Nonlinear Optical Engineering

Principles of Lasers (1)

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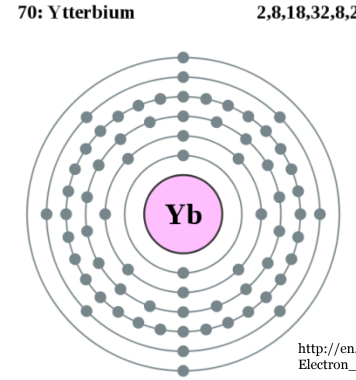
Spontaneous Transitions between Atomic Levels



http://en.wikipedia.org/wiki/File:Electron_shell_070_Ytterbium.svg

Atomic systems:

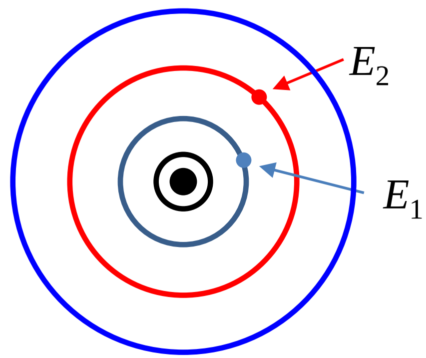
Predetermined set of energy states: Eigenstates

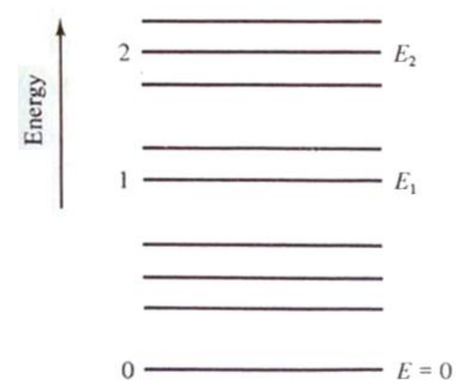
Spontaneous emission:

Emitting process of a photon of energy without the inducement of a radiation field: $h\nu = E_2 - E_1$

Spontaneous emission rate:

$$\frac{dN_2}{dt} = -A_{21}N_2 \equiv -\frac{N_2}{\tau_{21,sp}}$$



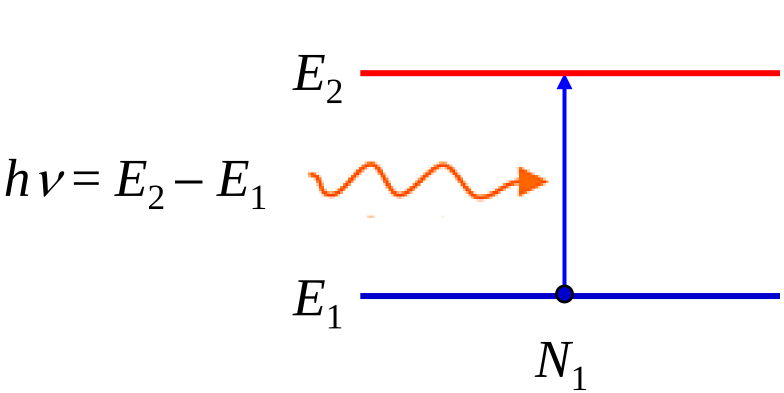


A. Yariv, *Optical Electronics*, 4th ed. Saunders, 1991.

Induced Transitions (1)

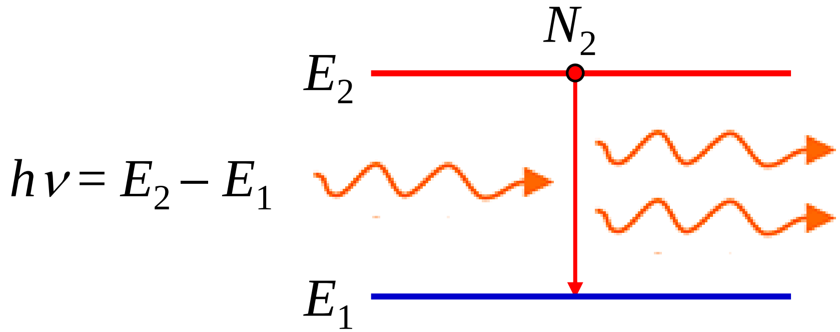
Induced transitions and their rates per atom:

Absorption



$$(W'_{12})_{induced} = B_{12}\rho(\nu)$$

Stimulated emission



$$(W'_{21})_{induced} = B_{21}\rho(\nu)$$

Energy density per unit frequency

Total downward (2→1) transition rate:

