노트 제목 2010-04-27

- 2-pt. corr. $R_{ij}(\underline{r}, t) = \langle u_i(\underline{x}, t)\, u_j(\underline{x}+\underline{r}, t)\rangle$

$$R_{ij}(\underline{r}) = \iiint_{-\infty}^{\infty} \phi_{ij}(\underline{k})\, e^{i\underline{k}\cdot\underline{r}}\, d\underline{k}$$

$$\phi_{ij}(\underline{k}) = \hat{u}_i^*(\underline{k})\, \hat{u}_j(\underline{k})$$

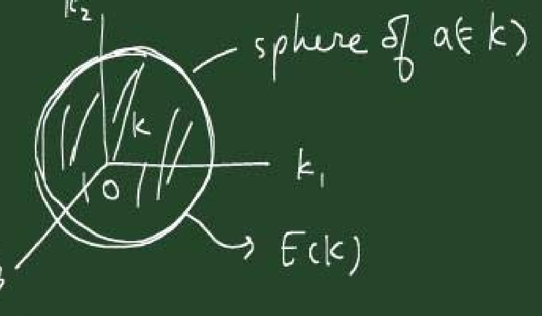
$$\phi_{ij}(\underline{k}) = \frac{1}{(2\pi)^3} \iiint_{-\infty}^{\infty} R_{ij}(\underline{r})\, e^{-i\underline{k}\cdot\underline{r}}\, d\underline{r}$$

$$\varepsilon = \iiint_{-\infty}^{\infty} 2\nu k^2 \frac{1}{2}\phi_{ii}(\underline{k})\, d\underline{k}$$

integration over the surface of the sphere

Energy spectrum ft. $E(k) \equiv \oint \frac{1}{2}\phi_{ii}(\underline{k})\, dS(k)$

$S(k)$ : sphere in wavenumber space ($k = |\underline{k}|$)




$$R_{ij}(0) = \langle u_i u_j\rangle = \iiint_{-\infty}^{\infty} \phi_{ij}(\underline{k})\, d\underline{k}$$

turbulent kinetic energy $k = \frac{1}{2}\langle u_i u_i\rangle = \int_0^{\infty} E(k)\, dk$

dissipation $\varepsilon = \int_0^{\infty} 2\nu k^2 E(k)\, dk$

- In isotropic turbulence, $\phi_{ij}$ can depend only on $\underline{k}$.

$$\phi_{ij}(\underline{k}) = A(k)\delta_{ij} + B(k)k_i k_j$$

incompressibility $\updownarrow$ $k_i\phi_{ij} = 0$

$$k_i\phi_{ij} = Ak_j + Bk^2k_j = 0$$

$$\rightarrow B = -A/k^2$$

$4\pi k^2$

$$E(k) = \oint \tfrac{1}{2}\phi_{ii}(\underline{k})\,dS(\underline{k})$$

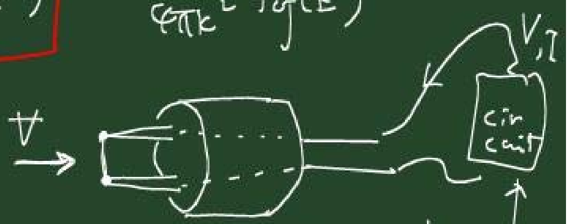
$$= \tfrac{1}{2}(3A + Bk^2)\cdot 4\pi k^2 = 6A\pi k^2 + 2B\pi k^4$$

$$= 4\pi k^2 A$$

$$\rightarrow A = \frac{E(k)}{4\pi k^2}$$

$$\boxed{\phi_{ij}(\underline{k}) = \frac{E(k)}{4\pi k^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)} = \frac{E(k)}{4\pi k^2}P_{ij}(\underline{k})$$

- Taylor's hypothesis

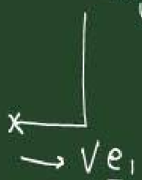



Hot-wire probe

Hot-wire anemometer

How to get $\phi_{ij}(\underline{k})$ ?

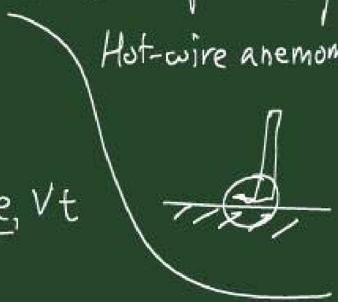
flying hot wire moving at V




probe position $\underline{X}(t) = \underline{x}_0 + \underline{e}_1 Vt$

velocity measured

$$\underline{U}^{(m)}(t) = \underline{U}(\underline{X}(t), t) - \underline{e}_1 V$$



Then, the temporal correlation

$$R_{ij}^{(M)}(s) = \left\langle \left(U_i^{(M)}(t) - \langle U_i^{(M)}(t)\rangle\right)\left(U_j^{(M)}(t+s) - \langle U_j^{(M)}(t+s)\rangle\right)\right\rangle$$

$$= \langle u_i(\underline{X}(t), t)\, u_j(\underline{X}(t+s), t+s)\rangle$$

$$= \langle u_i(\underline{X}(t), t)\, u_j(\underline{X}(t) + \underline{e}_1 r_1,\ t + r_1/V)\rangle$$

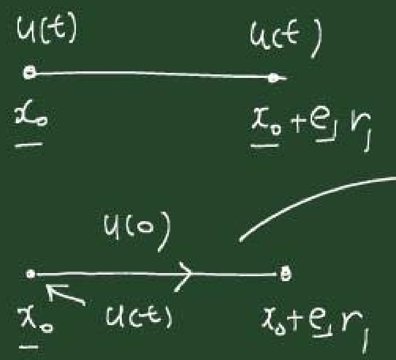
where $r_1 = Vs$.

If turbulence is homogeneous,

$$R_{ij}^{(M)}(s) = \langle u_i(\underline{x}_0 + \underline{e}_1 Vt, \overset{t}{\cancel{0}})\, u_j(\underline{x}_0 + \underline{e}_1 Vt + \underline{e}_1 r_1, \overset{t}{\cancel{0}})\rangle$$

$$= \langle u_i(\underline{x}_0, 0)\, u_j(\underline{x}_0 + \underline{e}_1 r_1, 0)\rangle$$

$$= R_{ij}(\underline{e}_1 r_1, \underline{x}_0, 0)$$ : Taylor hypothesis




$\rightarrow R_{uu}(\underline{e}_1 r_1)$

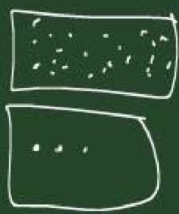
turbulence is frozen assumption

$$\frac{\partial}{\partial t}(\cdot) = -V\frac{\partial}{\partial x_1}(\cdot)$$

In grid turbulence with $u'/\langle U_1\rangle \ll 1$,
Taylor hypothesis is accurate.
In free shear flows, " " inaccurate.

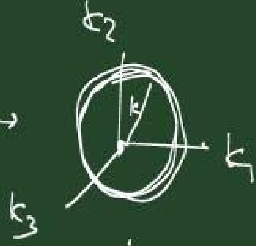
$\underline{R_{ij}(\underline{r})}$ → PIV →

LDV →



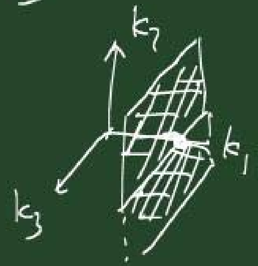
→ numerical simulation (DNS, LES)

• 3-D energy spectrum $E(k)$ →



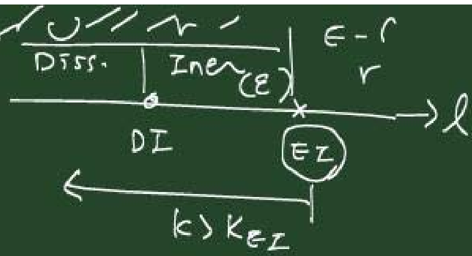

1-D " " $E_{ij}(k_1)$

$\Uparrow$




$E(k=0) = 0$ $\quad$ $\underline{E_{ij}(k_1=0) \neq 0}$

• Kolmogorov spectra $\quad \varepsilon$ & $\nu$




For $k > k_{EI}$, $E(k)$ is a universal ft. of $k$, $\varepsilon$ & $\nu$.

→ $E(k) = (\varepsilon \nu^5)^{\frac{1}{4}} \underbrace{\varphi(k\eta)}$ from dimensional analysis.

↑ a universal non-dimensional function : Kolmogorov spectrum ft.

In the inertial subrange,

$\boxed{E(k) = C \varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}}}$ : Kolmogorov $-\frac{5}{3}$ spectrum

$E_{11}(k_1) = C_1 \varepsilon^{\frac{2}{3}} k_1^{-\frac{5}{3}}$ $\qquad C_1 = 0.49, \; C = 1.5$

- Model spectrum

$$E(k) = C\varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}} f_L(kL)\, f_\eta(k\eta)$$

$f_\eta$: shape of dissipation range; tends to unity for small $kL$.

$f_L$: shape of energy-containing range; tends to unity for large $kL$.

In the inertial subrange, $f_L$ and $f_\eta$ are unity.

$$f_L(kL) = \left[ kL / \{(kL)^2 + c_L\}^{\frac{1}{2}} \right]^{\frac{5}{3}+p_0}$$

$p_0 = 4$ : von Karman spectrum, $E(k) \sim k^4$ for small $k$.

$$f_\eta(k\eta) = \exp\left\{ -\beta \left[ \{(k\eta)^4 + c_\eta^4\}^{\frac{1}{4}} - c_\eta \right] \right\}$$

$\beta = 5.2$, $c_L = 6.78$, $c_\eta = 0.40$