457.309.02 Hydraulics and Laboratory
.12 Pipe problems

Prepared by Jin Hwan Hwang
(2019.03.31)

Today's objectives

- Applying what you learned to interpret more practical pro blem
- Based on single problem, it will be extended to the multip le pipe network

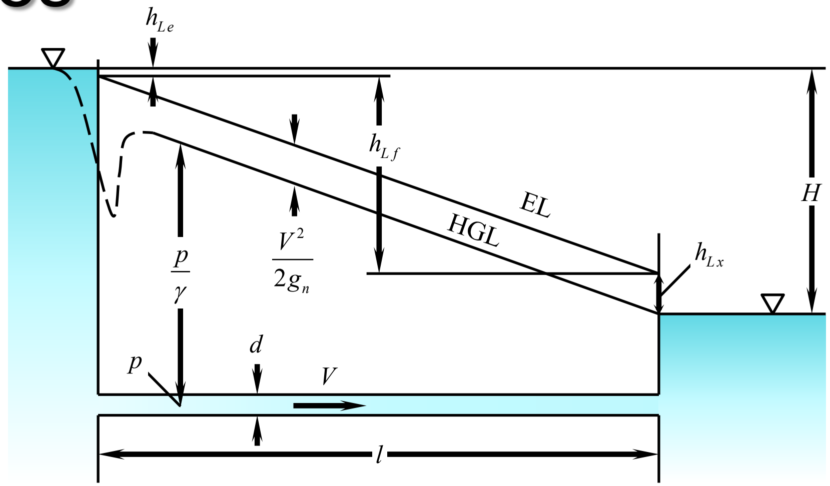
Review

- Fundamental equations
  - Based on work-energy relation for incompressible flow (from chapter 7), we got head loss, and shear velocity
- Laminar flow
  - Employ simple laminar flow, we drove head loss and friction factor for la minar flow
- Turbulent flow
  - Firstly, we learned smooth pipes and got some wall velocity profile
  - Based on this velocity profile we learned friction factor which only can b e solved by trial-errors
  - Secondly, rough pipe cases considering roughness height.
  - Here, we also knew that friction factor
  - Thirdly, we can figure out whether a pipe has smooth or rough wall.
- Engineering problem
  - What we learned so far, only works in laboratory, so generalized it for th e more practical problem.
  - Finally, we extended to empirical formula.

Pipeline problems- single pipes

- To solve engineering problem, we learned that the work-energy e quations set with continuity equations.
- More easily to access the problem, we may consider the some gr ade lines of energy and hydraulic.
- Engineering pipe-flow problems consists of
  - Calculation of head loss and pressure variation
  - Calculation of flowrate
  - Calculation of size of pipe
- Such calculation may be somehow trivial, even with great amoun t of efforts to solve problem, in real situations, we allow the signifi cant amount of safety margin.
- And this makes problem simple.
- Therefore, the using energy grade and hydraulic gradient lines m ust be helpful for you to solve in real field.

Pipeline problems- single pipes



- Let's say h_{L_e} : entrance loss

  h_{L_x} : exit loss

  h_{L_f} : pipe friction loss

  $$H = h_{L_e} + h_{L_f} + h_{L_x}$$

- Sharp edge entrance (p.363), pipe friction (p.324), open to reser voir (p.359)
- Then

$$\left(0.5 + f\frac{l}{d} + 1\right)\frac{V^2}{2g_n} = H$$

- Usually, l is much longer than d, and let's say 100 m length and 1 m diameter, then l/d = 100, in this case, we can assume f to be 0 .03. In such case, fl/d ~3, and this is twice of the extra terms.
- Also, if l/d increases, the contribution of other terms get smaller.