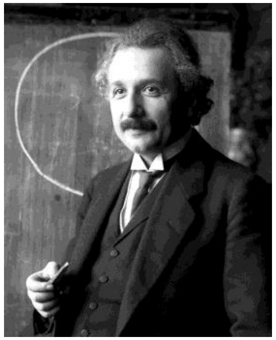
$$W'_{21} = B_{21}\rho(\nu) + A_{21}$$

Total upward (1→2) transition rate:

$$W'_{12} = (W'_{12})_{induced} = B_{12}\rho(\nu)$$

Induced Transitions (2)

Ludwig Boltzmann
(1844 – 1906)



Albert Einstein
(1879 – 1955)

Energy density with a blackbody radiation:

$$\rho(\nu) = \frac{8\pi n^3 h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

For thermal equilibrium:

$$N_2 W_{21}' = N_1 W_{12}' \rightarrow N_2[B_{21}\rho(\nu) + A_{21}] = N_1 B_{12}\rho(\nu)$$

Boltzmann distribution:
(with no degeneracy)

$$\frac{N_i}{N} = \frac{e^{-E_i/kT}}{\sum_j e^{-E_j/kT}} \rightarrow \frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} = e^{-h\nu/kT}$$

$$\rightarrow \frac{A_{21}}{B_{12}e^{h\nu/kT} - B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

Einstein's A and B coefficients:

$$\rightarrow B_{12} = B_{21} \quad \rightarrow \frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$$

$$\rightarrow W_i' = B_{12}\rho(\nu) = B_{21}\rho(\nu)$$

$$= \frac{A_{21}c^3}{8\pi n^3 h\nu^3}\rho(\nu) = \frac{c^3}{8\pi n^3 h\nu^3\tau_{sp}}\rho(\nu)$$

Induced Transitions (3)

"Atomic" lineshape function $g(\nu)$:

Spectral distribution of emitted intensity vs. frequency $\rightarrow \int_{-\infty}^{+\infty} g(\nu)d\nu = 1$

Monochromatic transition rate:

$$W_i(\nu) = \frac{c^3 \rho(\nu)}{8\pi n^3 h \nu^3 \tau_{sp}} g(\nu)$$

Total transition rate:

$$W_i' = \sum_{\nu_k} W_i(\nu_k) = \frac{c^3}{8\pi n^3 h \tau_{sp}} \sum_{\nu_k} \frac{\rho(\nu_k)}{\nu_k^3} g(\nu_k)$$

$$\rightarrow W_i' = \frac{c^3}{8\pi n^3 h \tau_{sp}} \int_{-\infty}^{+\infty} \frac{\rho(\nu)}{\nu^3} g(\nu) d\nu$$

$$\cong \frac{c^3 \rho(\nu)}{8\pi n^3 h \nu^3 \tau_{sp}} \int_{-\infty}^{+\infty} g(\nu) d\nu = \frac{c^3 \rho(\nu)}{8\pi n^3 h \nu^3 \tau_{sp}}$$

Transition rate in terms of the intensity:

$$W_i(\nu) = \frac{c^2 I_\nu}{8\pi n^2 h \nu^3 \tau_{sp}} g(\nu) = \frac{\lambda^2 I_\nu}{8\pi n^2 h \nu \tau_{sp}} g(\nu) \leftarrow I_\nu = \frac{c}{n} \rho(\nu)$$

Absorption and Amplification

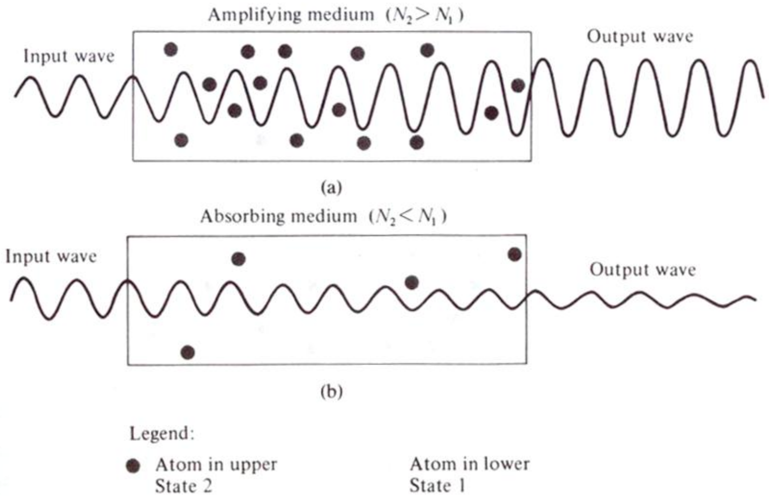
Net power generated per unit volume:

