y-mtm
x-mtm

노트 제목 2010-05-04

$\tau(y) = \tau_w \left(1 - \frac{y}{\delta}\right)$ $\quad \tau = \mu \frac{d\langle U\rangle}{dy} - \rho\langle uv\rangle$

@ $y=0$, $\tau = \tau_w$: wall shear stress

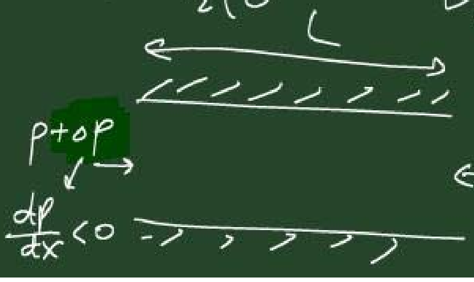
@ $y=\delta$, $\tau = 0$

$c_f = \frac{\tau_w}{\frac{1}{2}\rho U_0^2}$, $\quad C_f = \frac{\tau_w}{\frac{1}{2}\rho \bar{U}^2}$

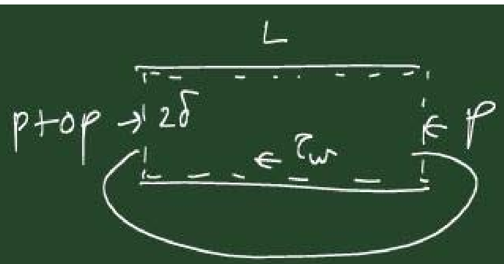
$\bar{U}$: bulk vel.
U_0 : center line vel.

$Re = (2\delta)\bar{U}/\nu$

$Re_0 = U_0\delta/\nu$



$-\frac{dp_w}{dx} = \frac{\tau_w}{\delta}$



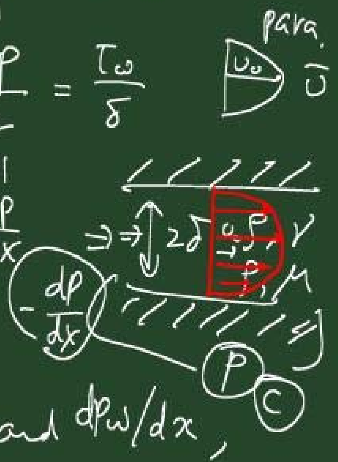
$\Sigma F_x = (p+\Delta p)\,2\delta - p\,2\delta - \tau_w \cdot L \cdot 2 = 0$

$-(2\delta)\Delta p = \tau_w \cdot L \cdot 2$

$\rightarrow \frac{\Delta p}{L} = \frac{\tau_w}{\delta}$

$-\frac{dp_w}{dx} \simeq \frac{\Delta p}{L}$ ‖ $\frac{dp}{dx}$ ← y-mtm eq.

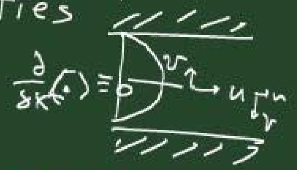
$\bar{U} = \frac{1}{\delta}\int_0^\delta \langle U\rangle dy$



In turbulent flow,
if flow is defined by ρ, ν, δ and dp_w/dx,
U_0 and $\bar{U}$ are not known a priori.

if $\bar{U}$ is imposed, dP_w/dx is again unknown a priori.

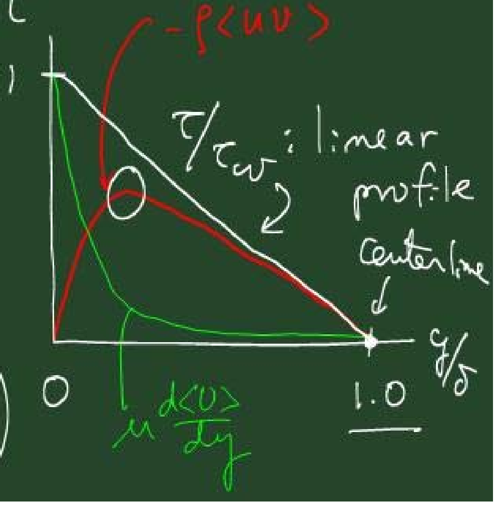
For laminar flow, all of these qunatities are determined.



- Near-wall shear stress

$$\tau = \mu \frac{d\langle U \rangle}{dy} - \rho \langle uv \rangle$$

viscous stress (~~laminar stress~~) — Reynolds stress



$$\tau_w = \mu \frac{d\langle U \rangle}{dy}\Big|_{y=0} \quad \begin{pmatrix} u = v = 0 \\ @\ y = 0 \end{pmatrix}$$

close to the wall, ν and τ_w are important parameters.

