Nonlinear Optical Engineering

Dispersion (1)

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Dispersion in Optical Fibers

Types of dispersion:

Chromatic dispersion:

Material dispersion

Waveguide dispersion: usually *smaller* than material dispersion

Short wavelength: The effective index is close to n_{core}.

Long wavelength: The effective index is close to $n_{cladding}$.

Modal dispersion:

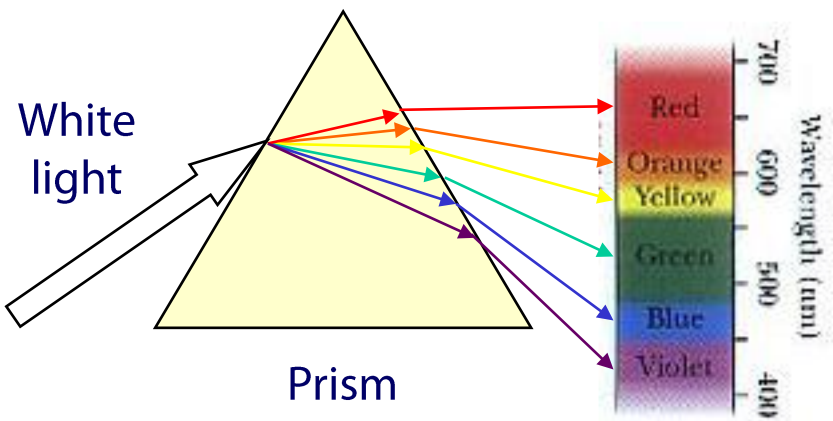
Pulse spreading in a multimode fiber

Dispersion is a problem in fiber communications, limiting the bandwidth of the fiber.

Material Dispersion (1)

White light that is a mixture of colors can be separated into different wavelengths.

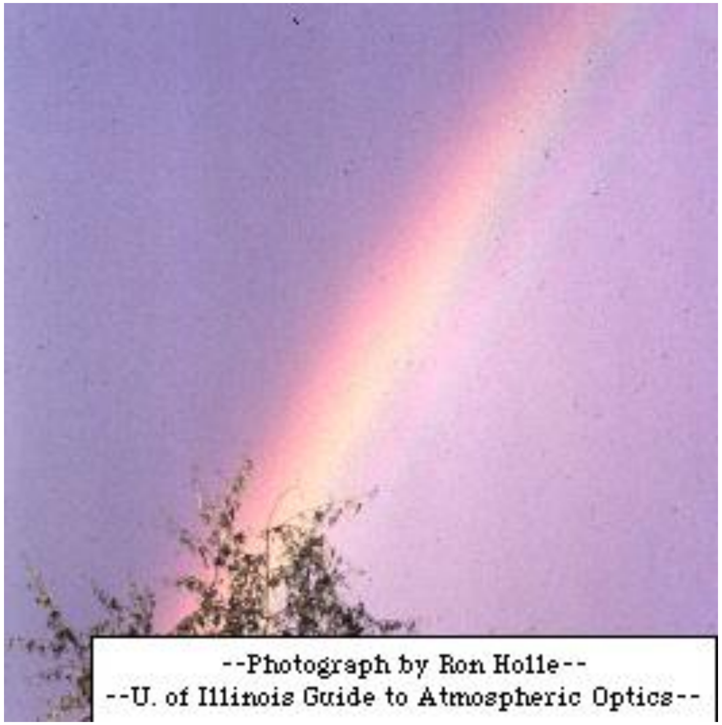
Refractive index *n* is inherently a function of wavelength.



