Nonlinear Optical Engineering

Dispersion (2)

Yoonchan Jeong

School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

Email: yoonchan@snu.ac.kr

Dispersion Relation

Mode-propagation constant β: $\rightarrow$ Taylor series expansion

$$\beta(\omega) = n(\omega)\frac{\omega}{c} = \beta_0 + \beta_1(\omega-\omega_0) + \frac{1}{2}\beta_2(\omega-\omega_0)^2 + \cdots,$$

$$\text{where} \quad \beta_m = \left(\frac{d^m\beta}{d\omega_m}\right)_{\omega=\omega_0} \quad (m = 1, 2, 3, \ldots)$$

$$\rightarrow \beta_1 = \frac{1}{c}\left(n + \omega\frac{dn}{d\omega}\right)$$

$$\rightarrow \beta_2 = \frac{1}{c}\left(2\frac{dn}{d\omega} + \omega\frac{d^2n}{d\omega^2}\right)$$

Wave packet:

$$\rightarrow \psi(z,t) = \int_{-\infty}^{\infty} A(\omega) e^{i[\beta(\omega)z - \omega t]} d\omega$$

Group Velocity and Group Velocity Dispersion

Wave packet:

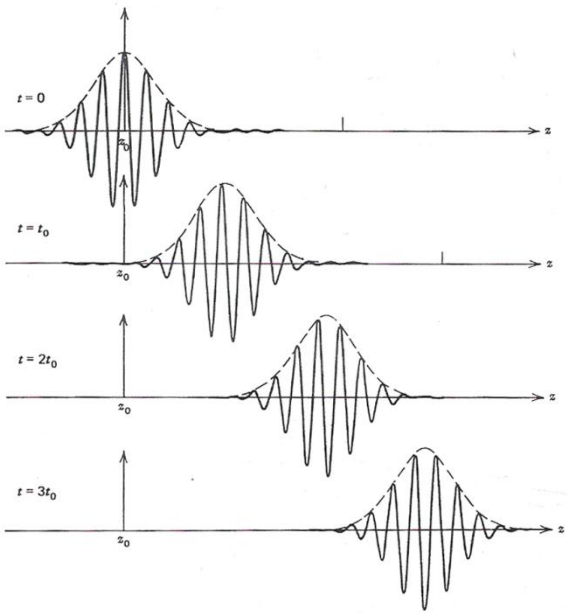
$$\psi(z,t) = \int_{-\infty}^{\infty} A(\omega) e^{i[\beta(\omega)z - \omega t]} d\omega$$

$$\cong \int_{-\infty}^{\infty} A(\omega) e^{i\{[\beta_0 + \beta_1(\omega-\omega_0)]z - \omega t\}} d\omega \quad \leftarrow \text{First-order perturbation}$$

$$= e^{i(\beta_0 z - \omega_0 t)} \int_{-\infty}^{\infty} A(\omega) e^{i[\beta_1(\omega-\omega_0)z - (\omega-\omega_0)t]} d\omega$$

$$= e^{i(\beta_0 z - \omega_0 t)} f(\beta_1 z - t)$$

Carrier Envelop



A. Yariv and P. Yeh, *Optical Waves in Crystals: John Wiley & Sons, 1984.*

Group velocity: Velocity of the envelop

$$\rightarrow v_g = \frac{1}{\beta_1} = \frac{c}{n_g}$$

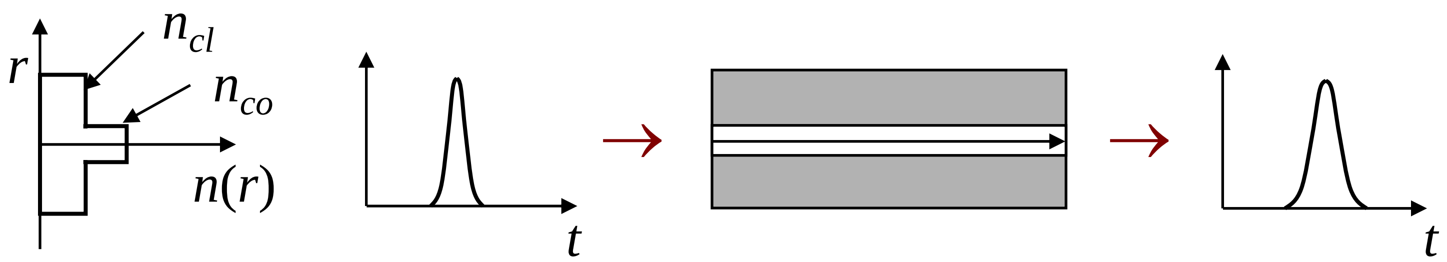
Group velocity dispersion: Change in GV

$$\rightarrow \beta_2 = \frac{d^2\beta(\omega)}{d\omega^2} = \frac{d\beta_1}{d\omega}$$

