Compressible flow

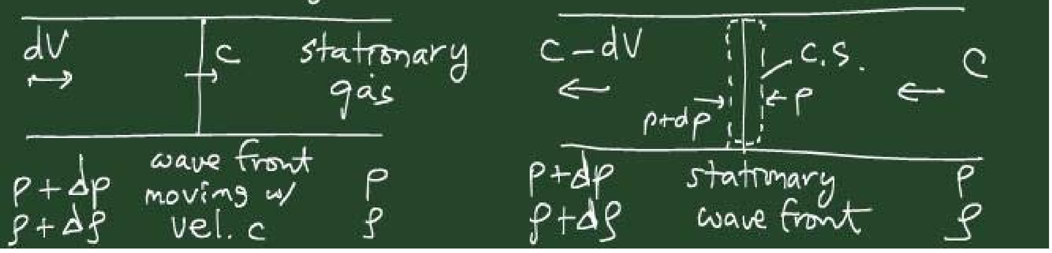
노트 제목 2010-05-13

"The Dynamics and Thermodynamics of Compressible Fluid Flow" by Ascher H. Shapiro

⊙ The velocity of sound

• Velocity of propagation of a plane pressure pulse

vel. of sound for a plane, infinitesimal press. wave



cont: $\cancel{\rho c} = (\rho+d\rho)(c-dV)$

$= \cancel{\rho c} - \rho dV + c\,d\rho - \cancel{d\rho\, dV}^{\,0}$

$\Rightarrow \dfrac{d\rho}{\rho} = \dfrac{dV}{c}$

mtm: $\cancel{pA} - (\cancel{p}+dp)A = \rho A c\left[(\cancel{c}-dV)-\cancel{c}\right]$

$\Rightarrow dp = \rho c\, dV$

$c^2 = \left.\dfrac{\partial p}{\partial \rho}\right|_s$ s : isentropic process

∵ dp, $d\rho$, dT is very small and very rapid process

reversible + adiabatic ← no heat exchange.

For perfect gas, $P/\rho^k = \text{const}$
& isentropic process

$$\ln p - k \ln \rho = \text{cons}.$$

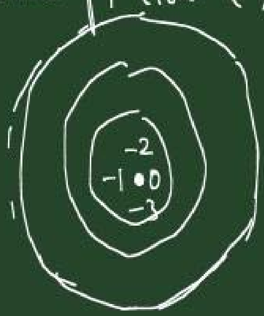
$$\rightarrow \frac{dp}{p} - k\frac{d\rho}{\rho} = 0$$

$$\frac{dp}{p} = k\frac{d\rho}{\rho} \rightarrow \left.\frac{\partial p}{\partial \rho}\right|_s = k\frac{p}{\rho} = kRT$$

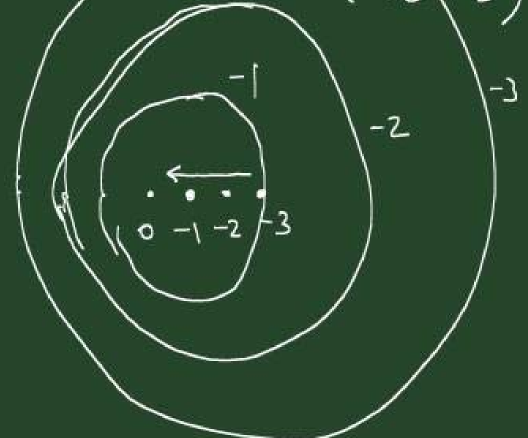
$$\therefore \boxed{c = \sqrt{kRT}}$$

- pressure field created by a moving ~~point~~ disturbance

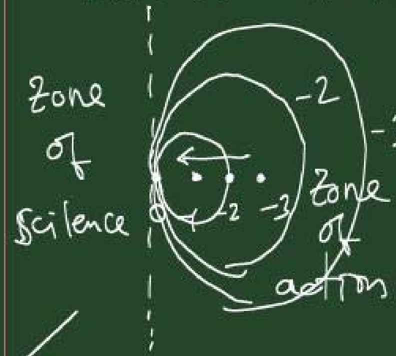
Incomp. flow ($V/c = 0$)