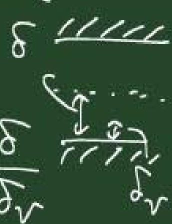
✓ viscous scales: vel. and length scales in the near-wall region.

friction velocity (wall shear velocity) $\quad u_\tau \equiv \sqrt{\tau_w/\rho}$

viscous length scale $\quad \delta_\nu \equiv \nu\sqrt{\frac{\rho}{\tau_w}} = \frac{\nu}{u_\tau}$

$$\left(\rightarrow \frac{u_\tau \delta_\nu}{\nu} = 1 \right)$$

Friction Reynolds number $\quad Re_\tau \equiv \frac{u_\tau \delta}{\nu} = \frac{\delta}{\delta_\nu}$



$$Re = 5600 \rightarrow Re_\tau = 180$$
$$13{,}750 \qquad\qquad 395$$

$$y^+ \equiv \frac{y u_\tau}{\nu} \quad \text{(wall units)}$$

$$u^+ \equiv \frac{\langle U \rangle}{u_\tau}$$

viscous wall region ($y^+ < 50$): direct effect of molecular viscosity on the shear stress

outer layer ($y^+ > 50$): direct effect of viscosity is negligible

viscous sublayer ($y^+ < 5$): Reynolds shear stress is negligible.

$$y^+ \equiv \frac{y u_\tau}{\nu}$$

$\underrightarrow{U_0 = 5\,m/s}$ air, $2\delta = 90\,cm \rightarrow \delta = 45\,cm = 0.45\,m$

$$Re_0 = \frac{U_0 \delta}{\nu} = \frac{5 \times \overset{0.3}{\cancel{0.45}}}{1.5\times10^{-5}} = \underline{1.5\times10^{5}}$$

$$\frac{U_0}{u_\tau} \simeq 20 \longrightarrow u_\tau = \frac{5}{20} = 0.25\ m/s$$

$$y^+ = 50 = \frac{y u_\tau}{\nu} \rightarrow y = \frac{y^+ \nu}{u_\tau} = \frac{y^+ \cdot 1.5\times10^{-5}}{0.25}$$

for $y^+ = 50$, $y = \frac{\overset{200}{\cancel{50}} \times 1.5\times10^{-5}}{\cancel{0.25}} = 3\times10^{-3}\,m = 3\,mm$

for $y^+ = 5$, $y = 0.3\,mm$. $\quad \because \frac{\delta_\nu}{\delta} \sim Re_\tau^{-1}$

As Re increases, fraction of viscous wall region decreases

- Mean velocity profiles.

Channel flow is completely specified by

ρ, ν, δ and dp_w/dx

or ρ, ν, δ and $u_\tau = \sqrt{\tau_w/\rho} = \left(-\frac{\delta}{\rho}\frac{dp_w}{dx}\right)^{\frac{1}{2}}$

$$\rightarrow \quad \frac{\langle U \rangle}{u_\tau} = u^+ = F_0\left(\frac{y}{\delta}, Re_\tau\right)$$

universal non-dimensional ft.

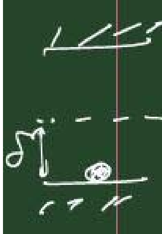
Let us consider $d\langle U \rangle / dy$

$$\frac{d\langle U \rangle}{dy} \Big/ \frac{u_\tau}{y} = \Phi\left(\frac{y}{\delta_\nu}, \frac{y}{\delta}\right)$$

$\Phi\left(\frac{y}{\delta}, Re_\tau\right)$

- Law of the wall

Prandtl (1925): at high Re, close to the wall ($y/\delta \ll 1$),



there is an inner layer in which $\langle U \rangle$ is determined by the viscous scales.

$$\rightarrow \quad \frac{d\langle U \rangle}{dy} \Big/ \frac{u_\tau}{y} = \Phi_1\left(\frac{y}{\delta_\nu}\right) \text{ for } \frac{y}{\delta} \ll 1.$$

$$\rightarrow \quad \frac{du^+}{dy^+} = \frac{1}{y^+}\Phi_1(y^+)$$

$$\xrightarrow{\text{integration}} \quad \boxed{u^+ = f_w(y^+)} \quad \text{law of the wall}$$

$u^+ = \frac{\langle U \rangle}{u_\tau}$

univ. ft.

• Viscous sublayer ($y^+ < 5$) → ~~laminar~~ sublayer

at wall, $\langle U \rangle = 0$ → $f_w(0) = 0$

$\tau_w = \mu \frac{d\langle U \rangle}{dy}\Big|_{y=0}$ → $f_w'(0) = 1$

⇒ $f_w(y^+) = \underbrace{f_w(0)}_{0} + \underbrace{y^+}_{f_w'(0) = 1} + O(y^{+2})$

$u(y) = u(0) + y \frac{du}{dy}\Big|_0 + \frac{1}{2} y^2 \frac{d^2u}{dy^2}\Big|_0 + \cdots$

⇒ $\boxed{u^+ = y^+ + O(y^{+2})}$ for $\underline{y^+ < 5}$.

• Log law

At high Re, outer part of $\underbrace{\text{inner layer}}_{\frac{y}{\delta} \ll 1}$

→ large y^+

i.e. $y^+ \simeq 0.1 \frac{\delta}{\delta_\nu} = 0.1\, Re_\tau \gg 1$

→ viscosity has little effect.

→ $\Phi_1\left(\frac{y}{\delta_\nu}\right)$ does not depend on ν.

→ Φ_1 is const.

→ $\Phi_1 = \frac{1}{\kappa}$ for $\frac{y}{\delta} \ll 1$ and $y^+ \gg 1$

→ $\frac{du^+}{dy^+} = \frac{1}{\kappa y^+}$

→ $\boxed{u^+ = \frac{1}{\kappa}\ln y^+ + B}$

log law

logarithmic law of the wall

von Karman (1930)

κ: von Karman constant

$\left(\begin{array}{l} \kappa = 0.41 \\ B = 5.2 \end{array} \right.$ $\left(\begin{array}{l} \kappa = 0.41 \\ B = 5.0 \end{array} \right.$ $\left(\begin{array}{l} \kappa = 0.4 \\ B = 5.5 \end{array} \right.$