Nonlinear Optical Engineering

Nonlinear Pulse Propagation in Optical Fibers (2)

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Nonlinear Pulse Propagation in Optical Fibers

NLSE:

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} - i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A = -i\gamma |A|^2 A$$

$$\rightarrow \; A(z,t) = \int \Phi(z,\Omega) \underline{\exp\{[-i(\beta_1\Omega + \frac{\beta_2}{2}\Omega^2) - \frac{\alpha}{2})]z\}} \exp(i\Omega t) d\Omega$$

$$= \int T(z,\Omega)\cdot\Phi(z,\Omega)\exp(i\Omega t)d\Omega,$$

$$\leftarrow \; \Phi(0,\Omega) = \frac{1}{2\pi}\int A(0,t)\exp(-i\Omega t)dt$$

$$\rightarrow \; \gamma|A|^2 A = \int T(z,\Omega) G(z,\Omega)\exp(i\Omega t)d\Omega,$$

$$\leftarrow \; G(z,\Omega) = \frac{1}{2\pi}\int T^{-1}(z,\Omega)\gamma|A|^2 A \exp(-i\Omega t)dt$$

In result:

$$\rightarrow \; \frac{\partial}{\partial z}\Phi(z,\Omega) = -iG(z,\Omega),$$

$\leftarrow$ *Predictor-corrector scheme or split-step Fourier method*

Predictor-Corrector Scheme

For a first-order differential equation:

$$\frac{dy}{dz} = f(z, y)$$

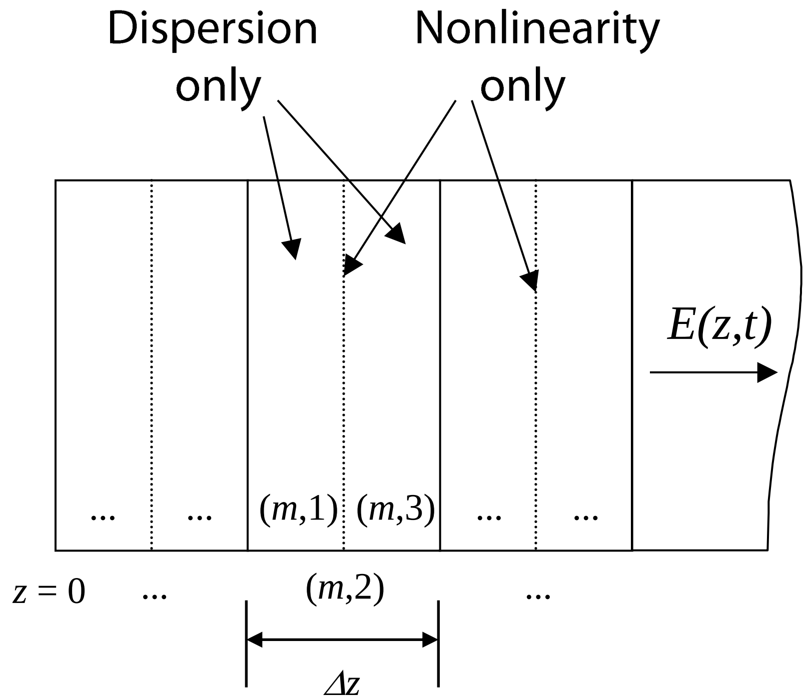
$$\rightarrow \; y_p(z + \Delta z) = y(z) + f[z, y(z)]\Delta z$$

$$\rightarrow \; y(z + \Delta z) = y(z) + \frac{1}{2}\{ f[z, y(z)] + f[z, y_p(z + \Delta z)]\}\Delta z$$

$\rightarrow$ *The value predicted by an initial rough estimation is to be corrected by iterations.*

Split-Step Fourier Method

Separation of dispersion and nonlinear effects:



NLSE:

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} - i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2}A = -i\gamma |A|^2 A$$

