Homework # 4 Due by June 1. 6.1, 6.2, 6.5, 6.6, 6.9

노트 제목 2010-05-18

$$\frac{w}{A} = \sqrt{\frac{k}{R}}\frac{P_0}{\sqrt{T_0}}\frac{M}{\left(1+\frac{k-1}{2}M^2\right)^{\frac{k+1}{2(k-1)}}} \rightarrow \frac{w\sqrt{T_0}}{P_0} \sim f(M)$$

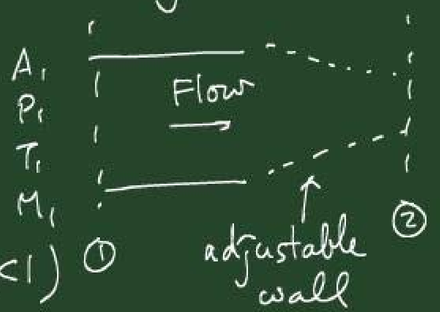
Maximum flow rate per unit area

$$\frac{\partial}{\partial M}\left(\frac{w}{A}\right) = 0 \quad \text{given } P_0 \text{ and } T_0$$

$\rightarrow M = 1$: the cross-sectional area for isentropic flow passes through a minimum at Mach # = 1.

$$\rightarrow \left(\frac{w}{A}\right)_{max} = \frac{w}{A^*} = \sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}\frac{P_0}{\sqrt{T_0}}$$

- Choking in isentropic flow



$(M_1<1)$

Provide certain values of P_1, T_1, M_1 & A_1.

Fix the flow rate w.

$$\frac{T_0}{T} = \left(1+\frac{k-1}{2}M^2\right)$$

$$\frac{P_0}{P} = (\cdots\ \text{"}\ )^{\frac{k}{k-1}}$$

If $A_1 = A_2$, $\bigcirc_2 = \bigcirc_1$

If $A_1 = A_2 + \varepsilon$, $M_2 \uparrow$, $P_2 \downarrow$, $T_2 \downarrow$

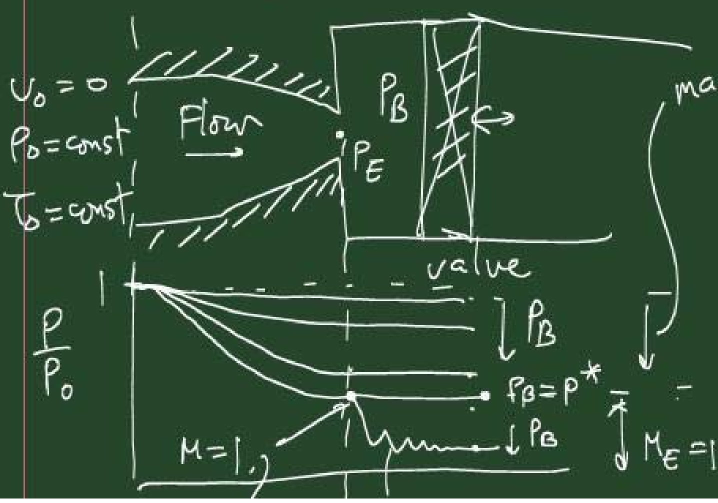
$\rightarrow$ slight reduction of A_2 is accompanied by a reduction in P_2.

Further reductions in A_2 may be made in the same way until the value of M_2 reaches unity.

Choking: After this point has been reached, there is no way of reducing the area further w/o a simultaneous change in the steady-state conditions at section 1.
For example, for const. P_1 & T_1,
further reduction in A_2/A_1 results in a reduced steady state M_1 $\rightarrow$ decrease w.
therefore, the max. possible M_1 is obtained when $M_2 = 1$.

For a given area reduction, there is in subsonic flow a maximum initial Ma # which can be maintained steadily, and in supersonic flow a min. initial Ma # which can be maintained steadily.
At either of these limiting conditions, the flow at section 2 is sonic, and is said to be choked.

