arm of the free surface effect (free surface correction): $l_F = \dfrac{\rho \cdot i}{\rho \cdot \nabla_A} = \dfrac{41.667}{187.5} = 0.222 m$

The changed vertical center of buoyancy: $KB_A = \dfrac{T_A}{2} = \dfrac{1.875}{2} = 0.938 m$

[Example] Damage of a Box-Shaped Ship (Immersion) (9/9)



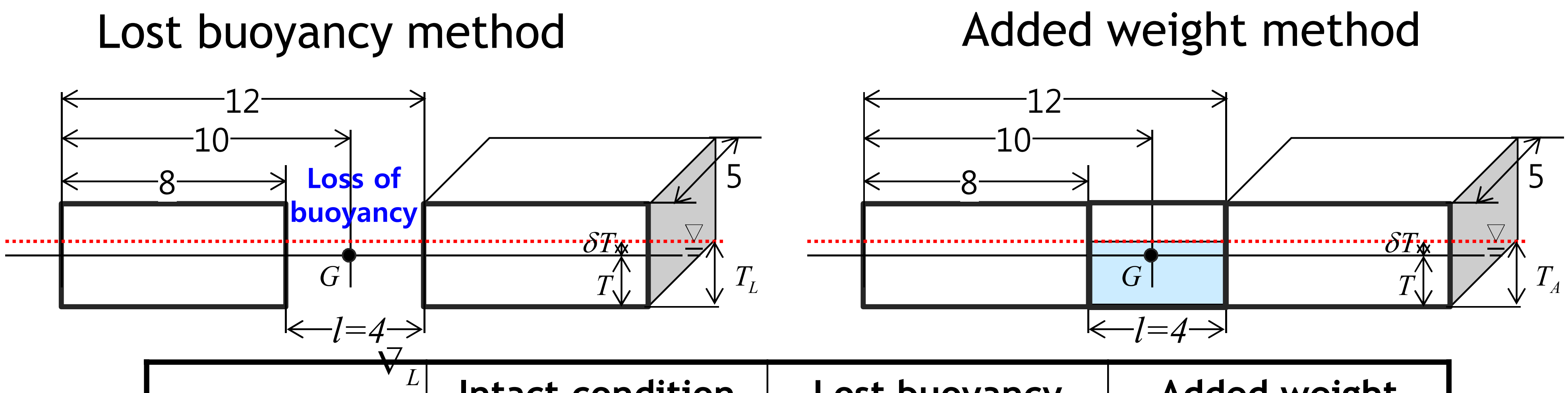
Metacentric height: $GM_A = KB_A + BM_A - KG_A - l_F$
$= 0.938 + 1.111 - 1.388 - 0.222 = 0.439m$

The changed displacement: $\Delta_A = \rho \nabla_A = 1.025 \times 187.5 = 192.188 ton$

The righting moment for small angle of heel by added weight method:

$$M_{RA} = \Delta_A \cdot GM_A \cdot \sin\phi = 192.188 \times 0.439 \sin\phi = 84.349 \sin\phi (ton \cdot m)$$

Comparison of Two Methods



	Intact condition (Initial state)	Lost buoyancy method	Added weight method
Draft(m)	1.500	1.875	1.875
∇(m^3)	150.000	150.000	187.500
Δ(ton)	153.750	153.750	192.188
KB(m)	0.750	0.938	0.938
BM(m)	1.389	1.111	1.111
KG(m)	1.500	1.500	1.388
GM(m)	0.639	0.549	0.439
Δ·GM(ton·m)	98.229	84.349	84.349

[Appendix] An Example of Finding Immersion and Heel of a Boxed-Shaped Ship with a Flooded Cargo

Governing Equations of Computational Ship Stability (1/2)

When the ship is in intact state.

$$\begin{bmatrix} \cancel{F^{(k+1)}} - F^{(k)} \\ \cancel{M_T^{(k+1)}} - M_T^{(k)} \\ \cancel{M_L^{(k+1)}} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g T_{WP}^{(k)} \cdot \cos\theta & \rho g L_{WP}^{(k)} \\ -\rho g T_{WP}^{(k)} & element(2,2) & \rho g I_P^{(k)} \\ \rho g L_{WP}^{(k)} & \rho g I_P^{(k)} \cdot \cos\theta & element(3,3) \end{bmatrix}_{{}^n z^{(k)}, \phi^{(k)}, \theta^{(k)}} \begin{bmatrix} \delta\, {}^n z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

Element (2, 2): $\left[-\rho g \left({}^n z_{B^{(i)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_G - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext}^{(k)} \right] \cdot \cos\theta$

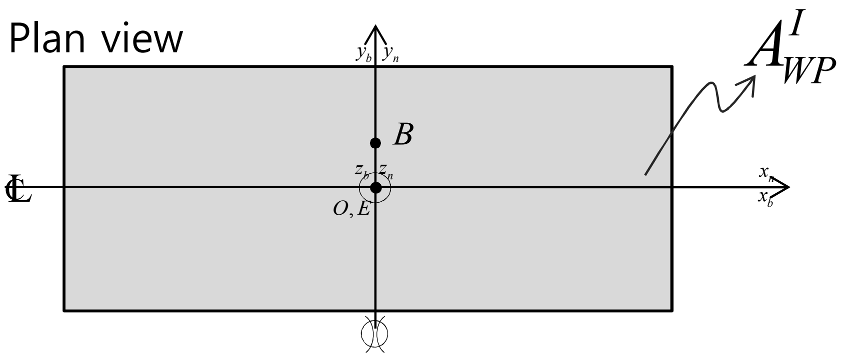
Element (3, 3): $-\rho g \left({}^n z_{B^{(i)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_G - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext}^{(k)}$

When the ship is in intact state,
the water plane area is as follows.

$$A_{WP} = A_{WP}^{I}$$

A_{WP} : Water plane area of the intact ship

Plan view



When the ship is flooded,
the water plane area is as follows.

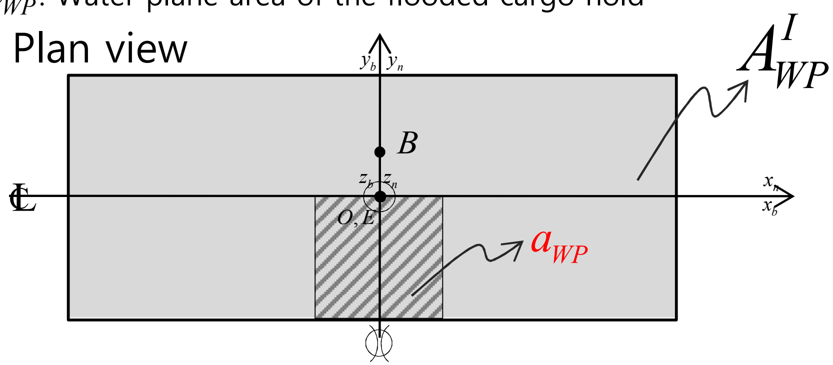
$$A_{WP} = A_{WP}^{I} - \mu_F \cdot a_{WP}$$

μ_F: Surface permeability of a compartment

A_{WP}^{I}: Water plane area of the intact ship

a_{WP}: Water plane area of the flooded cargo hold

Plan view



Governing Equations of Computational Ship Stability (2/2)

When the ship is flooded. (Damaged state)

$$\begin{bmatrix} \cancel{F^{(k+1)}} - F^{(k)} \\ \cancel{M_T^{(k+1)}} - M_T^{(k)} \\ \cancel{M_L^{(k+1)}} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g T_{WP}^{(k)}\cdot\cos\theta & \rho g L_{WP}^{(k)} \\ -\rho g T_{WP}^{(k)} & element(2,2) & \rho g I_P^{(k)} \\ \rho g L_{WP}^{(k)} & \rho g I_P^{(k)}\cdot\cos\theta & element(3,3) \end{bmatrix}_{{}^{n}z^{(k)},\phi^{(k)},\theta^{(k)}} \begin{bmatrix} \delta\, {}^{n}z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

Element (2, 2): $[-\rho g\left({}^{n}z_{B^{(i)}/E}\nabla^{(k)} + I_T{}^{(k)}\right) - {}^{n}z_{G^{(k)}/E}\cdot F_G - {}^{n}z_{G_{ext}^{(k)}/E}\cdot F_{ext}^{(k)}]\cdot\cos\theta$

Element (3, 3): $-\rho g\left({}^{n}z_{B^{(i)}/E}\nabla^{(k)} + I_L{}^{(k)}\right) - {}^{n}z_{G^{(k)}/E}\cdot F_G - {}^{n}z_{G_{ext}^{(k)}/E}\cdot F_{ext}^{(k)}$

$$A_{WP} = A_{WP}^{I} - \mu_F\cdot a_{WP}$$

$$I_T = I_T^{I} - \mu_F\cdot i_T$$

$$I_L = I_L^{I} - \mu_F\cdot i_L$$

$$I_P = I_P^{I} - \mu_F\cdot i_P$$

$$T_{WP} = T_{WP}^{I} - \mu_F\cdot t_{WP}$$

$$L_{WP} = L_{WP}^{I} - \mu_F\cdot l_{WP}$$

μ_F: Surface permeability of a compartment
A_{WP}^{I} : Water plane area of the intact ship
a_{WP} : Water plane area of the flooded cargo hold
I_T^{I} : Transverse moment of inertia of the water plane area of the intact ship about the $x_{b'}$ axis
i_T : Transverse moment of inertia of the water plane area of the flooded cargo hold about the $x_{b'}$ axis
I_L^{I} : Longitudinal moment of inertia of the water plane area of the intact ship about the $y_{b'}$ axis
i_L : Longitudinal moment of inertia of the water plane area of the flooded cargo hold about the $y_{b'}$ axis
I_P^{I} : Centrifugal moment of the water plane area of the intact ship about the $x_{b'}$ and $y_{b'}$ axis
i_P: Centrifugal moment of the water plane area of the flooded cargo hold about the $x_{b'}$ and $y_{b'}$ axis
T_{WP}^{I} : Transverse moment of water plane area of the intact ship about the $x_{b'}$ axis
t_{WP} : Transverse moment of water plane area of the flooded cargo hold about the $x_{b'}$ axis
L_{WP}^{I} : Longitudinal moment of water plane area of the intact ship about the $y_{b'}$ axis
l_{WP} : Longitudinal moment of water plane area of the flooded cargo hold about the $y_{b'}$ axis