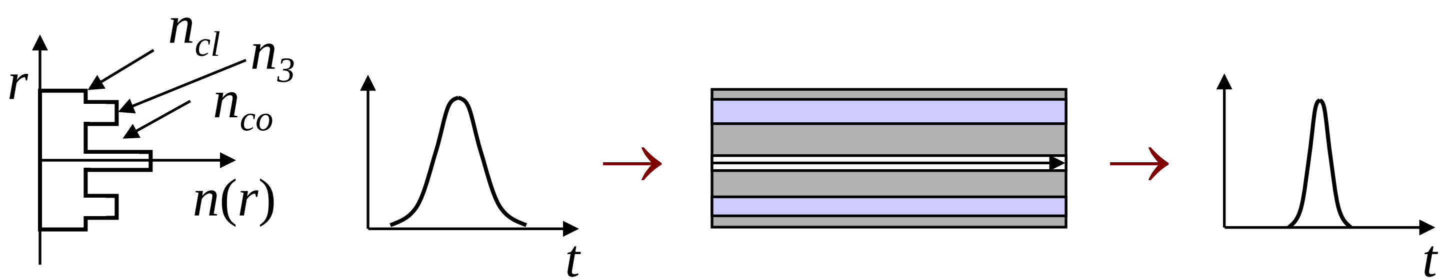
Waveguide Dispersion

Effective mode index: Also dependent on the fiber geometry and design

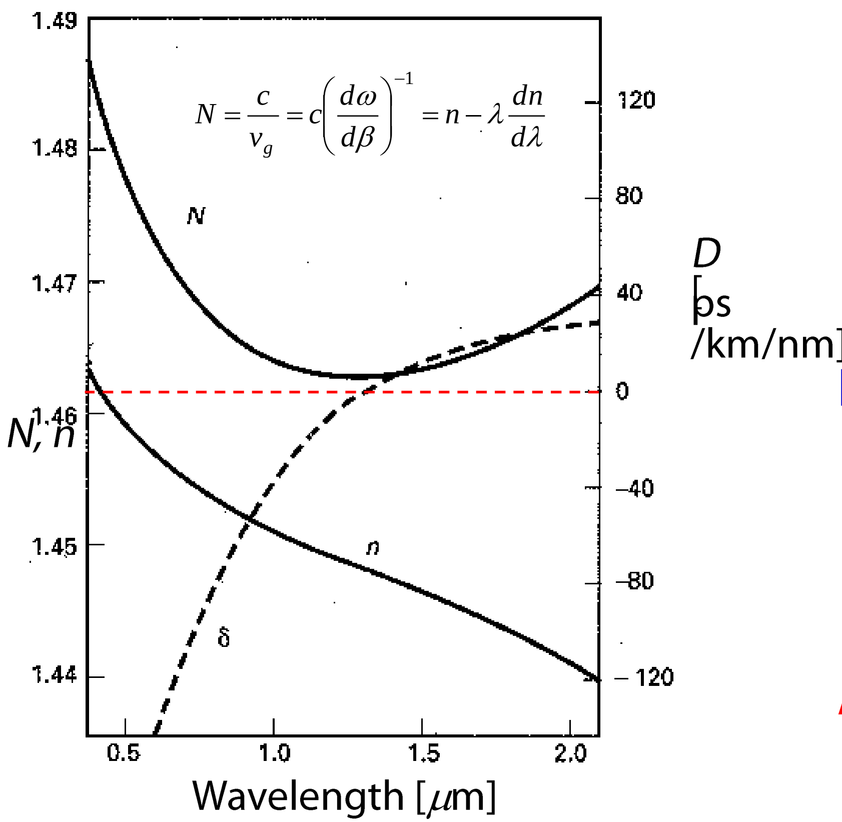
Single-mode fiber (SMF)



Dispersion-compensation Fiber (DCF)



