Ch. 2 – Relativity II

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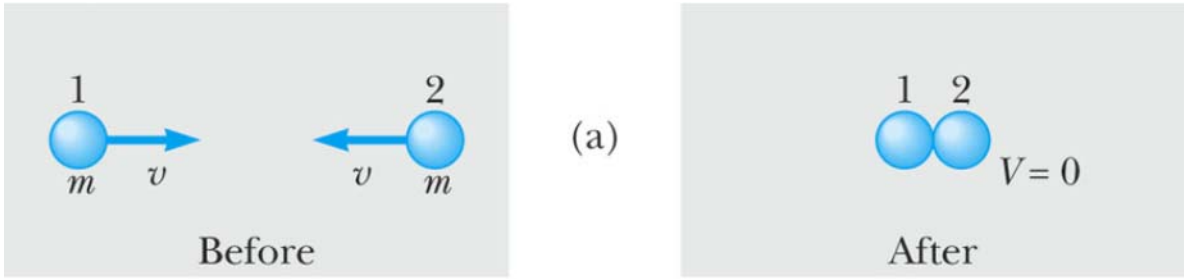
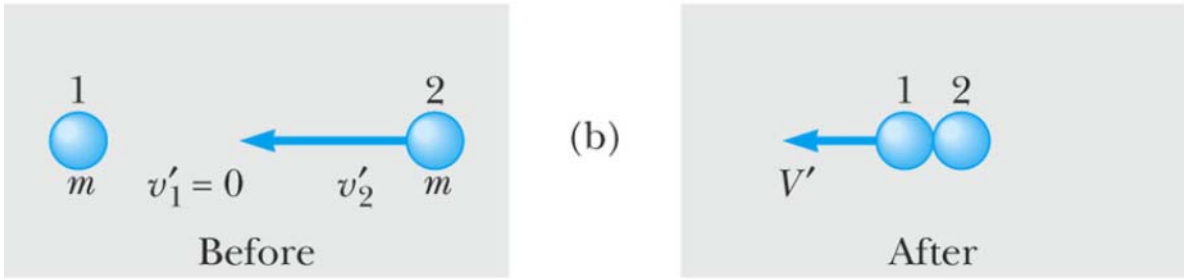
Momentum Conservation?



Momentum is conserved according to S

$p_{\text{before}} = mv + m(-v) = 0$

$p_{\text{after}} = 0$



Momentum is *not* conserved according to S′

$$p'_{\text{before}} = \frac{-2mv}{1 + v^2/c^2}$$

$p'_{\text{after}} = -2mv$

Fig. 2-1, p. 42

Momentum Conservation!

Momentum

$$\mathbf{p} = \frac{m\mathbf{u}}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma m\mathbf{u}$$

where

u is the velocity of the particle and

m is the proper mass

(in some books it is called rest mass and denoted by m_0.)

Newton's Law

Relativistic form of Newton's second law

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(\gamma m\mathbf{u})$$

Relativistic Energy

Work $$W = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} \frac{dp}{dt} dx = \int_{x_1}^{x_2} \frac{d(\gamma mu)}{dt} dx$$

$$= \frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}} - mc^2$$

Relativistic kinetic energy

$$K = \frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}} - mc^2 = \gamma mc^2 - mc^2$$

Relativistic Energy

Total energy

$$E = \gamma mc^2 = K + mc^2$$

(in some books $E = mc^2 = \gamma m_o c^2$.)

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

For a photon, the proper mass (some call it rest mass) is zero, and hence,

$$E = pc$$

Mass-Energy Equivalence

Conservation of mass-energy

General Relativity

Two postulates of the general theory of relativity:

- The laws of nature have the same form for observers in any frame of reference, whether accelerated or not.

- In the vicinity of any point, a gravitational field is equivalent to an accelerated frame of reference in the absence of gravitational effects.
  (Principle of Equivalence)

General Relativity

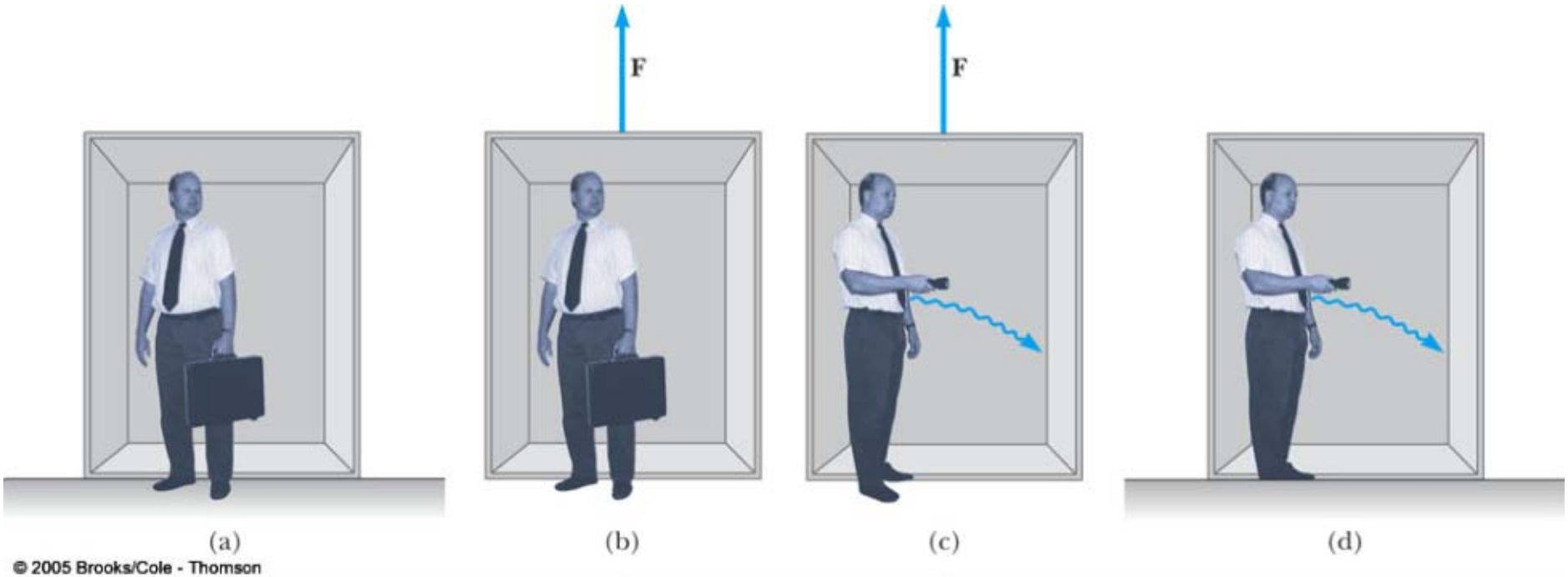


Fig. 2-4, p. 54

General Relativity

Some comments of interest:

- Time is altered by gravity.
- Curvature of spacetime
- Light can be bent by gravity.
- Black hole

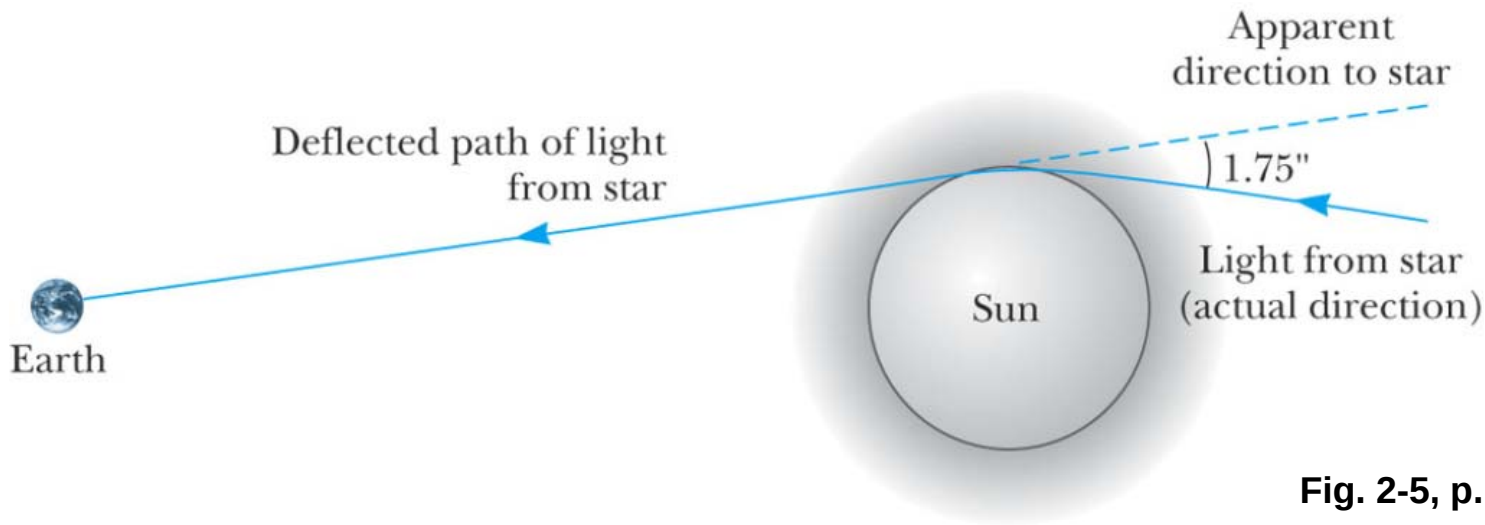


Fig. 2-5, p. 56