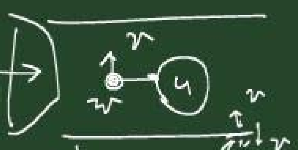
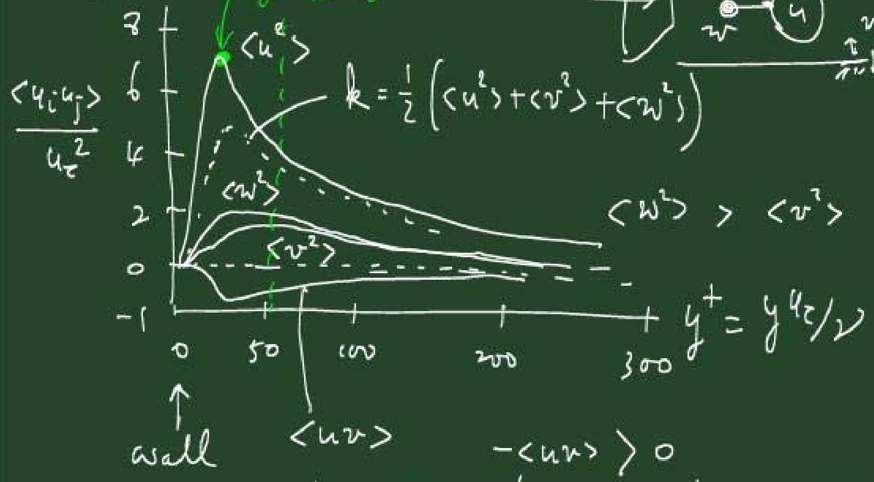
노트 제목 2010-05-11

- Reynolds stresses



$\langle v \rangle$
$\langle w \rangle = 0$



→ viscous wall region ($y^+ < 50$) contains vigorous turbulent activity.

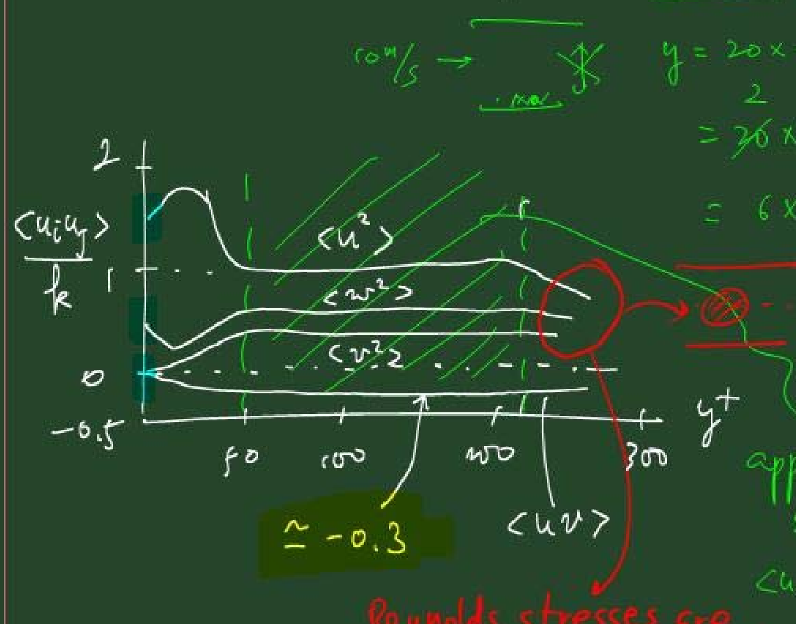
$y^+ = 20 = y \cdot \frac{u_\tau}{\nu} = y \frac{u_\tau}{u_\infty} \cdot \frac{u_\infty}{\nu}$

$y = 20 \times \frac{u_\infty}{u_\tau} \cdot \frac{\nu}{u_\infty}$

$= \cancel{20}\,2 \times 20 \times \frac{1.5\times10^{-5}}{\cancel{10}}$

$= 6\times10^{-4}\,\mathrm{m} = 0.6\,\mathrm{mm}$

very near the wall!



approximate self-similarity

$\langle u_i u_j \rangle / k$ are uniform.