Example

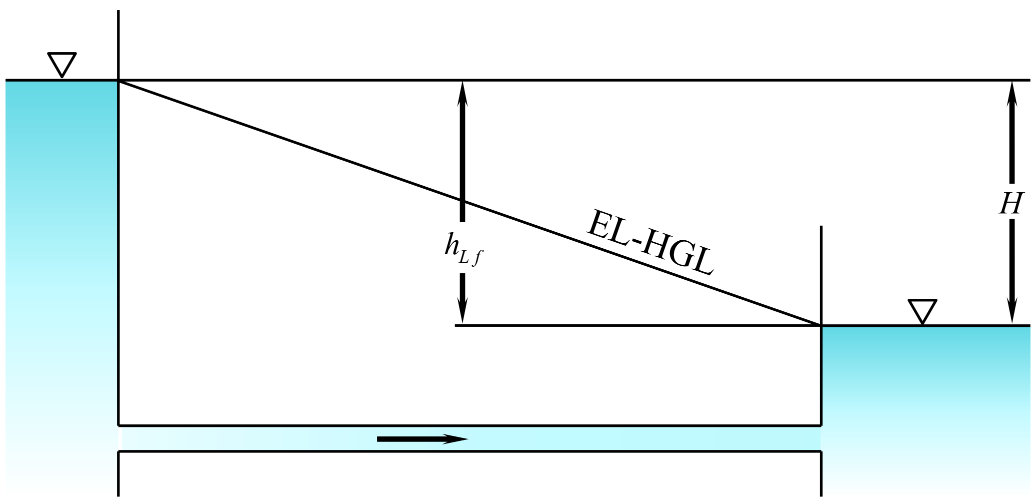
- A clean cast iron pipeline 0.30 m in diameter and 300 m long connects two reservoirs having surface elevations 60 m and 75 m. Calculate the flowrate through this line, assuming water at 10°C and a square-edged entrance.

- Sol)
  1. Reynolds number and assuming velocity
  2. e/d and determine friction factor (moody diagram)
  3. Calculate the local losses and whole pipe frictions
  4. Then the head difference (pressure head) between reservoir is loss of energy
  5. Finally, you can determine again velocity, also friction factor
  6. With the determined values, calculate again velocity.

Pipeline problems- single pipes

- Therefore, as length of pipe increases, we can ignore the effect o f local loss.
- Then we can simplify the equation based only on

$$h_L = f\frac{l}{d}\frac{V^2}{2g_n} = H$$



Example

- Sol) $$\mathrm{Re} = \frac{V}{d} = \frac{V \times 0.3}{1.306 \times 10^{-6} m^2 / s} = 229{,}000V$$
  Assuming $V \approx 2m/s$,
  $$\mathrm{Re} = 458{,}000$$

- Clean iron casting fives e in figure 9.11 and you can get e/d=0.0008 3.
- e/d and Reynolds number give friction factor as 0.02 (from moody di agram).
- Also, entrance is square edge and exit into reservoir for local loss,

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g_n} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g_n} + h_L$$

Example

- Sol)

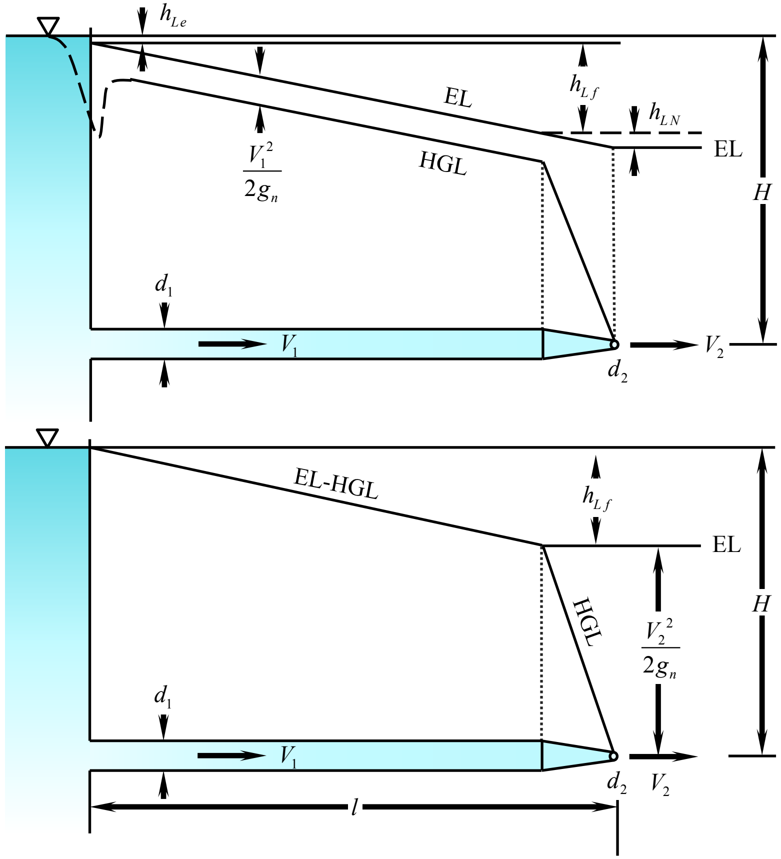
$$z_1+\frac{p_1}{\gamma}+\frac{V_1^2}{2g_n}=z_2+\frac{p_2}{\gamma}+\frac{V_2^2}{2g_n}+\left(0.5+f\frac{l}{d}+1\right)\frac{V^2}{2g_n}$$

$$75+0+0=60+0+0+\left(0.5+0.02\frac{300}{0.3}+1\right)\frac{V^2}{2\times 9.81}=1.096V^2$$

$$V=3.70m/s$$

- Now with this velocity recalculate Reynolds number (847,250) and f (0.0193). Then *V=3.76m/s*.

Example



- The case of the pipe terminated
- In this case, reservoir's velocity Is negligible but, nozzle's velocity Need to be considered

$$z_0 + \frac{p_0}{\gamma} + \frac{V_0^2}{2g_n} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g_n} + h_L$$

- Also, exit loss, can be ignored.
- Then

$$h_L = \left(h_{L_e} + f \frac{l}{d_1} \frac{V_1^2}{2g_n} \right)$$

Exit velocity Pipe velocity

$$H + 0 + 0 = 0 + 0 + \frac{V_2^2}{2g_n} + \left(K_{L_e} \frac{V_1^2}{2g_n} + f \frac{l}{d_1} \frac{V_1^2}{2g_n} \right)$$

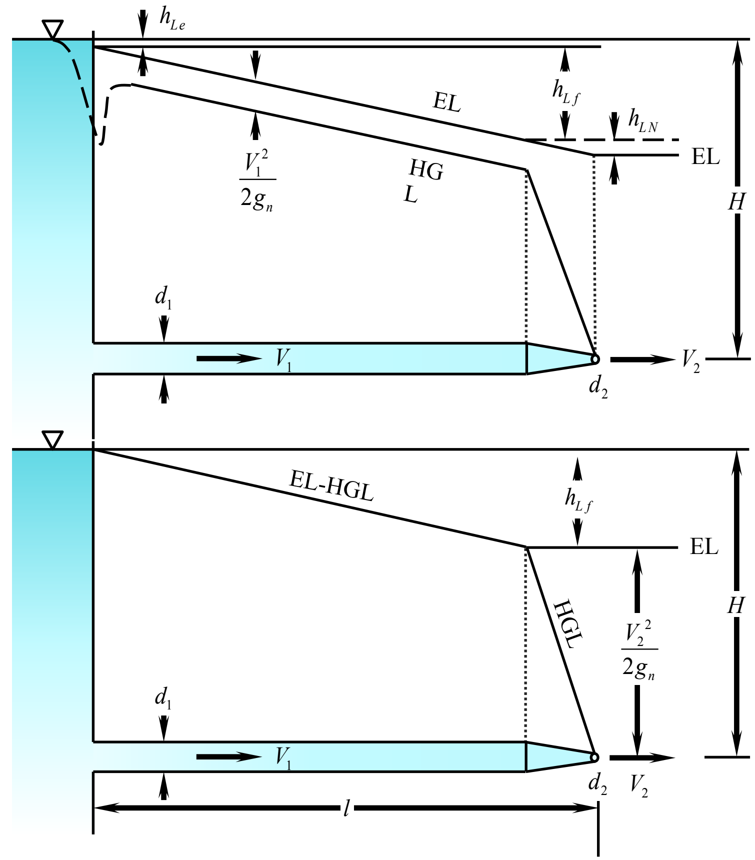
Example

- Using continuity equation

$$A_1V_1 = A_2V_2 \qquad V_2 = \left(\frac{d_1}{d_2}\right)^2 V_1$$

$$H = \left(\frac{d_1^2}{d_2^2} + K_{L_e} + f\frac{l}{d_1}\right)\frac{V_1^2}{2g_n}$$

- Now think about the case of generating electricity
- Power is work done in given time
- or product of pressure and flow rate.
- Here actual pressure is head (with specific weight)



$$Power = Energy\ \ flow\ rate = weight\ \ flow\ rate \times energy\ per\ unit\ weight$$

$$= (Q\gamma)\left(\frac{V^2}{2g_n}\right)$$

Example

- Neglecting local loss

$$\frac{V_2^2}{2g_n} = H - f\frac{l}{d_1}\frac{V_1^2}{2g_n} = H - f\frac{l}{d_1}\frac{Q^2}{2g_n A_1^2}$$

- Substitute into power

$$Power = \gamma Q\left(H - \frac{flQ^2}{2g_n d_1 A_1^2}\right)$$

- Find maximum jet power by differentiating

$$\frac{flQ^2}{2g_n d_1 A_1^2} = \frac{H}{3}$$

$$\Rightarrow \frac{V_2^2}{2g_n} = \frac{2H}{3} \quad and \quad \frac{flV_1^2}{2g_n d_1} = \frac{H}{3}$$

Actual limitation of the flow rate.

Example

$$\frac{V_2^2}{2g_n} = \frac{2}{3}H$$

- In hydropower practice, $V_2^2/2g_n$ will be found to be considerably larger than 2/3 H, depending on the economic values of the water.
- Also, HGL drops to negative, cavitation can occur. When cavitation occurs, dissolved gases are vaporized and makes bubbles.
- Such bubbles can inhibit flow and reduce the capacity.
- But, cavitation cannot be avoided, so engineers try to design preventing the negative pressure from exceeding about two thirds of the difference between barometric and vapor pressures.

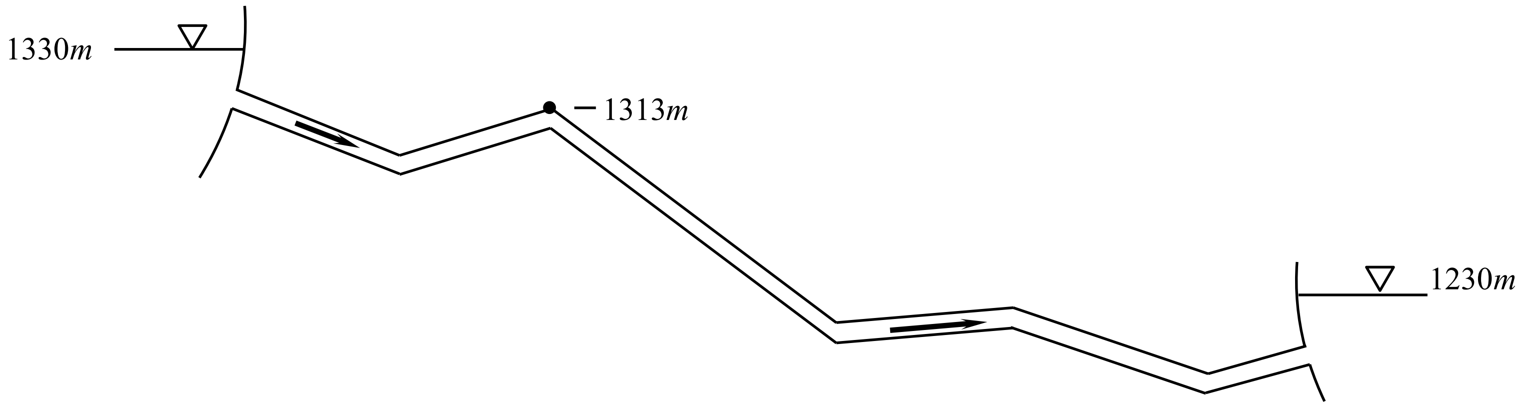
Cavitation



- When flow is too fast, the absolute pressure may fall to the vapor pressure of the liquid, at which point cavitation sets in.
- Large negative pressure in pipe should be avoided, if happens, then prevent from exceeding about two thirds of the difference between barometric and vapor pressures.
- However, most engineering liquids contain dissolved gases which will come out of solution well before the cavitation point is reached.
- Such gases become bubbles collect in the high points of the line, reduce the flow cross section and tend to disrupt flow.

Example

- A pipeline is being designed to convey water between two reservoirs whose elevations are shown below. The pipeline is 20km long and the preliminary pipeline profile has the line passing over a ridge where the pipeline elevation is 1,313 m at a distance of 4km from the upstream reservoir.
- There is concern that the ridge is too high and will create an unacceptably low pressure in the pipeline. What is your recommendation on as to the feasibility of the proposed location of the pipeline?



Example

- Let's neglect the local losses and consider the energy line and the h ydraulic grade line to be coincident.
- Then EL-HGL will fall uniformly 100 m over 20km.
- Since the ridge is 4 km, then EL-HGL elevation at the ridge will be

$$EL - HGL = 1330 - 1/5 \times 100 = 1310$$

- Since we assume EL=HGL, it means that velocity head difference is negligible if there is no big change in pipe diameter. Therefore, the di fference of elevation and EL height , will result in pressure drop.

$$z_0 = z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g_n}(\approx 0 \text{ EL=HGL}) + h_{f1}$$

$$1330 - 1313 - h_{f1}(=20) = \frac{p_1}{\gamma}$$

$$\frac{p_1}{\gamma} = -3$$

Example

- Which is smaller than water vaporized pressure (-10 of water), so se ems to be OK.
- But, in practical, the air entrained and dissolved gas will be vaporize d and it will reduce the capacity of pump.
- Also, this bubbles cannot be released by a valve since valve works f or positive pressure to force out the gases.
- Therefore, we may need to think another route for this pipe.

Designing pump

- Source pump and booster pump.
- The determination of the power required to meet flow rate and press ure demands, for pressure

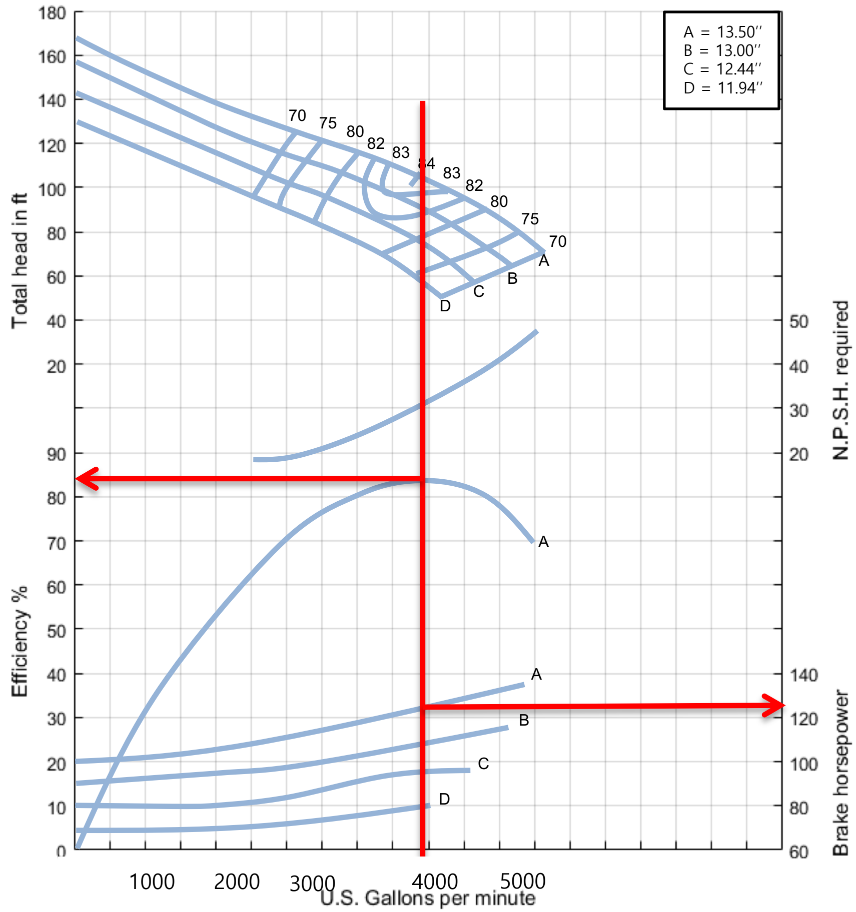
$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g_n} + E_p = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g_n} + h_L$$

- E_p is work per unit weight added to the fluid by the pump,
- For power

$$WHP = \frac{Q\gamma E_p}{550} \quad \text{(U.S. customoary)}$$

$$WKW = \frac{Q\gamma E_P}{1{,}000} \quad \text{(SI)}$$

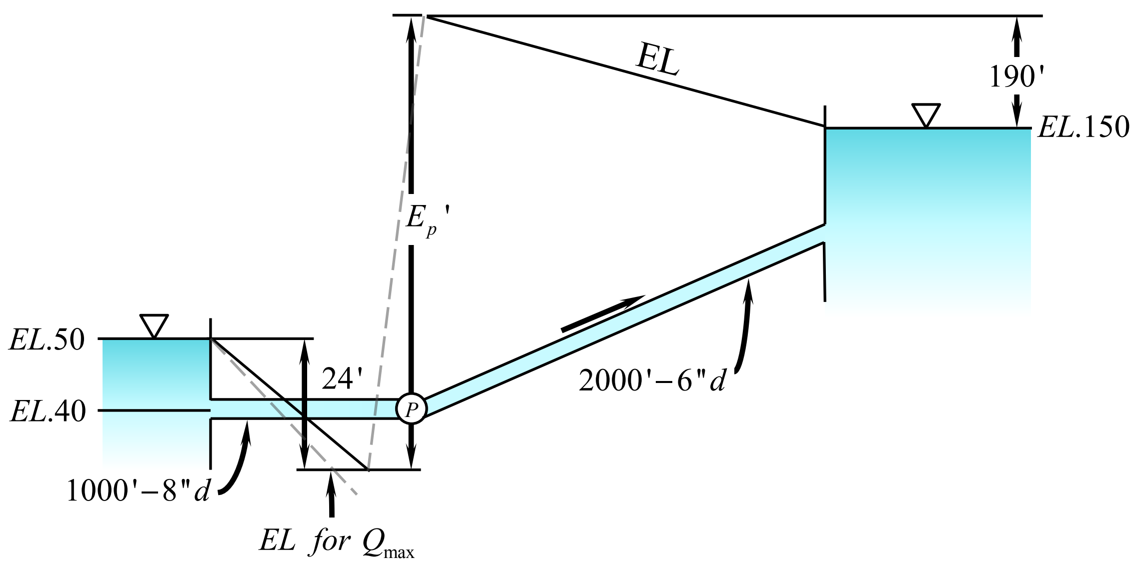
Typical pump characteristic diagram



- 13.50 inch impeller (curve A) is the most efficient impe ller at 84%. It pumps 3800 gal/m at this efficiency and requires 124 hp to accompl ish this.

Example

- Calculate the horsepower that the pump must supply to the water (50°F) in order to pump 2.5ft^3/s through a clean cast iron pipe from the lower reservoir to the upper reservoir. Neglect local losses and velocity heads.
- Using the criteria for minimum allowable pressure suggested earlier in this section, compute the maximum dependable flow which can be pumped through this system.



Example

- To get the pressure

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g_n} + E_p = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g_n} + h_L$$

- Power can be determined by

$$WHP = \frac{Q\gamma E_p}{550} \quad \text{(U.S. customoary)}$$

- Lower pipe has 8 inch and upper pipe has 6 in, and each case's velo city

$$V = \frac{Q}{A} \quad V_8 = 7.16 \quad and \quad V_6 = 12.72$$

- And Reynolds numbers for each

$$\mathrm{Re}_8 = 338{,}500 \qquad \mathrm{Re}_6 = 451{,}000$$

Example

- Since we already know this pipe is cast iron pipe then we can use figure 9.11 for determining the roughness height, and with this and Reynolds numbers, we can find the friction factor.

$$\left(\frac{e}{d}\right)_8 = 0.00128 \qquad f = 0.021$$

$$\left(\frac{e}{d}\right)_6 = 0.00171 \qquad f = 0.022$$

- The head loss in each of the pipes can now be calculated.

$$h_{L_8} = f\frac{L}{d}\frac{V^2}{2g_n} = 0.021\frac{1000\,ft}{8in./12}\frac{(7.16\,ft)^2}{2\times 32.2} = 25\,ft$$

$$h_{L_6} = f\frac{L}{d}\frac{V^2}{2g_n} = 0.022\frac{2000\,ft}{6in./12}\frac{(12.72\,ft)^2}{2\times 32.2} = 221\,ft$$

Example

- Consider the total losses from the both pipes

$$50 + 0 + 0 + E_P = 150 + 0 + 0 + 25 + 221$$

$$E_P = 346\,ft$$

$$WHP = \frac{Q\gamma E_P}{550} = \frac{2.5\,ft^3/s \times 62.4\,lb/ft^3 \times 346\,ft}{550} = 98\,hp$$

- But, we need to check the pressure drop, which is not lower than 20 ft (cavitation occurs).
- Let's check at the suction site (neglecting the local loss)

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g_n} = z_s + \frac{p_s}{\gamma} + \frac{V_s^2}{2g_n} + h_{L_8}$$

$$50 + 0 + 0 = 40 + \boxed{(-20)} + 0 + h_{L_8}$$

Minimum pressure drop

Example

- Therefore, $$h_{L_8} = 30\,ft = f\frac{L}{d}\frac{V_8^2}{2g_n}$$

  $$V_8 = 7.8\,ft/s$$

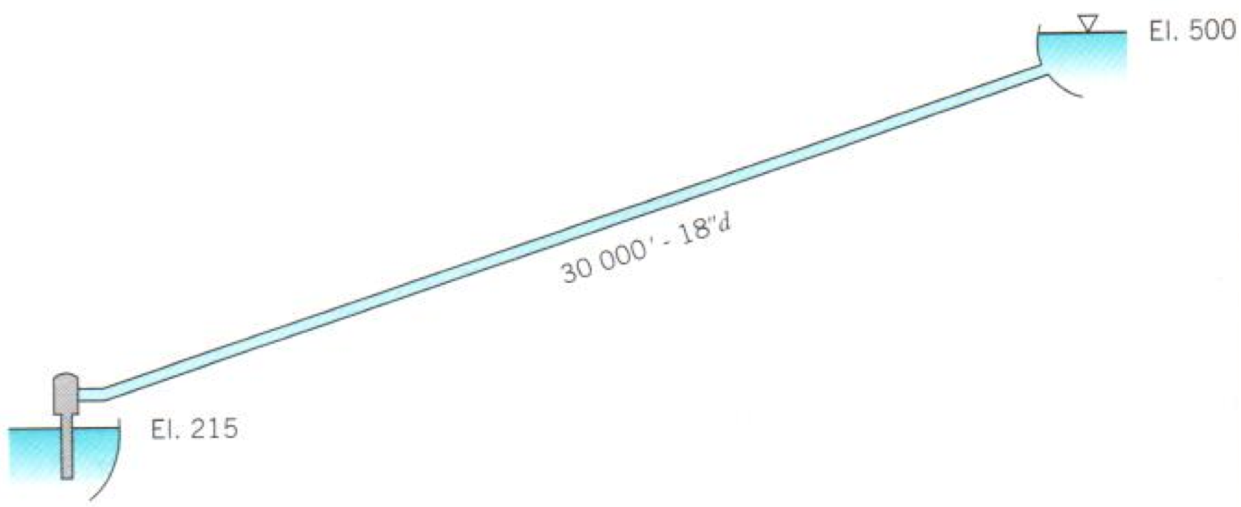
- So this is the maximum flow speed can be pumped through the syst em. The maximum dependable flowrate that can be pumped through the system

$$Q_{max} = V_8 A = 7.8\,ft/s \times \pi/4\left(8/12\right)^2 = 2.7\,ft^3/s$$

- Check the first calculation, then you may know the velocity is enoug h lower to prevent cavitation.

Example 2

- The pump whose characteristics are given in figure 9.25 (your text book) is proposed for use in the pipe system shown below. In order to provide the head necessary to deliver the required flowrate to the upper reservoir, a four-stage pump is planned. Using the graphical technique of fig 9.26, determine the flowrate produced by the proposed pumping configuration and estimate the efficiency of the pump. Neglect local losses and use a Darcy-Weisbach f-value of 0.018



Example 2

- System demand curve

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g_n} + E_p = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g_n} + f\frac{L}{d}\frac{V^2}{2g_n}$$

$$215 + 0 + 0 + E_p = 500 + 0 + 0 + 0.018\frac{3000\,ft}{18in/12}\frac{V^2}{2\times 32.2}$$

