Ch. 6 Scales of turbulent motion

노트 제목 2010-04-15

6.1 Energy cascade and Kolmogorov hypotheses

High Re number, $Re = \mathcal{U}\mathcal{L}/\nu$ $\quad \mathcal{U}$: char. vel. scale
$\mathcal{L}$: " length "

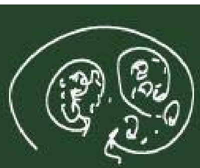
- Energy cascade

Richardson

turbulence is composed of eddies of different sizes.

$u(\ell)$ $\quad \tau(\ell) = \ell / u(\ell)$: eddy turn-over time

eddy $\quad$ large eddy contains small eddies



Eddies in the largest size : $\quad \ell_0 \sim \mathcal{L}$ (comparable to the flow scale)

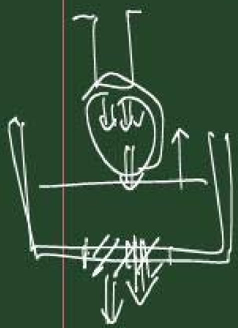
$u_0 = u(\ell_0) \sim u' \equiv \left(\frac{2}{3}k\right)^{\frac{1}{2}} \sim \mathcal{U}$ $\quad k = \frac{1}{2}(u^2+v^2+w^2) \doteqdot \frac{3}{2}u^2$

$Re_0 = u_0 \ell_0 / \nu \sim Re \gg 1$

$\therefore$ direct effect of viscosity is negligibly small.

mean flow $\Rightarrow$ (large eddies) $\xrightarrow{\text{transfer energy}}$ smaller eddies $\longrightarrow$ smaller eddies $\cdots\cdots \longrightarrow$ eddies are so small and stable

unstable break up



energy cascade → ↑ viscosity is effective

⇓ ↓

Dissipation is determined ⇐ dissipation
by the transfer of energy from the
largest eddies

→ largest eddies have energy of u_0^2
timescale of $\tau_0 = l_0/u_0$

$\therefore$ rate of energy transfer $\sim u_0^2/\tau_0 = u_0^3/l_0$

$\therefore$ $\varepsilon \sim u_0^3/l_0 \Rightarrow \varepsilon$ is indep. of ν.

⊙ Kolmogorov hypothesis

What is the size of the smallest eddies that are responsible for dissipating the energy?

As l decreases, do $u(l)$ and $\tau(l)$ increase, decrease or remain the same?

⇒ Kolmogorov (1941)'s hypotheses.
" theory

→ $u(l)$ and $\tau(l)$ decrease as l decreases.

- Kolmogorov hypothesis of <u>local isotropy</u>

At sufficiently high Re #, the small-scale turbulent motions ($l \ll l_0$) are statistically isotropic

$l_{EI} \simeq \frac{1}{6} l_0$

$l > l_{EI}$: anisotropic large eddies

$l < l_{EI}$: isotropic small eddies

- Kolmogorov's first similarity hypothesis

In every turb. flow at sufficiently high Re #, the statistics of small-scale motions ($l < l_{EI}$) have a universal form that is uniquely determined by ν and ϵ.

$l < l_{EI}$: universal equilibrium range

time scale $l/u(l) < l_0/u_0$

→ small eddies adapt quickly to maintain a dynamic equilibrium with energy transfer from large eddies.

