Nonlinear Optical Engineering

Second-Harmonic Generation (2)

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Second-Harmonic Generation (1)

Plane waves:

$$E^{\omega_j}(t,z) = E_j(z)\exp[i(\omega_j t - k_j z)] \quad (j=1,\,2,\,\omega_2 = 2\omega_1)$$

Second-order nonlinear coupled-wave equations:

$$\frac{dE_1}{dz} = -i\frac{\omega_1}{\varepsilon_o n_1 c} d_1 E_2 E_1^* \exp(-i\Delta kz) \quad \leftarrow \; \Delta k = k_2 - 2k_1$$

$$\frac{dE_2}{dz} = -i\frac{\omega_2}{2\varepsilon_o n_2 c} d_2 E_1^2 \exp(i\Delta kz)$$

Recall:

$$\rightarrow \frac{d}{dx}\tanh u = \mathrm{sech}^2 u \frac{du}{dx}$$

$$\rightarrow \mathrm{sech}^2 u + \tanh^2 u = 1$$

Phase-matched case: $\Delta k = k_2 - 2k_1 = 0$

$$\rightarrow A_l = \sqrt{\frac{n_l}{\omega_l}} E_l \quad \rightarrow A_2 = -iA_2'$$

$$\rightarrow \frac{dA_1}{dz} = -\kappa A_2' A_1$$

$$\rightarrow \frac{dA_2'}{dz} = \frac{1}{2}\kappa A_1^2$$

$$\rightarrow \frac{d}{dz}(A_1^2 + 2A_2'^2) = 0$$

$$\rightarrow A_1^2 + 2A_2'^2 = A_1^2(0)$$

$$\rightarrow \frac{dA_2'}{dz} = \frac{1}{2}\kappa[A_1^2(0) - 2A_2'^2]$$

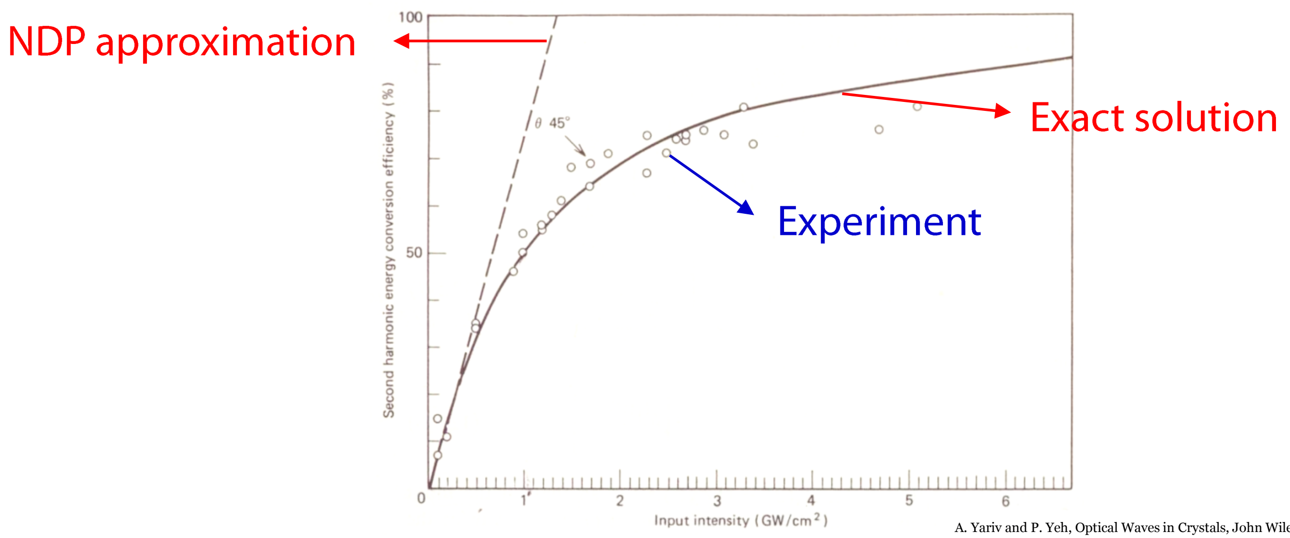
$$\rightarrow A_2'(z) = \frac{1}{\sqrt{2}} A_1(0)\tanh[\frac{A_1(0)}{\sqrt{2}}\kappa z]$$

Conversion efficiency:

$$\rightarrow \frac{P^{(2\omega)}}{P^{(\omega)}} = \frac{2|A_2(z)|^2}{|A_1(z)|^2} = \tanh^2[\frac{A_1(0)}{\sqrt{2}}\kappa z]$$