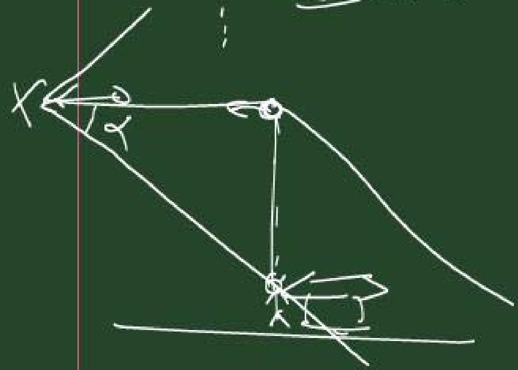


Subsonic motion ($V/c = \frac{1}{2}$)

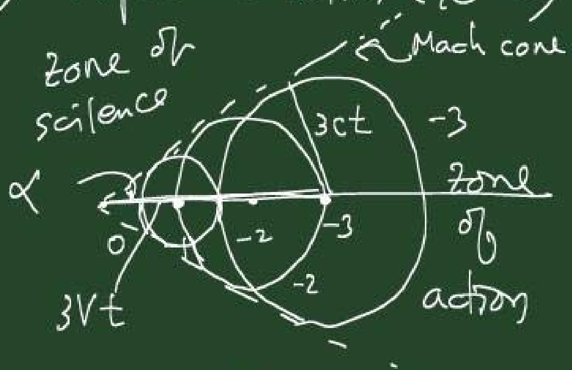


Transonic motion ($V/c = 1$)





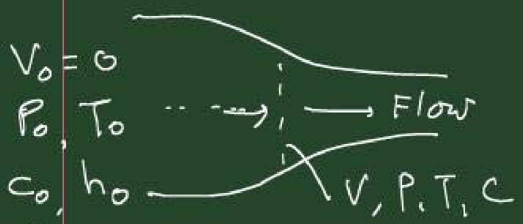
Supersonic motion ($V/c = 2$)



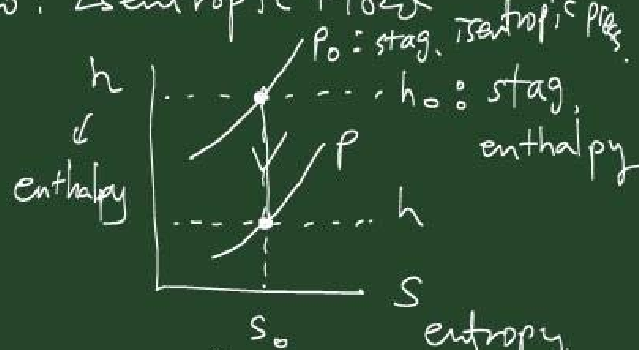
$$\sin\alpha = \frac{3ct}{3Vt} = \frac{c}{V} = \frac{1}{Ma}$$

↑ Mach angle

One-Dimensional Flow: Isentropic Flow



↑ stagnation properties



- energy eq. (1st law) $\quad h_0 = h + \frac{V^2}{2}$
- 2nd law $\quad s = s_0$
- Continuity $\quad G \equiv w/A = \rho V \quad$ w: mass flow rate
- Eq. of state $\quad h = h(s, p)$
  $\rho = \rho(s, p)$

energy eq.
$\rightarrow \quad 0 = dh + VdV \rightarrow dh = -VdV$

$Tds = dh - \frac{1}{\rho}dp \rightarrow dh = \frac{1}{\rho}dp$

$\Rightarrow dp = -\rho V dV$: Euler eq.