Reynolds stresses are anisotropic even on the centerline.

very near the wall,

$$u = \cancel{u_w} + b_1 y + b_2 y^2 + \cdots = b_1 y + \cdots$$

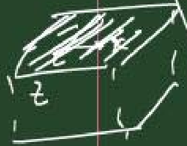
$$c_1 = \frac{\partial v}{\partial y}\Big|_w \qquad v = \cancel{v_w} + \cancel{c_1 y} + c_2 y^2 + \cdots = c_2 y^2 + \cdots$$

$$w = \cancel{w_w} + d_1 y + d_2 y^2 + \cdots = d_1 y + \cdots$$

very close to the wall, $v \simeq 0$

$\downarrow$

$u,\ w \neq 0$



$$\frac{\partial v}{\partial y}\Big|_w = -\frac{\partial u}{\partial x}\Big|_w - \frac{\partial w}{\partial z}\Big|_w = 0$$

(continuity)

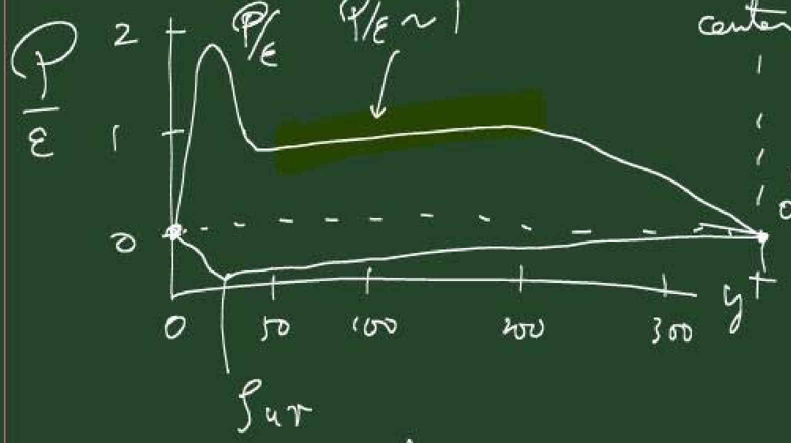
∴ two-component flow exists very close to the wall