- ε and ν $\rightarrow$ Kolmogorov scales

$$\eta = (\nu^3/\varepsilon)^{\frac{1}{4}}, \quad u_\eta = (\varepsilon\nu)^{\frac{1}{4}}, \quad \tau_\eta = (\nu/\varepsilon)^{\frac{1}{2}}$$

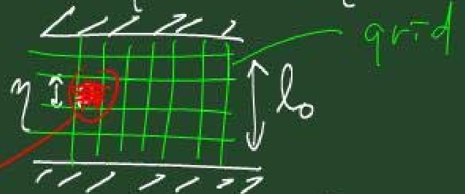
$\rightarrow \frac{u_\eta \eta}{\nu} = 1 \Rightarrow$ very dissipative eddies

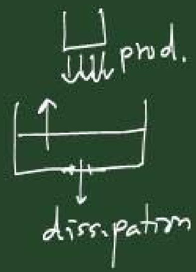
- On the small scales, all high-Re-number turbulent velocity fields are statistically similar; i.e., they are statistically identical when they are scaled by the Kolmogorov scales.
- $\varepsilon \sim u_0^3 / l_0$

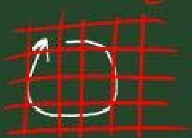
$$\eta / l_0 \sim Re^{-\frac{3}{4}}, \quad u_\eta / u_0 \sim Re^{-\frac{1}{4}}, \quad \tau_\eta / \tau_0 \sim Re^{-\frac{1}{2}}$$

$\rightarrow$ At high Re #, $u_\eta \ll u_0$, $\tau_\eta \ll \tau_0$.

$\eta / l_0 \sim Re^{-\frac{3}{4}}$







$$\Delta \lesssim \eta \simeq l_0 Re^{-\frac{3}{4}}$$

$$N = \frac{l_0}{\Delta} \gtrsim \frac{l_0}{\eta} \sim Re^{\frac{3}{4}}$$

$N \uparrow$ number of grid pts.

3D : $N^3 \sim \underline{\underline{Re^{\frac{9}{4}}}}$ (very big!)

Car: $\updownarrow$ $\left.\begin{array}{l} l \sim 1.5\,m \\ V \sim 30\,m/s \end{array}\right]$ $Re = \dfrac{30 \times \cancel{1.5}}{\cancel{1.5} \times 10^{-5}} = 3 \times 10^{6}$

10^{6} $9 \| 2$ 15

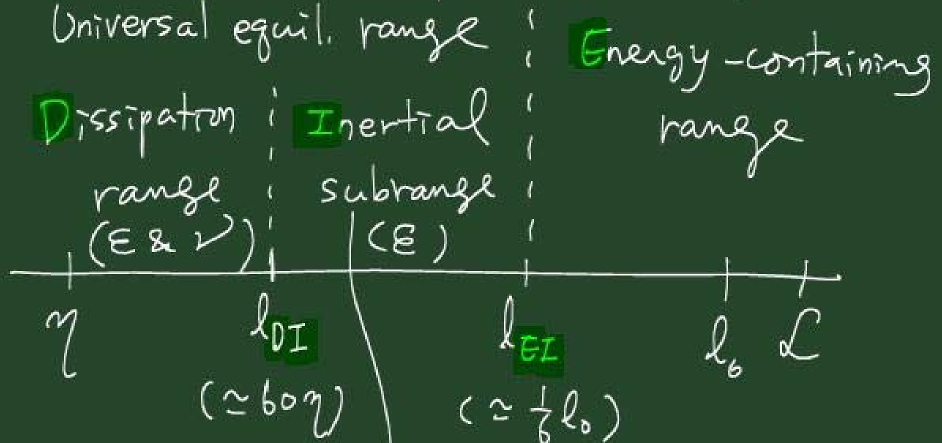
$N^{3} \sim Re^{\frac{9}{4}} = (3 \times 10^{6})^{\frac{9}{4}} = 3^{\frac{9}{4}} \times 10^{\frac{27}{2}}$

$= 10^{\frac{29}{2}} = 10^{15}$ $\simeq 10$ very big!

- Kolmogorov's second similarity hypothesis

In every turb. flow at sufficiently high Re #, the statistics of the motions of scale l in the range $l_0 \gg l \gg \eta$ have a universal form that is uniquely determined by ε, indep. of ν.

(because the eddies in this range are much bigger than the dissipative eddies)



motions are determined by inertial effects (viscous effects are negligible).