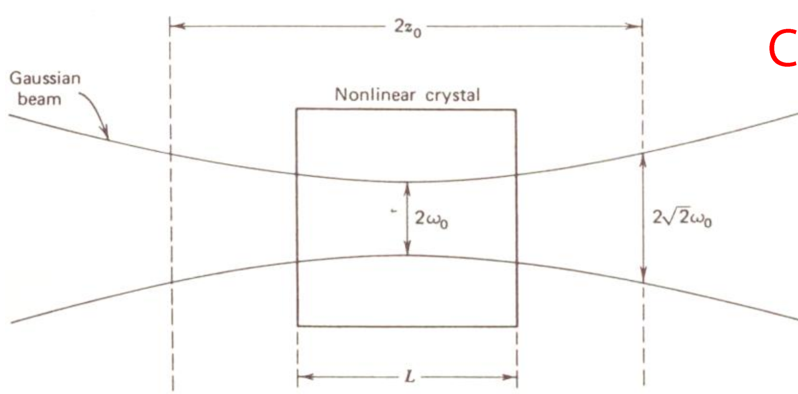
Second-Harmonic Generation (2)

Second-harmonic conversion efficiency:



A. Yariv and P. Yeh, Optical Waves in Crystals, John Wiley & Sons, 1984.

Gaussian beam focused inside a nonlinear crystal:



A. Yariv and P. Yeh, Optical Waves in Crystals, John Wiley & Sons, 1984.

Confocal beam parameter:

$$\rightarrow \ z_0 = \frac{\pi \omega_o^2 n}{\lambda}$$

CM Equations with 2nd and 3rd Nonlinearities (1)

Coupled equations:

$$\nabla\cdot(\mathbf{E}'_{\omega}\times\mathbf{H}^{*}_{\omega,p}+\mathbf{E}^{*}_{\omega,p}\times\mathbf{H}'_{\omega})=-i\omega\mathbf{E}^{*}_{\omega,p}\cdot\Delta\mathbf{P}_{\omega},\quad (p=1,2,\ldots),$$

$$\begin{aligned}\Delta\mathbf{P}_{\omega_1}&=\Delta\varepsilon_{ij}(\omega_1)E'_{\omega_1,j}+2d_{ijk}(-\omega_1,\omega_2,-\omega_1)E'_{\omega_2,j}E'^{*}_{\omega_1,k}\\&\quad+\left\{3\chi_{ijkl}(-\omega_1,\omega_1,-\omega_1,\omega_1)E'_{\omega_1,j}E'^{*}_{\omega_1,k}+6\chi_{ijkl}(-\omega_1,\omega_2,-\omega_2,\omega_1)E'_{\omega_2,j}E'^{*}_{\omega_2,k}\right\}E'_{\omega_1,l},\end{aligned}$$

$$\begin{aligned}\Delta\mathbf{P}_{\omega_2}&=\Delta\varepsilon_{ij}(\omega_2)E'_{\omega_2,j}+d_{ijk}(-\omega_2,\omega_1,\omega_1)E'_{\omega_1,k}E'_{\omega_1,k}\\&\quad+\left\{3\chi_{ijkl}(-\omega_2,\omega_2,-\omega_2,\omega_2)E'_{\omega_2,j}E'^{*}_{\omega_2,k}+6\chi_{ijkl}(-\omega_2,\omega_1,-\omega_1,\omega_2)E'_{\omega_1,j}E'^{*}_{\omega_1,k}\right\}E'_{\omega_2,l}.\end{aligned}$$

Coupled equations for the complex amplitudes:

$$\frac{dA_{\omega_1}}{dz}=-i\kappa_{\varepsilon,\omega_1}A_{\omega_1}-i\kappa_{d,\omega_1}A_{\omega_2}A^{*}_{\omega_1}\exp(-i\Delta\beta z)-i(\kappa_{\chi,\omega_1,\omega_1}\left|A_{\omega_1}\right|^2+\kappa_{\chi,\omega_1,\omega_2}\left|A_{\omega_2}\right|^2)A_{\omega_1},$$

$$\frac{dA_{\omega_2}}{dz}=-i\kappa_{\varepsilon,\omega_2}A_{\omega_2}-i\kappa_{d,\omega_2}A^{2}_{\omega_1}\exp(i\Delta\beta z)-i(\kappa_{\chi,\omega_2,\omega_1}\left|A_{\omega_1}\right|^2+\kappa_{\chi,\omega_2,\omega_2}\left|A_{\omega_2}\right|^2)A_{\omega_2},$$

$$\Delta\beta=\beta_{\omega_2}-2\beta_{\omega_1}.$$

CM Equations with 2nd and 3rd Nonlinearities (2)

Coupled equations for the amplitudes and phases:

$$\frac{du}{dz} = -\kappa_{d,\omega_1}\sqrt{P_o}\,uv\sin\theta, \quad \frac{dv}{dz} = \kappa_{d,\omega_2}\sqrt{P_o}\,u^2\sin\theta,$$

$$\frac{d\phi_f}{dz} = \kappa_{\varepsilon,\omega_1} + \kappa_{d,\omega_1}\sqrt{P_o}\,v\cos\theta + P_o(\kappa_{\chi,\omega_1,\omega_1}u^2 + \kappa_{\chi,\omega_1,\omega_2}v^2),$$

$$\frac{d\phi_s}{dz} = \kappa_{\varepsilon,\omega_2} + \kappa_{d,\omega_2}\sqrt{P_o}\,\frac{u^2}{v}\cos\theta + P_o(\kappa_{\chi,\omega_2,\omega_1}u^2 + \kappa_{\chi,\omega_2,\omega_2}v^2),$$

$$\frac{d\theta}{dz} = \frac{\cos\theta}{\sin\theta}\cdot\frac{1}{u^2 v}\cdot\frac{d}{dz}(u^2 v) + f_o + g_o v^2.$$

$$\rightarrow\ \Gamma_o = u^2 v\cos\theta + \sigma_f v^2 + \sigma_g v^4$$

$$\rightarrow\ \left(\frac{dv^2}{dz}\right)^2 = (2\kappa_{d,\omega_2}\sqrt{P_o})^2\cdot\left\{(1-v^2)^2 v^2 - (\Gamma_o - \sigma_f v^2 - \sigma_g v^4)^2\right\}$$

CM Equations with 2nd and 3rd Nonlinearities (3)

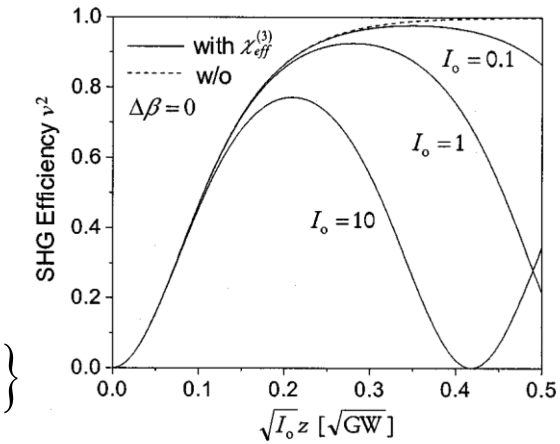
Analytic solutions:

$$u^2 = 1 - \frac{v_b^2(v_c^2 - v_a^2) - v_c^2(v_b^2 - v_a^2)sn^2(\alpha z' + \varsigma_o)}{v_c^2 - v_a^2 - (v_b^2 - v_a^2)sn^2(\alpha z' + \varsigma_o)},$$

$$v^2 = \frac{v_b^2(v_c^2 - v_a^2) - v_c^2(v_b^2 - v_a^2)sn^2(\alpha z' + \varsigma_o)}{v_c^2 - v_a^2 - (v_b^2 - v_a^2)sn^2(\alpha z' + \varsigma_o)},$$

$$\phi_f = \phi_{f,0} + \psi_{f,0} z + \frac{1}{\alpha}\Big(\psi_{f,1}\left\{\Pi(\varphi, \rho_{f,1}, \gamma^2) - \Pi(\varphi_o, \rho_{f,1}, \gamma^2)\right\}$$
$$+ \psi_{f,2}\left\{\Pi(\varphi, \rho_{f,2}, \gamma^2) - \Pi(\varphi_o, \rho_{f,2}, \gamma^2)\right\}\Big),$$

$$\phi_s = \phi_{s,0} + \psi_{s,0} z + \frac{1}{\alpha}\Big(\psi_{s,1}\left\{\Pi(\varphi, \rho_{s,1}, \gamma^2) - \Pi(\varphi_o, \rho_{s,1}, \gamma^2)\right\}$$
$$+ \psi_{s,2}\left\{\Pi(\varphi, \rho_{s,2}, \gamma^2) - \Pi(\varphi_o, \rho_{s,2}, \gamma^2)\right\}\Big).$$



Y. Jeong et al., IEEE J. Quant. Electron., vol. 37, pp. 1292-1300, 2001.

Refs.) J. A. Armstrong et al., Phys. Rev., vol. 127, pp. 1918-1939, 1962.
Y. Jeong et al., IEEE J. Quant. Electron., vol. 37, pp. 1292-1300, 2001.