→ wall-parallel flow

$$\Rightarrow \langle u^2\rangle \sim y^2,\ \langle v^2\rangle \sim y^4,$$

$$\langle w^2\rangle \sim y^2,\ -\langle uv\rangle \sim y^3$$

$$k \sim y^2$$



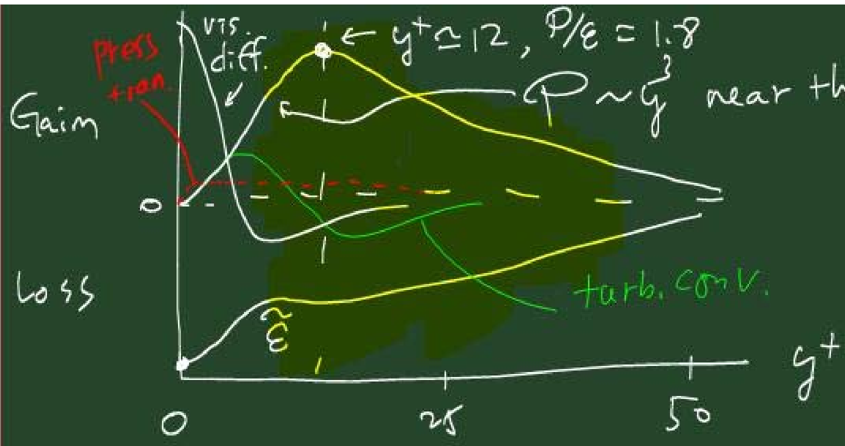
$$P = -\overline{uv}\,\frac{dU}{dy}$$

$\simeq 0 \quad \simeq 0$

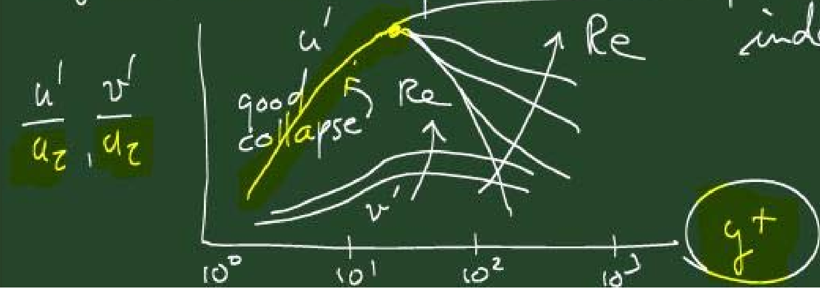
- Balance eq. for k

$$0 = P - \tilde{\varepsilon} + \nu\frac{d^2 k}{dy^2} - \frac{d}{dy}\left\langle \frac{1}{2} v u_j u_j \right\rangle - \frac{1}{\rho}\frac{d}{dy}\langle v p' \rangle$$

production, pseudo-dissipation, viscous diffusion, turb. convection, press transport



• Reynolds number dependence — peak location is indep. of Re.



• Length scales and mixing length

In the log-law region,

• $S = \frac{d\langle U\rangle}{dy} = \frac{u_\tau}{\kappa y}$ or $\frac{du^+}{dy^+} = \frac{1}{\kappa y^+}$

• $\mathcal{P}/\varepsilon \simeq 1$

• $-\langle uv\rangle/k \simeq 0.3$

• $\frac{Sk}{\varepsilon} = \frac{\mathcal{P}}{-\langle uv\rangle}\cdot\frac{k}{\varepsilon} = \frac{\mathcal{P}}{\varepsilon}\cdot\frac{k}{-\langle uv\rangle} \simeq 1\cdot\frac{1}{0.3} \simeq 3$

turb. lengthscale $L \equiv k^{3/2}/\varepsilon$

$$\Rightarrow L = \kappa y \frac{|\langle uv\rangle|^{\frac{1}{2}}}{u_\tau}\cdot\frac{\mathcal{P}}{\varepsilon}\cdot\left|\frac{\langle uv\rangle}{k}\right|^{-\frac{3}{2}} = C_L y$$

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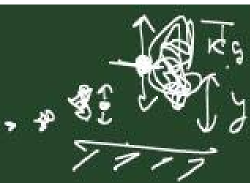


• Mixing length

$$-\langle uv\rangle = \nu_T \frac{d\langle U\rangle}{dy}$$

$$P = -\langle uv\rangle \frac{d\langle U\rangle}{dy}$$

$$\nu_T = u^* l_m \quad \text{(vel. scale} \times \text{length. scale)}$$

$$u^* = \sqrt{-\langle uv\rangle}$$

$$u^{*2} = u^* l_m \frac{d\langle U\rangle}{dy} \rightarrow u^* = l_m \frac{d\langle U\rangle}{dy}$$

At high Re., $-\langle uv\rangle \sim u_\tau^2$ in the overlap region.

$$\frac{d\langle U\rangle}{dy} = \frac{u_\tau}{\kappa y} = \frac{u^*}{l_m} = \frac{\sqrt{-\langle uv\rangle}}{l_m} = \frac{u_\tau}{l_m}$$

$$\therefore u_\tau = l_m \frac{u_\tau}{\kappa y} \Rightarrow \boxed{l_m = \kappa y}$$

varies linearly with y.

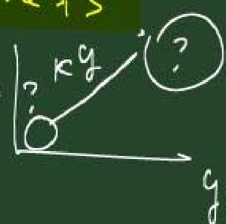
$$\Rightarrow \boxed{\nu_T = u^* l_m = l_m^2 \frac{d\langle U\rangle}{dy}}$$

: Prandtl's mixing-length hypothesis

l_m : mixing length

in overlap region, $l_m \sim \kappa y$

other region → §7.3



$u^* \sim \sqrt{-\langle uv\rangle}$

$u^* \sim k^{\frac{1}{2}} \rightarrow$ k-eq

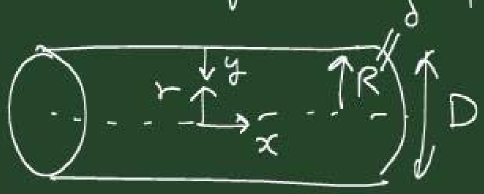
$l \sim k \,\&\, \varepsilon \rightarrow \left(\begin{array}{l} k\text{-eq} \\ \varepsilon\text{-eq} \end{array} \right.$

$u^* = u_i u_j \rightarrow$ RSM

$$(\nu + \nu_T)\frac{d\langle U\rangle}{dy}$$

7.2 Pipe flow

- Friction law for smooth pipes



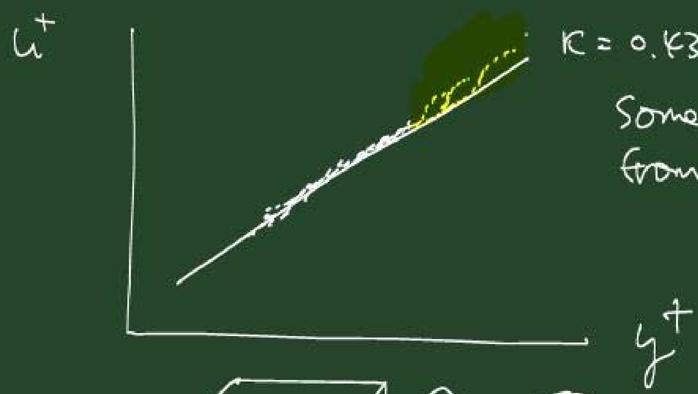
mean centerline vel.

$U_0 \equiv \langle U(x, r=0, \theta) \rangle$

$y = R$

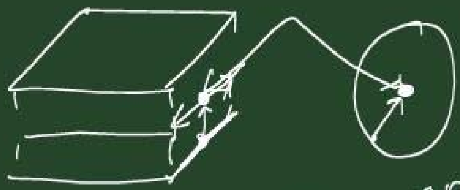
bulk vel. $\bar{U} = \frac{1}{\pi R^2} \int_0^R \langle U \rangle \cdot 2\pi r\,dr$

$Re = \frac{\bar{U} D}{\nu} = \frac{2 \bar{U} \delta}{\nu}$ bulk Reynolds number



$\kappa = 0.436$, $B = 6.13$

Somewhat different from those of channel flow



Friction factor $f \equiv \frac{\Delta p \cdot D}{\frac{1}{2} \rho \bar{U}^2 \mathcal{L}}$

Δp : the press. drop over an axial distance $\mathcal{L}$.

Use log law to get friction law

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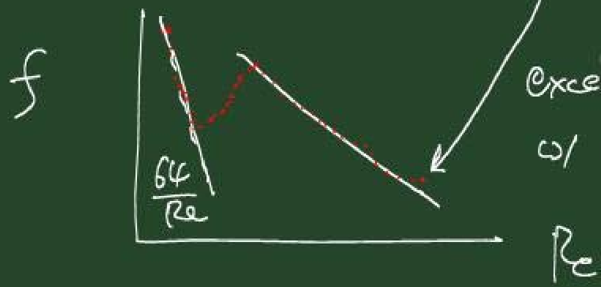
$$\rightarrow \frac{1}{\sqrt{f}} = 1.99 \log_{10}(Re\sqrt{f}) - 0.95$$

$$\rightarrow \frac{1}{\sqrt{f}} = 2.0 \log_{10}(\sqrt{f}\,Re) - 0.8$$: Prandtl's friction law for smooth pipe



excellent agreement w/ exp. data.