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} - i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2}A = 0$$

$$\frac{\partial A}{\partial z} = -i\gamma |A|^2 A$$

Nonlinear Phase Shift

Recall NLSE:

Loss

Nonlinear phase shift due to $\chi^{(3)}$

$$A(z,t) = \int \tilde{A}(z,\Omega)\exp(i\Omega t)d\Omega$$

$$= \int \Phi(\Omega)\exp[i(\Omega t - \mathrm{B}z) - \frac{\alpha}{2}z - i\gamma|A|^2 z]d\Omega$$

$$= \int \Phi(\Omega)\exp[i\Omega t - i(\beta_1\Omega + \frac{\beta_2}{2}\Omega^2 + \cdots)z - \frac{\alpha}{2}z - i\gamma|A|^2 z]d\Omega$$

$$\rightarrow \frac{\partial A}{\partial z} + \beta_1\frac{\partial A}{\partial t} - \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2}A + \cdots = -i\gamma|A|^2 A$$

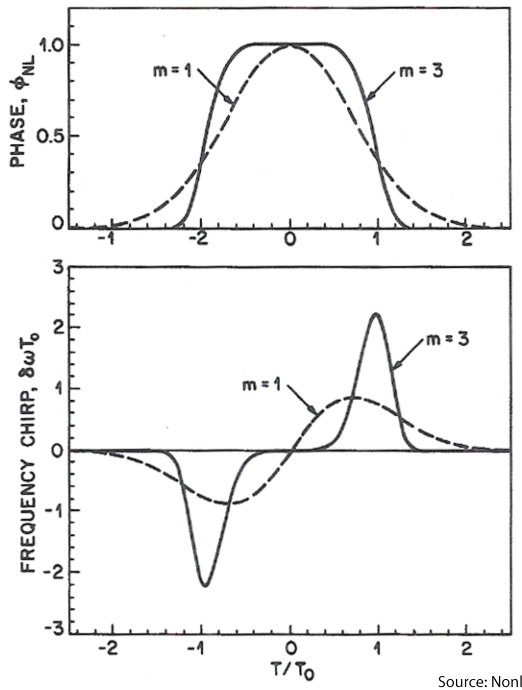
Nonlinear phase shift:

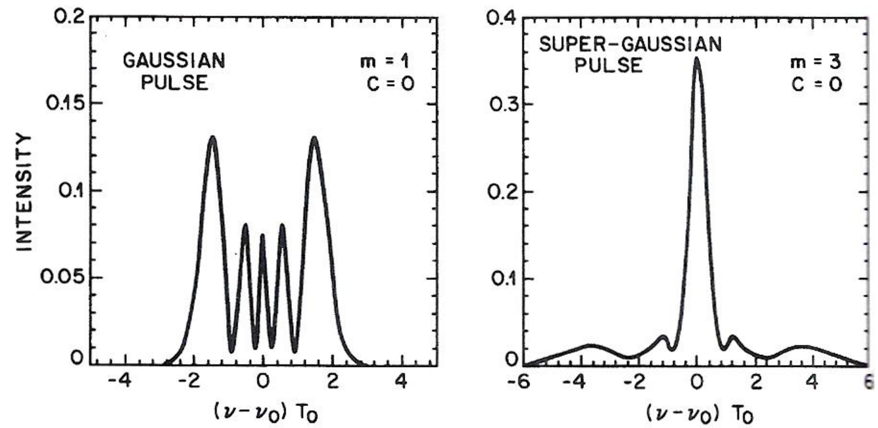
$$\phi_{NL}(z,t) = -\gamma|A(z,t)|^2 z$$

$$\rightarrow \delta\omega(z,t) = \frac{\partial}{\partial t}\left(-\gamma|A(z,t)|^2 z\right)$$

Self-Phase Modulation (SPM)

For Gaussian and super-Gaussian pulses:





Source: Nonlinear Fiber Optics, G. P. Agrawal

$$\phi_{NL,\max} = 4.5\pi$$

$\rightarrow$ *Frequency chirp incurred*