Dispersion in SMF



Source: Nonlinear Fiber Optics, G. P. Agrawal

Dispersion parameter:

$$D = \frac{d\beta_1}{d\lambda} = -\frac{2\pi c}{\lambda^2}\beta_2 \approx \frac{\lambda}{c}\frac{d^2 n}{d\lambda^2}$$

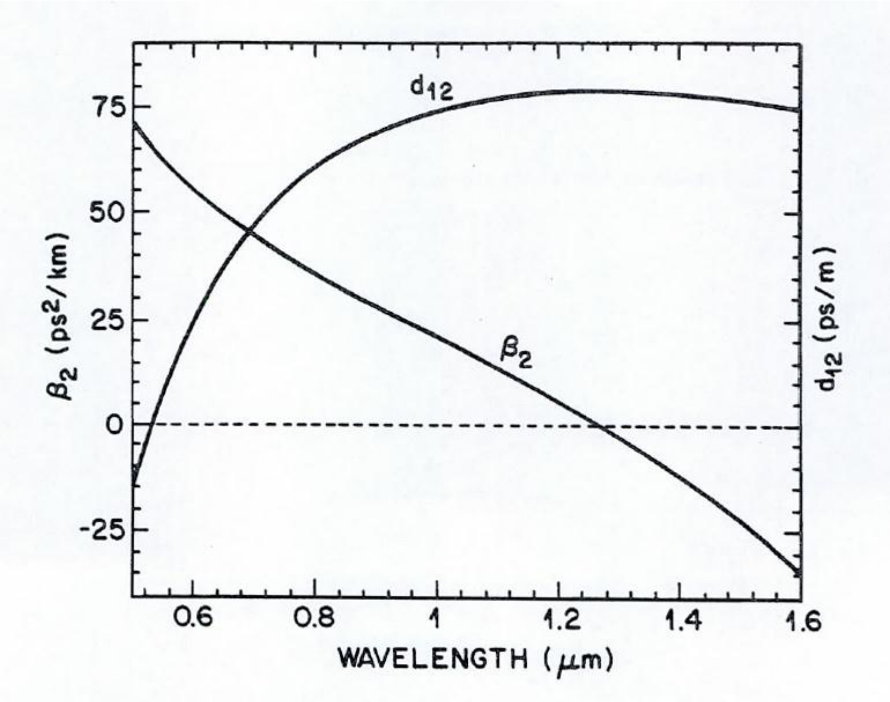
Normal dispersion: $\beta_2 > 0$ *or* $D < 0$

High-freq. components slower than low-freq. components

Anomalous dispersion: $\beta_2 < 0$ *or* $D > 0$

High-freq. components faster than low-freq. components

Walk-Off in SMF



Source: Nonlinear Fiber Optics, G. P. Agrawal

Walk-off parameter:

$$d_{12} = \beta_1(\lambda_1) - \beta_1(\lambda_2) = \frac{1}{v_g(\lambda_1)} - \frac{1}{v_g(\lambda_2)}$$

Walk-off length:

$$L_W = \frac{T_0}{|d_{12}|}$$

e.g. $\lambda_1 = 532\ nm,\ \lambda_2 = 1064\ nm$

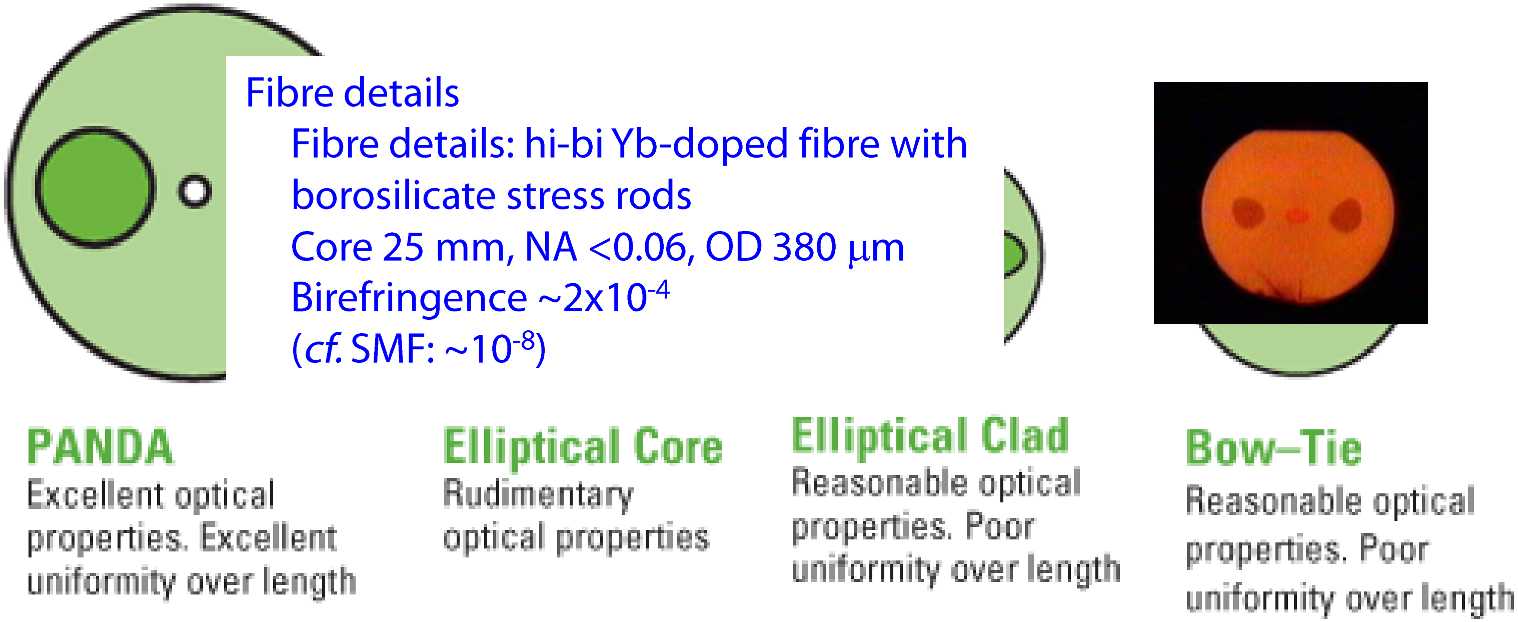
$d_{12} \approx 80\ ps/m$

$L_W = 25\ cm \quad for \quad T_0 = 20\ ps$

Polarization-Mode Dispersion (PMD)

Fibre details

- Fibre details: hi-bi Yb-doped fibre with borosilicate stress rods
- Core 25 mm, NA <0.06, OD 380 μm
- Birefringence ~2×10^{-4}
- (*cf.* SMF: ~10^{-8})



Source: www.nufern.com

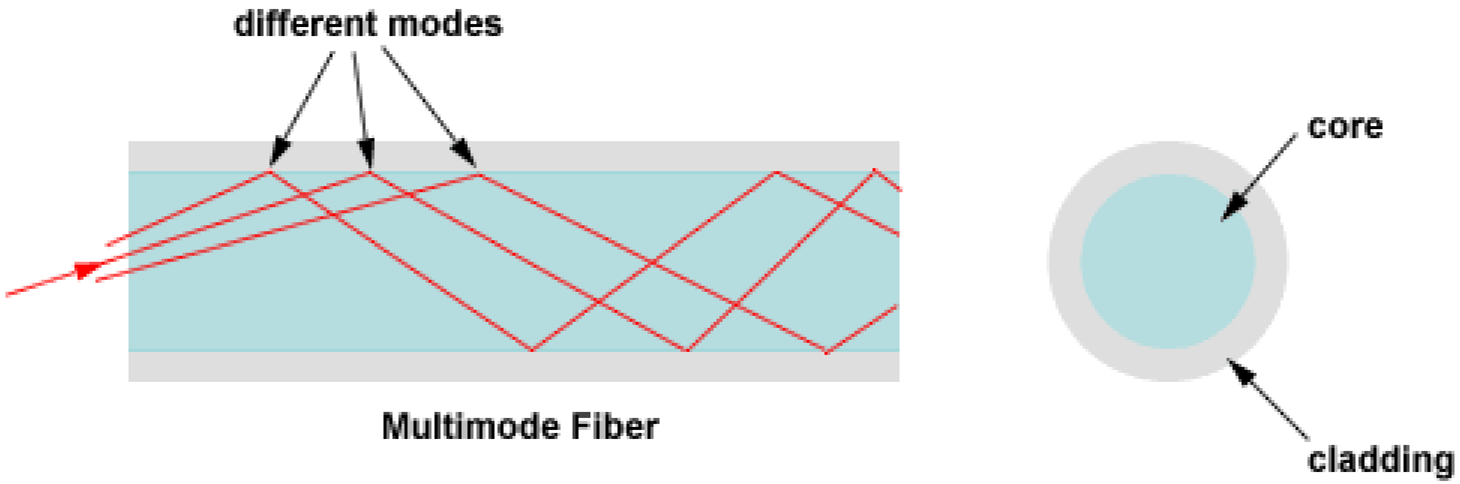
Birefringence:

$$B_m = \frac{|\beta_x - \beta_y|}{k_0} = |n_x - n_y|$$

Beat length:

$$L_B = \frac{2\pi}{|\beta_x - \beta_y|} = \frac{\lambda}{B_m}$$

Modal Dispersion



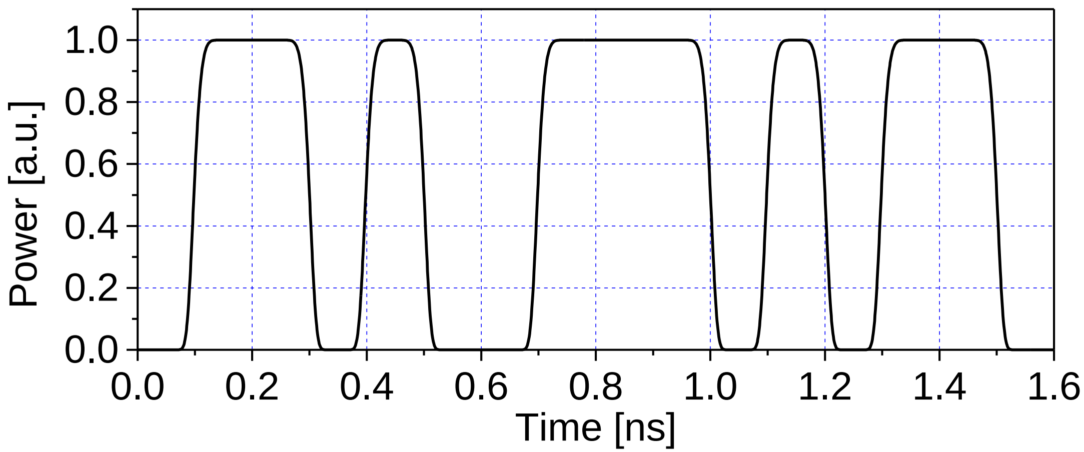
Source: www.fiberoptics4sale.com

Optical path differences among modes $\rightarrow$ Difference in group velocity

$$v_{g,m} = \frac{d\omega}{d\beta_m}$$

Data Transmission in SMF

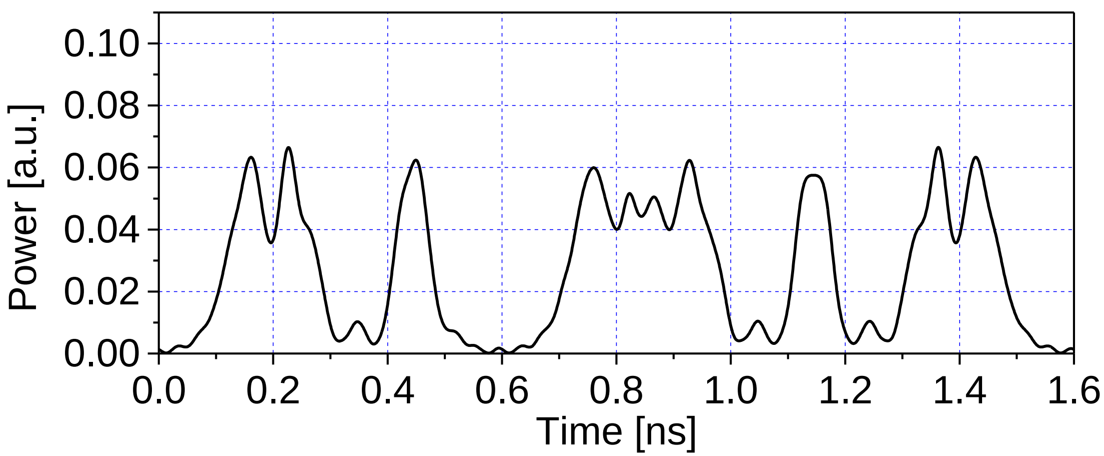
Initial optical pulses (10 Gbps, 0 dBm):



Group velocity dispersion:
→ Frequency chirp

Nonlinear effect:
→ SPM, XPM, FWM, etc.

After 50-km Transmission:



Power spectrum: