457.562 Special Issue on River Mechanics (Sediment Transport)
.08 Mechanics of Bedload Transport

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1. Equations of particle motion

- We already learned the appropriate equations governing particle motion in the previous class
- For simplicity, the z-coordinate is taken to be approximately upward vertical. Variation in the n-coordinate is taken to be negligible. The mean near-bed flow filed is

$$\bar{u}(s,n,z,t) = 2.5u_* \ln\left(30\frac{z}{k_s}\right)$$

$$\bar{v}(s,n,z,t) = \bar{w}(s,n,z,t) = 0$$

1. Equations of particle motion

- Getting back to the old equations of particle

$$(\rho_s + c_m\rho)V_p \frac{du_p}{dt} = -\frac{1}{2}\rho c_D A_p |u_r|(u_p - u_f) + \rho(1+c_m)V_p \frac{du_f}{dt}$$

$$(\rho_s + c_m\rho)V_p \frac{dw_p}{dt} = -\frac{1}{2}\rho c_D A_p |u_r| w_p - (\rho_s - \rho) g V_p + \frac{1}{2}\rho c_L A_p \left(u_{rT}^2 - u_{rB}^2\right)$$

- In the previous relations

$$u_f(t) = \bar{u}\left(s_p(t), n_p(t), z_p(t), t\right) = 2.5u_* \ln\left(30\frac{z}{k_s}\right)$$

- denotes the fluid velocity extrapolated to the particle centroid, and $|u_r|$ denotes the magnitude of the particle velocity relative to the fluid $|u_r|^2 = (u_p - u_f)^2 + w_p^2$

1. Equations of particle motion

- Getting back to the old equations of particle

$$\left(\rho_s + c_m \rho\right) V_p \frac{du_p}{dt} = -\frac{1}{2} \rho c_D A_p \left|u_r\right| \left(u_p - u_f\right) + \rho \left(1 + c_m\right) V_m \frac{du_f}{dt}$$

$$\left(\rho_s + c_m \rho\right) V_p \frac{dw_p}{dt} = -\frac{1}{2} \rho c_D A_p \left|u_r\right| w_p - \left(\rho_s - \rho\right) g V_p + \frac{1}{2} \rho c_L A_p \left(u_{rT}^2 - u_{rB}^2\right)$$

u_{rT} and u_{rB} represent the magnitude of the relative particle velocity at the top and bottom

$$u_{rT}^2 = \left(u_p - u_{fT}\right)^2 + w_p^2 \qquad u_{rB}^2 = \left(u_p - u_{fB}\right)^2 + w_p^2$$

- Where

$$u_{fT} = 2.5 u_* \ln\left(30 \frac{z_p + 1/2D}{k_s}\right); \qquad u_{fB} = 2.5 u_* \ln\left(30 \frac{z_p - 1/2D}{k_s}\right)$$

1. Equations of particle motion

- Remember, in the previous relations

$$u_f(t) = \overline{u}\left(s_p(t), n_p(t), z_p(t), t\right) = 2.5u_* \ln\left(30\frac{z}{k_s}\right)$$

- In streamwise directional equation, there is the fluid acce leration term and this must be evaluated following particl e (Lagrangian),

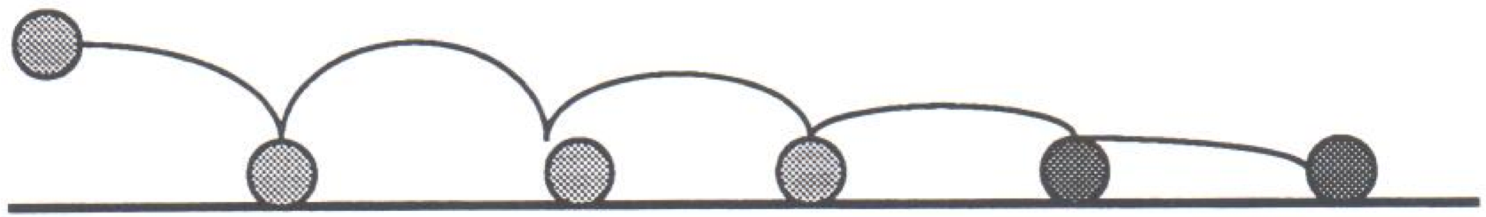
$$\frac{du_f}{dt} = \frac{\partial u_f}{\partial t} + u_p\frac{du_f}{ds_p} + w_p\frac{du_f}{dz_p} = 2.5w_p\frac{u_*}{z_p}$$

$$\left(differentiate \ \ln\left(30\frac{z_p(t)}{k_s}\right) = w_P\frac{1}{z_p}\right)$$

2. Collision at the bed: Splash function

- As will be found in the homework assignment, the effect of lift and fluid acceleration in momentum equations can often be neglected in a consideration of the motion of sal tating bedload particles.
- If this is taken to be the case, it is clear that there is no u pward force in vertical direction momentum equation acti ng on the particle during its entire saltation trajectory.
- Since collisions with the bed can be expected to be only partially elastic (e<1), the particle will rebound from the b ed with less energy and thus momentum, than it collided.
- A particle would saltate with progressively lower height, u ntil the trajectory devolved into a rolling or sliding motion s.

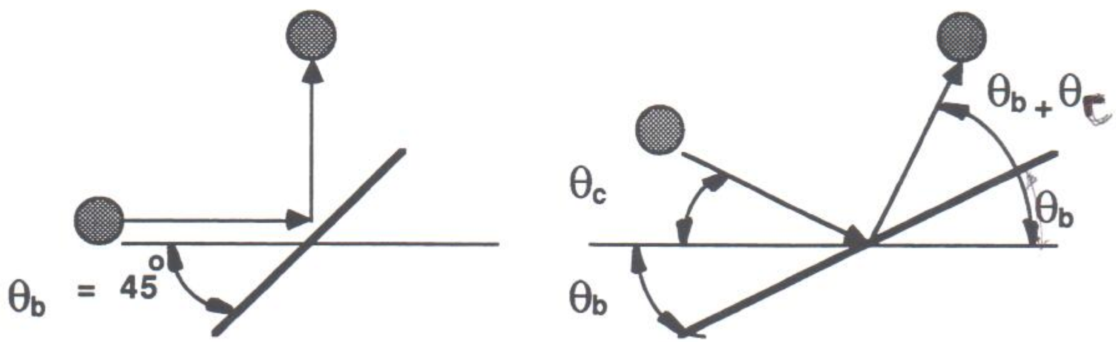
2. Collision at the bed: Splash function



- This devolution does indeed occur when particles saltate over a ***smooth bed***. The key to maintenance of a stable saltation path is ***bed roughness***.
- In order to see this, it is useful to start with a simple mod el, in which a particle collides at angle θ_c to the horizonta l against a surface that is always facing an angle θ_b in th e upstream direction to the horizontal

2. Collision at the bed: Splash function

- For example θ_c=0 and θ_b=45°. In this case, the incoming forward momentum of the particle is converted solely int o upward normal momentum,



- It is seen that collision against an upward-facing surface can provide a mechanism to convert forward momentum to upward vertical momentum.
- The forward momentum lost in the collision is replaced a s the particle rebounds upward into the flow; fluid drag a gain accelerates the particle downstream.

2. Collision at the bed: Splash function

- This model can be generalized.

$$u_{pT}\Big|_{in} = \left(u_p^2 + w_p^2\right)_{in}^{1/2} \cos\left(\theta_c + \theta_b\right)\Big|$$

$$u_{pN}\Big|_{in} = -\left(u_p^2 + w_p^2\right)_{in}^{1/2} \sin\left(\theta_c + \theta_b\right)\Big|$$

- As in the previous class,

$$u_{pT}\Big|_{out} = u_{pT}\Big|_{in}; \qquad u_{pN}\Big|_{out} = -e \cdot u_{pN}\Big|_{in}$$

- If follows from the above relations that the angle of rebound between the particle path and the bed surface is given by θ_r, where

$$\tan\left(\theta_r\right) = \frac{u_{pN}\big|_{out}}{u_{pT}\big|_{out}} = \frac{e \cdot u_{pN}\big|_{in}}{u_{pT}\big|_{in}} = e \cdot \tan\left(\theta_b + \theta_c\right)$$

2. Collision at the bed: Splash function

- The values of out bound velocities are

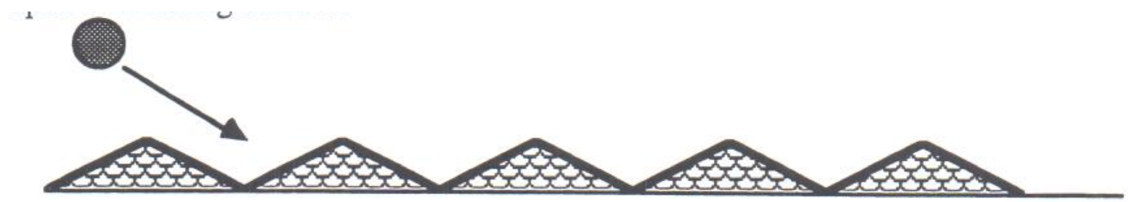
$$u_p\Big|_{out} = \left(u_{pT}^2 + u_{pN}^2\right)_{out}^{1/2} \cos\left(\theta_b + \theta_r\right)$$

$$w_p\Big|_{out} = \left(u_{pT}^2 + u_{pN}^2\right)_{out}^{1/2} \sin\left(\theta_b + \theta_r\right)$$

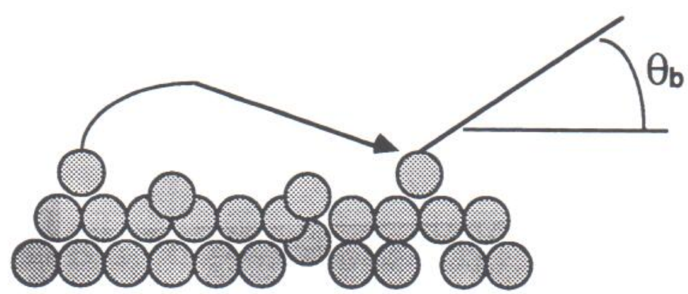
- Basic picture of the mechanics of saltation
  - The streamwise fluid drag force provides the source of particle m omentum.
  - Gravity pulls the particle toward the bed, causing collision.
  - Particle energy is lost due to the collision
  - Forward momentum is partially converted to upward normal mo mentum
  - The energy lost in the collision is then replaced by the work of th e fluid drag force as the particle rebounds in to the high-velocity

2. Collision at the bed: Splash function

- The limitation of the simplified model
  - Not true that a particle collides with an upstream-facing sur face, even if upstream- and downstream-facing surface ar e equally distributed in space, a particle striking the bed ob liquely is far more likely to strike an upstream-facing surfac e



  - Angle probability distribution of upstream-facing surfaces. Since the bed is ıerally distrib uted randomly.



2. Collision at the bed: Splash function

- In a full model of saltation, the probability distribution of bed angle (the ***splash function***) must be specified.
- The previous many researches showed that particle preferentially strike upward-facing surfaces.
- In any actual computation of saltation, then it is necessary to include both a deterministic element (particle trajectory) and a stochastic element (collision and splash). Even at “equilibrium” conditions, each saltation differs from every other due to this random element; the equilibrium is defined in terms of an average over many saltations.

3. Results of Sample Numerical Solution

- This may be helpful for your mini-project.
- In an actual calculation of saltation, the parameters c_m a nd e are normally set to constant values of each 0.5 and 0.6. The parameters c_D and c_L are specified as functions of particle Reynolds number

$$\frac{|u_r| D}{\nu} \qquad and \qquad k_s = a_k D \quad (a_k \text{ is one order number})$$

- Appropriate initial conditions are specified, the details ar e not too important since the asymptotic equilibrium state is not dependent of them.
- As shown in the project assignment, the final results can be expressed in dimensionless forms.