$\Rightarrow \frac{dV}{dp} < 0$: pressure always decreases in an accelerating flow and increases in a decelerating flow

$\rho A V = \text{const.} \rightarrow \frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$

$\rightarrow \frac{dA}{A} = -\frac{d\rho}{\rho} - \frac{dV}{V} = -\frac{d\rho}{\rho} + \frac{dp}{\rho V^2} = \frac{dp}{\rho}\left(\frac{1}{V^2} - \frac{d\rho}{dp}\right)$, with $\frac{d\rho}{dp} = \frac{1}{c^2}$

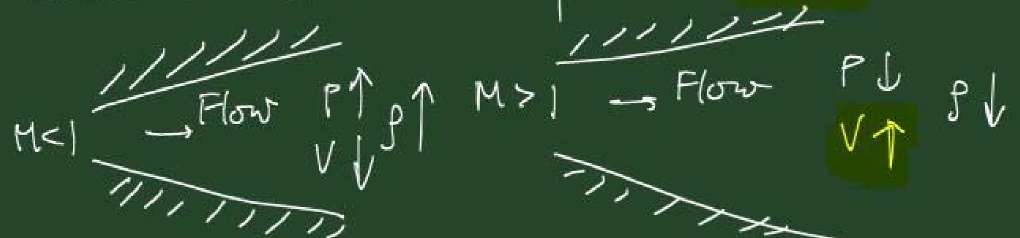
$= \frac{dp}{\rho V^2}\left(1 - \frac{V^2}{c^2}\right) = \frac{1-M^2}{\rho V^2}dp$

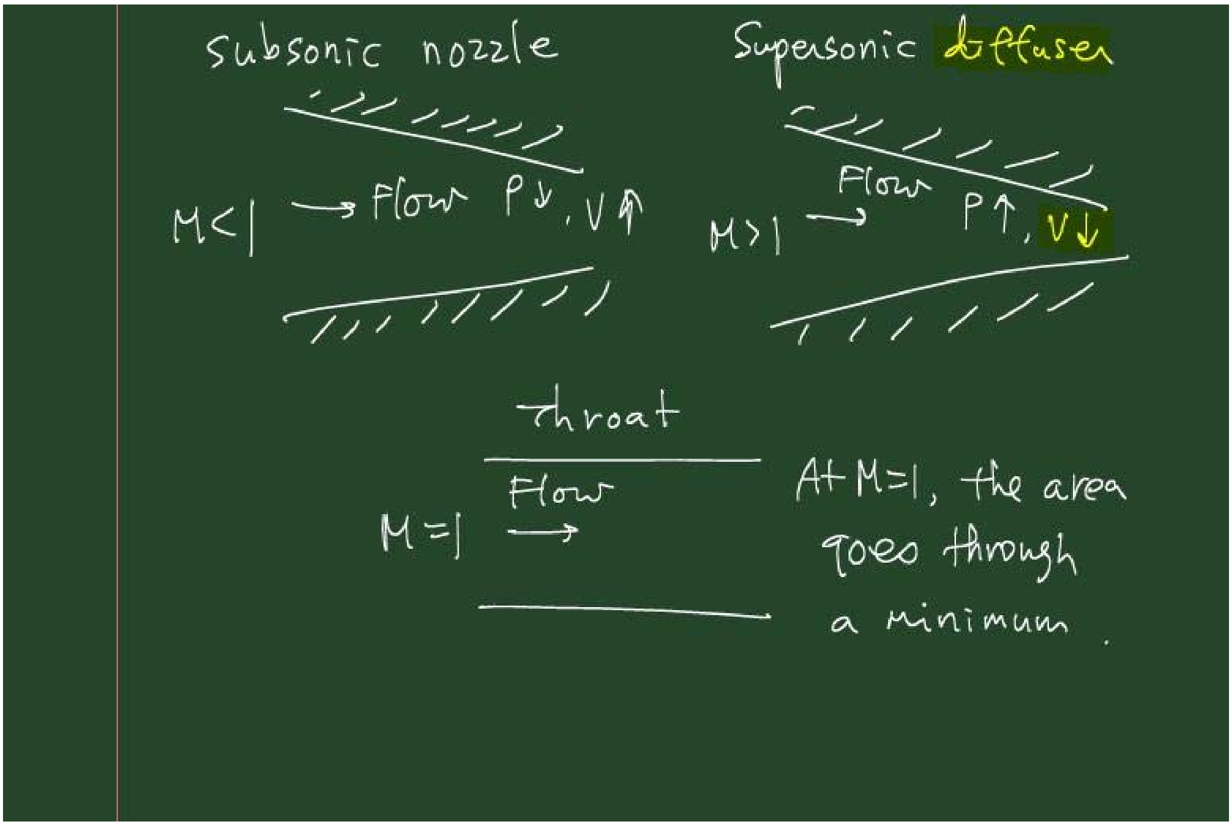
i) for subsonic speeds $(M<1)$, $\frac{dA}{dp} > 0$, $\frac{dA}{dV} < 0$

for supersonic speeds $(M>1)$, $\frac{dA}{dp} < 0$, $\frac{dA}{dV} > 0$

for sonic speeds $(M=1)$, $\frac{dA}{dp} = 0$, $\frac{dA}{dV} = 0$

subsonic diffuser   Supersonic nozzle





• Adiabatic Flow of a Perfect Gas.

For a perfect gas, $\Delta h = c_p \Delta T$

$$c_p - c_v = R$$

$$c_p / c_v = k$$

$$c_p = \frac{k}{k-1} R$$

$$h_0 = h + \frac{V^2}{2} \rightarrow V = \sqrt{2c_p (T_0 - T)} = \sqrt{\frac{2k}{k-1} R (T_0 - T)}$$

@ $M=1$, $V^* = c^*$: $\sqrt{\frac{2k}{k-1} R (T_0 - T^*)} = \sqrt{kRT^*}$

$*$: properties at $M=1$

$$\rightarrow \frac{T^*}{T_0} = \frac{2}{k+1}, \quad V^* = c^* = \sqrt{\frac{2k}{k+1} R T_0}$$

$$V = \sqrt{2c_p(T_0 - T)}$$

$$\rightarrow \frac{T_0}{T} = 1 + \frac{V^2}{2c_pT} = 1 + \frac{V^2}{kRT}\frac{kR}{2c_p} = 1 + \frac{k-1}{2}\frac{V^2}{c^2}$$

$$\rightarrow \underline{\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2}$$

Flow per unit area

$$G \equiv \frac{w}{A} = \rho V = \frac{P}{RT}V = \frac{PV}{\sqrt{kRT}}\sqrt{\frac{k}{R}}\sqrt{\frac{T_0}{T}}\frac{1}{\sqrt{T_0}}$$

$$= \sqrt{\frac{k}{R}}\frac{P}{\sqrt{T_0}}M\sqrt{1 + \frac{k-1}{2}M^2}$$

For isentropic flow, $\frac{P}{P_0} = \left(\frac{\rho}{\rho_0}\right)^k$, $\frac{T}{T_0} = \left(\frac{P}{P_0}\right)^{\frac{k-1}{k}}$

Then, $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$: adiabatic flow

$$\left.\begin{aligned} \frac{P_0}{P} &= \left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}} \\ \frac{\rho_0}{\rho} &= \left(1 + \frac{k-1}{2}M^2\right)^{\frac{1}{k-1}} \end{aligned}\right\} \text{isentropic flow (rev. adiabatic)}$$

$$\therefore \underline{\frac{w}{A} = \sqrt{\frac{k}{R}}\frac{P_0}{\sqrt{T_0}}\frac{M}{\left(1 + \frac{k-1}{2}M^2\right)^{\frac{k+1}{2(k-1)}}}} \rightarrow \frac{w}{A}\frac{\sqrt{T_0}}{P_0} \sim f(M)$$