$$\frac{P}{Volume} = (N_2 - N_1) W_i h\nu$$

$$\rightarrow \frac{dI_\nu}{dz} = (N_2 - N_1) \frac{c^2 g(\nu)}{8\pi n^2 \nu^2 \tau_{sp}} I_\nu$$

$$\rightarrow I_\nu(z) = I_\nu(0) e^{\gamma(\nu) z}$$

$$\leftarrow \gamma(\nu) = (N_2 - N_1) \frac{c^2 g(\nu)}{8\pi n^2 \nu^2 \tau_{sp}}$$



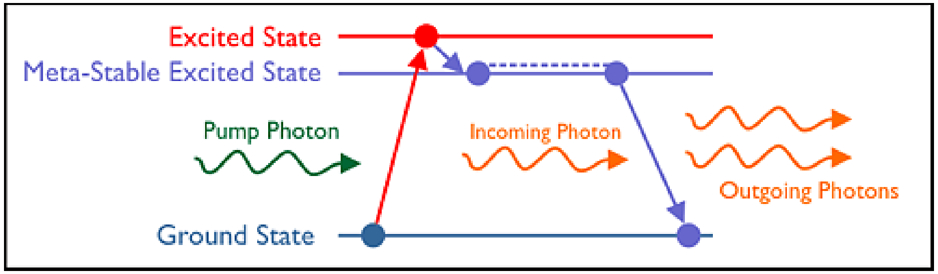
A. Yariv, *Optical Electronics*, 4th ed. Saunders, 1991.

Absorption:

$$\frac{N_2}{N_1} = e^{-h\nu/kT} \rightarrow N_2 < N_1 \leftarrow \text{Thermal equilibrium}$$

Amplification: $N_2 > N_1 \leftarrow$ Population inversion

via pumping through a metastable state



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$$\tau_{sp} >> 0$$

Induced Transitions: Degeneracy (1)

Energy density with a blackbody radiation:

$$\rho(\nu) = \frac{8\pi n^3 h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

For thermal equilibrium:

$$N_2 W'_{21} = N_1 W'_{12} \;\rightarrow\; N_2[B_{21}\rho(\nu) + A_{21}] = N_1 B_{12}\rho(\nu)$$

Boltzmann distribution:

(with degeneracy)

$$\frac{N_i}{N} = \frac{g_i e^{-E_i/kT}}{\sum_j g_j e^{-E_j/kT}} \;\rightarrow\; \frac{N_2}{N_1} = \frac{g_2 e^{-E_2/kT}}{g_1 e^{-E_1/kT}} = \frac{g_2}{g_1} e^{-h\nu/kT}$$

$$\rightarrow \frac{A_{21}}{(g_1/g_2)B_{12}e^{h\nu/kT} - B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

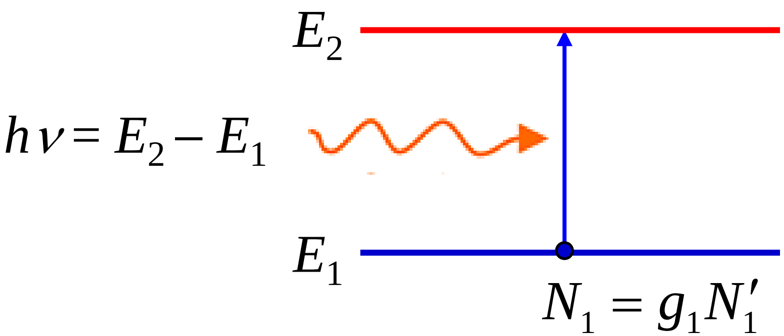
Einstein's A and B coefficients:

$$\rightarrow \frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3} \;\rightarrow\; B_{12} = \frac{g_2}{g_1} B_{21}$$

Induced Transitions: Degeneracy (2)

Induced transitions and their rates per atom:

Absorption

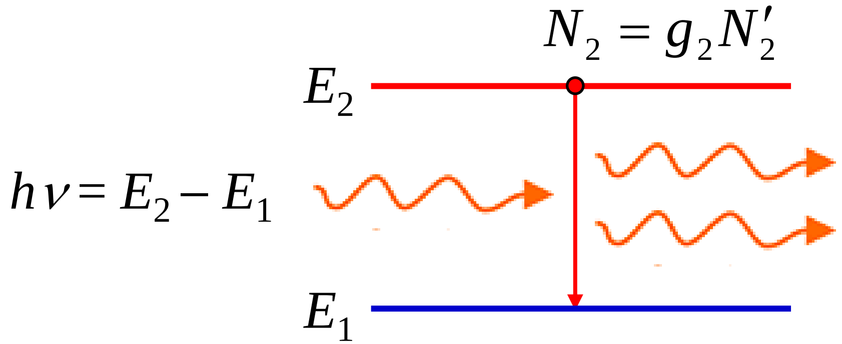


$$(W_{12}')_{induced} = B_{12}\rho(\nu)$$

$$\rightarrow W_{i,a}' = \frac{g_2}{g_1} B_{21}\rho(\nu)$$

$$= \frac{g_2}{g_1} \frac{c^3}{8\pi n^3 h\nu^3 \tau_{sp}} \rho(\nu)$$

Stimulated emission



$$(W_{21}')_{induced} = B_{21}\rho(\nu)$$

$$\rightarrow W_{i,e}' = \frac{A_{21}c^3}{8\pi n^3 h\nu^3} \rho(\nu)$$

$$= \frac{c^3}{8\pi n^3 h\nu^3 \tau_{sp}} \rho(\nu)$$

Transition rate in terms of the intensity:

$$\rightarrow W_{i,a}(\nu) = \frac{g_2}{g_1} \frac{\lambda^2 I_\nu}{8\pi n^2 h\nu \tau_{sp}} g(\nu)$$

$$\rightarrow W_{i,e}(\nu) = \frac{\lambda^2 I_\nu}{8\pi n^2 h\nu \tau_{sp}} g(\nu)$$

Absorption and Emission Crosssections

Transition rate in terms of the intensity:

$$\rightarrow W_{i,a}(\nu) = \frac{g_2}{g_1} \frac{\lambda^2 I_\nu}{8\pi n^2 h\nu \tau_{sp}} g(\nu) \rightarrow W_{i,e}(\nu) = \frac{\lambda^2 I_\nu}{8\pi n^2 h\nu \tau_{sp}} g(\nu)$$

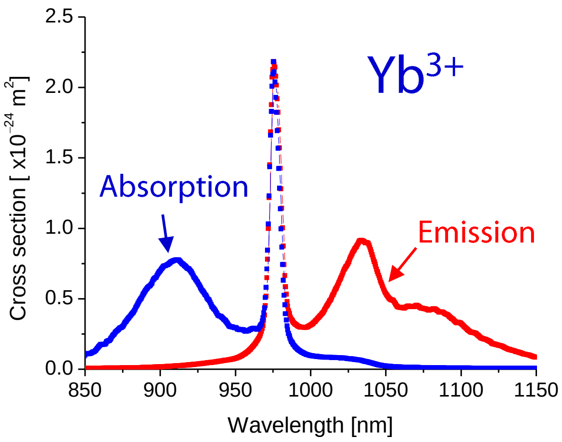
Emission and absorption crosssections:

$$\rightarrow \sigma_e(\nu) = \frac{\lambda^2}{8\pi n^2 \tau_{sp}} g(\nu)$$

$$\rightarrow \sigma_a(\nu) = \frac{g_2}{g_1} \frac{\lambda^2}{8\pi n^2 \tau_{sp}} g(\nu) = \frac{g_2}{g_1} \sigma_e$$

$$\rightarrow W_{i,e}(\nu) = \frac{\sigma_e I_\nu}{h\nu}$$

$$\rightarrow W_{i,a}(\nu) = \frac{\sigma_a I_\nu}{h\nu}$$



Absorption and Amplification

Net power generated per unit volume:

$$\frac{P}{Volume} = (N_2 W_{i,e} - N_1 W_{i,a}) h\nu \quad \leftarrow \quad W_{i,e}(\nu) = \frac{\sigma_e I_\nu}{h\nu}, \; W_{i,a}(\nu) = \frac{\sigma_a I_\nu}{h\nu}$$

$$\rightarrow \frac{dI_\nu}{dz} = (N_2 - \frac{\sigma_a}{\sigma_e} N_1) \sigma_e I_\nu$$

$$\rightarrow I_\nu(z) = I_\nu(0) e^{\gamma(\nu) z} \quad \leftarrow \quad \gamma(\nu) = (N_2 - \frac{\sigma_a}{\sigma_e} N_1) \sigma_e$$

Absorption:

$$N_2 - \frac{\sigma_a}{\sigma_e} N_1 < 0$$

Amplification:

$$N_2 - \frac{\sigma_a}{\sigma_e} N_1 > 0$$

"Effective population inversion"