- Converging nozzles



mass flow rate ↑

$\frac{w\sqrt{T_0}}{A_E P_0}$ depends on $\frac{P_B}{P_0}$

$P_B \simeq P_E$, $\frac{P_B}{P_0} > \frac{P^*}{P_0}$

$M_E = 1$,

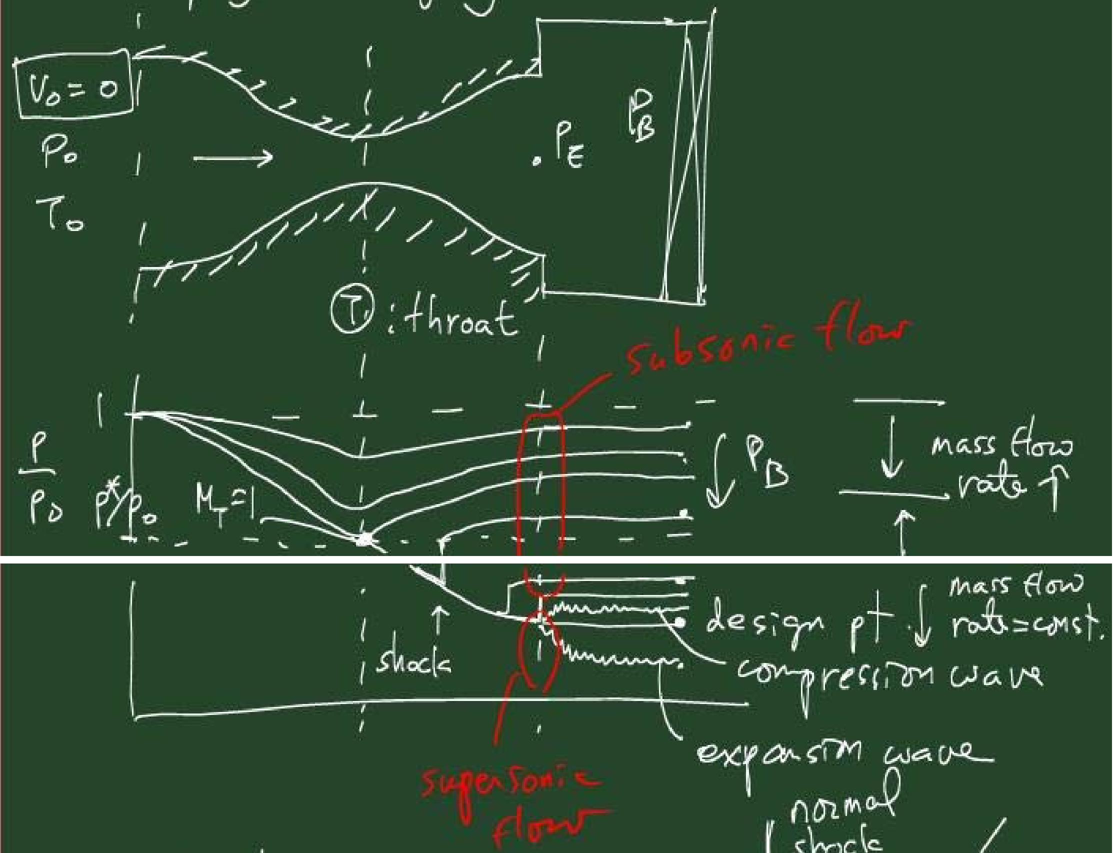
$M_E = 1$, $\frac{P_B}{P_0} < \frac{P^*}{P_0}$, $P_E = P^*$

$A_E = A^*$ ↓ expansion waves

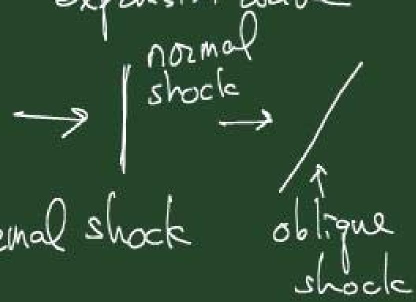
$\frac{\dot{m}\sqrt{T_0}}{A_E P_0}$ indep of $\frac{P_B}{P_0}$

mass. flow rate = const.

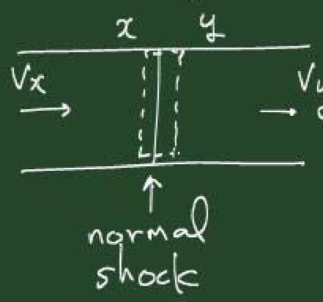
• Converging - diverging nozzles



⑤ Normal shock waves



Governing relations of the normal shock



• energy eq.

$$h_x + \frac{V_x^2}{2} = h_y + \frac{V_y^2}{2} = h_0 \quad -①$$

stag. enthalpy

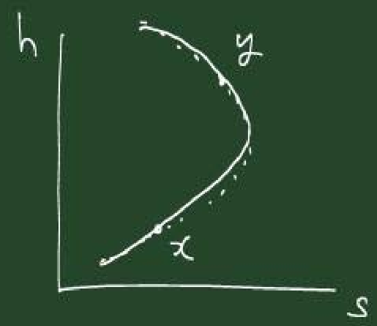
- cont. $\frac{w}{A} = \rho_x V_x = \rho_y V_y$ — ②
- mtm. $P_x - P_y = \frac{w}{A}(V_y - V_x)$ — ③

② & ③ → $P_x + \rho_x V_x^2 = P_y + \rho_y V_y^2$ — ④

cf. Bernoulli eq.

- eq. of state $h = h(s, \rho)$ — ⑤

$s = s(p, \rho)$



* Fanno line

1. Fix all conditions at x.
2. Choose a particular V_y.
3. Compute ρ_y from ②
4. " h_y " ①
5. " s_y " ⑤
6. repeat 2-5 to draw a line → Fanno line.

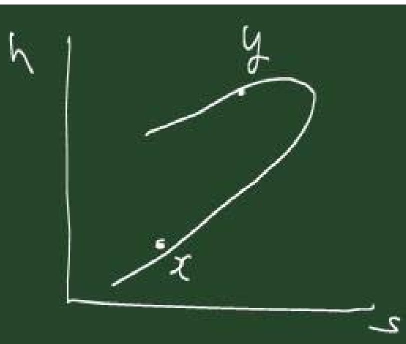
→ mtm eq. is not used.

→ Each pt. on a Fanno line can have each own mtn. flux.

→ Frictional effects are required to pass continously along the Fanno line from x to any state on the line.

- Rayleigh line

1. Fix all conditions at x.
2. Choose a particular V_y.
3. Compute ρ_y from ②
4. " P_y " ④
5. " s_y " ⑤
6. repeat 2–5 to draw a line → Rayleigh line

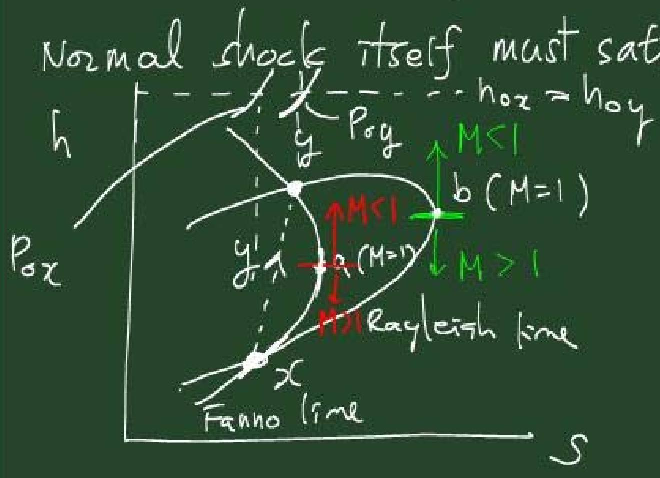


→ energy eq. is not used.

→ each pt. on a Rayleigh line has diff. stag. enthalpy.

→ States on the Rayleigh line are reachable from each other by continuous changes along the Rayleigh line only through heat transfer effects.

Normal shock itself must satisfy ①–⑤.



direction of shock wave : s should increase.

point a? ✓ $dh + VdV = 0$

$\underline{ds = 0}$ ✓ $d(\rho V) = 0 \rightarrow \rho dV + V d\rho = 0$

✓ $Tds = dh - \frac{1}{\rho} dp = 0$

$\rightarrow V = \sqrt{\left.\frac{\partial p}{\partial \rho}\right|_s}$ ✓

$M=1$ @ pt a,

Similarly, $M=1$ @ pt b.

@ pt x, $ds>0$ + ... $\rightarrow M>1$

pt y, $ds<0$ + ... $\rightarrow M<1$