3. Results of Sample Numerical Solution

$$\bar{u}_p = \frac{1}{T_s}\int_0^{T_s} u_p(t)\,dt; \qquad \bar{u}_f = \frac{1}{T_s}\int_0^{T_s} u_f(t)\,dt$$

(mean streamwise particle and fluid velocities, where T_s time duration)

- Ensemble averaging over many realization at equilibrium yields the following relationships;

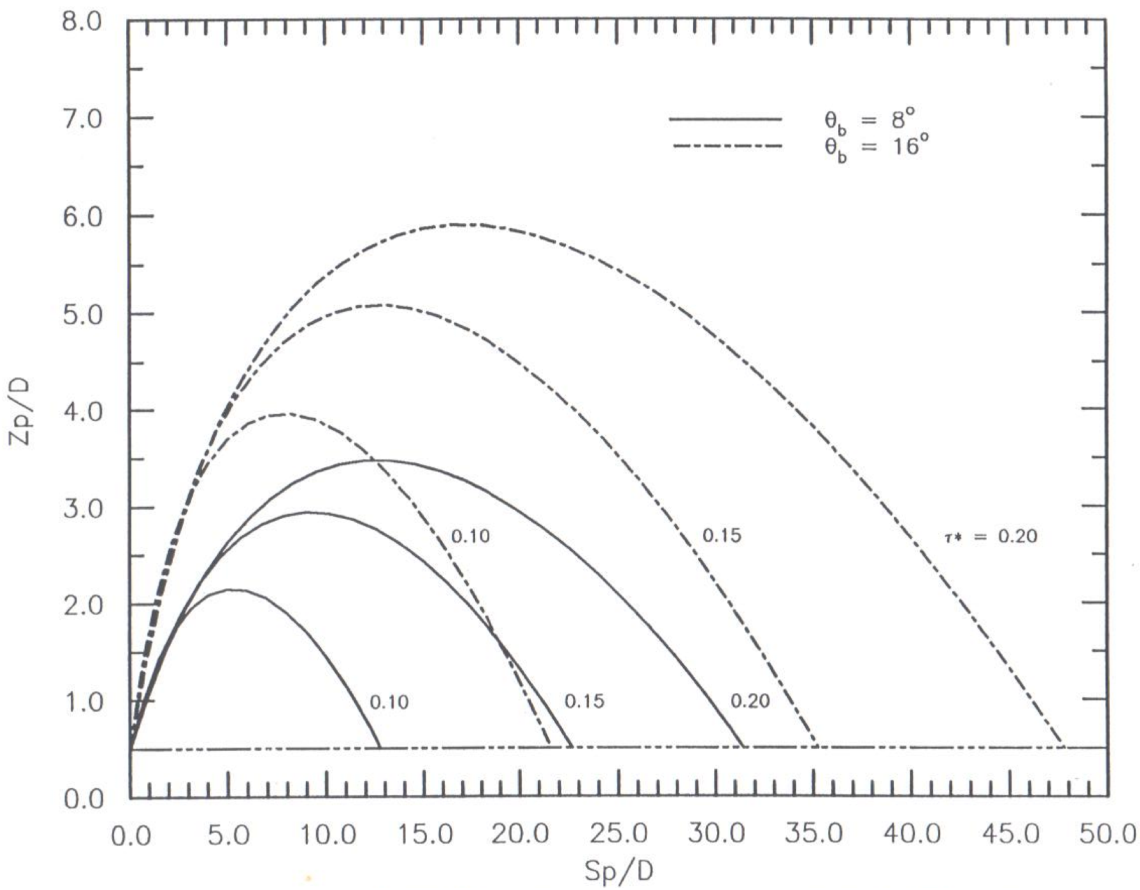
$$\frac{\lambda_s}{D} = f_\lambda\left(\tau^*, R_{ep}, R\right); \quad \frac{u_* T_s}{D} = f_T\left(\tau^*, R_{ep}, R\right); \quad \frac{h_s}{D} = f_h\left(\tau^*, R_{ep}, R\right);$$

$$\frac{\bar{u}_p}{D} = f_p\left(\tau^*, R_{ep}, R\right); \quad \frac{\bar{u}_f}{D} = f_f\left(\tau^*, R_{ep}, R\right)$$

- Here $\tau^* = \dfrac{u_*^2}{RgD}$ (Shields stress), $R_{ep} = \dfrac{\sqrt{RgD}\,D}{\nu}$, $R = \left(\dfrac{\rho_s}{\rho} - 1\right)$

Example

- The results show the following figure.



Example

- The results show the following figure.