



Linear System Theory and Design

Text : Linear System Theory and Design,
Chi-Tsong Chen,
Oxford, 3rd Ed.

Evaluation: Homework 30%, Midterm 30%, Final 30%
Attitude 10%,
F for 3 times absence w/o pre-notice



1. Introduction

- ✓ Definition of System
- ✓ Examples of System
- ✓ Linear and Nonlinear System
- ✓ Why study Linear System
- ✓ System Design Procedure
- ✓ Related Courses
- ✓ Scope of the Lecture

Definition of System

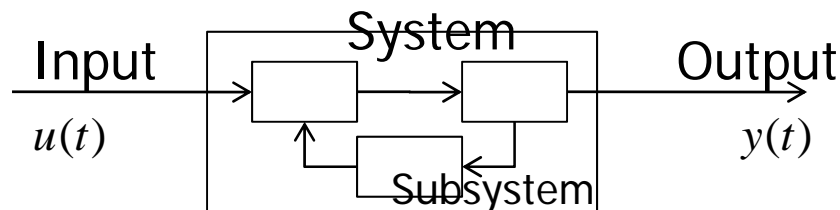
Definition(Webster): **System** is defined as a collection of objects united by some form of interaction or interdependence.

Static System

$$y(t) = f(u(t))$$

Dynamic System (RLC, Spring-Damper system)

$$y^{(n)}(t) = f(y^{(n)}(t), y^{(n-1)}(t), \dots, y(t), u^{(n)}(t), u^{(n-1)}(t), \dots, u(t))$$





Example of System

- ✓ Communication systems
- ✓ Circuits & Systems
- ✓ Control systems
- ✓ Power systems
- ✓ Automotive systems
- ✓ Surveillance systems
- ✓ Human (neural systems)
- ✓ Ocular motor systems
- ✓ Economic systems
- ✓ Biological systems
- ✓ Chaotic systems
- ✓ Etc. ...



Linear and Nonlinear Systems

- ✓ Linear systems

- Satisfy “Principle of Superposition”

Additivity: $f(u_1 + u_2) = f(u_1) + f(u_2)$

Homogeneity: $f(\alpha u) = \alpha f(u)$

- Category: stable, unstable systems

- ✓ Nonlinear Systems

- Does not Satisfy “Principle of Superposition”
- Category: stable, unstable, periodic, chaotic systems
- Most real world systems

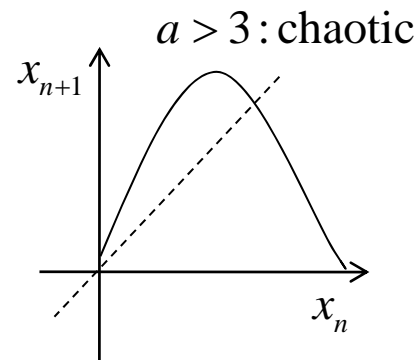
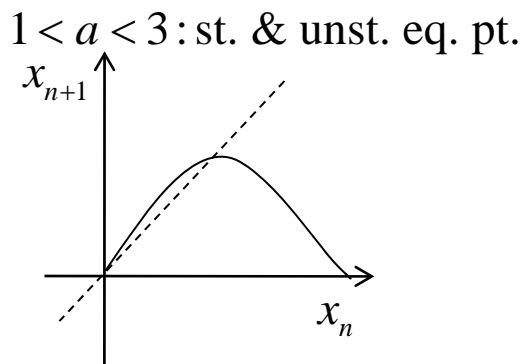
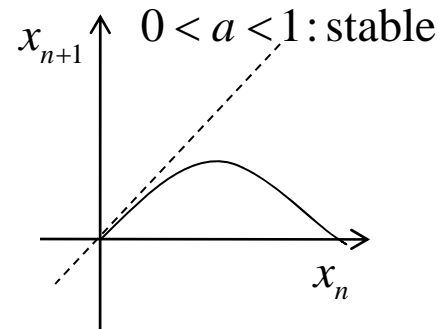
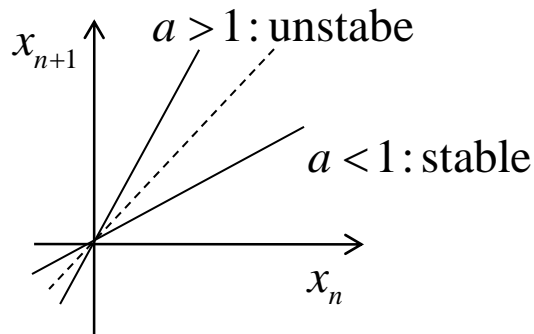
Pendulum: $\ddot{x} + \sigma \dot{x} + g \sin x = a + b \cos \omega t$

Logistic map (Ecosystem): $x_{n+1} = ax_n(1 - x_n)$

Linear and Nonlinear Systems

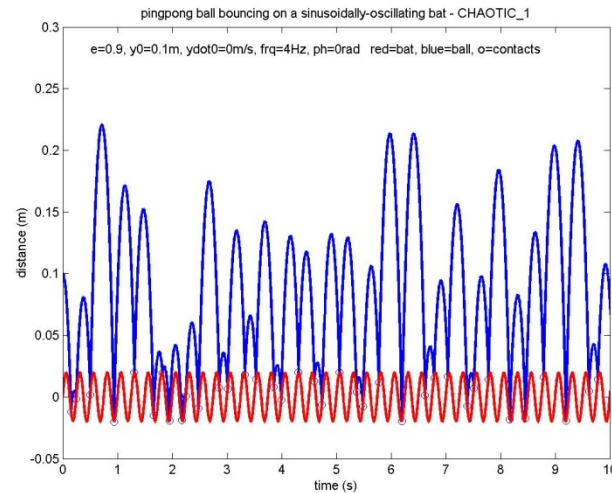
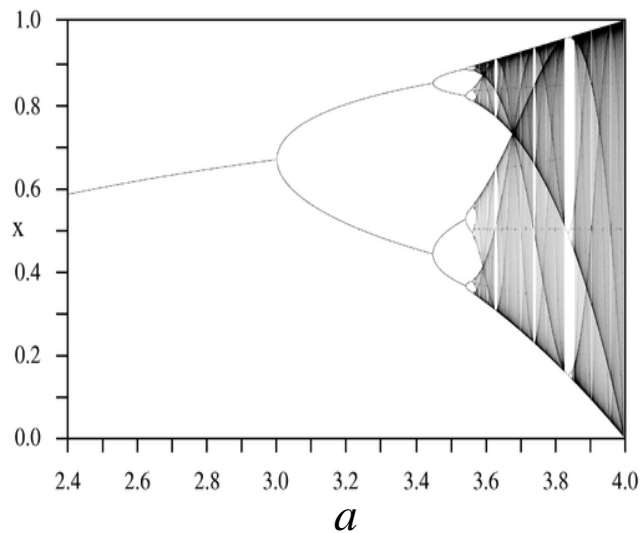
Ecosystem: $x_{n+1} = ax_n(1 - x_n)$

Linear system: $x_{n+1} = ax_n$



Linear and Nonlinear Systems

Ecosystem: $x_{n+1} = ax_n(1 - x_n)$





Why study Linear Systems

- ✓ Many physical systems can be modelled by linear systems over their normal operating ranges
- ✓ The theory of linear systems is complete and well organized
- ✓ The theory is the basis for the study of nonlinear systems



(Control) System Design Procedure

- ✓ Modelling
- ✓ Setting up mathematical description
- ✓ Analysis
- ✓ Design



(Control) System Design Procedure

- ✓ Modelling
 - Determine the model order, structure, parameters ...
 - Nonlinear or linear systems

- ✓ Setting up mathematical description

- ✓ Analysis

- ✓ Design



(Control) System Design Procedure

- ✓ Modelling

- ✓ Setting up mathematical description
 - Difference or differential equations
 - Impulse functions (convolution integral equations)
 - Transfer functions
 - State space descriptions

- ✓ Analysis

- ✓ Design



(Control) System Design Procedure

- ✓ Modelling
- ✓ Setting up mathematical description

- ✓ Analysis
 - Quantitative analysis
 - Time response(rising time, settling time, overshoot, ss error ...)
 - Frequency response(bandwidth, phase ...)
 - Qualitative analysis
 - Stability, controllability, observability, stabilizability

- ✓ Design



(Control) System Design Procedure

- ✓ Modelling
- ✓ Setting up mathematical description
- ✓ Analysis
- ✓ Design
 - Improve the system response
 - Time response: rising/settling time, overshoot, ss error ...
 - Frequency response: bandwidth, phase ...
 - Improve the qualitative property
 - Stabilization, robust, gain margin, phase margin
 - Design techniques
 - State feedback, state estimator (observer, Kalman filter ...)
 - Compensator
 - Optimal/adaptive/nonlinear control



Scope of the Lecture

- ✓ Mathematical descriptions of systems (2 lectures)
- ✓ Linear operator theory for linear system analysis (4 lectures)
- ✓ State space solutions and realizations (2 lectures)
- ✓ Stability (3 lectures)
- ✓ Controllability and observability (4 lectures)
- ✓ Minimal realizations and coprime fractions (4 lectures)
- ✓ State feedback and state estimators (4 lectures)
- ✓ Pole placement and model matching (2 lectures)