Recall: Snell's law

$$n_i \sin\theta_i = n_t \sin\theta_t$$

Natural Dispersion:
RAINBOW



Material Dispersion (2)

Nonlocality in time:

$$\mathbf{D}(\mathbf{x},\omega)=\varepsilon(\omega)\mathbf{E}(\mathbf{x},\omega)$$ → For a monochromatic input

$$\rightarrow\ \mathbf{D}(\mathbf{x},t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\varepsilon(\omega)\mathbf{E}(\mathbf{x},\omega)e^{-i\omega t}d\omega$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\varepsilon(\omega)\left(\int_{-\infty}^{\infty}\mathbf{E}(\mathbf{x},t')e^{i\omega t'}dt'\right)e^{-i\omega t}d\omega$$

$$\leftarrow\ \varepsilon(\omega)=\varepsilon_0[1+\chi_e(\omega)]$$

$$\leftarrow\ G(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}[\varepsilon(\omega)/\varepsilon_0-1]e^{-i\omega t}d\omega$$

$$\rightarrow\ \mathbf{D}(\mathbf{x},t)=\varepsilon_0[\mathbf{E}(\mathbf{x},t)+\int_{-\infty}^{\infty}G(\tau)\mathbf{E}(\mathbf{x},t-\tau)d\tau]$$

Causality and analyticity domain of $\varepsilon(\omega)$:

$$\rightarrow\ \mathbf{D}(\mathbf{x},t)=\varepsilon_0[\mathbf{E}(\mathbf{x},t)+\int_{0}^{\infty}G(\tau)\mathbf{E}(\mathbf{x},t-\tau)d\tau]$$

$$\rightarrow\ \varepsilon(\omega)/\varepsilon_0=1+\int_{0}^{\infty}G(\tau)e^{i\omega\tau}d\tau\ \rightarrow\ \varepsilon(-\omega)/\varepsilon_0=\varepsilon^*(\omega^*)/\varepsilon_0$$

Material Dispersion (3)

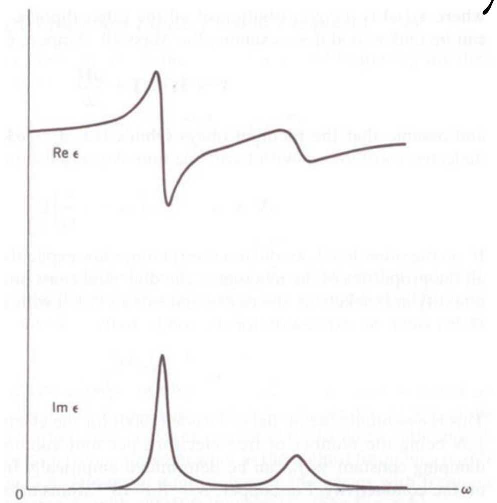
Analyticity domain of $\varepsilon(\omega)$:

$$\varepsilon(\omega)/\varepsilon_0 = 1 + \frac{1}{2\pi i}\oint_C \frac{[\varepsilon(\omega')/\varepsilon_0 - 1]}{\omega' - z} d\omega' \quad \leftarrow \text{Cauchy's theorem}$$

$$= 1 + \frac{1}{\pi i} P\int_{-\infty}^{\infty} \frac{[\varepsilon(\omega')/\varepsilon_0 - 1]}{\omega' - \omega} d\omega' \quad \leftarrow \text{Principal part}$$

$$\rightarrow \ \mathrm{Re}[\varepsilon(\omega)/\varepsilon_0] = 1 + \frac{1}{\pi} P\int_{-\infty}^{\infty} \frac{\mathrm{Im}[\varepsilon(\omega')/\varepsilon_0]}{\omega' - \omega} d\omega'$$

$$\rightarrow \ \mathrm{Im}[\varepsilon(\omega)/\varepsilon_0] = -\frac{1}{\pi} P\int_{-\infty}^{\infty} \frac{\mathrm{Re}[\varepsilon(\omega')/\varepsilon_0 - 1]}{\omega' - \omega} d\omega'$$



J. D. Jackson, Classical Electrodynamics

→ Kramers-Kronig relations:

The real and imaginary parts are related to each other!

Material Dispersion (4)

Sellmeier equation:

$$n^2(\omega) = 1 + \sum_{j=1}^{m} \frac{B_j \omega_j^2}{\omega_j^2 - \omega^2}$$

The origin of chromatic dispersion is related to the characteristic resonance frequencies at which the medium absorbs the electromagnetic radiation through oscillations of bound electrons.

← Recall: Kramers-Kronig relations