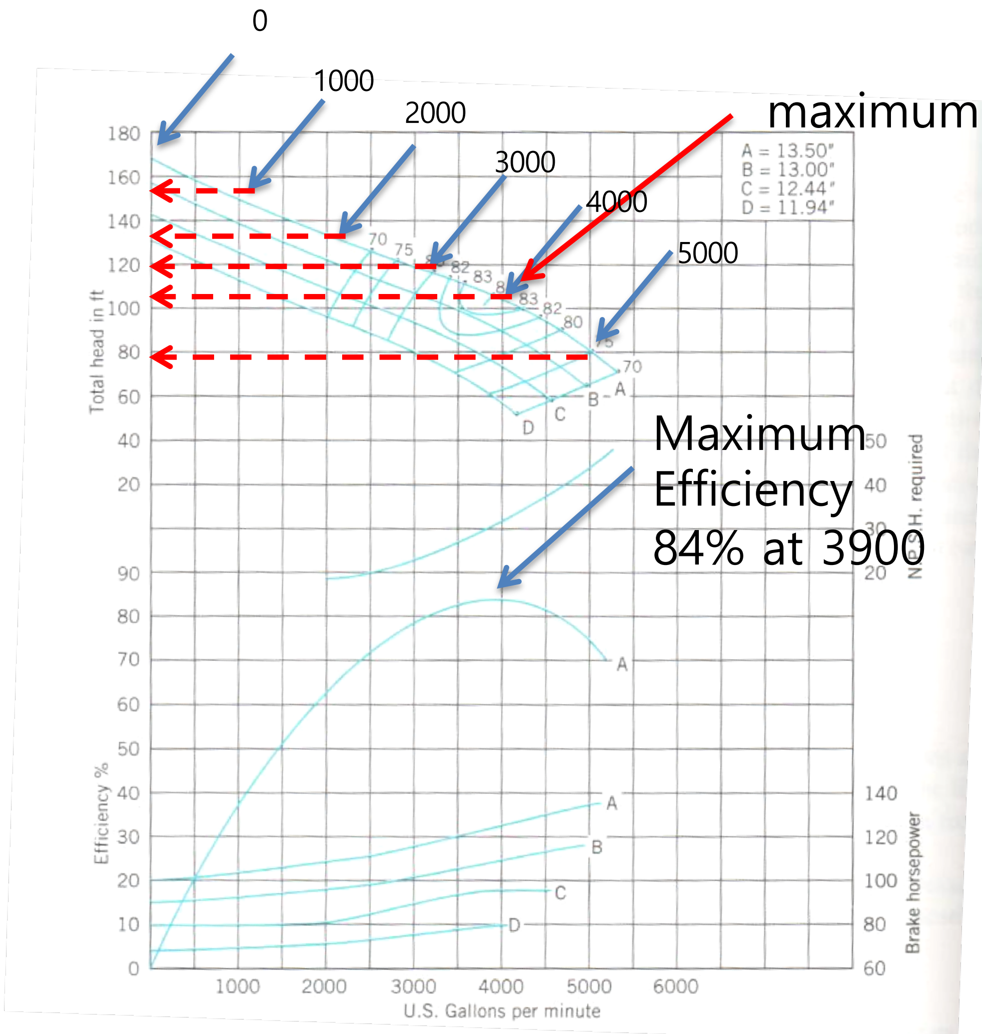
$$E_p = 285 + 5.59V^2$$

Q (ft^3/s)	V	$5.59V^2$	$285+5.59V^2$
0	0	0	285
2	1.13	7.1	292
4	2.26	28.6	314
6	3.40	64.6	350
8	4.53	114.7	400
10	5.66	179.1	464

Example 2

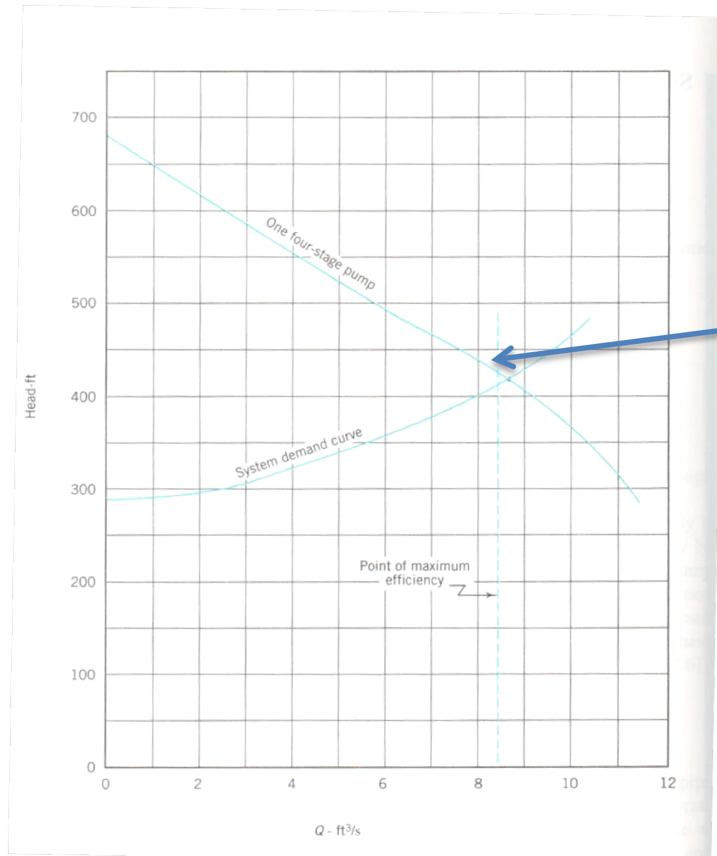
- Pump Supply Curve

Q (gal/min)	Q(ft^3/sec)	1 stage Head	4 stage head
0	0	168	672
1000	2.23	150	600
2000	4.45	133	532
3000	6.68	119	476
4000	8.91	103	412
5000	11.14	78	312



Example 2

- Pump's maximum efficiency is found at the similar value to the cross point of two curves.



Maximum Efficiency 84% at 3900